Quota bonuses in a principle-agent setting

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Abstract

Theoretical articles on incentive systems almost excusively focus on linear compensations, while in practice, nonlinear elements, eg. quota bonuses are not uncommon. Our article tries to bridge that gap and show how the use of quotas can increase the owners' profit.

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1 Introduction

Theoretical articles on incentive schemes, following Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987) almost excusively focus on linear compensation systems. Besides analytical tractability, the main reason cited is the argument put forth by Holmstrom and Milgrom (1987), who show that in a dynamic situation linear compensation schemes are quite robust and under certain conditions could lead to optimal solutions of the principal-agent problem.

A somewhat similar point is made by Basu and Kalyanaram (1990), who emphasize that linear contracts can be much more easily understood. They obtain a result that for their exponential utility function, parametrization assuming more risk averse agents leads to the superiority of linear compensation plans, while using parameters positing less risk averse agents, nonlinear plans are preferable. A comparison of linear and piecewise linear compensation plans

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by Chen and Miller (2009) tells us that while linear schemes might be better using exponential utility functions, in the case of agents with power utility functions piecewise linear compensation systems are preferable.

However, in practice, nonlinear elements, e.g. quota bonuses are not uncommon. According to an empirical study by Joseph and Kalwani (1998), 5 percent of the firms participating in the survey used fixed salaries for their salespeople, 24 percent used only commissions, while the overwhelming majority included some kind of bonus payment in their compensation packages. By far the most important factor in determining bonus payments was the comparison of actual sales and a predetermined quota. As Oyer (1998) remarks, executive contracts also often include quota-like features. The behavior of salespeople – eg. their attitudes toward risk, as demonstrated by Ross (1991) – is strongly influenced by how quotas are set.

Oyer (1998) also points out a potential dynamic problem with the use of quotas. This could lead to uneven effort during the year, since agents increase their effort when the deadline for determining quota bonus is near. Executives or salespeople could behave in an opportunistic manner and engage in "timing games", ie. rushing sales or using creative accounting to ensure the quota bonus. However, the findings of Steenburgh (2008), based on analysing individual-level salesforce data, seem to indicate that timing games are less common in practice and the main effect of quotas are to increase the efforts of the agents.

In this paper, we try to link the two strands of literature and to incorporate the use of quotas in a theoretical model of incentives.

2 The model

The owner of the firm employs two agents (salespeople) to sell the product.[1] Their "downstream market" is a differentiated Bertrand duopoly, thus the (normalized) demand for the products sold by the k_{th} agent is $1-p_k-bp_{-k}$, where p_k is the price charged by the k^{th} agent, p_{-k} is the price charged by the other agent, and $0 < b < 1$ represents the homogeneity or differentiation of the downstream market.[2]

The two agents also differ in their efficiency. The agent might have to make a certain effort to make a sale, which can be either an internal or external cost born by the agent themself and efficiency is understood as a difference of these inherent costs.

The agents' want to maximize their own utility, ie. the difference of their salary and their effort. The owner wants to maximize their profit.[3]

[1] Another interpretation of our model could be an owner with two firms offering incentives to the executives.

[2] Of course, this can be easily generalized for the case when an "outside" competitor exists, assuming constant marginal costs and competitive or markup pricing.

[3] In line with the literature, we only focus on the profit from sales and omit the deduction of the salaries paid. This can be justified if the agents' alternative utility is low enough or the fixed part of their salary is high enough, since in that case the variable bonuses could be negligible when comparing outcomes.

The owner thus first sets the compensation scheme in this two-stage game. The compensation (disregarding a possible fixed part of the salary) depends on the profit from the sales of the agent and – if the owner chooses to include it – a reward if sales exceed a certain quota[4].

In the second stage of the game, the salespeople – knowing the relevant incentives – set their prices and competition occurs. However, we assume that there is some uncertainty about finalized sales within the given period[5]. For simple analytical treatment, we describe this uncertainty with a uniformly distributed error term, thus when an agent expects to sell q unit (based on the demand they face), then the actual number of completed sales is a realization from the $[q - \varepsilon, q + \varepsilon]$ interval.[6]

3 Results

3.1 Optimal choice of the agents

The salespeople want to maximize they expected bonuses – their commisions from the profit and the quota bonus. Due to the uncertainty described above, the existence of the quota is going to affect their behavior only if their planned sales are close to the quota; if they deem the quota unreachable, or they are utterly confident about quota fullfilment then they do not need to include the quota in their calculations.

More formally, assume that the quota (Q) is fixed. In this case, there are four possible cases for each agent.

1. Planned optimal output is less than $Q - \varepsilon$.
2. Planned optimal output is between $Q - \varepsilon$ and $Q + \varepsilon$.
3. Planned optimal output is equal to $Q + \varepsilon$ (corner solution).
4. Planned optimal output exceeds $Q + \varepsilon$.

In the first case, $P(bonus) = 0$, so the k_{th} agent maximizes $(1 - p_k - bp_{-k})(p_k - e_k)$, where e_k is the necessary effort on the behalf of the k_{th} agent, so their first-order condition is $1 + e_k - 2p_k - bp_{-k} = 0$. Thus they will set the following price: $p_k = \frac{1}{2}(1 + e_k - bp_{-k})$.

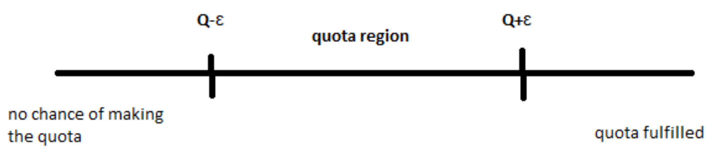
In the second case, $P(bonus) = \frac{q_k - (Q-\varepsilon)}{2\varepsilon}$, so the k_{th} agent maximizes $(1 - p_k - bp_{-k})(p_k - e_k) + \lambda \frac{(1-p_k-bp_{-k})-(Q-\varepsilon)}{2\varepsilon}$, where λ is the quota bonus. The first-order condition for the agent is therefore $1 + e_k - bp_{-k} - 2p_k - \frac{\lambda}{2\varepsilon} = 0$. The optimal choice for the agent is to set the price as: $p_k = \frac{1}{2}(1 + e_k - bp_{-k}) - \frac{\lambda}{4\varepsilon}$

[4] According to Joseph and Kalwani (1998), approximately 35 percent of the firms in the sample studied used both commissions and quotas in their compensation schemes.

[5] Thus, as in Oyer (1998), we also pay some attention to the timing of the finalized sales; however, we focus on unintentional timing effects.

[6] Again, a possible alternative interpretation of our model could be mentioned: this is equivalent to a quota bonus with upper and lower cap. Cf. Figure II in Oyer (1998).

The corner solution implies that $1 + e_k - bp_{-k} - 2p_k - \frac{\lambda}{2\varepsilon} < 0$ and $1 + e_k - 2p_k - bp_{-k} > 0$ whilst $q_i = Q + \varepsilon$. The discussion of the "corner solution", however, will be omitted since it can be easily shown that this is not going to be optimal for the owner.[7]



In the last case, $P(bonus) = 1$, so the first-order condition and the optimal price for the agent are similar to those of the first case. Hence in the following, I will refer to the first and fourth case as "outside the quota range", to the second case as "within the quota range". Furthermore, if both salespeople are within the quota range, I will use the term "pooling quota", while if only one of them is in the quota range, I will talk about a "separating quota".

Furthermore, from now on, I will denote the more efficient agent as agent i and the less efficient agent as agent j. Efforts are normalized so that $e_i = 0$ and $e_j = e$.

3.2 Comparison of quota regimes

Now we have to solve a two-stage game, when in the first period the owner chooses a quota setup and in the second period the agents influenced by the quota make their pricing decisions and sales take place. We have already taken a look at the agents' decisions, so now we have to expand our focus to include the owner's problem.

As usual in the literature, we posit a risk-neutral owner, so their task is to maximize the expected profit. We further make the assumption that uncertainty shocks to salespeople are independent, hence we can simply use the expected quantities in our profit calculations.

The case when both agents' output is outside the quota range[8], is simple to calculate and will be the basis of comparisons to understand the effect of quotas on the owner's profit.

Proposition 1. *If both agents' optimal outputs are outside the quota range,*

[7]The owner can use a lower quota bonus, where the relevant first order condition ($1 + e_k - bp_{-k} - 2p_k - \frac{\lambda}{2\varepsilon} = 0$) still leads to the same output level.

[8]Of course, this is equivalent to the baseline case when there are no quotas at all.

then the expected profit of the owner is:

$$\pi = \frac{2 - be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} \tag{1}$$

Proof. It follows from lemma 1. See appendix. $\square$

The next case we look at is the one where both agents are within the quota range. Here we can state the following:

Proposition 2. *If the owner introduces a pooling quota, then the expected profit of the owner is:*

$$\pi = \frac{2 - be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} + \frac{(b+e+be)^2}{2(1+b)(2+b)^2} \tag{2}$$

Proof. It is the immediate consequence of lemmas 2 and 3. See appendix. $\square$

We can clearly see that introducing a pooling quota increases the profit of the owner. The case when the focus of the agents moves towards fullfiling their quota is analogous with the setup when managers of firms competing in prices are offered sales bonuses. Our proposition is therefore an extension to the result of Sklivas (1987), showing that quotas in this case can act similarly as quantity bonuses.

Now let us consider the separating quotas, firstly the one aimed at the more efficient agent.

Proposition 3. *If the owner introduces a separating quota with the more efficient agent within the quota range and the less efficient agent outside the quota range, then the expected profit of the owner is:*

$$\pi = \frac{2 - be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} + \frac{((2-b)^2 b + 2b^3 e)^2}{4(4-b^2)^2(4-3b^2)} \tag{3}$$

Proof. It is implied by lemmas 4 and 5. See appendix. $\square$

Examining our latest result, we now find that a separating quota aimed at the more efficient agent increases profit compared to the baseline case, however, it provides less profit then the pooling quota.

The last case to discuss is the separating quota aimed at the less efficient agent.

Proposition 4. *If the owner introduces a separating quota with the more efficient agent outside the quota range and the less efficient agent within the quota range, then the expected profit of the owner is:*

$$\pi = \frac{2 - be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} + \frac{(b((2-b)^2 - 6be) + 8e)^2}{4(4-b^2)^2(4-3b^2)} \tag{4}$$

Proof. The result follows from lemmas 6 and 7. See appendix. $\square$

Again, the profit is going to be clearly higher than in the baseline case and also higher than in the case of the separating quota aimed at the more efficient agent. However, the comparison with the pooling case is not clear-cut. Our results could be summarised as follows:

Theorem 1. *The separating quota aimed at the less efficient agent leads to higher profit if the efficiency difference is sufficiently high ($e > \frac{1}{2}$). If the efficiency difference is lower than the abovementioned threshold, then for any value of e, one can find b^*, when for any value of $b < b^*$, the separating quota aimed at the less efficient agent leads to higher profit. In other words: if the efficiency difference is not that high, it is sufficient to have more differentiated agents. In other cases the pooling quota provides the highest profit, if it is feasible.*

We can also predict the effect of different variables on the size of the quota bonus.

Theorem 2. *In the case of an effective quota[9], the optimal quota bonus is increasing in the difference in efficiency and the uncertainty of the agents and decreasing in the differentiation of the agents.*

Proof. See appendix. □

4 Summary

Although nonlinear elements of compensation schemes are quite widespread, theoretical study so far focused on linear incentives. We have shown that if uncertainty exists regarding the timing of sales, quotas affect the behavior of agents not unlike quantity bonuses and therefore can increase the profit of the owners compared to the case when only commissions are used. We have further shown that although quotas are useful in symmetric cases as well, their significance increases if the abilities of the agents are different.

Appendix

Using the derived reaction functions for the different cases from subsection 3.1, we can easily find the equilibrium prices and expected quantities for the possible pairings of the agents.

A No effective quota

In this case the quota is irrelevant in either agent's optimization problem. The first-order conditions are as follows:

$$\left.\begin{array}{l} 1 + e_i - 2p_i - bp_j = 0 \\ 1 + e_j - 2p_j - bp_i = 0 \end{array}\right\} \tag{5}$$

[9] Ie. at least one agent is within the quota range.

Solving this system of equations for p_i and p_j, then using those results to calculate the quantities, we can work out the outcome of this case.

Lemma 1. *If both agents' optimal outputs are outside the quota range, then the equilibrium prices and expected outputs are:*

$$p_i = \frac{2-b-be}{4-b^2} \tag{6}$$

$$p_j = \frac{2-b+2e}{4-b^2} \tag{7}$$

$$q_i = \frac{2-b-be}{4-b^2} \tag{8}$$

$$q_j = \frac{2-b-2e+b^2e}{4-b^2} \tag{9}$$

B Pooling quota

Now both first-order conditions include the the expected value of the quota bonus:

$$\left.\begin{aligned} 1+e_i-bp_j-2p_i-\frac{\lambda}{2\varepsilon}=0 \\ 1+e_j-bp_i-2p_j-\frac{\lambda}{2\varepsilon}=0 \end{aligned}\right\} \tag{10}$$

Again, we can solve for the equilibrium prices and then derive the equilibrium quantities, conditional on the quota bonus.

If both agents' optimal outputs are within the quota range, ie. the owner introduces a pooling quota, then the equilibrium prices and expected outputs are:

$$p_i = \frac{2-b-be}{4-b^2} - \frac{\lambda}{2\varepsilon(2+b)} \tag{11}$$

$$p_j = \frac{2-b+2e}{4-b^2} - \frac{\lambda}{2\varepsilon(2+b)} \tag{12}$$

$$q_i = \frac{2-b-be}{4-b^2} + \frac{\lambda(1+b)}{2\varepsilon(2+b)} \tag{13}$$

$$q_j = \frac{2-b-2e+b^2e}{4-b^2} + \frac{\lambda(1+b)}{2\varepsilon(2+b)} \tag{14}$$

Using the fact that according to our assumption the owner maximizes $\pi = p_iq_i + p_jq_j$, we can calculate the profit of the owner conditional on the quota bonus.

Lemma 2. *If the owner introduces a pooling quota, and further the owner gives lambda bonus for fulfilling the quota, then the expected profit of the owner is:*

$$\pi = \frac{2-be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} - \frac{\lambda((1+b)\lambda - 2(b+e+be)x)}{2(2+b)^2\varepsilon^2} \tag{15}$$

Now, the owner chooses the bonus which maximizes their profit. The first-order condition is

$$\frac{(b+e+be)\varepsilon - (1+b)\lambda}{(2+b)^2\varepsilon^2} = 0 \tag{16}$$

The solution of the above equation gives us:

Lemma 3. *If the owner introduces a pooling quota, the owner-optimal bonus for fulfilling the quota is:*

$$\lambda^* = \frac{(b+e+be)\varepsilon}{1+b} \tag{17}$$

C Separating quota aimed at the more efficient agent

The relevant first-order conditions this time are:

$$\left.\begin{aligned} 1 + e_i - bp_j - 2p_i - \frac{\lambda}{2\varepsilon} = 0 \\ 1 + e_j - bp_i - 2p_j = 0 \end{aligned}\right\} \tag{18}$$

The equilibrium prices and the quantities can be easily derived from this system of equations for a fixed quota bonus.

If the optimal output of agent i is within the quota range and that of agent j is outside the quota range, ie. the owner introduces a separating quota, aimed at the more efficient salesperson, then the equilibrium prices and expected outputs are:

$$p_i = \frac{2-b-be}{4-b^2} - \frac{\lambda}{\varepsilon(4-b^2)} \tag{19}$$

$$p_j = \frac{2-b+2e}{4-b^2} + \frac{\lambda b}{2\varepsilon(4-b^2)} \tag{20}$$

$$q_i = \frac{2-b-be}{4-b^2} + \frac{\lambda(2-b^2)}{2\varepsilon(4-b^2)} \tag{21}$$

$$q_j = \frac{2-b-2e+b^2e}{4-b^2} + \frac{\lambda b}{2\varepsilon(4-b^2)} \tag{22}$$

Simple calculation gives us the profit conditional on the quota bonus:

Lemma 4. *If the separating quota is aimed at the more efficient agent, and further the owner gives λ bonus for fulfilling the quota then the expected profit of the owner is:*

$$\pi = \frac{2-be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} - \frac{\lambda((-4+3b^2)\lambda + 2b(4+b(-4+b+2be))\varepsilon)}{4(4-b^2)^2\varepsilon^2} \quad (23)$$

Since the owner wants to maximize their profit, they choose the quota bonus according to the following first-order condition:

$$\frac{b(4+b(-4+b+2be))\varepsilon - (4-3b^2)\lambda}{2(4-b^2)^2\varepsilon^2} = 0 \quad (24)$$

Thus we can state the following:

Lemma 5. *If the owner introduces a separating quota that is focusing on the more efficient agent, the owner-optimal bonus for fulfilling the quota is:*

$$\lambda^* = \frac{b(4-4b+b^2+2b^2e)\varepsilon}{4-3b^2} \quad (25)$$

D Separating quota aimed at the more efficient agent

In the last case, we start with the following first-order conditions:

$$\left.\begin{aligned} 1+e_i-bp_j-2p_i &= 0 \\ 1+e_j-bp_i-2p_j-\frac{\lambda}{2\varepsilon} &= 0 \end{aligned}\right\} \quad (26)$$

Solving first for prices, then inferring the quantities, we can again obtain the outcome given the quota bonus.

If the optimal output of agent i is outside the quota range and that of agent j is within the quota range, ie. the owner introduces a separating quota, aimed at the less efficient salesperson, then the equilibrium prices and expected outputs are:

$$p_i = \frac{2-b-be}{4-b^2} + \frac{\lambda b}{2\varepsilon(4-b^2)} \quad (27)$$

$$p_j = \frac{2-b+2e}{4-b^2} - \frac{\lambda}{\varepsilon(4-b^2)} \quad (28)$$

$$q_i = \frac{2-b-be}{4-b^2} + \frac{\lambda b}{2\varepsilon(4-b^2)} \quad (29)$$

$$q_j = \frac{2-b-2e+b^2e}{4-b^2} + + \frac{\lambda(2-b^2)}{2\varepsilon(4-b^2)} \quad (30)$$

Using this, the profit of the owner is:

Lemma 6. *If the separating quota is targeted at the less efficient agent, and further the owner gives λ bonus for fulfilling the quota then the expected profit of the owner is:*

$$\pi = \frac{2-be}{(2+b)^2} - \frac{(4-3b^2)\varepsilon^2}{(4-b^2)^2} - \frac{\lambda((4-3b^2)\lambda - 2(8e+b((2-b)^2-6be))\varepsilon)}{4(4-b^2)^2\varepsilon^2} \quad (31)$$

The following first-order condition ensures profitmaximization for the owner:

$$\frac{(8e+b((-2+b)^2-6be))\varepsilon-(4-3b^2)\lambda}{2(4-b^2)^2\varepsilon^2} = 0 \quad (32)$$

Solving this for λ, we arrive to:

Lemma 7. *If the owner introduces a separating quota that is focusing on the less efficient agent, the owner-optimal bonus for fulfilling the quota is:*

$$\lambda^* = \frac{(b((2-b)^2-6be)+8e)\varepsilon}{4-3b^2} \quad (33)$$

E Proof of Theorem 2

If quotas are effective, three cases are to be considered:

- pooling quota
- separating quota aimed at the more efficient agent
- separating quota aimed at the less efficient agent

E.1 Pooling quota

We have obtained the owner-optimal bonus in lemma 3. Its derivatives show us how certain changes affect the size of the quota bonus.

$$\frac{\partial\lambda}{\partial e} = \varepsilon > 0 \quad (34)$$

$$\frac{\partial\lambda}{\partial\varepsilon} = \frac{b+e+be}{1+b} > 0 \quad (35)$$

$$\frac{\partial\lambda}{\partial b} = \frac{(1+e)x}{1+b} - \frac{(b+e+be)x}{(1+b)^2} > 0 \quad (36)$$

E.2 Separating quota aimed at the more efficient agent

The optimal bonus is given by lemma 5. The changes are shown by following derivatives:

$$\frac{\partial\lambda}{\partial e} = \frac{2b^3\varepsilon}{4-3b^2} > 0 \tag{37}$$

$$\frac{\partial\lambda}{\partial \varepsilon} = \frac{b(4-4b+b^2(1+2e))}{4-3b^2} > 0 \tag{38}$$

$$\frac{\partial\lambda}{\partial b} = \frac{(16+b(-32+24b(1+e)-3b^3(1+2e)))\varepsilon}{(4-3b^2)^2} > 0 \tag{39}$$

E.3 Separating quota aimed at the less efficient agent

The size of the quota bonus is shown in lemma 7. The relevant derivatives are as follows:

$$\frac{\partial\lambda}{\partial e} = \frac{(8-6b^2)\varepsilon}{4-3b^2} > 0 \tag{40}$$

$$\frac{\partial\lambda}{\partial \varepsilon} = \frac{8e+b((-2+b)^2-6be)}{4-3b^2} > 0 \tag{41}$$

$$\frac{\partial\lambda}{\partial b} = \frac{(16+b(-32+24b-3b^3))\varepsilon}{(4-3b^2)^2} > 0 \tag{42}$$

The theorem follows from the above inequalities. □

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