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ANALYSIS OF A MULTI-ECHELON PRODUCTION-INVENTORY SYSTEM WITH RANDOM SUPPLY*

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ABSTRACT

In an earlier paper a multi-echelon production-inventory system has been described and a heuristic suggestion has been developed. The purpose of this paper is to give further analysis of the same system; namely, its dynamic behavior is discussed.

1. THE SYSTEM

A detailed description of the multi-echelon production-inventory system and its operation is given in ref. [1]. Here only the main features of the system are outlined.

The system is a somewhat simplified model of a real company with several fairly independent plants. These plants operate as separate profit centers, for which only the "rules of the game" are determined, and under these rules they make their own decisions. The production process includes a number of plants which are interconnected basically in a convergent way but with one divergent phase included. (The system can be seen in Fig. 1).

The plants receive the production task from the Master Production Schedule (MPS) which is based on market forecast for the end item of the company. The MPS is determined for a three month period — within that the plants schedule their production independently.

The company is strong in the market which means that it can dictate the delivery conditions of its finished product. However the external supply of raw materials and spares of the company is subject to random factors and this fact influences the operation of the whole system. The independent production schedul-

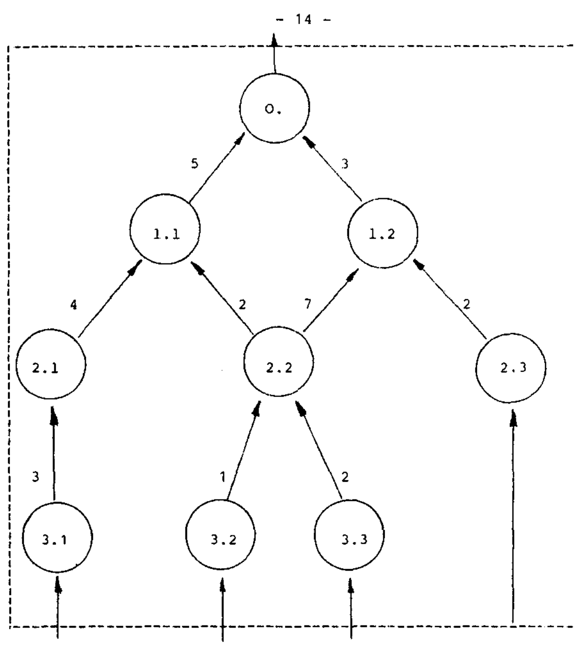

Fig. 1. The multi-echelon production-inventory system; structure of the plants and the unit requirements.

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ing of the various plants and the random raw material supply together lead to the randomness of supply at each plant. Our task is to determine the production lot sizes and the safety stocks in the system so as to ensure the operation at minimal costs.

In the previous paper which we have referred to above, a heuristic solution has been suggested. This solution gave fairly good results for a static model, but couldn't handle the dynamic operation of the system. In this paper we deal with the dynamic version of our heuristics.

2. THE OPERATION

The company can forecast its demand with high reliability for about two years. The basic information for the operation of the system is the time series of demand (the long-range production plan) for two years.

The Master Production Schedule is determined for a three month period. (This means that the time horizon of the production plan is eight periods). Based on this MPS, an MRP procedure determines the net requirements for the various items in the system. (A one item–one plant correspondence is supposed, which seems acceptable in the real system as well as for our purposes.)

Each plant j on echelon i has a practically continuous production pattern, and determines its delivery schedule independently.

Given the net requirement for the finished product of each plant ij (which in fact are semi-finished from the point of view of the whole company) the following questions have to be answered:

(a) What should be the delivery lot sizes, or (which is obviously the same question) how many deliveries should be made during the scheduling period?

(b) How can it be achieved that the interest of the whole system is considered when preparing the plants own independent delivery plan?

(c) How can the various plants protect themselves against the uncertainty arising from the fact that although the lot size of their supplying plant is fixed, the delivery time is random in consequence of the uncertain character of the raw material input?

Various answers can be provided to these questions, with which the system can operate similarly well. The three basic approaches to similar multi-stage lot-sizing problems are described for example in ref. [2] as follows:

(i) The first approach focuses on algorithms which yield production schedules and order quantities for the entire product structure.

(ii) The Hierarchical Production Planning following the organizational lines links high-level aggregate planning with detailed operational planning.

(iii) The most common approach is to decompose the multi-level problem into a series of single-stage lot-sizing problems. In this case the levels are analyzed separately from each other — the BOM represents the only connection among them.

Based on principle (iii) we used the following approach to the questions:

(a) The lot sizes can be determined by some variant of the economic order quantity (EOQ model).

(b) The connection between the subsequent plants is made through the inventory between the two echelons. We assume that once an item is produced at a workshop on echelon i it is kept in stock at this workshop until the production of the whole delivery lot is finished and then it is delivered to the input stocking point of a plant on the $(i-1)$ echelon. This procedure leads to the fact that the lot size of plant i influences the input stock of plant $(i-1)$ and need to be considered when determining lot sizes. This leads to an, at least, partly common interest between plants i and $(i-1)$.

(c) Since the delivery schedule of plant i is to be considered random from the point of view of plant $(i-1)$, a safety stock must be held at the input stocking point of plant $(i-1)$ to ensure that its production schedule can be met (at least with a high probability).

A reliability type model is used to determine the safety stock needed in consequence of the random supply. This reliability model determines the initial stock at the beginning of the scheduling period needed to meet the demand with high probability considering the random character of supply (which in our case means the randomness of the arrival time of equal lots coming from the plant at the preceding level). Since this safety stock depends on the frequency of arrivals of the next period it must be revised before each period, well in advance. Since our assumption is that all order arrives within one scheduling period (only the arrival time is random), this means that a decision on the initial stock of a period must be made one period earlier. This decision will of course influence the actual net requirement of the previous period, namely the requirement stemming from the MRP must be adjusted to achieve the target initial stock of the next period.

We do not handle the occurence of shortage. This means that we suppose that the safety stock is large enough to ensure a safe operation. Considering that, according to the assumptions, the total quantity ordered by the higher level arrive before the end of the period and only its time pattern is unknown, this assumption is satisfying from a practical viewpoint.

This procedure is a recursive one, i.e. we must know how much ending inventory is planned by the company for the end of the whole planning horizon. The procedure can be used for a rolling schedule, and also can handle the change of the parameters within the planning period.

3. THE MODEL

The following notations will be used:

- r_{ij}^t demand (production quantity scheduled) for the plant ij in period t;
- n_{ij}^t number of deliveries from plant ij (the number of production lots; after finishing each lot delivery is performed);
- β_{ij} the unit value of the product ij (independent of t);
- h_{ij} the unit holding cost of the period ij independent of t, and the location of the product (i.e. whether it is stored in a store of level i, the producer or level $(i-1)$, the user). In this version of the model it is assumed that $h_{ij}=0.3\beta_{ij}$;
- w_{ij} the constant cost of one delivery from plant ij, independent of t;
- m_i the number of the products at level i;
- δ the coefficient with which the safety stock of the $(T+1)$th period is determined (originally it was 0.8);
- T number of the periods in the time horizon of the production plan;
- $1-\epsilon_{ij}^t$ safety factor, the probability of meeting all demand in the scheduling period. In this version it is assumed that $1-\epsilon_{ij}^t=1-\epsilon=0.99$ throughout the system;
- $I_{ij}^{(s)t}$ safety stock of the product of plant ij in period t;
- $I_{ij}^{(c)t}$ cycle stock of the product of plant ij in period t;
- $I_{ij}^{(T)t}$ total inventory, $I_{ij}^{(T)t}=I_{ij}^{(s)t}+I_{ij}^{(c)t}$;
- $C(n_{ij}^t)$ the total system cost associated with the number of deliveries n_{ij}^t from plant ij in period t. Part of this cost will actually occur at plant ij and the other part (namely, the cost of holding safety stock) at the subsequent plant at level $(i-1)$.

Using these notations the general form of the cost function at plant ij in period t can be written as follows:

$$C(n_{ij}^t)=h_{ij}I_{ij}^{(c)t}(r_{ij}^t,n_{ij}^t)+w_{ij}(n_{ij}^t)+h_{ij}I_{ij}^{(s)t}(n_{ij}^t,r_{ij},\epsilon) \quad (1)$$

As it has already been mentioned, the EOQ formula is used to determine the lot size at each plant. The value of demand used in the formulas is determined as follows:

$$r_{ij}^{*t} = r_{ij}^{t} + I_{ij}^{(s)(t+1)} - I_{ij}^{(s)t} \qquad (2)$$

where

r_{ij}^{t} the demand for the product of plant ij determined by the MRP procedure;

$I_{ij}^{s(t)}$ the safety stock actually held in order to protect against the uncertainty stemming from the random delivery of plant ij. This stock is physically held at the plant(s) of level $(i-1)$. It is expressed in units of plant ij.

The value of r_{ij}^{*t} is the corrected demand for the product, where the correction is made in order to adjust the safety stock level. The calculation of the EOQ at any plant is the following:

$$q_{ij}^{*t} = \sqrt{\frac{2w_{ij}r_{ij}^{*t}}{h_{ij}}} \qquad (3)$$

For the calculation of the safety stock we also need the number of deliveries, which can be obtained by using the following formula:

$$n_{ij}^{*t} = \sqrt{\frac{r_{ij}^{*t}h_{ij}}{2w_{ij}}} = \sqrt{r_{ij}^{*t}}\sqrt{\frac{h_{ij}}{2w_{ij}}} \qquad (4)$$

The safety stock can be determined by a formula based on a reliability equation expressing the condition that the probability of shortage do not exceed a given level (in our case 0.01%). We use the following formula (for the derivation see ref. [3]).

$$I_{ij}^{(s)t} = r_{ij}^{*t} \cdot \sqrt{\frac{1}{n_{ij}^{*t}} \cdot \tfrac{1}{2} \cdot \ln\frac{1}{\epsilon}} \qquad (5)$$

This formula is valid only if there is only one user of the product of plant ij. In our case this condition is met except plant 2.2. We handle the production of plant 2.2 as it is described in ref. [1] — the essence is that plant 2.2 delivers to both users at the same time and proportionates the quantity delivered to the overall demand of the two users. Then the plant can be handled the same way as any other.

The above formula, eqn. (5) can be written in the following form:

$$I_{ij}^{(s)t} = (r_{ij}^{t} + I_{ij}^{(s)(t+1)} - I_{ij}^{(s)t}) \cdot \sqrt{\frac{1}{n_{ij}^{*t}}} \cdot z \qquad (6)$$

where

$z = \sqrt{\tfrac{1}{2}\ln\frac{1}{\epsilon}}$ is a constant for all plants and periods.

It is easy to see that we can use formulas (4) and (6) to determine the operating parameters of the system. For that we need to know the time series of forecasted demand (r_{ij}^{t}), the ending inventory, the cost parameters and the reliability level.

The description and the steps of the algorithm we applied are as follows:

Input data: $r_{ij}^{t}, h_{ij}, w_{ij}, m_i, \epsilon, \delta$
  $(t=1,2,\ldots,T; i=1,2,3; j=1,2,\ldots,m_i)$;

Further notations: $z := \sqrt{\tfrac{1}{2}\ln\frac{1}{\epsilon}}$; $\gamma_{ij} := \sqrt{\frac{h_{ij}}{2w_{ij}}}$;

  $I_{ij}^{(s)(T+1)} =: \delta \cdot r_{ij}^{T+1}$ for $\forall i$ and $\forall j$;

Step 1: For $t=T$ $\alpha_{ij} := r_{ij}^{t} + I_{ij}^{(s)(t+1)}$
  $(i=1,2,3; j=1,2,\ldots,m_i)$

Step 2: Solve the following equation for n_{ij}^{*t}
  (n_{ij}^{*t} is an integer variable)

$$(n_{ij}^{*t})^2 + z\cdot(n_{ij}^{*t})^{3/2} - \alpha_{ij}^{t}\cdot\gamma_{ij}^2 = 0$$

(This equation is a transformation of eqns. (4) and (5)).

Step 3: Having solved the above equation, determine the value of $I_{ij}^{(s)t}$:

Step 4: $r_{ij}^{*t} := \alpha_{ij}^{t} - I_{ij}^{(s)}$

Step 5: If t equals to 1 then Stop, else decrease t by one and GO TO Step 1.

4. COMPUTATIONAL RESULTS

We have made a lot of calculations to test our algorithm. A model based on the "Multi-

TABLE 1

Sample calculation testing the algorithm

	Periods	1	2	3	4	5	6	7	8	9	Aver.	St. Dev.	Cost		
Prod. 0	Reqs.	113	92	280	203	322	88	313	128	301	204	100	Prod. 0		
	Corr. reqs.	113	92	280	203	322	88	313	128		192	100			
	Del. numb.	10	9	16	14	17	9	17	11		12	3	800	Set up	9600
	Lot size	11	10	18	15	19	10	18	12		14	3			
	C. stock	5.5	5	9	7.5	9.5	5	9	6		7	1	1422	Hold.	133842
safety	Prod. 1.1	307	350	581	507	600	359	630	562	1204	566	267			
stock	Prod. 1.2	171	188	322	277	327	194	354	319	722	319	165		Total	143442
Prod. 1.1	Reqs.	565	460	1400	1015	1610	440	1565	640	1505	1022	503	Prod. 1.1		
	Corr. reqs.	608	691	1326	1108	1369	711	1497	1282		1074	352			
	Del. numb.	9	9	12	11	12	9	13	12		10	1	400	Set up	4000
	Lot size	67	76	110	100	114	115	106		95	18				
						t 79									
	C. stock	33.5	38	55	50	57	39.5	57.5	53		47	9	87	Hold.	58278
safety	Prod. 2.1	1304	1474	2382	1988	2275	809	2612	2315	4816	2219	1138			
stock	Prod. 2.2	435	467	804	693	834	502	959	1220	7464	1486	2255		Total	62278
Prod. 2.1	Reqs.	2260	1840	5600	4060	6440	1760	6260	2560	6020	4088	2013	Prod. 2.1		
	Corr. reqs.	2430	2748	5206	4347	4974	3563	5963	5061		4286	1257			
	Del. numb	8	8	11	11	11	6	12	11		9	2	100	Set	900
	Lot size	303	343	473	395	452	593	496	460		439	91			
	C. stock	151.5	171.5	236.5	197.5	226	296.5	248	230		219	45	5	Hold.	14661
ss	Prod. 3.1	3830	4189	6951	6021	7000	4313	7559	6740	14448	6783	3193		Total	15561
Prod. 3.1	Reqs.	6780	5520	16800	12180	19320	5280	18780	7680	18060	12266	6039	Prod. 3.1		
	Corr. reqs.	7139	8282	15870	13159	16633	8526	17961	15388		12869	4280			
	Del. numb.	8	9	12	11	13	9	13	12		10	1	100	Set up	1000
	Lot size	892	920	1322	1196	1279	947	1381	1282		1152	199			
	C. stock	446	460	661	598	639.5	473.5	690.5	641		576	99	2	Hold.	11590
ss	Prod. 3.1	2895	3359	6436	5337	6746	3458	7284	6241		5219	1736		Total	12590
Prod. 1.2	Reqs.	339	276	840	609	966	264	939	384	903	613	301	Prod. 1.2		
	Corr. reqs.	356	410	795	659	833	424	904	787		646	218			
	Del. numb.	10	11	14	13	15	11	15	14		12	1	450	Set up	5400
	Lot size	35	37	56	50	55	38	60	56		48	10			
	C. stock	17.5	18.5	28	25	27.5	19	30	28		24	5	234	Hold.	88809
safety	Prod. 2.2	1087	1167	2009	1734	2086	1254	2399	3050	7464	2472	1977			
stocks	Prod. 2.3	448	494	804	713	823	517	890	772	1444	767	299		Total	94209
Prod. 2.2	Reqs.	3503	2852	8680	6293	9982	2728	9703	3968	9331	6337	3120	Prod. 2.2		
	Corr. reqs.	3615	4031	8294	6786	8818	4330	10615	14626		7639	3786			
	Del. numb.	13	14	20	18	21	14	23	27		18	4	300	Set up	5400
	Lot size	178	287	414	377	419	309	461	541		385	91			
	C. stock	139	143.5	207	188.5	209.5	154.5	230.5	270.5		192	45	29	Hold.	78090
safety	Prod. 3.2	1786	2005	3330	2863	3374	2005	3660	3298	7464	3309	1707			
stock	Prod. 3.3	3043	3268	5624	4843	5799	3343	6133	5689	14929	5852	3613		Total	83490
Prod. 3.2	Reqs.	3503	2852	8680	6293	9982	2728	9703	3968	9931	6337	3120	Prod. 3.2		
	Corr. reqs.	3722	4177	8213	6804	8613	4384	9341	8134		6673	2254			
	Del. numb	10	10	14	13	15	11	15	14		12	2	120	Set up	1440
	Lot size	372	417	586	523	574	398	622	581		509	98			
	C. stock	186	208.5	293	261.5	287	199	311	290.5		254	49	6	Hold.	17754
ss	Prod. 3.2	1509	1693	3330	2759	3492	1777	3787	3298		2705	914		Total	19194
Prod. 3.3	Reqs.	7006	5704	17360	12586	19964	5456	19406	7936	18662	12675	6240	Prod. 3.3		
	Corr. reqs.	7231	8060	16579	13542	17508	8246	18962	17176		13413	4859			
	Del. numb.	13	14	20	18	21	14	22	21		17	3	200	Set up	3400
	Lot size	556	575	828	752	833	589	861	817		726	130			
	C. stock	278	287.5	414	376	416.5	294.5	430.5	408.5		363	65	9	Hold.	52209
ss	Prod. 3.3	2932	3268	6722	5491	7099	3343	7689	6964		5438	1970		Total	55609
Prod. 2.3	678	552	1680	1218	1932	528	1878	768	1806	1226	603		Prod. 2.3		
	Corr. reqs.	724	862	1589	1328	1626	901	1760	1440		1278	396			
	Del. numb.	6	7	9	8	9	7	9	8		7	1	150	Set up	1058
	Lot size	120	123	176	166	180	128	195	180		158	29			
	C. stock	60	61.5	88	83	90	64	97.5	90		79	14	15	Hold.	8955
ss	Prod. 2.3	293	349	644	538	659	365	713	583		518	160		Total	10005
														Total	519868

TABLE 2

Sensitivity of total cost to changes of various model parameters

Demand parameters	Increase of delivery cost $A=\frac{w'_{ij}}{w_{ij}}$	$\delta1=0$	$\delta2=1/3$	$\delta3=2/3$	$\delta4=.8$	$\delta5=1.0$
D1	A1=1	308786	352295	398015	406236	430073
(137, 47)	A2=1/2	368894	415049	457718	472066	496290
	A3=1/3	442091	457181	499972	516903	543807
D2	A1=1	371268	410370	451609	470932	490177
(174, 73)	A2=1/2	443267	480235	529856	533422	570446
	A3=1/3	494725	543470	578850	595488	622038
D3	A1=1	385613	473634	550500	579868	629276
(195, 96)	A2=1/2	477156	553991	631061	664667	729269
	A3=1/3	503244	599928	684915	715699	749803

plan" spreadsheet program has been built to evaluate how the algorithm works.

The calculations show that the heuristic works, since it has given empirically meaningful results, and the computations carried out with different parameters provided acceptable and explainable changes in the cost structure. As for as operational variables of the system, we have found the following:

(1) The lot sizes are rather stable quantities, their standard deviations are relatively low. This allows for a stable production schedule and good capacity utilization.

(2) The safety stocks fluctuate with the demand almost proportionally. (This consequence can be expected because of the model applied.) In most cases the safety stock shows a somewhat smaller fluctuation, due to absorption of some of the demand fluctuation by changes in the lot size.

To analyse the effects of the changes in the various parameters of the system, the following computations have been carried out.

We have used three different uniformly distributed demand samples with the following means and standard deviations: D1=(137, 47); D2=(174, 73); D3=(195, 96).

Three variations of the cost parameters have been considered: we decreased the original ratio of the holding and set-up costs to its half and third by multiplying the set-up cost by two and three. These variations, in the decreasing value of the ratio, are denoted by A1, A2, A3.

As it is described in the model, we have to estimate the safety stock of the last period. This safety stock will be given in percentage (δ) of the demand in the last period: $\delta1=0$; $\delta2=1/3$; $\delta3=2/3$; $\delta4=0.8$; $\delta5=1.0$.

Such for example Table 1 shows the results of using the values of D3, A1 and $\delta4$. In this case the total cost of the operation of the system is 579,868 Ft.

Having made all the computations we have $3\times3\times5=45$ tables showing the results of the different demand patterns, cost parameters and estimations. A summary of the results is shown in Table 2.

The results show that the system works as can be expected.

(a) If the demand is increasing the total cost is also increasing.

(b) Having increased the set-up cost (decreased the ratio of the holding and set-up cost parameter), the total cost is increasing.

(c) A higher safety stock of the last period increases the total cost of the operation.

(d) The joint effect of simultaneous changes in more than one of the parameters has also been examined. There were no surprising results, in fact, both the operational variables and the cost behave as expected.

SUMMARY AND CONCLUSION

The combination of two elementary models have been used to describe the operation of a realistic multi-stage production-inventory system of partially independent plants. A simple heuristics was used to determine the system's operational variables, namely the lot sizes and the safety stocks. The computations provided well explanable results. The conclusion is that in the case examined, as in many other cases as well, heuristical application of simple decision rules lead to well operationable system parameters.

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