PATTERNS IN THE LOTTERY GAME

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Abstract. The present study is based on the analysis and results of a close to 5 years' study in the frame which we used a "Lottery Game" in the "Decision Making Skills" subject taught at Corvinus Business School, Corvinus University of Budapest. In the frame of the "game", the students (Hungarians (n=231) and foreigners (n=267) alike) have to mark 6 numbers on a 7x7 lottery ticket. The winner is the student whose numbers differ the most from those of all the other students'. Upon analyzing the results (irrespectively of nationality) the authors have noted something notable: the winning combinations - rather than being located randomly on the ticket, characteristically resemble a geometric form. In our study we wanted to detect the relevance of geometry in this kind of choices. It is hypothesized that in such games (lottery type, related to numeric combination choice), where the players decide upon their strategy (choice of numbers) by also taking into consideration others' expected choices, the winning strategy is characteristically some consciously chosen scheme or pattern as opposed to a random one. The study presents the results of the available samples (Hungarian students: n=231, foreign students: n=267), the winning combinations, the most often designated numbers, as well as the least "popular" numbers and their presentation on a "heat map". In the case of the majority of the winning tickets we found the use of conscious strategic choice to be more useful. These conscious strategic decisions were reflected in identifiable geometric forms. Based on the results, we hypothesize that in the "hidden lottery" game – in contrast with random choice – the most effective strategy of choice is the conscious ordered one in which the player marks the numbers on the lottery ticket in some modified geometric pattern. The goal of the paper is to propose further research on the field.

Key words: lottery game, patterns, students.

Introduction

The modified lottery game was elaborated by László Mérő, a Hungarian mathematician and psychologist to provide empirical support for his theory of collective rationality (Mérő 2007). In the frame of the game, players have to mark 6 out of a total of 49 numbers on a 7x7 symmetrical lottery ticket. In contrast with the traditional lottery, the players play not against chance, but against other, consciously thinking players. While in the traditional version, the winner is the one who is most successful in guessing the randomly selected (either mechanically or by hand) numbers, in the modified game the winner is the person "whose numbers least resemble those of the other players" (Mérő 2007: 287)[1]. The administration of the game is as follows: When all the participants have chosen their numbers, the lottery tickets and the choices are evaluated. In the case of each single number (1 to 49), we determine the exact number of players who chose the given number. After this for each and every player we check, how many other participants had marked the numbers that he/she specifically had chosen. The winner is the one, whose overall sum is the smallest (i.e. whose numbers overall had the least number of other choosers).

Mérő has shown that in the "hidden lottery" game random choice is the so-called "evolutionarily stable strategy", as if it spreads amongst the players, no other strategy can be more effective. In short, the most rational solution is random choice. Based on the 236 person sample studied by Mérő, he indicated that the numbers chosen by the players (i.e. showing all the choices on a 7x7 ticket) really do indicate a more random frequency. At the same time – based on further research – it has become evident that the choices of the individual players are very rarely truly random. The choices for the major part correlate to some sort of logical thinking: avoiding the well-known lucky numbers (e.g. dates of birth) or those in outstanding positions (i.e. at the corners), or marking these precisely because others would think the same way. In short, while the majority of the individual choices were based more or less on some sort of logical rather than random choice, and were thus irrational from the point of view of evolutionary stable strategy, together with the results deduced from the common choices, they can be considered to be rational (Mérő 2007).

In this paper we not only present the results of the experiments conducted and the conclusions, but also share propositions which require further future studies. After presenting the theoretical and practical aspects of the modified "Lottery Game" used, we briefly review the literature on lottery games and the players' choices highlighting the results relevant to our study. Furthermore, we also present the methodology and the results of our study together with our conclusions and hypotheses. The paper ends with a summary of the proposed further research to be conducted.

[1] It is important to know that in keeping with the structure of the different lottery games (such as the Hungarian lottery), the prize money for a given number of successfully chosen numbers is divided between all the winners who have achieved the same given number of hits. This means that the players are playing not only against luck, but also against all the other players, as well, i.e. in order to maximize the prize money, their choice of numbers must not only coincide with those drawn, but at the same time these numbers must be ones that hopefully only they themselves will have marked from amongst all the players.

1. Theoretical background – strategies of choice: random vs. conscious choice

People have always been interested in random events. Our forefathers considered coincidence to be more or less destiny, while later on the term also became connected to games of fortune (e.g. dice games). Naturally men have always tried and continue to try to unravel and explain the nature of random events. Random events occur in an anomalous way without method or conscious choice (Oxford Dictionaries 2015). At the same time, further to the individual, random, and unforeseeable events we can also describe the random outcome of repeatedly conducted studies. As a result – based on an appropriately large number of trials – we must study the frequency of the various random outcomes (e.g. the random heads or tails result of flipping a coin), based on which the "probability" of the possible outcomes can be determined (as Bernoulli recognized in the "law of large numbers"). In keeping with the above, randomness can be interpreted as the index of uncertainty, and as such, also as its probability.

The concept of uncertainty (and together with this, that of probability and risk) is closely related to games of chance. It is sufficient to refer to the "problem of points" game, which first appeared in 1494[2] and in the case of which the solution for the fair distribution of the bets was elaborated in Pascal and Fermat's correspondence (1654), thus laying down the basis of the modern mathematical theory of probability.

Lottery, like all games of chance, relies on randomness. In this oldest and most widespread game all numbers have an equal chance to be drawn, all have an equal chance to be winners. It is the players' aim to choose as many as possible of the winning numbers to win the largest possible prize, which, in case of a full hit, can even be a fortune. As a result, consciously or not, the players apply different strategies of choice from random marking to what appears to be a logical choice.

People usually find it hard or impossible to interpret randomness and together with this the probability of the occurrence of certain events. They use heuristics to make understanding easier, which in effect simplify and speed up decision making, but which, as has been proven, lead to decision traps (Tversky and Kahnemann 1974). A number of papers on the choice of lottery numbers deal with the study and description of the phenomena of "gambler's fallacy" and "hot hand fallacy" which belong to the heuristics of representativeness (Clotfelter and Cook 1991, 1993; Terrel 1994; Papachristou 2004; Jorgensen et al. 2011). In the case of "gambler's fallacy" people believe that the probability of a random event is affected by the outcome of

[2] The game was first described in Lucas Pacioli: *Summa de arithmetica, geometrica, proportioni et proportionalità*.

a similar (but otherwise independent) event, in fact, that the probability of the occurrence of these events is in negative correlation with each other. The effect seen in the case of "gambler's fallacy" is opposite to that of the "hot hand fallacy". In a study conducted by Gilovich and Vallone (Gilovich et al. 1985), people perceive a positive correlation between the occurrences of independent random events (the authors show the cognitive illusion of "hot hand fallacy" through the perception of the randomness of successful shots in basketball). While the two events are theoretically contradictory, there are a number of correlations between them.

According to some both "fallacy" illusions originate from the "law of small numbers", in other words a trap of insensitivity to the sample number: based on the characteristics of the small sample people expect to be able to deduce the characteristics of the whole sequence (whole population) (Rabin 2002; Rabin és Vyanos 2010). Studying the choices of lottery players, a number of authors (Clotfelter at al. 1993; Kendall 2010; Jorgensen et al. 2011) have proven that the two events often go hand-in-hand, i.e. the length of the streaks influences the perception of randomness. In general – in the case of short winning streaks – the illusion of "gambler's fallacy" is stronger (the players are less apt to choose the winning lottery numbers of previous weeks), while as the streaks strengthen, the phenomenon of "hot hand fallacy" gains dominance (the players are more inclined to choose the numbers that had won in longer streaks previously) (Jorgensen et al. 2011). Based on an in-depth study of the Dutch lottery system, other authors have confirmed that those who play the lottery frequently avoid the previous winning numbers, while the occasional players are more inclined to choose them (Potter van Loon et al. 2015). Further studies show that the illusion of "gambler's fallacy" or "hot hand fallacy" is determined by whether the draw is achieved electronically or by hand – in the case of the former people perceive a negative correlation, in the case of the latter a positive correlation (Kong et al. 2013).

In studying the lottery, a number of researchers have shown that players usually apply a conscious selection rather than simply marking the numbers randomly[3]. Players believe that with conscious choice, they can "influence" random output (Goodman and Irwing 2006). The structures of the lottery games are unique in that the players are gambling not only against chance, but also against each other: by having the prize money designated for a winning class divided between the winners within that class, in addition to chance, the players are also affected by the choices of the other players.

It has been proven through various lottery systems (e.g. Hauser-Rethaller and König (2002) – the Austrian), Henze (1997) – the German, Roger and Broi-

[3] The results of the lottery related studies are reviewed in detail in the papers of Perez (2009) and Grote et al. 2011.

hanne (2007) – the French, Walker et al. (1998) and Ayton et al. (2015) – the British, Potter van Loon et al. (2015) – the Dutch lottery) that players like to choose "lucky numbers", such as dates of birth, favorite numbers (e.g. 3, 7 or their multiples), zip codes or prime numbers. Players are also inclined to mark numbers that they can easily remember, such as the date the lottery ticket was filled in, the draw date or the lottery ticket's number (Potter van Loon et al. 2015).

Geometric patterns often play a role in the applied strategies of choice. It is possible to note that the numbers located in the middle of the lottery ticket are usually preferred, while those on the edge or close to each other are avoided (i.e. "edge aversion" and "proximity aversion", respectively). In their choices, the players tend to choose symmetric and "aesthetic" forms and patterns, often looking at the whole range of available numbers (i.e. based on a misperception of "chance", in the 6/45 game the six numbers are usually strewn across the 1 to 45 range, rather than marking ones closer to each other, or ones located in the lower, middle, or upper part of the ticket) (Potter van Loon et al. 2015; Ayton et al. 2015).

2. Hidden lottery game: a case study

"Decision Making Skills" is a compulsory elective course for students in the master programs of Corvinus Business School, Corvinus University of Budapest. Each semester 150 to 200 students with a sound knowledge in business and economic foundation subjects take the course. In the frame of the seminars, when covering the material on the psychology of decision making, students are usually asked whether they play the lottery. Naturally, everyone knows the game. They know the various types of lotteries available in Hungary and their respective rules: the most popular lotteries are the 5/90 (marking 5 numbers out of a possible 90) and the 6/45 one (marking 6 numbers out of a possible 45). They all know that in order to win in these games, the player must have as many hits as possible (in the case of the 5/90 min. 2 hits, in the case of the 6/45 min. 3) of the numbers drawn. The draw can take place electronically with a machine or by hand, randomly choosing from all the possible numbers.

As to the question on whether or not they play the lottery, the students – mostly with self-satisfied smiles and relaxed that they will not walk into a "trap" set by the professor – all reply: "of course not". No, playing the lottery makes no sense, they think, seeing the excellent business and economics students that they are, i.e. practically "homo economicus". To support their view, they take into consideration all the possible outcomes (benefits), the related probabilities and simply calculate the profit to be expected from the game. When taking a decision, they compare the expected profit with the cost incurred (the price of the lottery ticket) and conclude (unless the expected prize is enormous) that in general it is not worthwhile, thus, they do not play.

This reply is comforting on the one hand, but is, on the other hand, also thought provoking. What is fascinating is that – knowing that games of chance (lottery, gambling, etc.) provide a significant income countries' budgets, and that hundreds of thousands await the draw with the lottery tickets in their hand – of the close to 500 students interviewed to date is it really true that not one of them is enticed to forget the formal, rational way of thinking in hope of a prize whether it be small or large? At the same time it is nice to see that the university students apply the basic theories of economics so well and avoid the heuristic traps related to estimations and feelings of probability. It appears that they see the advertisements related to lottery promising easily acquired wealth, the news on lottery millionaires, and are aware of the role of chance and the notion of probability. They do not take part in a game where they have to play against the laws of nature (fate), against an "opponent", whose "steps" and laws are known and can be described, but who is at the same time unpredictable and where the prize (profit) to be expected is smaller than the actual investment (the price of the lottery ticket).

We accepted the students' decision, i.e. not to participate in a traditional lottery game, however, we asked them to play a lottery, albeit a modified one – a "hidden" lottery game – with us. Our aim was twofold: first that at least once in their life they fill in a lottery ticket, and second to determine their logical thinking, their problem-solving abilities, their affinity to fortuitousness and probability by analyzing the decision making strategies – primarily those of the winners – and the reasons behind them through the modified lottery game.

3. Methodology

The study was conducted with the Hungarian and foreign students participating in the Hungarian and English programs of the Corvinus University of Budapest. The students were all taking the "Decision Making Skills" master level subject at Corvinus Business School. Since 2010, twice a year (in the fall and spring semesters, respectively), we ask the students to fill in a "hidden lottery" ticket.

No changes have been made to the strategic game elaborated by László Mérő, i.e. in contrast to the traditional lottery game where players must choose the randomly draw winning numbers and their prize depends on the number of choices that were successful[4], in the "hidden lottery", players are playing against *consciously thinking* competitors, where their success depends basically on the choice and strategy of the other players.

[4] In the case of some lottery games the prize also depends on how many players have the same number of hits, as the prize money designated for that many hits is divided between them. Based on this, players are not merely playing a "game of chance", but also to some extent a game of strategy, whose logic resembles that of the "hidden lottery" – they need to mark numbers that are drawn, but which were not marked by other players.

In the frame of the study we used the 7x7 matrix with numbers ranging from 1 to 49 and gave the following instructions:

"In the frame of the game you must fill in a lottery ticket. You have to mark 6 numbers in the range of 1 to 49, located in the 7x7 quadrant lottery ticket. The winner will be the person whose chosen numbers least resemble those of the other players. For of each and every student we shall individually determine exactly how many players marked his/her numbers and the player whose aggregated sum is the lowest will be the winner".

The students had to fill in the lottery ticket during class within the same amount of time, but independently of each other, by clearly marking the chosen 6 numbers on their "lottery" ticket. Independent choice was assured by the fact that in general, the students did not know each other. The foreign students (aged: 21 to 25 years) usually represented 8-10 different countries, while the Hungarian students (aged: 22 to 24 years) usually came from different programs. In general, they were not aware of the game or its logic, thus we were able to ensure that they did not have any previously conceived concept of the strategic choice to be made. Proper participation was further motivated by the fact that we awarded some extra study points to the winner – as a "prize" – in addition to the overall points achieved throughout the semester.

The filled in lottery tickets were collected after each game and the total number of times each numeric value from 1 to 49 was marked was noted on an empty ticket. The overall points of the individual players were determined by adding together the number of times each of the individual numbers marked by the player had been also marked by others. The winner was the person whose overall points was the lowest, i.e. his/her numbers had been marked the least by the other players. The results were shown to the students.

We hypothesized that in such games (lottery type, related to numeric combination choice), where the players decide upon their strategy (choice of numbers) by also taking into consideration others' expected choices, the winning strategy is characteristically some consciously chosen scheme or pattern as opposed to a random one.

In the frame of our study, we analyzed and evaluated (with special attention paid to the winning choices) the results of the 18 individual games jointly. We have determined the "hot-numbers" (most often marked) and the "cold-numbers" (least marked) both on a per game basis, as well as, on an overall basis. The two "extremes" were marked on a 7x7 matrix with color coding to enable an easier determination of the characteristics and distribution of the choices. We displayed the winning combinations on the summary tables, thus ensuring that we be able to differentiate between random and conscious strategic choices. The "hot" and "cold" numbers of the Hungarian and foreign students were analyzed both separately and jointly which provided us with the possibility of identifying possible culture-based differences.

4. Results

The number of participants in the study varied from semester to semester. In the case of the Hungarian students the lowest number of participants in the lottery game was 14 (Fall 2014), the highest 50 (Fall 2013), while as regards foreign students the lowest number of participants was 10 (Spring 2010) and the highest 37 (Fall 2011). Since 2010, we have conducted the lottery game study in a total of 18 courses with an overall participation of 498 students (average: 27 students per semester). The distribution of players was: 231 Hungarians (7 studies, average 33 students per study) and 267 foreigners (11 studies, average 24 students per study).

4.1. Study of the frequency of the markings

Figure 1 shows the aggregated results of the 18 studies conducted with Hungarian and foreign students (n=498) as regards the 6 choices made 1 to 49 number range in the "hidden lottery". In the lower left-hand corner of each cell on the 7x7 ticket, we have given the number of times the given number was chosen (i.e. number 1 was chosen a total of 139 times). The color coding facilitates the visual identification of the frequently and less frequently chosen numbers. The cells marked with different shades of red indicate, the more frequently, while those marked with different shades of green indicate, the less frequently chosen numbers. The darker the shade, the more or less frequently the given number was chosen.

Figure 1.Aggregated frequency of choices (n=498)

1 139	2 92	3 79	4 69	5 72	6 60	7 85
8 58	9 63	10 45	11 63	12 47	13 66	14 44
15 45	16 53	17 80	18 55	19 68	20 44	21 40
22 49	23 70	24 41	25 65	26 55	27 48	28 35
29 53	30 66	31 55	32 51	33 43	34 42	35 33
36 45	37 88	38 43	39 61	40 53	41 65	42 32
43 91	44 56	45 44	46 55	47 71	48 67	49 90

The table shows that considering the overall results of the 18 studies, '1' was chosen most often (139 times), while '42' was chosen the least (32 times). The numbers situated in the corners of the lottery ticket were the most popular ('1' – 139x, '43' – 91x, '49' – 90x, '7' – 85x), while the non-corner numbers of the last column were the least popular ('42' – 32x, '35' – 33x, '28' – 35x, '21' – 40x).

Overall, students marked numbers most frequently in the corners, the lottery ticket's upper and lower rows, and the diagonal numbers, while the central numbers of the ticket's furthest left and right columns and those located on the right side of the ticket were chosen the least. The frequent marking of the combination of neighboring numbers (whole rows, whole columns) is striking. The frequency of the patterns of the "hot" and "cold" numbers on the lottery ticket are more easily determined in Figure 2.

Figure 2.The distribution of the "hot" and "cold" numbers on the lottery ticket (n=498)

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

Looking at the Hungarian and international samples individually, we can safely say that there is no difference in the frequency of the choices of the two large "players" groups. Figure 3 shows side by side the frequency with which given numbers were chosen by the two individual groups.

In the case of the foreign students (n=267), the most frequently chosen lottery number was '1' (83x), the least frequently chosen was '42' (13x). In the case of Hungarian students (n=231) the most "popular" lottery number was '1' (56x), while the least chosen one (16x) was '12'. In both samples the corner numbers were the most popularly marked ones: in the foreign sample '1' – 83x, '7' – 41x, '43' – 46x, '49' – 50x, while in the Hungarian sample these same numbers were marked 56, 44, 45, and 40 times, respectively.

Figure 3.The frequency of choices made by Hungarian (n=231) and international (n=267) students

1 56	2 43	3 35	4 30	5 35	6 24	7 44
8 25	9 21	10 20	11 31	12 16	13 28	14 19
15 19	16 21	17 39	18 25	19 37	20 17	21 23
22 29	23 32	24 25	25 26	26 20	27 20	28 19
29 23	30 37	31 18	32 21	33 18	34 17	35 18
36 18	37 47	38 19	39 23	40 23	41 31	42 19
43 45	44 30	45 20	46 25	47 38	48 33	49 40

1 83	2 49	3 44	4 39	5 37	6 36	7 41
8 33	9 42	10 25	11 32	12 31	13 38	14 25
15 26	16 32	17 41	18 30	19 31	20 27	21 17
22 20	23 38	24 16	25 39	26 35	27 28	28 16
29 30	30 29	31 37	32 30	33 25	34 25	35 15
36 27	37 41	38 24	39 38	40 30	41 34	42 13
43 46	44 26	45 24	46 30	47 33	48 34	49 50

Based on the number of times the individual numbers were marked and their geometric positions, we *presume* that in choosing their strategy, the players relied less on mere chance and more on *conscious strategies, applying cognitive patterns of choice.* While in analyzing traditional lottery games the researchers (Potter van Loon et al. 2015; Ayton et al. 2015) defined as characteristic, conscious strategy the avoidance of choosing the corner numbers ("edge-aversion") and neighboring numbers "proximity-aversion", in the case of the "hidden lottery" it is exactly these combinations that are the most popular. While the results appear to be contradictory, considering the structure of the two types of games, this contradiction can be easily resolved knowing the conscious strategy applied by players in the "hidden lottery" game. Students reason that as lottery players usually avoid the corner numbers, those in the first and last rows and those close to each other (as they calculate that these have a lower probability of being drawn), it is worthwhile to choose these. At the same time, as a number of players think similarly, these numbers are chosen frequently all the same. The same situation applies to the numbers in proximity to each other, which, as most players think that they will be chosen by few others at most, are marked rather frequently.

The relatively less frequent choice of the numbers on the right-side of the lottery ticket reflects cognitive patterns, schemes of choice. The learnt automatism of reading and writing (left to right and up to down) can be seen in the choice of the six numbers as shown by the Figures. The player looks at the lottery ticket from left to right, and – unless he/she already has a specific strategy – will be apt to choose from the most easily accessible numbers, thus, not reaching the higher numbers located on the right-hand side of the ticket. This is true even if the players attempt to think in terms of the whole range of numbers given on the ticket.

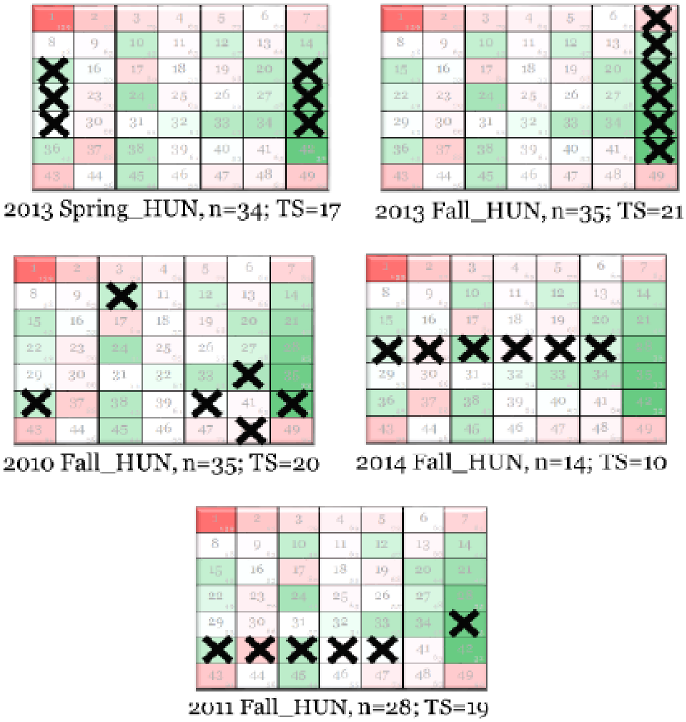
The conscious choice of the "*favorite numbers"* is also visible in the Figures. The conscious choice of *dates of birth* may play a role in the fact that numbers in the lower right-hand corner are marked less frequently. As months range from 1 to 12 and days of the month from 1 to 31, players applying this strategy rarely, if ever, reach the numbers in the 32-49 range. Prime numbers – though in the present study these often coincide with one of the more frequently chosen geometrical forms (first row, diagonal row) – are also chosen frequently. All the prime numbers in the 1 to 49 range (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43) are amongst the more frequently chosen lottery numbers.

4.2. Evaluation of the winning tickets

The winning tickets were evaluated separately for the Hungarian and foreign samples. In the winning combinations we sought to find the "strategies" and patterns used by the traditional lottery players as described by the studies covered under "Theoretical Background". We *presumed* that – based on the previously determined geometric characteristics observed during the evaluation of the results – we *would be able to identify in the winning combinations* either (geometric) *patterns,* or the characteristics of *conscious choice, rather than random ones.*

Analyzing the Hungarian sample, a geometric pattern was identifiable in 5 of the 7 studies. These are shown in Figure 4.

Figure 4.Winning combinations (patterns) in the Hungarian samples[5]

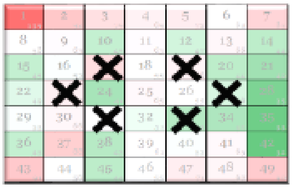


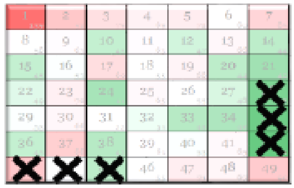
In addition to the whole row (No. 4) and column (No. 2) patterns, we also identified the "H-pattern" (No. 1), and the modified whole patterns, i.e. partial patterns (No. 3 and No. 5). In the case of the Hungarian participants the minimum expected total score (sum of the overall number of points achieved based on the total number of times the six choices of a given player were also chosen by others) was 15.23.

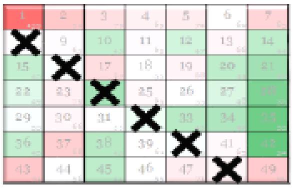
In the case of the foreign samples we identified geometrical patterns in 9 of the 11 studies (5 of these are shown in Figure 5) which are very similar to those used by the Hungarian students. Here, too, we found the "H-pattern" (No. 4), the modified and the partial patterns (e.g. offset diagonal – No. 3., the mirrored diagonal – No. 5, the broken row/column combination – No. 2). In the case of the foreign participants the minimum expected total score (sum of the overall number of points achieved based on the total number of times the six choices of a given player were also chosen by others) was 13.96.

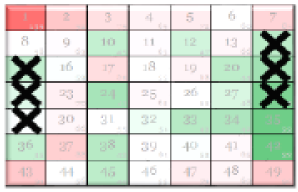
[5] TS – Total Score (the overall number of points of the winning combination), HUN – Hungarian students.

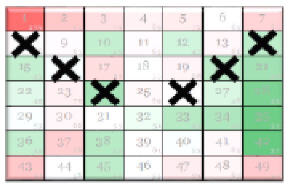
Figure 5.Winning combinations (patterns) in the foreign samples[6]


2015 Spring_INT; n=18; TS=12


2014 Fall_INT, n=22; TS=10


2014 Spring_INT, n=22; TS=15


2013 Fall_INT; n=23; TS=12


2012 Spring_INT; n=24; TS=18

5. Discussion and hypotheses evaluation

It seems that all the winning combinations reflecting geometric patterns are the outcome of conscious choice. The students filling in the winning tickets all used number combinations which reflects thinking outside of the average: conscious avoidance of the corners, marking of the inner numbers, uncharacteristic patters (e.g. broken rows). Interestingly enough, on the majority of the winning tickets (HUN No. 3 is an exception) neighboring numbers or ones located rather close to each other were chosen. It can be *presumed that in the case of the winning combinations the choice of the appropriate geometric pattern played a greater role than disregarding the strategy of "edge-" or "proximity-aversion".* In short, from the point of view of winning the game the way – the special geometric pattern used by the player when marking the numbers is more important than avoiding the numbers on the edge of the ticket or in close proximity to each other (combinations and positions often marked by other players).

In determining their strategy, the winning players apparently took into consideration two important factors: First, they presumed that the other players would mark random numbers (in which case the probability of the others' num-

6 INT – international students.

bers coinciding with those in geometric patterns is rather small). Second, if all the other players also mark their numbers in geometric patterns, then the probability of them applying the similarly modified (broken) patterns is minimal. Thus we can deduce that in their minds, the probability of other students choosing their numbers based on the same geometric pattern was lower than the probability of the overlapping of randomly chosen numbers.

Based on the results we can assume that *the role of the numbers and the location of their combinations play an important role.* Some of the "losing" players told us in the course of the evaluation of the results that they had concentrated more on the numbers themselves than on their actual location. In contrast, the owners of the winning tickets placed a larger emphasis on the pattern and place of the numbers marked on the lottery ticket.

We *presume* therefore that in the case of the "hidden lottery" – playing against intelligent, consciously thinking competitors, in keeping with the predefined rules of the game and in the frame of the symmetrical layout of the 7x7 lottery ticket – the most effective strategy is that of the conscious, ordered choice in which the player marks the numbers in some sort of *modified geometric pattern* as opposed to simple random choice.

Our hypotheses are based on the results of the evaluation of the 18 "hidden lottery" games played to date, thus at this stage they are hunches, the results of "more than interesting" coincidences. Of the millions of possible combinations[7] in the 6/49 type game, the almost 500 person sample is still too small to enable us to draw significant deductions from the winning strategies. In addition to determining the correctness of our presumptions, further data and analysis can also be used to decide whether the identified *geometric patterns – especially the modified (broken) patterns – are present in addition to otherwise random choice or whether they are really the results of conscious decisions.* A further question to be posed is whether the symmetrical structure of the lottery ticket used (7x7 layout) attracts the geometric patterns, i.e. whether or not the *ordered pattern influences the strategy chosen.*

We shall conduct further research to prove our presumptions and hypotheses. Future participants shall be asked – after having filled in the lottery ticket – to explain (in writing or orally) the thought process that resulted in the choice of the numbers marked. We also plan to change the layout of the lottery ticket (e.g. by using an asymmetrical layout) and the order of the numbers to obtain further answers to the question whether players concentrate more on the numbers themselves or rather their position, i.e. whether it is the pattern or the number that is more important. By removing the geometric frame, we wish to determine to what extent the strategy applied in choosing the numbers is influenced by the ticket as a structuring principle. In order to study this, in the

7 In the 6/49 lottery game, the total number of possible combinations is 13,983,816.

case of one team of participating students we shall simplify the game: we will not use the ticket, but will simply ask them to choose six numbers from 1 to 49 and shall note these down.

Of course, we must also study all the other tickets, primarily those of the "loser" participants' (those who received the highest overall score). In the case of the "losing" combination we must determine whether there is any sign of conscious choice and/or whether we can identify any geometrical patterns on these, as well.

Conclusions

Lottery – as one of the oldest and most popular game of chance throughout the world – serves as an excellent instrument in studying random and conscious strategies of choice. We know that the winning lottery numbers cannot be predicted, at the same time there are – not so successful – "sure" theories on the marking of the winning numbers. In our study, we merged the characteristics of the traditional lottery game with the elements of strategic games and using a modified lottery game – the "hidden lottery" we studied and analyzed the strategies of choice applied by the players with a special emphasis on the winning combinations.

Students participating in the game had to mark 6 numbers from 1 to 49 on a 7x7 lottery ticket. The winner was the student whose chosen numbers were the least in accord with those chosen by other players. In contrast to the traditional lottery, players of the "hidden lottery" must hit not randomly chosen numbers, but rather, they have to play against other, consciously thinking and intelligent players.

Analyzing the results of Hungarian and foreign students we found an interesting phenomenon. In contrast with our expectations that students are best off with a random choice of numbers, in the case of the majority of the winning tickets we found the use of conscious strategic choice to be more useful. These conscious strategic decisions were reflected in identifiable geometric forms. Based on the results, we hypothesize that in the "hidden lottery" game – in contrast with random choice – the *most effective strategy of choice is the conscious ordered one* in which the player marks the numbers on the lottery ticket in some modified geometric pattern.

The number of studies performed and the number of samples at this stage is still too low to enable us to prove our hypotheses without doubt. We shall extend the study, increase the sample number, expand the applied research methods to test our hypotheses and answer further research questions.

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