

## HEDGING UNDER LIQUIDITY CONSTRAINTS<sup>1</sup>

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### ABSTRACT

Although risk management can be justified by financial distress, the theoretical models usually contain hedging instruments free of funding risk. In practice, management of the counterparty risk in derivative transactions is of enhanced importance, consequently not only is trading on exchanges subject to the presence of a margin account, but also in bilateral (OTC) agreements parties will require margins or collateral from their partners in order to hedge the mark-to-market loss of the transaction. The aim of this paper is to present and compare two models where the financing need of the hedging instrument also appears, influencing the hedging strategy and the optimal hedging ratio. Both models contain the same source of risk and optimisation criterion, but the liquidity risk is modelled in different ways. In the first model, there is no additional financing resource that can be used to finance the margin account in case of a margin call, which entails the risk of liquidation of the hedging position. In the second model, the financing is available but a given credit spread is to be paid for this, so hedging can become costly.<sup>2</sup>

*JEL codes:* G17, G32

*Keywords:* risk management, hedging, financing liquidity

### 1. INTRODUCTION

The rationale of corporate risk management is justified in financial theories by market imperfections or incentives. Models analysing the effects of taxes, transaction costs, information asymmetry or the costs of financial distress all conclude that a perfect hedge is optimal in most cases<sup>3</sup> (Dömötör, 2014). Theories

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<sup>3</sup> The process of corporate risk management and the Hungarian practice is detailed in WALTER (2014).

vindicating corporate risk management for funding or incentive-based reasons are supported by empirical experience, while explanations connected to taxation or transaction costs have not been corroborated (*Hommel, 2005*).

The financial crisis that began in 2007 proved that inadequate management of liquidity can be a source of serious problems, and that no market participant has access to unlimited financing. Consequently, the issue of financing cannot be neglected in models of market risk management, while liquidity constraints also apply to hedging positions, given that losses on derivatives generate a credit risk exposure.

This paper initially provides a short overview of the appearance of financing in models of corporate risk management, then presents two models focusing on the effects of funding risk on hedging positions. Finally the models and their results are compared.

## 2. THE EFFECT OF FINANCING ON RISK MANAGEMENT

The starting point for theories that find an explanation for the value of hedging in corporate financing is that if a firm lacks internal funds, it must make use of an external funding source, which is, in contrast to the *Miller-Modigliani* theorem (1958), expensive or not even possible at all. Funding-related costs can be either direct (administrative) costs or agency costs arising from information asymmetry (*Myers, 1984, Tirole, 2006*). Hedging decreases corporate cash flow dispersion, and as a result, the probability of financial distress is also reduced. The costs of financial distress can appear in the form of transaction costs, while higher expected bankruptcy costs can cause a direct decrease in the firm's value (*Smith and Stulz, 1985*). Financial distress can result in a firm's partial or complete inability to realize its positive net present value investments, which likewise reduces the value of the firm (*Lessard, 1990, Froot et al, 1993*).

Two basic models explaining the value of corporate hedging from the point of view of limited financing resources are the models of *Froot et al (1993)* and *Tirole (2006)*. Both models assume a risk-free corporate utility function (linear in profit) and costly external financing. The cost function is given exogenously in the model of *Froot et al*, but in the *Tirole* model it derives from agency-based considerations, as the provider of the external financing requires a certain level of own funds from the investor, in the absence of which it does not take the risk of financing even for a credit spread.

Both models conclude that the costs or even the unavailability of external financing justify the rationale of hedging, as through hedging a certain level of internal financing resources can be ensured to implement the investments with positive net present value.

In both models, corporate production and hedging is described as a two-period decision, where hedging itself is made for a single period, so a hedging decision has to consider exclusively the production and price distribution at maturity. As there are no interim periods in hedging, the hedging position creates no cash-flow and hence no further risk.

In practice hedging positions need financing for several reasons: upfront fees have to be paid for derivatives with an asymmetric payout function (such as options); or there is a mismatch of the hedging position with the underlying risk (basis risk); or the daily mark-to-market settlement of futures<sup>4</sup> has cash-flow consequences. In the case of exchange trading, a certain amount of initial margin is required and also a minimal level – the so-called maintenance margin – has to be ensured during the entire lifetime of the transaction. Although profit or loss on derivatives on the over-the-counter (OTC) market does not need to be settled on a daily basis, in practice in most cases the partners require some initial or interim collateral to reduce the counterparty risk (*Korn*, 2003). The new European Market Infrastructure Regulation (EMIR, 2012) prescribes central clearing for even OTC transactions above a certain level, in order to reduce counterparty risk and so the vulnerability of the financial system.

Contracts of the ISDA (International Swaps and Derivatives Association), which provides the legal framework of derivatives trading, are essentially credit contracts, their annexes also containing credit risk-mitigating elements such as collateral obligations or covenants. As a consequence of the crisis, these documents were supplemented by the Credit Support Annex (CSA), which contains mutual collateralization obligations even in the case of the largest and supposedly safest counterparties or banks. Furthermore, even if a firm has no financing obligation connected to its derivative transactions, the non-realized loss of the position increases the exposure of the bank toward the firm and restricts the availability of further financing. Consequently not only exchange-traded derivatives, but also OTC positions are path-dependent, their profit depending on price evolution during the term.

As a consequence of all this, the availability of financing is critical for a hedging position as well. The maturity of derivatives used for hedging can often be measured in years, and their financing needs affect a firm's financing opportunities.

Although the analysis of *Froot et al.* (1993) mentions the trade-off between the variability of future cash-flow and the fluctuation of cash in the interim period if the hedging position is to be financed, they do not analyse this problem further. The issue of financing needs for hedging positions appears in the analysis of *Anderson and Danthine* (1983). In their multi-period model, hedging occurs on several dates and the mark-to-market value of the hedging position (futures) is

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<sup>4</sup> The differences in trading and hedging of forwards and futures are detailed in *BERLINGER et al* (2005).

settled in each interim period. Nevertheless the model does not include any financing constraint or credit spread, so that cash-flow can be converted simply to the date of maturity at the risk-free rate.

The liquidity risk of the hedging position appears in theoretical models in the 2000s. *Mello and Parsons* (2000) investigate optimal hedging strategies by considering liquidity aspects, concluding that financial constraints lead to the sub-optimality of both cash-flow variance-minimizing and corporate value variance-minimizing hedging strategies. Optimal hedging minimizes the variance in the marginal value of corporate cash; it switches cash to the outcomes where the marginal utility is the highest.

In the models detailed in this paper, the optimisation criterion is not the maximization of the expected profit, but the maximization of the (concave) corporate utility function. Liquidity risk appears in two forms: firstly, through the modelling of the margin account, providing that the firm has no or only a limited source of financing in case of a margin call (*Deep*, 2002). The unavailability of financing derives from the fact that internal resources are too expensive to hold for this purpose while external investors are not willing to provide financing or require a spread because of information asymmetry, as it cannot be ascertained from outside whether the losses on derivative positions are caused by prudent hedging or irresponsible speculation.

The other way of modelling liquidity risk is based on the financing costs deriving from the credit spread to be paid to collateralize the loss of the position (*Korn*, 2003).

### 3. HEDGING IN CASE OF LIMITED MARGIN AVAILABILITY – DEEP MODEL

The risk investigated by *Deep* (2002) can be perfectly eliminated by futures hedging. The future corporate output ( $\pi$ ) is given; the risk derives from the uncertainty of the future price of the product. The price is assumed to follow geometric Brownian motion with a drift equal to the risk-free rate.

$$dS_t = rS_t dt + \sigma S_t dw_t, \quad (1)$$

where  $S_t$  is the spot price in time  $t$ ,  $r$  stands for the risk-free interest rate,  $\sigma$  is the volatility of the price change and  $dw_t$  – the change in the Wiener process – denotes the stochastic part of the price movement.

As the expected growth of the price is the risk-free rate, the evolution of the forward rate – using Itô's lemma – is a martingale process.<sup>5</sup>

<sup>5</sup> The drift of the forward rate process is the difference between the drift of the underlying asset and the risk-free rate, which is zero in our case.

$$dF_t = \sigma F_t dw_t. \quad (2)$$

With this simplification, the speculative motive of the hedge can be eliminated and the profit or loss of the hedging position has no impact on the optimal hedging.<sup>6</sup> The firm hedges its exposure with short futures, where the hedged amount ( $\theta_t$ ) can be adjusted on any interim date. The value of the hedge (futures) position is settled on the margin account ( $X_t$ ) on each interim date, so the value of the margin is also stochastic:

$$dX_t = rX_t dt + \theta_t dF_t. \quad (3)$$

The firm has a certain amount of cash ( $X_0$ ) to use as a margin in order to open the hedging position, but it cannot get further financing if the margin account drops below a minimal ( $K$ ) level that has to be maintained and the firm receives a *margin call*. The inability to meet the margin obligation leads to liquidation of the position, and so the original exposure becomes unhedged. Although the model assumes the unavailability of financing, a credit line can be built into the model by adjusting the values of  $K$  and the initial margin of  $X_0$ .

Table 1 shows the probability of liquidation of the hedging position for different maturities, initial margin amounts and price volatility. The price follows the geometric Brownian motion described in Equation (1).

**Table 1**  
**The probability of liquidation of the hedging position**

Volatility	Probability of liquidation		
	Initial margin ( $X_0/F$ )	T = 26 weeks	T = 52 weeks
15%	0.05	55%	65%
	0.10	29%	44%
	0.25	1%	9%
	0.50	0%	0%
	1.00	0%	0%
20%	0.05	62%	69%
	0.10	41%	48%
	0.25	8%	13%
	0.50	0%	1%
	1.00	0%	0%

Source: own calculation based on Deep (2002)

6 The expected return of a derivative position is analysed in BERLINGER (1998).

The above results are based on a Monte Carlo simulation by running 1,000 realizations. In the case of an initial margin requirement of 10%, the probability of liquidation of a 1-year position within term is 44%. As the initial margin is in reality below 10%, the hedging firm has to calculate a future financing need that has to be managed without financial distress.

The goal of corporate management is to maximize the expected utility of corporate value – the sum of the production and the margin account – at maturity. Assuming constant relative risk aversion (CRRA)<sup>7</sup> in the corporate utility function, the optimal hedge maximizes the following equation:

$$\max_{\theta_t} E_t \left[ \frac{(X_T + \pi F_T)^\gamma}{\gamma} \right]; \quad 0 < \gamma < 1^8, \quad (4)$$

subject to  $X_t \geq K$ .

Deep solves the optimisation with the help of stochastic dynamic programming. The optimal hedging problem is a stochastic control problem that has only a numerical solution because of the non-linearity of the partial differential equation to be solved.

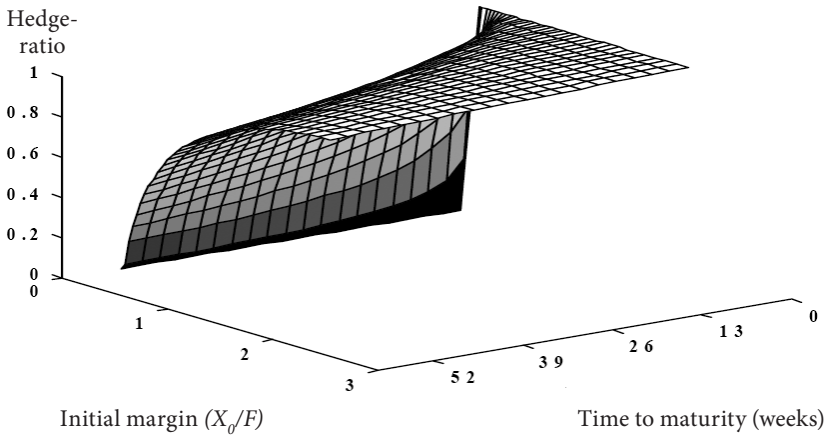
Factors influencing the optimal hedging strategy are: corporate exposure, maturity of the hedge, volatility of the risk factor, available financial resources and corporate risk attitude.

When deciding about hedging, corporations have to choose between two types of risk: the lesser the uncertainty over the future price of production (*value risk*), the greater the risk of liquidation of the position within term (*cash-flow or liquidity risk*). The latter risk, i.e. the probability of falling beneath the minimum level of the margin account, clearly diminishes as the time to maturity decreases, and grows as the volatility of the price increases. It can also be seen intuitively that a higher margin account balance means less constrained liquidity.

*Chart 1* shows the effect of available financing sources and the time to maturity, assuming the risk factor follows geometric Brownian motion with annual volatility of 15% and a drift rate equalling the risk-free rate (5%), while the level of corporate risk aversion ( $1-\gamma$ ) is 0.5.

<sup>7</sup> This means that the attitude toward risking a certain ratio of assets is constant. The measures of investors' risk preferences are formalized in the works of ARROW (1970) and PRATT (1964).

<sup>8</sup> Here  $\gamma$  denotes not the measure of risk aversion, but  $(1 - \text{risk aversion})$ . As DEEP uses a risk aversion of 0.5, the two values are equal.

**Chart 1****Optimal hedge ratio as a function of initial margin and maturity**

Source: Deep (2002), Chart 5.1

The optimal hedge ratio is thus a negative function of the *time to maturity*, and a positive function of the *available financing resources*. The corporate risk aversion is in inverse proportion to the level of financing resources. Higher risk aversion has a similar effect as higher liquidity constraints; namely, both reduce the optimal hedge ratio, since the utility reduction deriving from the termination of the hedge is larger.

Deep's model explains underhedging as a financing decision. This model of optimal hedging leads to the conclusion that financial difficulties in maintaining the position result in the reduction of the corporate hedge ratio of the predetermined output.

#### 4. FINANCING COST OF THE HEDGE POSITION – KORN MODEL

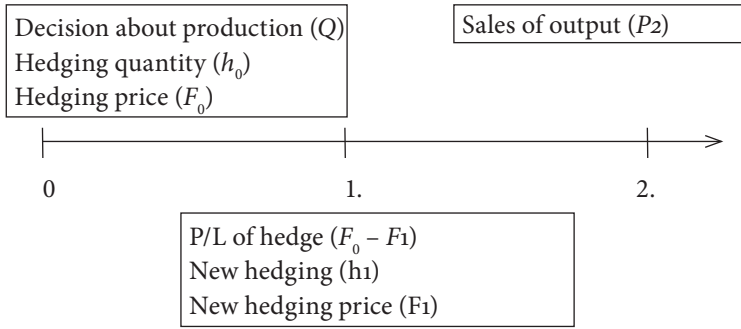
The liquidity risk of the hedging transaction is due in the model of Korn (2003) not to the potential liquidation of the position, but to the extra cost of financing. The model assumes that the firm is able to secure financing on the market, albeit at a cost since it cannot obtain such financing at the risk-free interest level ( $r$ ). The higher the corporate-specific credit spread ( $s$ ), the higher the cost of financing and hence the liquidity risk of hedging. Although the model contains a constant credit spread, it can be extended with a need-dependent credit spread, while unavailable financing can also be simulated by increasing the spread to infinity.

In *Korn's* basic model, the firm decides about the quantity of production ( $Q$ ) that will be realized in 2 periods. The selling price of the output ( $P$ ) is stochastic, generating the risk to be hedged. Forward agreements are used for the hedge, with the evolution of the forward rate presumed to be a martingale (as in the previous model). The firm can conclude hedging deals at both dates, initially and also in the interim period.

Chart 2 depicts the process, with indices representing time.

### Chart 2

#### The process of corporate operation in the model of Korn



Source: own design, based on *Korn* (2003)

The corporate profit realized at the end of the second period ( $\Pi$ ) consists of three parts: the operating profit, the profit or loss of hedging positions, and the cost of collateral of the hedging deal.

$$\Pi = \underbrace{P_2 Q - c(Q)}_{\text{Operating profit}} + \underbrace{h_0 (F_0 - P_2) + h_1 (F_1 - P_2)}_{\text{P/L of the hedge}} + \underbrace{s \min \left[ h_0 \frac{(F_0 - F_1)}{1+r}; 0 \right]}_{\text{Cost of the hedge}} \quad (5)$$

The parameters of the equation are defined above.

The optimal hedging strategy, maximizing expected utility ( $E[U(\Pi)]$ ), is to be determined in two steps retrogressively.

In order to calculate the amount to be hedged at the first (interim) date ( $h_1$ ), equation (5) is to be maximized at the already known levels of  $Q$ ,  $F_1$  and  $h_0$ :

$$\max_{h_1} E_1[U(\Pi) | F_1, Q, h_0] \quad (6)$$



The prime condition ensuring the existence of the maximum, as the utility function is concave:

$$E_1[U'(\Pi)(F_1 - F_2)] = 0. \quad (7)$$

As the expected value of the forward rate at maturity is presumed to equal the forward rate at time 1, ( $E_1(F_2)=F_1$ ), the equation is held if the covariance of the two terms of the product is zero, namely the profit function is independent of  $F_2$ .

Accordingly, the optimal hedging amount at time 1 is:

$$h_1^* = Q - h_0. \quad (8)$$

At the first date the entire output is to be hedged, irrespective of the corporate financing cost ( $s$ ), as no further collateral obligation will arise. At time zero, substituting equation (8), the profit function is the following:

$$\Pi^* = F_1 Q - c(Q) + h_0 (F_0 - F_1) + s \min \left[ h_0 \frac{(F_0 - F_1)}{1+r}; 0 \right]. \quad (9)$$

In the absence of financing costs ( $s=0$ ), the expected utility can be maximized, if:

$$E_0[U'(\Pi^*)(F_0 - F_1)] = 0. \quad (10)$$

Similarly to the above-presented hedging at time 1, the independency of the profit function from the forward rate ( $F_1$ ) can be ensured by hedging the entire production, so:

$$h_0^* = Q^* \quad \text{and} \quad h_1^* = 0. \quad (11)$$

In Korn's model the production quantity is also an endogenous variable that can be determined through equation (9) and the corporate utility function:

$$E_0[U'(\Pi^*)(F_1 - c'(Q))] = 0, \quad (12)$$

which holds if:

$$E_0[U'(\Pi^*)(F_0 - c'(Q)) + \text{cov}_0[U'(\Pi^*), F_1]] = 0. \quad (13)$$

In the optimum the covariance term is zero, if equation (11) holds, so the optimal output is at the level where the marginal cost of production equals the initial forward price ( $F_0$ ). This result suggests a perfect financial hedge, similarly to the model of Froot et al.

If financing is costly ( $s > 0$ ), hedging increases value by reducing the uncertainty of corporate profit, but on the other hand it also has a cost that affects the expected profit negatively. Consequently, the optimal output will be less than in the cost-free case and the optimal hedging ratio is below 1. Korn proves that a hedge ratio less than zero – that is, an exposure in the same direction as the original risk – cannot be optimal due to the costs of any derivative position. The optimal hedge ratio can be calculated in knowledge of the corporate utility function and the evolution of the forward price. The analysed model assumes the forward rate

is lognormally distributed and the utility function reflects constant relative risk aversion (CRRA). Based on these assumptions, *Korn* indirectly proves the following limits of the optimal hedge ratio<sup>9</sup>:

$$\frac{1+r}{1+r+s} \geq h_0^* / Q^* \geq \bar{c} / F_0, \quad (14)$$

where  $\bar{c}$  stands for the average cost of a unit produced.

In order to calculate the optimal hedge ratio, *Korn* adopts the same parameters as *Deep*: a risk-free interest rate of 5% and volatility of the forward price ( $\sigma$ ) of 15%. The cost function is not defined, the average cost is 0.1, and both periods of the model are 1 year.

The optimal hedge ratio is given by maximizing the expected value of the utility (15).

$$U = \frac{(\Pi^*)^{(1-\gamma)}}{1-\gamma}, \quad (15)$$

where:

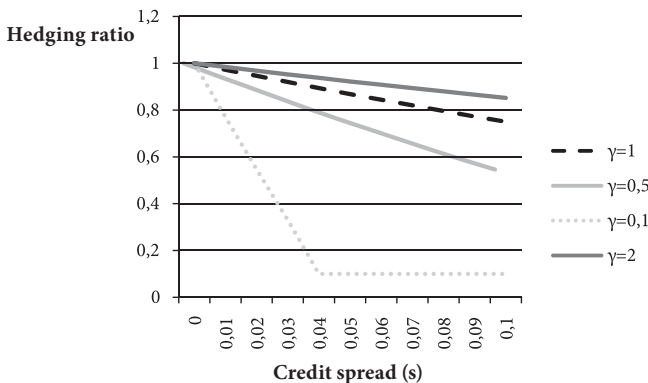
$U$ : utility

$\Pi^*$ : profit in case of optimal hedging

$\gamma$ : risk aversion.

Solving the optimisation numerically with the above parameters, *Chart 3* shows the optimal hedge ratio as a function of the corporate credit spread ( $s$ ) and risk aversion ( $\gamma$ )<sup>10</sup>.

**Chart 3**  
**Optimal hedging in Korn's model**



Source: own simulation, with antithetic variates based on *Korn* (2003)

<sup>9</sup> If the hedge ratio falls outside these limits, a loss (negative profit) occurs with positive probability, which cannot be optimal for a risk-averse company.

<sup>10</sup> Risk aversion is zero in the case of risk neutrality, while the upper extreme value of 2 is the individual risk aversion determined by BLUME and FRIEND (1975).

As *Chart 3* illustrates, a perfect hedge is optimal if the firm secures financing at the risk-free interest rate. A one percentage-point increase in the credit spread leads to a five percentage-point reduction in the optimal hedge ratio in case of a corporate risk aversion of 0.5 ( $\gamma = 0.5$ ).

As the risk aversion decreases (decreasing  $\gamma$ ), the optimal hedge ratio also declines, since the utility of the hedge offsetting the interest cost of the hedge is lower.

The costs of production impact the hedging policy significantly. The ratio of the average cost of production to the forward price represents the lower limit of the optimal hedge ratio, as this level of hedge ensures that the revenue at least covers the costs of operation.<sup>11</sup>

The increase in average cost raises the minimal level of the hedge ratio, since profit will be lower and the slope of the utility function is higher with smaller values (the firm is more sensitive to negative outcomes), so that the utility achieved by the hedge is also higher. However, it is important to note that the above relationship refers to the hedge ratio, while the optimal level of output – and hence the amount of the hedge – can substantially decrease in the presence of financing costs.

The volatility (standard deviation) of the risk factor has a dual effect on the optimal hedge. On the one hand, the higher the volatility of the forward rate (higher risk), the greater the optimal hedge ratio of a risk-averse firm. On the other hand, higher volatility also increases the expected value of the financing costs of the hedge, which has a negative effect on the optimal hedging level. The result of these two contradictory effects is not obvious. In the case of the parameter set examined by Korn ( $r=5\%$ ,  $\gamma=0.5$ , average cost=10%,  $F_0=1$ ,  $F$  lognormal with a mean of 1, and three different values of volatility:  $\sigma=0.1; 0.15; 0.2$ ), increasing volatility causes a rise in the optimal hedge ratio.

The question arises of how the optimal hedging strategy evolves if options transactions are also available, as bought options do not induce a financing need during their lifetime. However, the upfront fee on options makes this strategy too expensive and hence suboptimal for a financially constrained firm.

## 5. COMPARISON OF LIQUIDITY-ADJUSTED HEDGING MODELS

The two models detailed above describe the funding liquidity risk deriving from the financing need of the hedge position in different ways, and their conclusions also partially differ.

<sup>11</sup> Assuming the initial forward price exceeds the average cost; otherwise it is not worth investing in the project.

The selling price of production ( $P$ ) is a risk factor in both models, and therefore corporate revenue and profit are also stochastic. The product is traded on the market and can be sold by (short) forward or (short) futures agreements at any time, in any quantity, at the prevailing market price ( $F_t$ ). Another aspect common to the two models is that the spot and forward rates of the underlying asset follow geometric Brownian motion and the drift of the spot price equals the risk-free rate of return, consequently the progress of the forward rate is a martingale. This assumption simplifies the calculations, as the expected value of the hedge position is zero, so that forward or futures sales have no speculative motive. Both models investigate optimal hedging based on a corporate utility function that reflects constant relative risk aversion (CRRA).

The main difference between the models is in the hedging position. *Deep* uses futures for the hedge, the value of which is settled on a daily basis on the margin account, so that the liquidity risk derives from the financing limits in case of a margin call. In the *Korn* model, hedging occurs through forward agreements that have to be collateralized (in cash) in case of a loss at a single interim date during the term. The liquidity risk appears in the form of the credit spread of the loan taken out to meet the collateral obligation.

The model of *Deep* does not include *production costs*; optimization is based on profit that is the sum of operating income (price of the output at maturity) and financial income (value of the margin account). The produced quantity is an exogenous variable of the model; while in the *Korn* model *production costs* affect both optimal output and the minimal hedging ratio.

Table 2 summarizes the ceteris paribus impact of the parameters influencing the optimal hedge ratio, which can differ in the two models, using the parameter values specified above.

**Table 2**  
**Ceteris paribus effect of factors influencing optimal hedging in the models of Deep and Korn**

	Deep	Korn
Risk aversion ( $\gamma$ )	↓	↑
Volatility of the risk factor ( $\sigma$ )	↓	↑
Credit spread ( $s$ )	--	↓
Financing resource ( $X$ )	↑	--
Hedging period ( $t$ )	↓	↑
Production costs ( $c(Q)$ )	--	↑

Source: own design, based on *Deep* (2002) and *Korn* (2003)

Increasing *risk aversion* leads to a decreasing optimal hedge ratio in the model of *Deep*, as the risk of liquidation of the hedge can be lowered by a smaller derivative exposure. However, in the *Korn* model, a higher risk aversion leads to higher utility achieved by the hedge; consequently, in spite of the enhanced financing costs, the optimal hedge ratio will be higher.

The *volatility of the risk factor* affects both the potential loss from the underlying exposure and the costs of the hedge. These contrary effects have different result in the two models: in *Deep*'s model, the latter is more significant, so that increasing volatility leads to a lower optimal hedge ratio, while in the *Korn* model the enhanced utility of the hedge exceeds the costs, so that the optimal hedge ratio increases in tandem with volatility.

The parameters of *liquidity risk* have the same effect in both models: a higher credit spread or lower margin amount results in the reduction of the optimal hedge ratio.

The *time to maturity* has an impact similar to that of volatility, increasing the risk of liquidation of the hedge position. It therefore lowers the optimal hedge ratio in the model of *Deep*, while in *Korn*'s model the enhanced utility of the hedge due to volatility increases the level of the optimal hedge. Longer maturity means a longer hedging period, but the financing need appears only at a single date in the *Korn* model.

## 6. SUMMARY

As a consequence of the crisis, regulations and the risk management of financial institutions increasingly focus on monitoring and controlling counterparty risk. The daily settlement of derivative positions is being introduced even on OTC markets, so that such positions are exposed to liquidity risk. Consequently risk managers have to consider the financing need of hedging derivatives in their hedging decisions, which may explain the wide range of derivative instruments offered and the common practise of overhedging and underhedging.

The present article presents and compares two theories incorporating the financing need of hedging. The optimal hedge ratio – which is the ratio of the hedging position to the exposure – is determined in both models by the trade-off between the increased utility arising from volatility reduction and the costs of financing of the hedge. However, the approaches to modelling liquidity risk are fundamentally different, so that the conclusions and the impact of certain influencing parameters are partly contradictory.

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