The effect of focusing on loan decisions

Barna Bakó∗ Linda Dezső† Gábor Nesveda‡

November 10, 2015

Abstract

In this paper we investigate some implications of recent results about salience on loan decisions. Using the framework of focus-weighted utility we show that consumers might take out loans even when that yield them negative utility. We claim, however, that consumers are more prudent in their decisions and might be less likely to take out such loans when the usual fixed- and increasing-installment plans are coupled with an equivalent decreasing-installment option. We argue that harmful loan consumption, especially in the case of loans with increasing-installments (e.g. alternative mortgage loans), could be decreased if a policy would prescribe presentation of loan repayment schedules in a way that employs this effect. Moreover, using the model of focus-weighted utility we give a possible explanation for the unpopularity of decreasing-installment plans, the success of increasing-installment plans and their higher default rate observed during the recent financial crisis.

Keywords: focus weighted utility, loan decisions, welfare analysis

1 Introduction

Suppose you are about to purchase a laptop, worth $1000. A shop offers it with a loan option which lets you to pay it off in two equal installments of $500 each. An other shop offers you the same item with two different possible ways to pay for: either of two equal-size installments of $500 each (same as the first shop’s offer), or a decreasing installments of

∗Department of Microeconomics, Corvinus University of Budapest and MTA-BCE „Lendület” Strategic Interactions Research Group, Fővám tér 8, 1093 Budapest, Hungary, e-mail: barna.bako@uni-corvinus.hu
†Department of Applied Psychology, University of Vienna, Universitätsstraße 7, 1010 Wien, Austria, e-mail: linda.dezsoe@univie.ac.at
‡Department of Finance, Tilburg University, Warandelaan 2, 5037 AB Tilburg, The Netherlands, e-mail: g.neszveda@uvt.nl
$750 and $250, respectively. Assuming you would purchase the laptop at the first shop, would you buy it at the second shop if you happened to see their offer first? Based on the classical discounted utility model, rational decision-makers would consider the second shop’s offer at least as good as the first’s one. That is, if one would choose to purchase the item based on the first shop’s offer, she would definitely purchase it when she is faced with the options offered by the second shop.

However, a growing body of evidence from laboratory and field experiments suggests that this might not necessarily be the case. Provocative examples are presented for example in Schkade and Kahneman (1998) or in Dunn et al. (2003). In these articles the authors find strong support for the observation that decision-makers when faced with multi-dimensional decisions tend to overweight few attributes of a decision relative to the others leading to counterintuitive results.\(^1\)

Disproportionate weighting of attributes has been intensely researched and the notion recently was formalized for both risky decisions (Bordalo et al., 2012a) and intertemporal choices (Kőszegi and Szeidl, 2013).\(^2\) The main assumption in these models is that people tend to assign greater weight to the importance of an attribute in which their alternatives differ more. These approaches have been successful in explaining a range of puzzling observations in different fields of economic decisions. More specifically, the model of Bordalo and his colleagues can account for the decoy effect (Bordalo et al., 2013a), the endowment effect (Bordalo et al., 2012b) the anchoring effect (Bordalo et al., 2014), provides an explanation on how salience leads to a transformation of objective probabilities into probability weights (Bordalo et al., 2012a) and explains several puzzles associated with asset prices (Bordalo et al., 2013b). Furthermore, the model of Kőszegi and Szeidl (2013) explains time-inconsistent behavior, both present bias and overcommitment to future goals at the same time, price sensitivity in health decisions (Abaluck, 2011), loan financing without budget constraints (Bertaut et al., 2009; Stango and Zinman, 2009) and lump-sum preferences compared to annuity in retirement and health decisions (Brown et al., 2008).

While the model of Kőszegi and Szeidl (2013) shows obvious similarities to the one presented in Bordalo et al. (2012a,b, 2013a,b, 2014) it assumes a weight function which is not option-specific in contrast to the option-specific characterisation suggested by Bordalo et al. and thus is more suitable to draw welfare conclusions and regulatory implications. The model builds on a time-separable utility function where each attribute measures the consumption in a given time period and welfare is defined as the sum of the utilities of the respective consumption. In this framework the decision-maker maximizes her focus-

\(^{1}\)More related examples can be found in Huber et al. (1982), Simonson (1989), Tversky and Simonson (1993) or Roelofsma and Read (2000). For a detailed review of related experimental findings see for example Camerer et al. (2004). More recently, Bertrand et al. (2010) presented field experiment evidence about how context specific information changes the decision-maker’s behaviour.

\(^{2}\)For earlier works on this literature see Tversky (1969), Tversky and Simonson (1993), González-Vallejo et al. (1996), Roelofsma and Read (2000), González-Vallejo (2002), Scholten and Read (2010) or González-Vallejo et al. (2012).
weighted utility based on which the utility of a time period is weighted with a focus function. This framework, however, does not specify the focus function in the presence of discounting and such its applicability is somewhat limited in cases when time is an important factor of the decision.

In this paper we extend this framework and present a model about loan decisions. To make the framework suitable for analyzing intertemporal decisions we introduce discounting and consider a more general case of the model of focusing. We analyze two different specification of the focus function: decision-makers either focus on the nominal values or on the discounted values of the utility. We show that a decision-maker’s disproportionate focus on the initial benefit a loan entails (e.g. when receiving money or a purchased good) can lead to decisions which yield negative utility. However, as we will show in this paper this can be counterbalanced by introducing a specific alternative repayment schedule. In particular, we claim that the introduction of a decreasing-installments plan in addition to an existing fixed-installments plan makes the decision-maker less likely to take out loans which would yield her negative utility. That is, adding well designed new alternatives to the choice-set decreases the bias towards taking out harmful loans and as a consequence increases welfare. This might have important implications for policy making regarding loan consumption. We also find that even tough decreasing repayment plans have a positive effect on loan decisions they are always dominated by fixed-installment plans. This result is consistent with the empirical findings of Cox et al. (2014) that decreasing repayment plans (e.g., equal principal repayment plan) are the least favored in the loan market when other plans are also available to the consumers. Furthermore, we show that in some specific cases the introduction of an increasing-installments plan can further increase the focusing bias. This may explain why alternative mortgages had gained a large market share both in the US and Europe (Demyanyk and Hemert, 2011; Cox et al., 2014). Based on this result one may claim that the high focusing bias of increasing-installments plans may account for the higher default rate among alternative mortgages.\footnote{Mayer et al. (2009) findings suggest that the sharp increase of mortgage loans with initial low payments followed by a period of higher payments, could be one of the main cause for the rise in mortgage defaults in the late 2000s. As the authors show, a vast majority of these loans had become seriously delinquent, especially after the initial low payment period had ended, and many cases eventually resulted in default. Similar results are presented by Demyanyk and Hemert (2011) and Amromin et al. (2011).}

We argue that due to their strong focusing bias such loans should be coupled with an equivalent decreasing repayment plan to counterbalance the negative effect of the focusing bias, which otherwise may motivate harmful loan consumption.

In what follows we present our model, derive its propositions and finally interpret our results.

\footnote{This result is in line with the empirical observation that people tend to underestimate the burden of a loan (Hoelzl et al., 2009; Akers, 2014).}
2 The Model

Let the consumption choice-set be given by a finite set $C \subset \mathbb{R}^{T+1}$, where $T > 1$. Adapting the framework of Kőszegi and Szeidl (2013) we assume that the consumer maximizes her focus-weighted utility given by $U(c) = \sum_{t=0}^{T} g_t u_t(c_t)$, where $g_t \equiv g(\max_c u_t(c_t) - \min_c u_t(c_t))$ is a focus weight of period (attribute) $t$ with $g(\cdot)$ a positive, strictly increasing function. However, by consuming $c = (c_0, c_1, \ldots, c_T) \in C$ the decision-maker realizes her consumption-based utility of $U(c) = \sum_{t=0}^{T} u_t(c_t)$.

In order to make the model more specific and relevant to intertemporal choices, we introduce discounting and consider focus-weighted utility given by $U(c) = \sum_{t=0}^{T} \delta_t g_t u_t(c_t)$, with $\delta_t \equiv \delta^t$ as the common discount function, where $\delta \in (0, 1]$ is the per period discount factor. Furthermore, we consider consumption-based utility or personal welfare as $U(c) = \sum_{t=0}^{T} \delta_t u_t(c_t)$. It is important to note that with this specification of the focus weights the decision-maker’s focus is based on the nominal values rather than on discounted values of the utilities. In section 3 we will show the consequences of the model when the focus weights are defined on the discounted values.

We make the following assumptions:

**Assumption 1** $u_t(c_t) = c_t$

**Assumption 2** $0 \in C$

**Assumption 3** $(L, -x, \ldots, -x) \in C$

These assumptions are not restricting the explanatory power of the model in any relevant case. As it is shown by Kőszegi and Szeidl (2013) the model remains valid for any arbitrary functional form of the focus function. By assuming linear utility functions we follow the literature in this regard and assume that the utility function is a money metric measure. Furthermore, we assume that the loan free status-quo is always in the choice-set, that is, not taking out a loan is always an option for the decision-maker. Since the fixed-installments plan is the most typical observed loan in practice we assume that the flat plan as a possible repayment plan is always available in the choice-set.

Now, consider the following two consumption profiles: $c^A = (L, -x, -x, \ldots, -x)$ where $L, x \geq 0$ and $c^0 = (0, 0, 0, \ldots, 0)$. One can think of $c^A$ as a loan with fixed-installments (i.e., flat plan or annuity) and $c^0$ as the loan-free status-quo.\(^5\)

Based on the aforementioned framework, a consumer is going to choose $c^A$ instead of $c^0$, whenever $U(c^A) \geq U(c^0)$, i.e when:

$$g(L)L - \sum_{t=1}^{T} \delta^t g(x)x \geq 0$$ (1)

\(^5\)Notice that we do not restrict our analysis to the case of fair loans only. Throughout the analysis a loan is considered in the most general way as a consumption profile, which can yield negative or non-negative utility to the consumer.
Note, however, that consuming $c^A$ could lead to a negative consumption utility, while (1) is still satisfied. To illustrate that this might be the case, consider the following example.

**Example 1** Let $T = 3$, $\delta = 0.9$ and $g_t = \max_c u_t(c_t) - \min_c u_t(c_t)$, with $c^A$ and $c^0$ as follows:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^A$</td>
<td>1000</td>
<td>-600</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>$c^0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$g_t = \max_c u_t(c_t) - \min_c u_t(c_t)$

In this case the consumer chooses $c^A$, since $U(c^A) = 1000 \cdot 1000 - 0.9 \cdot 600 \cdot 600 - 0.9^2 \cdot 600 \cdot 600 - 0.9^3 \cdot 600 \cdot 600 = 121960 > 0$, however, $c^A$ yields a consumption-based utility of $U(c^A) = 1000 - 0.9 \cdot 600 - 0.9^2 \cdot 600 - 0.9^3 \cdot 600 = -463.4 < 0$. Hence, a consumer focusing on the closer-to-the-present attributes may be tempted to choose a consumption profile which yields a negative utility for her.

**Assumption 4** $x_t < L$ for $t = 1, \ldots, T$

We assume that all installments are strictly smaller than the lump-sum value, that is, we assume that a loan is always paid back in several installments.

**Definition 1** For a given $c$ repayment plan and discount factor the fair lump-sum value ($L_W$) is the value for which the consumption-based utility of the loan is zero, i.e., $u_0(L_W) + \sum_{t=1}^{T} \delta^t u_t(c_t) = 0$.

**Definition 2** For a given $c$ repayment plan and discount factor the fair focus-weighted utility lump-sum value ($L_{FWU}$) is the value for which the focus-weighted utility of the loan is zero, i.e., $g(L_{FWU})u_0(L_{FWU}) + \sum_{t=1}^{T} \delta^t g_t u_t(c_t) = 0$.

**Proposition 1** For a given $c$ repayment plan and discount factor the decision-maker is always willing to accept a lower lump-sum value based on her focus-weighted utility than based on her consumption utility, i.e. $L_{FWU} < L_W$.

**Proof:** Since $\frac{g_t}{g(L_{FWU})} \in (0, 1)$ we have that

$$\sum_{t=1}^{T} \delta^t \frac{g_t}{g(L_{FWU})} u_t(c_t) > \sum_{t=1}^{T} \delta^t u_t(c_t)$$

or $L_{FWU} < L_W$.

This proposition indicates that it is always possible to create a loan contract which seems to be beneficial for the decision-maker even though it yields negative utility for her.
Definition 3 For a given \( c \) repayment plan and discount factor the focusing bias in loan decisions (\( B \)) is the difference between the fair lump-sum values, i.e., \( B = L_W - L_{FWU} \)

To examine the effects of focusing, let us introduce one more consumption profile: \( c^B = (L, -x_1, -x_2, \ldots, -x_T) \), where \( x_i \leq x_j \) whenever \( i \geq j \) and \( x_t \geq 0 \), \( (i, j \in \{1, 2, \ldots, T\} \) and \( t = 1, 2, \ldots, T) \). We assume that \( \sum_{t=1}^{T} \delta^t x_t = \sum_{t=k}^{T} \delta^t x \), where \( x \) refers to the installments of \( c^A \).

One can think of \( c^B \) as a decreasing loan repayment plan with a present value equal as \( c^A \). In this case, the consumer’s maximization problem can be written as:

\[
\max_{c} \mathcal{U}(c) \quad \text{for} \quad c \in \{c^0, c^A, c^B\}.
\]

(2)

Proposition 2 Introducing a decreasing loan repayment plan in addition to a flat repayment plan decreases the focus-weighted utility of the flat plan.

Proof: Let \( k \equiv \min\{i | x_i \leq x\} \). If \( x_1 \geq x \) and \( x_T \leq x \), then \( k \in \{1, 2, \ldots, T\} \) is well defined, as it is the case for loan repayment plans. We shall prove that:

\[
g(L)L - \sum_{t=1}^{T} \delta^t g(x) x \geq g(L)L - \sum_{t=1}^{k-1} \delta^t g(x) x - \sum_{t=k}^{T} \delta^t g(x) x - \sum_{t=1}^{k-1} \delta^t g(x_t) x
\]

which is equivalent to

\[
\sum_{t=1}^{k-1} \delta^t [g(x_t) - g(x)] x \geq 0
\]

(3)

Since \( g(\cdot) \) is a positive, strictly increasing function and \( \delta > 0 \), this inequality holds if \( x_1, \ldots, x_{k-1} \geq x \), which is the case by definition. \( \blacksquare \)

Remark 1 Notice that Proposition 2 holds not only when a decreasing installments plan is introduced, but also when any other type of loan repayment plan is added to the choice-set in addition to the flat plan. This is because in the periods when the flat installment is greater then the alternative one the actual focus will be determined by the flat plan and hence can be eliminated, while in those periods when the installment of the alternative plan is higher

---

6Notice, that we do not restrict our attention to alternatives with the same duration as the original plan. Throughout the analysis we allow alternative repayment plans to have shorter duration than the flat plan as far as their present value is the same. In this regard, periods with no installments should be considered as periods with \( x_t = 0 \).
relative to the flat installment than the assigned focus is also going to be greater. Thus, inequality (3) holds for any type of equivalent repayment plan. Furthermore, with a similar argument one can easily show that the same is true if the loan originally is offered with an increasing-installments plan rather than with a flat plan.

Proposition 2 yields an interesting result. If the consumer’s profile-set consists only of \(c^A\) and \(c^0\), she chooses \(c^A\), whenever (1) is satisfied. Yet, if \(c^B\) is part of the set as well, she may prefer \(c^0\). To demonstrate this, consider the next example.

**Example 2** Let \(T = 3\), \(\delta = 0.9\) and \(g_t = \max_c u_t(c_t) - \min_c u_t(c_t)\) again and the consumption profiles as follows:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c^A)</td>
<td>1000</td>
<td>-600</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>(c^0)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(c^B)</td>
<td>1000</td>
<td>-780</td>
<td>-670</td>
<td>-300</td>
</tr>
</tbody>
</table>

\(g_t = \max_c u_t(c_t) - \min_c u_t(c_t)\) again and the consumption profiles as follows:

In this case \(U(c^A) = 1000 \cdot 1000 - 0.9 \cdot 780 \cdot 780 - 0.9^2 \cdot 670 \cdot 670 - 0.9^3 \cdot 300 = -9260 < 0\), \(U(c^0) = 0\) and \(U(c^B) = 1000 \cdot 1000 - 0.9 \cdot 780 \cdot 780 - 0.9^2 \cdot 670 \cdot 670 - 0.9^3 \cdot 600 \cdot 300 = -42389 < 0\). Therefore the optimal choice is \(c^0\).

Furthermore:

**Proposition 3** If \(C = \{c^0, c^A, c^B\}\), then \(c^A \succ c^B\), i.e., if a flat and a decreasing plan are both available to a consumer, then the former is always preferred.

**Proof:** We shall prove that:

\[
g(L)L - \sum_{t=1}^{k-1} \delta^t g(x_t)x - \sum_{t=k}^{T} \delta^t g(x)x \geq g(L)L - \sum_{t=1}^{k-1} \delta^t g(x_t)x_t - \sum_{t=k}^{T} \delta^t g(x)x_t \tag{4}
\]

Define \(y_i \equiv x_i - x\). Note that \(y_i \geq 0\) if \(i = 1, 2, \ldots, k - 1\) and \(y_i \leq 0\) otherwise. Thus (4) can be written as:

\[
\sum_{t=1}^{k-1} \delta^t g(x + y_t)x + \sum_{t=k}^{T} \delta^t g(x)x \leq \sum_{t=1}^{k-1} \delta^t g(x + y_t)(x + y_t) + \sum_{t=k}^{T} \delta^t g(x)(x + y_t)
\]

This simplifies to:

\[
0 \leq \sum_{i=1}^{k-1} \delta^i g(x + y_t)y_t + \sum_{t=k}^{T} \delta^i g(x)y_t \tag{5}
\]
or

\[ 0 \leq \sum_{t=1}^{k-1} \delta^t g(x + y_t) y_t + g(x) \sum_{t=k}^{T} \delta^t y_t \tag{6} \]

Since \( \sum_{t=1}^{T} \delta^t x = \sum_{t=1}^{T} \delta^t (x + y_t) \), we have that \( \sum_{t=1}^{T} \delta^t y_t = 0 \). Using this, (6) can be written as:

\[ 0 \leq \sum_{t=1}^{k-1} \delta^t g(x + y_t) y_t - g(x) \sum_{t=1}^{k-1} \delta^t y_t \]

that is

\[ 0 \leq \sum_{t=1}^{k-1} \delta^t [g(x + y_t) - g(x)] y_t \tag{7} \]

As \( y_t > 0 \) for \( t = 1, 2, \ldots, k-1 \) and \( g(\cdot) \) is strictly positive and increasing by definition, this inequality always holds. Moreover, the inequality is strict whenever \( \exists s \in \{1, 2, \ldots, T - 1\} \) for which \( y_s > 0 \), in other words when \( c^A \neq c^B \).

\[ \square \]

**Remark 2** Proposition 3 holds even if \( c^B \) is not a decreasing but any other type of loan repayment plan. In this case, inequality (6) can be rewritten as

\[ 0 \leq \sum_{t \in K} \delta^t g(x + y_t) y_t + g(x) \sum_{t \notin K} \delta^t y_t, \]

where \( K \) is the set of indices for which the non-flat installment is higher than the respective repayment of the flat schedule.

**Proposition 4** Introducing any type of repayment plan in addition to a fixed-installments plan decreases the focusing bias \( B \).

**Proof:** According to Proposition 2 introducing an alternative equivalent repayment plan makes the fixed-installment plan less attractive and \( L_{FWU} \) increases compared to the original setting. Thus, by introducing a new repayment plan the \( L_{FWU} \) is increasing while the \( L_W \) does not change. As a consequence, the focusing bias \( B \) decreases. \[ \square \]

**Proposition 5** Introducing any number of repayment plans in addition to a fixed-installments plan decreases the focusing bias \( B \).

**Proof:** Let the set of those periods in which at least one of the alternative repayment plans’ relevant installment is greater than the fixed installment be \( K \), i.e., \( K \equiv \{ t \mid x_t > x \} \) for any \( c \in C \). Since the focus weights in these periods are determined by the maximum effective installments, it follows that these are bigger that the weights effective when only
the flat plan is available. However, in those periods, when the installment of any alternative plan is equal or smaller then the fixed installment the focus weight is determined by the fixed installment. More formally, \( g_t = g(\max_c x_t) \) if \( t \in K \) and \( g_t = g(x) \) otherwise. Counterposing the two focus-weighted utility of the flat plan, we can eliminate all those periods' utilities where the focus weight is similar, i.e., when \( t \in T \setminus K \). The remaining periods are all characterized by greater focus weights than the weights effective when only the flat plan is available, since \( g_t(\cdot) = g(\max_c x_t) > g(x) \) if \( t \in K \) by definition. It follows that the focus-weighted utility of the flat plan is smaller when the flat plan is coupled with alternative repayment plans relative to the case when it is the only available repayment plan. In other words, coupling the flat plan with alternative repayment plans decreases the \( L_{FWU} \) of the flat plan. Thus, if it happens that the consumer prefers the flat plan over all the alternatives then the focusing bias \( B \) is decreasing following the introduction of new repayment plans.

If, however, at least one alternative dominates the flat plan the consumer would prefer to choose the loan with an alternative repayment rather than with the fixed-installments plan. Let the set of alternatives with the highest focus-weighted utility given that all alternatives are available be \( D \). More formally, let \( D = \{ d \in C | d \succ c, \forall c \in C \} \). Take an element of this set, say \( \tilde{d} \). We know that the \( L_{FWU} \) of \( \tilde{d} \) is strictly greater than the \( L_{FWU} \) of the fixed-installments plan when there are only these two repayment plans available in addition to the status-quo. We also know that the \( L_{FWU} \) of \( \tilde{d} \) is getting greater as new alternatives are added to the choice-set, since the effective focus weights are never getting smaller but potentially greater with the introduction of new repayment plans. However, since the consumer prefers \( \tilde{d} \) over any other repayment plans, the \( L_{FWU} \) of \( \tilde{d} \) should be the lowest when all alternatives are available. Yet, as we have shown, this later \( L_{FWU} \) is greater than the \( L_{FWU} \) of the fixed-installments plan when the choice-set consist only of these two repayment plans apart from the status-quo. Consequently, the \( L_{FWU} \) of \( \tilde{d} \) is always higher than \( L_{FWU} \) of the fixed-installments plan in the original setting. Since we have chosen \( d \) arbitrarily it follows that the argument holds for all \( d \in D \). As a consequence, the introduction of new repayment plans decreases the focusing bias \( B \).

■

3 Focus weights based on discounted values

So far, we considered cases with focus weights based on nominal values of the repayment plans. Let us now examine our results when focusing is based on discounted values, that is, when \( g_t \equiv g(\delta^t(\max_c u_t(c_t) - \min_c u_t(c_t))) \). In the following we will show that if we define focus weights in terms of discounted values Proposition 2 to 5 still hold for decreasing-installment plans, although not necessarily for other alternative plans such as increasing-installment plans.
Proposition 6 When the decision-maker focuses on the discounted values of the utilities, introducing a decreasing loan repayment schedule makes a flat plan less attractive for the consumer.

Proof: In this case (3) changes to:

$$\sum_{t=1}^{k-1} \delta^t g(\delta^t x_t) x - \sum_{t=1}^{k-1} \delta^t g(\delta^t x) x \geq 0$$

(8)

Since $x_1, \ldots, x_{k-1} \geq x$, $g(\cdot)$ positive and strictly increasing, while $\delta \in (0, 1]$ this inequality always holds.

Remark 3 Note that Proposition 6 holds not only for decreasing plans but for any other type of loan repayment schedule. Moreover, the same is true for an increasing-installments plan when it is coupled with new alternatives. To see this, one can use the same technic presented in Remark 1.

Proposition 7 When the decision-maker focuses on the discounted values and $C = \{c^0, c^A, c^B\}$ then $c^A \succ c^B$, i.e., if a flat and a decreasing plan are both available to a consumer, then the former is always preferred.

Proof: In this case (5) can be written as:

$$0 \leq \sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t + \sum_{t=k}^{T} \delta^t g(\delta^t x) y_t$$

(9)

Notice that $\sum_{t=k}^{T} \delta^t g(\delta^t x) y_t$ is always negative, since $y_t < 0$ for any $t = k, \ldots, T$. That is, by replacing $\delta^t$ with $\delta^k$ for each $t = k, \ldots, T$, we have that the right-hand side of (9) is never lower than $\sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t + \sum_{t=k}^{T} \delta^t g(\delta^k x) y_t$. Hence:

$$\sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t + \sum_{t=k}^{T} \delta^t g(\delta^t x) y_t \geq \sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t + \sum_{t=k}^{T} \delta^t g(\delta^k x) y_t$$

(10)

The right-hand side of (10) can be written as $\sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t + g(\delta^k x) \sum_{t=k}^{T} \delta^t y_t$ and since $\sum_{t=k}^{T} \delta^t y_t = 0$, this equals to $\sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t + g(\delta^k x) \sum_{t=1}^{k-1} \delta^t y_t$ or $\sum_{t=1}^{k-1} \delta^t g(\delta^t (x + y_t)) y_t - \sum_{t=1}^{k-1} \delta^t g(\delta^k x) y_t$. This, however, can be written as $\sum_{t=1}^{k-1} \delta^t [g(\delta^t (x + y_t)) - g(\delta^k x)] y_t$. As $y_1, \ldots, y_{k-1} \geq 0$ and $g(\cdot)$ is positive and strictly increasing, this latter expression is always non-negative, that is, (10) always holds, and as a consequence (9) is always true.

Proposition 7 is not necessarily true if $c^B$ is not a decreasing repayment plan, but for instance, an increasing one. To illustrate this consider the following example.
Example 3 Let $T = 3$, $\delta = 0.9$, $g_t = \delta^t [\max_c u_t(c_t) - \min_c u_t(c_t)]$ and the consumption profiles be as follows:

<table>
<thead>
<tr>
<th></th>
<th>0.</th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^A$</td>
<td>1000</td>
<td>-600</td>
<td>-600</td>
<td>-600</td>
</tr>
<tr>
<td>$c^0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$c^C$</td>
<td>1000</td>
<td>-542</td>
<td>-602</td>
<td>-669</td>
</tr>
<tr>
<td>$g_t$</td>
<td>1000</td>
<td>540</td>
<td>487.62</td>
<td>487.7</td>
</tr>
</tbody>
</table>

In this case $\bar{U}(c^A) = 1000 \cdot 1000 - 0.9 \cdot 540 \cdot 600 - 0.9^2 \cdot 487.62 \cdot 600 = 258096.26$, $\bar{U}(c^0) = 0$ and $\bar{U}(c^C) = 1000 \cdot 1000 - 0.9 \cdot 540 \cdot 542 - 0.9^2 \cdot 487.62 \cdot 602 - 0.9^3 \cdot 487.7 \cdot 669 = 260962.47$. Therefore the optimal choice is $c^C$.

We summarize this in the following

**Proposition 8** In case of focusing on discounted values the welfare effect of introducing a repayment plan is ambiguous, however, introducing a decreasing repayment plan still provides welfare improvement.

**Remark 4** Notice that for the propositions to hold we don’t need a real decreasing-installments plan in the sense that $x_i \geq x_j$ whenever $i \geq j$, we only need a repayment plan with a $k$ for which $x_i > x$ if $i < k$ and $x_i \leq x$, otherwise.

4 Discussion

Our propositions in both specifications suggest that introducing a new repayment plan increases the $L_{FWU}$ of a flat plan. One might argue that this may possibly deter the decision-maker from taking out a loan which could result in positive consumption utility. This, however, cannot be the case. Whenever the consumption utility of a loan is positive the focus-weighted utility of it is also positive.

**Remark 5** If $U(c) > 0$, then $\bar{U}(c) > 0$ also holds. To show this consider the following.

The focus-weighted utility of $c = (L, -x_1, -x_2, \ldots, -x_T)$ can be written as

$$g_0 L - \delta g_1 x_1 - \delta^2 g_2 x_2 - \cdots - g_T \delta^T x_T$$

where $g_t$ is the focus weight for period $t$. Since $x_i < L$ ($i = 1, \ldots, T$), we have that $g_0 \geq g_t$ for any $t = 1, \ldots, T$. Thus

$$g_0 L - \delta g_1 x_1 - \delta^2 g_2 x_2 - \cdots - g_T \delta^T x_T \geq g_0 L - \delta g_0 x_1 - \delta^2 g_0 x_2 - \cdots - g_0 \delta^T x_T = g_0 (L - \sum_{t=1}^{T} \delta^t x_t)$$

Yet, this is always positive, since $U(c) > 0$ and $g_0 > 0$. 

11
Proposition 9  Our results remain valid in the case of quasi-hyperbolic discounting (Laibson, 1997).

Proof: Using $\beta \delta(t)$ instead of $\delta(t)$ the relevant expressions increase $\beta$-fold, where $\beta \in (0, 1]$ is the parameter for present bias. By dividing them with $\beta > 0$ we obtain exactly the same inequalities we derived in the above given proofs.

Independently from the specification of the focusing function, our results suggest that introducing a decreasing repayment plan decreases the focusing bias. In other words, a loan is always less attractive based on its focus-weighted utility compared to its consumption utility when a new decreasing plan is introduced in addition to the original plan and as a consequence the bias $B = L_W - L_{FWU}$ decreases. We have also shown that the decrease of the bias caused by the introduction of new repayment plans cannot deter the decision-maker from taking out a loan with positive consumption utility. Moreover, introducing quasi-hyperbolic discounting for modeling present biased behavior does not affect our results.

It is important, however, to investigate which of the specifications is more descriptive for loan decisions. The main testable difference in implications is the attitude towards increasing repayment plans. This is especially relevant nowadays because of the various new types of mortgages. Interest-only mortgages and deferred amortization mortgages could all be examples of different types of increasing repayment plans. These financial innovations, however, have unclear impact on loan decisions and one might think that they have a negative effect on the decision-maker’s judgment. According to some experimental and empirical findings increasing repayment plans are less preferred by the decision-makers than other types of repayment plans which suggests that the specification of focusing on nominal values might be more adequate for loan decisions. For example, empirical findings by Hoelzl et al. (2011) suggest that subjects prefer fixed-installment plans over increasing-installment plans and this preference is robust both in presence and in absence of interest rate. On the other hand, the popularity of alternative mortgages (see Mayer et al., 2009) indicate that the model with focusing on discounted values might be more robust than the one with nominal values. Furthermore, one may reason that the higher default rate observed among those who had chosen increasing-installment plans (Amromin et al., 2011) might be due to their stronger focusing bias. This again supports the specification with focusing on discounted values.

7However, van Leeuwn and Bokeloh (2012) argue that the popularity of some specific increasing repayment plans (i.e., interest-only mortgages and deferred amortization mortgages) in different countries might be only due to tax refund possibilities. This is also supported by the empirical findings of Cox et al. (2014), who claim that in the Netherlands the increasing repayment plans were taken out by wealthier, less risk averse and more financially literate people suggesting that these loans are not preferred due to misperception, but for some other reasons.
5 Conclusion

In this paper we have investigated the effects of focusing in the presence of discounting considering two possible specifications. In the first case we defined the focus weights based on nominal values of the attributes, while in the second one we defined the focus weights on discounted values. In the first specification we have shown that any extension of the choice-set may improve consumer welfare. In the second specification, however, it is possible that an extension of the choice-set with increasing-installments plan may result in welfare loss. However, it is important to emphasize that even in this case the introduction of a decreasing-installments plan makes the focusing bias less powerful in distorting valuations and loan decisions.

From a policy standpoint, one may wonder what the practical consequences of the existence of focusing bias are. In this regard our results indicate that lenders may have strong, intended or unintended, influence on borrowers’ decisions just by offering them the loans with specific repayment plans. As we have shown, a loan presented with a repayment plan featured with great focusing bias may incentivize consumption of the loan even if that could result in negative utility. This, however, could be counterbalanced by the mechanism presented in this paper. Based on our results we conjecture that adding a decreasing-installments plan to the choice-set would make the decision-makers less likely to take out loans which yield them negative utility without affecting their attitude towards the ones with positive utility. Yet, we argue that people would still prefer to choose the fixed-installments plan from this extended choice-set. Moreover, we claim that by exploiting the aforementioned effect of focus, people could make more deliberate loan decisions. If banks, for example, would present a loan in fixed- and decreasing-installments options, they could end up getting more prudent decisions from their clients. This obviously boils down to policy making. Namely, a policy could prescribe that financial institutions present a loan repayment schedule also in a decreasing-installment option, and not only in fixed- or increasing-installment one. The induced focus on the decreasing-installments plan could dampen the increased focus on getting the loan, thereby discouraging decision-makers from taking out loans which might yield them negative utility. This may be especially important in the case of loans with increasing-installments plans (e.g. mortgages with initial 'teaser' rates), since these instruments could generate the highest focusing bias, and as a consequence, may motivate harmful loan consumption the most.

References


