THE ISSUE OF MACROECONOMIC CLOSURE REVISITED AND EXTENDED

Ernő Zalai and Tamás Révész

Content

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Abstract

Léon Walras (1874) already had realized that his neo-classical general equilibrium model could not accommodate autonomous investment. Sen analysed the same issue in a simple, one-sector macroeconomic model of a closed economy. He showed that fixing investment in the model, built strictly on neo-classical assumptions, would make the system overdetermined, thus, one should loosen some neo-classical condition of competitive equilibrium. He analysed three not neo-classical "closure options", which could make the model well determined in the case of fixed investment. Others later extended his list and it showed that the closure dilemma arises in the more complex computable general equilibrium (CGE) models as well, as does the choice of adjustment mechanism assumed to bring about equilibrium at the macro level. By means of numerical models, it was also illustrated that the adopted closure rule can significantly affect the results of policy simulations based on a CGE model.

Despite these warnings, the issue of macro closure is often neglected in policy simulations. It is, therefore, worth revisiting the issue and demonstrating by further examples its importance, as well as pointing out that the closure problem in the CGE models extends well beyond the problem of how to incorporate autonomous investment into a CGE model. Several closure rules are discussed in this paper and their diverse outcomes are illustrated by numerical models calibrated on statistical data. First, the analyses is done in a one-sector model, similar to Sen's, but extended into a model of an open economy. Next, the same analyses are repeated using a fully-fledged multi-sectoral CGE model, calibrated on the same statistical data. Comparing the results obtained by the two models it is shown that although, using the same closure option, they generate quite similar results in terms of the direction and – to a somewhat lesser extent – of the magnitude of change in the main macro variables, the predictions of the multi-sectoral CGE model are clearly more realistic and balanced.

1. Introduction

Beginning from the 1980s a large number of computable general equilibrium (CGE) models have been developed all over the world to study a wide range of economic policy areas and issues in which simpler, partial equilibrium or aggregate macro models would be unsatisfactory. CGE models have become standard tools in studying a variety of policy issues, including tax policies, energy and environmental policies, to evaluate the impact of EU cohesion policy and so on[2].

It had been realized already by Walras (1834-1910), the father of neoclassical general equilibrium models, that in his multi-sector model, built strictly on neo-classical assumptions, there was no room for autonomous investment. Despite this early appearance of the problem, the discussion of the issue is usually traced back only to Sen (1963), who analysed this problem in a simple one-sector model, in which he showed that in the neoclassical realm investment has to be a variable, adjusting freely. He then presented three not neo-classical adjustment mechanisms, to illustrate how one could fix investment in an otherwise neo-classical model.

Sen's analysis was later extended by Taylor and Lysy (1979) in the framework of CGE models, discussing a wider range of closure alternatives. Dewatripont and Michel (1987)

[2] From the vast literature on CGE models we refer to the books of Dervis, De Melo and Robinson (1982), Shoven and Whalley (1992), Bergman, Jorgenson and Zalai (1990), Hertel (1997).

analysed the microeconomic basis of the closure problem in the context of temporary competitive equilibrium. Some numerical investigations[3] made clear that the choice of macro closure can significantly affect the policy simulation results obtained from a CGE model.

Despite these early warnings, the issue of macro closure is mostly neglected in policy simulations. It is seldom analysed how sensitive the CGE simulation results are depending on what closure option is chosen. It is, therefore, worth revisiting the issue and demonstrating its importance by further theoretical and numerical examples. It is also important to point out that the closure problem extends beyond the treatment of autonomous investment. The problem of how to close a CGE model arises in other areas as well, because, as a rule, the number of potential variables exceeds the number of equations which can be formulated on the basis of trustworthy and operational theories.

Monetary and financial forces can be treated at most in an *ad hoc* fashion in the CGE models, real dynamic considerations are seldom included into them either. Attempts to extend the models in these directions make them less tractable and less reliable. It can be safer to set the expected magnitude of certain economic variables exogenously rather than determining them endogenously by using formulas questionable on conceptual or empirical ground. Thus, the model builder has to fix the value of some potential macroeconomic variables, i.e., has to choose which variables will be endogenous and which exogenous (fixed) in his model.

In our paper we generalize first Sen's model to a one-sector model of an open (rather than closed) economy, in which the single product has differentiated varieties, which are less than perfect substitutes for each other. This brings the one-sector model closer to the multisectoral CGE models. Then, based on the related literature, Sen's analysis is extended to different, characteristic closure alternatives. The diversity of outcomes yielded by different closures is illustrated by numerical simulations based on a model calibrated using Hungarian statistical data. In the second part of the paper the same analysis is extended to a fully-fledged CGE model, including taxes/subsidies, elaborated income distribution scheme, distinguishing five production sectors and three groups of households, calibrated on the same data set. Comparing the results of similar simulation runs, based on the same closure option and assuming the same shock, will highlight not only the qualitative similarities but also the significant quantitative differences between the results achieved by the aggregated macroeconomic and the multisectoral CGE model.

2. Prelude: the closure problem as encountered by Walras

Walras analysed the conditions of general equilibrium at different levels of abstraction in various simple models of a closed economy.[4] Here we reconstruct his one-period, multi-commodity model with investment and capital goods, in which the problem of closure arose for him. His model can be seen as the reduced form of a more detailed and complex model of general equilibrium, in the framework of which the closure problem will be analysed later. The survey of this simple and transparent model will help the reader understand better how

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[3] See, for example, Rattsø (1982), Taylor (1990), Decaluwe et al. (1988) and Robinson (1991, 2006), Thissen (1998).

[4] See Zalai (2004) for more details on Walras's models.

the closure problem arises and the structure of the multisectoral general equilibrium models too.

For the sake of simplicity, Walras distinguishes three types of commodities in his model: consumer goods, natural (e.g., labour) and produced factors of production (capital goods). He left out of consideration intermediate demand for products as well as final demand for primary goods. The supply of natural factors of production and consumers' demand for products are represented by functions of prices (homogeneous of degree zero), and fixed input coefficients are used to determine demand for the factors of production.

Unlike Walras, we do not split products into consumer and capital goods. We assume simply that any commodity could be consumed as well as accumulated as a capital good, which can be used only in the subsequent time periods, determining the capital stocks. The necessary conditions of equilibrium can be formulated as follows.

Equilibrium on the markets of products and factors of production:

$$x_i = y_i^c(\mathbf{p}, \mathbf{q}, \mathbf{w}) + y_i^b, \qquad\qquad i = 1, 2,\ldots, n, \qquad \text{(W-1)}$$

$$b_{i1}{\cdot}x_1 + b_{i2}{\cdot}x_2 +\ldots+ b_{in}{\cdot}x_n = K_i, \qquad\qquad i = 1, 2,\ldots, n, \qquad \text{(W-2)}$$

$$d_{k1}{\cdot}x_1 + d_{k2}{\cdot}x_2 +\ldots+ d_{kn}{\cdot}x_n = S_k(\mathbf{p}, \mathbf{q}, \mathbf{w}), \qquad\qquad k = 1, 2,\ldots, s. \qquad \text{(W-3)}$$

Equilibrium pricing rules (price equals cost):

$$p_j = w_1{\cdot}d_{1j} + w_2{\cdot}d_{2j} +\ldots+ w_s{\cdot}d_s + q_1{\cdot}b_{1j} + q_2{\cdot}b_{2j} +\ldots+ q_n{\cdot}b_{nj}, \qquad j = 1, 2,\ldots, n. \qquad \text{(W-4)}$$

$$q_i = (r_i^a + \pi_i){\cdot}p_i, \qquad\qquad i = 1, 2,\ldots, n. \qquad \text{(W-5)}$$

where

- x_i total (final) output of product i,
- p_i the price of product i ($i = 1, 2,\ldots, n$),
- q_i the rental price of capital good i,
- w_k the price of primary factor k ($k = 1, 2,\ldots, s$),
- $y_i^c(\mathbf{p}, \mathbf{q}, \mathbf{w})$ consumer's demand for product i, where $\mathbf{p} = (p_i)$, $\mathbf{q} = (q_i)$, $\mathbf{w} = (w_k)$,
- y_i^b the amount of product i invested,
- b_{ij} the input coefficients of capital goods (capital good i used per unit of product j),
- K_i the supply of capital good i (accumulated in the past),
- d_{kj} the input coefficients of primary factors of production (primary factor k used per unit of product j),
- $S_k(\mathbf{p}, \mathbf{q}, \mathbf{w})$ the supply of primary factors,
- r_i^a the rate of depreciation of capital good i,
- π_i the net rate of return on capital good i.

The unknowns in equations (W-1) − (W-5) are x_i, y_i^b, p_i, q_i, π_i and w_k, their number is thus $5n + s$. The number of the equations is however only $(4n + s)$, leaving n degrees of freedom for the above system of equations. Neither the rates of return (π_i) nor the investment demand (y_i^b) is yet determined according the conditions of long-run equilibrium in the above model. The rates of return should be equalized, whereas investment should match the future demand

for capital goods. Walras became aware of the fact that he could only set one of these conditions in order to close the model. He decided to prescribe the equality of the net rates of return (π) by adding to equations (W-1) – (W-5)

$$\pi_i = \pi, \qquad\qquad\qquad\qquad i = 1, 2,\ldots, n, \qquad \text{(W-6)}$$

by which the number of the unknowns (x_i, y_i^{b}, p_i, q_i, π_i, π, w_k) became ($5n + s + 1$), and the number of the equations is ($5n + s$).

The same effect could be achieved by substituting

$$q_i = (r_i^{\text{a}} + \pi)\cdot p_i, \qquad\qquad\qquad i = 1, 2,\ldots, n. \qquad \text{(W-5a)}$$

for equations (W-5) and dropping the variables π_i. In either case the degree of freedom of the resulting model will be one, which can be eliminated by the choice of the numeraire.

Walras was aware of the fact that investment demand became residual by this choice and therefore his model represents only part of the conditions of a long-run equilibrium. He was criticized later by Keynes for his choice, who would have introduced investment demand functions based on expectations instead. By doing so, however, he would have arrived at the conditions of a temporary equilibrium rather than that of a long-run equilibrium, which Walras tried to define, but could not do it in a consistent way.

Referring to the CGE models it is interesting to note that the definition of the rental price of capital (equation W-5) introduced by Walras, is used in the applied GE models as well. It is also worth of noting, that capital goods in Walras' model take the form of homogenous sectoral products, as in Leontief's theoretical input-output models. In the applied multisectoral models, including the CGE models, sectoral capital goods are as a rule composite commodities, put together from homogenous sectoral products without using directly any factor of production.

3. Analysis of the closure issue in the framework of a one-sector macroeconomic general equilibrium model

Sen (1963), as mentioned, analysed the closure problem in a simple one-sector model of a likewise closed economy. We extend, first of all, Sen's analysis to an open economy, which brings it closer to the CGE models.[5] We extend Sen's analysis also by adding to it further closure possibilities from the related literature. In addition, the results will be numerically illustrated by means of a computable model, based on statistical data.

Three basic and two composite commodity varieties of the same single product are differentiated in our model: commodity produced and sold at home, exported and imported, its composite domestic output and its composite supply on domestic market. The volume (use value) of the composites of differentiated basic products is measured by monotone increasing, linear homogenous (aggregation) functions. Unlike Sen, we take into account its intermediate use in production too.

Output capacity is defined by a nested production function of Johansen type, as common in CGE models. Labour (L) and capital (K) are assumed to be imperfect substitutes, jointly

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[5] Using one-sector models for didactic purposes is quite common and useful practice in the CGE literature too. See, for example, Devarajan et al. (1994) and Robinson (2006).

determining the output capacity by a linear homogeneous production function, $F(L, K)$. In our numerical model it will be represented by a constant elasticity of substitution (CES) function. The composite output (X) is divided between domestic (X^h) and export (Z) supply by means of a constant elasticity of transformation (CET) function, $X = X(X^h, Z)$. Reexport, as usual, is left out of consideration as usual. The composite home supply (X^{hm}) of the commodity produced at home (X^h) and imported (M) is defined by a CES aggregation function, $X^{hm} = X^{hm}(X^h, M)$.

This means that five commodity prices have to be introduced: the user's price on the domestic (p^h) and the world market (p^{we}), the world market price of the imported good (p^{wm}), the producer's price of the composite output (p^a), and the users' price of the composite domestic supply (p^{hm}). For the sake of simplicity *taxes/subsidies,* modifying potentially the prices, *are disregarded.* Therefore, the domestic equivalents of the world market export and import price are simply $v \cdot p^{we}$ and $v \cdot p^{wm}$, where v is the exchange rate.

In the case of import price we adopt the small open economy assumption, p^{wm} will be thus an exogenous variable. In the case of export, however, as often assumed in CGE models, its world market price depends on its volume, $p^{we} = p^{we}(Z)$. According to neo-classical theory this would mean that exports are differentiated on the world market by the area of their origin. Each country faces thus a less than perfectly elastic export demand function, and $p^{we}(Z)$ is the inverse of that demand function.

Neo-classical theory assumes that the wage rate (w) and the rental price of capital (q) has to be equal thus to the marginal revenue of labour and capital. The revenue is measured here by the value added, $p^a - p^{hm} \cdot A$, where A is the constant material input coefficient. In some of the discussed closure rules, however, factor prices will be allowed to adjust freely and depart from their marginal products, while the nonprofit pricing rule will be maintained. Therefore, the equilibrium conditions of production will be formulated in the following way:

$$X = F(L, K) \quad (1), \qquad w/\alpha_w = (p^a - p^{hm} \cdot A) \cdot \frac{\partial F(L, K)}{\partial L} \quad (2), \qquad p^a \cdot X = p^{hm} \cdot A \cdot X + w \cdot L + q \cdot K. \quad (3)$$

The new variable, α_w in equation (2) plays the role of a switch variable. If α_w is fixed at value 1, then the wage rate will be equal to the marginal revenue of labour, and by virtue of Euler's theorem and the assumed linear homogeneity of the production function from equations (1) – (3) follows that the rental price (cost) of capital (q) is equal to its marginal revenue:

$$q = (p^a - p^{hm} \cdot A) \cdot \frac{\partial F(L, K)}{\partial K}. \tag{3a}$$

In this case, equation (3) can be viewed as the equation which indirectly determines the equilibrium rental price of the capital.

If the factor prices are allowed to depart from the respective marginal products, α_w measures simply the ratio of the wage rate to the marginal revenue of labour.

For later reference it is also worth mentioning that one could also use

$$L = l(w/\alpha_w, q) \cdot X \qquad (1'), \qquad\qquad K = k(w/\alpha_w, q) \cdot X \qquad (2')$$

conditions instead of (1) and (2), where $l(w/\alpha_w, q)$ and $k(w/\alpha_w, q)$ are the factor demand functions in the case of unit output, derived from the above conditions. In the CGE models it

is common and useful to use such dual forms. We will also use them in the specification of our CGE model.

Following Walras's definition, the relation between the cost of capital (q) and the net rate of return on capital (π) is $q = (r^a + \pi) \cdot p^{hm}$. For convenience, *amortization is disregarded* ($r^a = 0$), thus, this relationship reduces to $q = \pi \cdot p^{hm}$.

The prices of the composite commodities are determined based on the assumption that their composition is always optimal. In the case of the domestic/export composite the total sales revenue ($p^h \cdot X^h + v \cdot p^{we} \cdot Z$) is maximized (*assuming price taker individual exporters*), in the case of the domestic/import composite, total costs ($p^h \cdot X^h + v \cdot p^{wm} \cdot M$) is minimized. These assumed optimizations can be represented by the following first order necessary conditions:

$$X = X(X^h, Z) \qquad (4), \qquad p^h = p^a \cdot \frac{\partial X(X^h, Z)}{\partial X^h} \qquad (5), \qquad v \cdot p^{we} = p^a \cdot \frac{\partial X(X^h, Z)}{\partial Z} \qquad (6),$$

$$X^{hm} = X^{hm}(X^h, M) \quad (7), \qquad p^h = p^{hm} \cdot \frac{\partial X^{hm}(X^h, M)}{\partial X^h} \quad (8), \qquad v \cdot p^{wm} = p^{hm} \cdot \frac{\partial X^{hm}(X^h, M)}{\partial M} \quad (9),$$

where the composite prices (p^a and p^{hm}) are, as a matter of fact, the Lagrangian multipliers corresponding to the respective optimization problems.

Condition (6), however, has to be modified, because we assume that export demand is less than perfectly elastic. p^{we} defined by $p^{we}(Z)$, the inverse demand function, will thus appear instead of p^{we}. The assumption would imply potential monopolistic position for the exporters, which could be exploited by means of so-called optimal tariffs (see, for example, Limão, 2008). In CGE models, designed for practical uses, it would not be realistic to take this theoretical possibility into consideration and price taker agents are assumed. The introduction of less than perfectly elastic export demand functions serve only the purpose to restrain price induced changes in the volume of export and *vice versa*.

Observe that – since the aggregation functions are by assumption linearly homogenous and by virtue of Euler's theorem – from equations (5)-(6) and (8)-(9) one can derive:

$$p^a \cdot X = p^h \cdot X^h + v \cdot p^{we} \cdot Z \qquad (6a), \qquad\qquad p^{hm} \cdot X^{hm} = p^h \cdot X^h + v \cdot p^{wm} \cdot M. \qquad (9a)$$

These indicate first, that the prices of the composite goods are the weighted averages of the prices of their components. Second, these derived forms could also be used as conditions of optimality, for example, instead of (6) and (9), respectively. Third, one can also derive the following conditional export supply and import demand functions from (5)-(6) and (8)-(9), respectively:

$$Z = r^{eh}(p^h, v \cdot p^{we}) \cdot X^h \qquad\qquad (5a), \qquad\qquad M = r^{mh}(p^h, v \cdot p^{wm}) \cdot X^h \qquad\qquad (8a),$$

where r^{eh} and r^{mh} are the optimal share coefficients, defined as functions of relative prices. One could thus use these latter forms instead of (5) and (8). We will use such derived (dual) forms later in presenting the equations of the multi-sectoral CGE model.

Equations (1) - (9) define the equilibrium conditions for the supply of commodities for export and domestic use, the demand for imports as well as the market clearing commodity prices. They have to be completed yet with equations describing the income (re)distribution and final demand side of the model.

We will make use of many simplifying assumptions in formulating the various budget and behavioural constraints.

$$p^{wm} \cdot M - p^{we}(Z) \cdot Z = D_e \tag{10}$$

$$p^{hm} \cdot G + S^g = \tau(w \cdot L + q \cdot K) \tag{11}$$

$$p^{hm} \cdot C + S^p = (1 - \tau) \cdot (w \cdot L + q \cdot K) \tag{12}$$

$$S^p = (1 - \tau) \cdot (\sigma_w \cdot w \cdot L + \sigma_k \cdot q \cdot K) \tag{13}$$

Equation (10) defines the trade balance deficit (D_e), in fact, the net savings of foreigners, since non-trade related transfers are neglected in our model. Equation (11) is the budget balance of the public household (government), where G denotes public consumption and S^g net public saving. In this model, for the sake of simplicity, government is assumed to collect revenue only from income tax[6], by means of a uniform tax rate (τ) applied to both labour and capital income. Equation (12) represents the budget balance of the households, where C denotes private consumption. On the left hand side one can see the sum of the value of private consumption ($p^{hm} \cdot C$) and net private savings (S^p), on the right one the formation of disposable (net) income. S^p is determined by equation (13), assuming different savings ratios (σ_w, σ_k) in the case of labour and capital income.

The commodity balance on the home market takes the following form:

$$X^{hm} = A \cdot X + C + G + I. \tag{14}$$

where I denotes investment.

By routine transformations it can be shown that equations (1)-(12) and (14) imply

$$p^{hm} \cdot (C + G + I) = w \cdot L + q \cdot K + v \cdot D_e.$$

This means that final expenditure will be always equal to total income (Walras law) and investment to savings. Therefore, in most related papers, including *Sen's* seminal paper itself, the reader finds equation $p^{hm} \cdot I = S^p + S^g + v \cdot D_e$ instead of (14).

The real rate of exchange is defined as the ratio of the foreign to the domestic value of the produced commodity, i.e., as the domestic cost of earning one unit of foreign exchange:

$$v_r = v \cdot p^{we}(Z)/p^h. \tag{15}$$

The above 15 equations define the skeleton of the general equilibrium model that will be used later and we will refer to it as the *basic model*. The model will be well-determined if the number of variables and equations are equal. The potential (endogenous) variables exceed the number of equations, since at least the following 25 could be chosen as variable, depending on our assumptions: I, L, K, X, X^h, X^{hm}, Z, M, C, G, p^a, p^h, p^{hm}, p^{wm}, w, $w_r = w/p^{hm}$, α_w, q, v, v_r, S^p, S^g, σ_w, σ_k, τ. We will therefore refer to them as variables, to distinguish them from the parameters, which will be always constant. The actual choice of model specification will decide which of them will be *endogenous* and/or *exogenous variable*.

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[6] Distinguishing consumer's price from general user's price, income tax could be replaced by consumption tax.

4. Macro closure options in the one-sector general equilibrium model

The actual choice of model specification, i.e., the *closure rule*, decides which critical macro variables will remain endogenous and which become exogenous in a given model. X, X^h, X^{hm}, Z, M, C, p^a, p^h, q, v, S^p and S^g (12 altogether) will be, as usual, endogenous in each closure to be discussed below. At the same time, available capital (K), public consumption (G), world market import price (p^{wm}) and the savings rates (σ_w, σ_k) will be fixed in all versions, although they could be endogenous variables in certain models. Either foreign saving (D_e) or the real rate of exchange (v_r) will be fixed in each closure version too. The first case is perhaps closer to a neo-classical setup, treating foreign exchange as a scarce resource, similarly to labour and capital. As will be seen, the two cases can lead to considerably different results.

The general price level has to be set exogenously yet. It will be fixed by $p^{hm} = 1$, in which case w will be the nominal as well as the *real wage rate* ($w_r = w/p^{hm}$), whereas, since $r^a = 0$, the cost of capital (q) will represent at the same time the rate of return (π), because $q = \pi p^{hm} = \pi$ (their dimensions remain though different, π refers to the value, q to the volume of capital).

Table 1: Summary of various closure rules in the one-sector model

	I. neo-classical	II. Keynesian	III. Johansen	IV. neo-Keynesian I.	V. neo-Keynesian II.	VI. structuralist I.	VII. structuralist II.	VIII. loan-able funds
I	**endog.**	exog.	exog.	exog.	exog.	**endog.**	**endog.**	**endog.**
L	exog.	**endog.**	exog.	exog.	**endog.**	**endog.**	**endog.**	exog.
w	**endog.**	**endog.**	**endog.**	**endog.**	exog.	exog.	**endog.**	**endog.**
α_w	1	1	1	**endog.**	**endog.**	**endog.**	**endog.**	1
v_r	exog./ **endog.**	exog./ **endog.**	exog./ **endog.**	exog./ **endog.**	exog./ **endog.**	exog./ **endog.**	exog./ **endog.**	exog./ **endog.**
D_e	**endog.**/ exog.	**endog.**/ exog.	**endog.**/ exog.	**endog.**/ exog.	**endog.**/ exog.	**endog.**/ exog.	**endog.**/ exog.	**endog.**/ exog.
τ	exog.	exog.	**endog.**	exog.	exog.	exog.	exog.	exog.
c^π	na	na	na	na	na	**endog.**	exog.	na
i	na	na	na	na	na	na	na	**endog.**

Notes: endog. = endogenous, exog. = exogenous, na = not applicable.

The basic model, discussed so far, contains 15 equations and only 12 variables have been chosen as endogenous ones so far. Three additional endogenous variables should be thus still chosen from among the remaining potential endogenous variables (I, L, w, α_w, v_r or D_e, τ) to close the model. We start the discussion of possible closure rules with the four alternatives

analysed by Sen in his simpler model of a closed economy. Table 1 provides a useful summary of various closure rules to be discussed.

I. The neo-classical (Walrasian) closure

In a neo-classical closure L would be the fixed, and the factors of production paid according to their marginal revenue.[7] Thus, w is endogenous variable and $\alpha_w = 1$. As mentioned above, either v_r or D_e will be also endogenous variable, whereas τ is a given parameter, influencing the disposable income of the households. Only investment (I) remains thus to be the still missing, 15th endogenous variable.

Observe that, if either D_e or v is exogenous variable, as assumed, as long as L is fixed and the factor prices equal their marginal revenues ($\alpha_w = 1$), equations (1)-(10) can be solved independently from the rest of model equations for variables X, X^h, X^{hm}, Z, M, w, q, p^h, p^a and p^{hm} (or v), which appear in them, by setting the price level as $v = 1$ (or $p^{hm} = 1$). In such a case, thus, the total supply of goods for final use is determined by the production and trade possibilities alone. The rest of the model determines only its distribution. The supply and demand side of the model depend on each other thus only if L is free to adjust and/or factor prices can depart from their marginal revenues.

The neo-classical closure rests thus on the assumption that whatever is saved will be invested, independently of the expected future demand for capital. By the same token, the capital inherited from the past (K), was formed with no concern for its present needs. This reveals clearly the roots of the closure problem.

Note also that it is not specified in this model, what sort of mechanism could make the investment adjust to savings. The interest rate is often assumed implicitly to equilibrate savings and investments, which option will appear later in the *loanable funds* closure.

II. Keynesian (General Theory) closure

As indicated earlier, if one wants to introduce autonomous investment into the model, he or she must relax the strict neo-classical conditions. Borrowing from Keynesian theory, Sen relaxed first the assumption of fixed labour constraint (full employment) by reinterpreting L as the variable level of employment and could thus fix the level of investment instead. Since the wage rate remains determined by the marginal revenue of labour, any increase in the level of employment will go hand in hand with its decrease and the increase of the rate of return on capital, and *vice versa*, establishing thus the equality between investment and savings. Any exogenous increase in investment (or government expenditure) would generate *Keynesian multiplier* effect in such a model.

III. Johansen closure

The third closure option discussed by Sen was inspired by Johansen's (1960) pioneering CGE model, in which it was assumed that the government could intervene by appropriate tax policy to secure full employment even in the case of exogenously fixed investment level. In Johansen's model, the supply of both labour and capital was fixed, and marginal pricing rule

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[7] It can be shown that as long as L is fixed equations (1) - (10), i.e., the production and trade possibilities determine alone the total supply of goods for final use and the rest of the model determines only its distribution.

prevailed. If one adopts these assumptions, output will be determined in the same way as in the case of the neo-classical closure. If public consumption (G) and investment is also fixed, only the level of consumption can adjust to reach equilibrium. In this closure the changing level of tax rate (τ) is assumed to bring about equilibrium. Johansen assumed that by setting tax level appropriately the government could achieve such level and generate the saving which matches the fixed investment demand. In this closure thus the tax rate (τ) becomes the adjusting variable instead of investment. Another possibility for the government to balance demand and supply would be to regulate its own spending (G) instead of the tax rate.

IV. Neo-Keynesian closures I. (forced savings)

The fourth and last option discussed by Sen was based on the "forced savings" models of Kaldor (1956) and Pasinetti (1962), which he termed as a neo-Keynesian closure. Forced savings is a macro closure scenario, in which α_w becomes endogenous variable instead of the investment level. It is thus assumed that changing real wage could make savings adjust to a fixed level of investment even in the case of full employment.

Forced savings is thus a macro closure scenario, in which investment becomes exogenous variable and α_w will be endogenous variable instead, but the wage rate is not equal to the marginal revenue of labour, measuring the ratio of the wage rate to the marginal revenue of labour. Compared to the Johansen closure, α_w and not τ will be the new endogenous variable compensating for fixing the investment level in this case.

If the general price level is set by $w = 1$, instead of $p^{hm} = 1$, as common in Keynesian models, the output price will be the equilibrating variable, which will set the real wage and through that consumer demand to such a level that brings about equilibrium between saving and fixed investment demand. Increasing investment, for example, would drive real wage down, as in the case in the Keynesian closure. Unlike in the latter, however, the level of both employment and output remains unchanged, and increasing investment crowds out consumption.

V. Neo-Keynesian closure II. (fixed real wage)

An alternative closure also with Keynesian flavour could be one, in which real wage rate is fixed and the employment level is the equilibrating variable, as in the so-called Keynesian closure. This would combine thus two Keynesian assumptions, the refusal of the neo-classical idea of full employment as well as the marginal revenue productivity theory of wages.

VI-VII. Structuralist closures

Structuralist CGE models, designed and used mainly for analysing economic policy options in developing economies[8], bring in institutional considerations in describing the behaviour of certain macroeconomic variables. Following Keynes and classical economists, they usually allow for fixed nominal or real wage rate, for unemployment (i.e., variable level of employment) and may depart in other aspects also from the neo-classical assumptions. For example, investment can depend on expected future returns, the level of capacity utilization

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[8] For a detailed discussion of the structuralist models see, first of all, Taylor (1990).

may vary, various market imperfections are taken into account, cost-plus-markup price formation is assumed.

We will analyse here a model, in which cost-plus-markup pricing rule is assumed and investment is a function of the net rate of return on capital (π):

$$I = I(\pi), \tag{16}$$

where π can be interpreted as an index of expected future returns.

Cost-plus-markup pricing means that equation (3) is replaced by the following one:

$$p^a \cdot X = (1 + \pi_m) \cdot (p^{hm} \cdot A \cdot X + w \cdot L), \tag{3b}$$

where π_m is the profit markup, which replaces the rate of return on capital.

Alternatively, one can simply introduce profit markup as a new variable, together with an additional equation that establishes the relationship between the rate of return on capital and the profit markup. Choosing this option, the following form will be used:

$$\pi p^{hm} \cdot K = p^a \cdot c^\pi \cdot X, \tag{17}$$

where $c^\pi = \pi_m / (1 + \pi_m)$, is the ratio of profit in total revenue, replacing π_m.

It is worth observing that equation (17) can be rewritten as

$$\pi p^{hm} \cdot K = \pi_m \cdot (p^{hm} \cdot A \cdot X + w \cdot L) = \frac{\pi_m}{1 + \pi_m} p^a \cdot X. \tag{17a}$$

These forms provide an interesting insight into the model. If profit markup (π_m) is fixed, the rate of return on capital moves practically in proportion to X, which in turn changes only, if L changes into the same direction too. Thus, for example, an increase in investment will lead to an increase in the rate of return on capital, triggering an *accelerator effect*, in addition to the *multiplier effect* caused by variable employment level, although the profit markup is fixed.

After suitable rearrangement, from (17a) we get

$$\kappa = \frac{X}{K} = \frac{1 + \pi_m}{\pi_m} \frac{p^{hm}}{p^a} \pi,$$

where $\kappa = X/K$ can be interpreted as a measure capacity utilization, at least as long as the level of output is less than achievable in case of full employment (at the fixed supply of capital).

From the above equation one can see that the investment function (16) is similar to, in fact, if $p^{hm} = p^a$ (e.g., in a closed economy) it is equal to

$$I = I(\kappa, \pi_m),$$

which is an investment demand function commonly used in econometric models. κ is here interpreted as an accelerator term, and π_m as an index of expected future returns. As will be seen later, the investment function will indeed bring forth upwards or downwards *accelerator effect* in the model, in addition to the *multiplier effect* caused by variable employment level.

In the model extended by equations (16) and (17) five further endogenous variables remain to be chosen and their potential list is extended by variable c^π. One cannot, of course, fix both π and c^π at the same time. In the case of the two structuralist closures introduced I, L

and α_w are endogenous variable, and τ remains exogenous. One has thus to choose two more from among w, π_m and v_r or D_e. We choose either v_r or D_e, as before, thus the remaining choice is either the (real) wage rate or the profit markup. In *Structuralist closure I.* the wage rate, in *Structuralist closure II.* the profit markup will be fixed.

VIII. The loanable funds closure

Based on the classical idea that savings can be viewed as the supply of loanable funds and investment the demand for them, one could introduce into the model the real rate of interest (i) as an additional variable, and assume that the saving rates, $\sigma_w(i)$ and $\sigma_k(i)$ are increasing, whereas investment, $I(i)$ a decreasing function of it. In such a model the rate of interest would be the equilibrating variable that makes investment and savings match one another. In that model the private saving identity and the investment function have to be modified as follows:

$$S^p = (1 - \tau)\cdot[\sigma_w(i)\cdot w\cdot L + \sigma_k(i)\cdot q\cdot K]. \tag{12a}$$

$$I = I(i). \tag{16a}$$

As explained by Taylor (1990), such a closure is subject to at least two serious objections (both emphasized by Keynes already). First, the interest rate, i.e., the rate of return to assets, is in principle determined by stock markets and not by saving and investment flows. Second, the influence of the interest rate on aggregate investment demand is limited by various institutional factors. It could affect only certain part of gross capital formation and purchases of durable goods. This mechanism is thus seldom used in CCE models, but Bourguignon et al. (1991) attempted to handle this problem by incorporating cumulative flows of funds into the model.

IX. The real balances (Pigovian) closure

Continuing the above comments, the loanable funds approach can be extended into a real balances closure by taking into account wealth effects, in the form of the Pigou (1943) effect or, based on the portfolio model developed by Tobin (1971), introducing interest payments and interest clearing financial asset markets. Assuming exogenously given nominal wage and introducing money supply (M) one could define the real balances of the wealth holders (M/p^{hm}). If the saving rate depends on the real balance, the real balance effect could work in the same way as the rate of interest in the loanable funds closure.

5. A numerical example and simulation results

We selected six characteristic closure rules (I., II., III., V., VI. and VII.) to demonstrate the adjustment mechanisms assumed to work in them by means of a numerical model. We used Hungarian statistical data for 2010 to calibrate the model (Hungarian Central Statistical Office 2012, 2013). The data base used for calibration is arranged in Table 2, which can be regarded as an Input-Output (I-O) table combined with a Social Accounting Matrix (SAM) (integrating national accounts and household budget survey data). Making use of the simple structure of our models, we managed to include the income redistribution into the fourth quadrant of the table.

Table 2: The initial data arranged into a combined I-O and SAM table
(values in thousand billion HUF)

Expenditures ⇒ Receipts ⇓	Commodities (composition of sources)	Consumers	Government	Investors	Rest of the World	Total receipts
Commodities (deliveries)	$p^{hm}\cdot A\cdot X =$ 31.69	$p^{hm}\cdot C =$ 10.85	$p^{hm}\cdot G =$ 6.17	$p^{hm}\cdot I =$ 4.58	$v\cdot p^{we}\cdot Z =$ 20.37	73.66
Consumers	$w\cdot L =$ 13.97			$(1-\tau)\cdot(1-\sigma_k)\cdot q\cdot K = 3.73$		17.70
Government		$\tau w\cdot L = 2.96$		$\tau q\cdot K = 2.01$		4.97
Investors	$q\cdot K = 9.46$	$S_w = \sigma_w\cdot(1-\tau)\cdot w\cdot L = 3.89$	$S^g = -1.20$		$v\cdot D_e = -1.83$	10.32
Rest of the world	$v\cdot p^{wm}\cdot M =$ 18.54					18.54
Total outlays	73.66	17.70	4.97	10.32	18.54	

The base level of all price indeces p^a, p^h, p^{hm}, p^{we}, p^{wm} and that of the *nominal* foreign exchange rate v were set to 1 in the calibration, which with the observed $L = 4$ and $K = 200$ factor employments led to the initial values of 3.493 and 0.047 for w and q. Calibrated values of some parameters:

- $\pi_m = 0.207$; $\tau = 0.212$; $\sigma_w = 0.353$; $\sigma_k = 0.5$;

- the elasticity of substitution (set exogenously) and the calibrated distribution parameters (a_*) in the CES-CET functions are as follows:

  CES $F(\varphi, a_L, a_K, r_L, r_K) = F(0.5, 0.596, 0.404, 0.0726, 3.628)$, where r_L, and r_K are the labour and capital coefficients in the base case;

  CET $X(\sigma, a_d, a_z) = X(-2, 6.330, 53.613)$;

  CES $X^{hm}(\mu, a_h, a_m) = X^{hm}(0.5, 0.425, 0.121)$.

- the price elasticity, ε in the $p^{we}(Z) = a_z\cdot Z^{-1/\varepsilon}$ export demand function, was set at -4, which means that 1 per cent increase in export volume leads to 0.25 per cent decrease in export price. In most CGE models somewhat smaller values are used, which yield larger price effects.[9]

As one can see, in the case of exports we assumed that domestic and export supply can be transformed into each other with relative ease (the elasticity of transformation is 2), whereas, in the case of imports, we assumed that they are rather complements than substitutes to domestic products (the elasticity of substitution set at 0.5), which means their supply will move in the same direction.

In our simulation exercises we stick always to the rule that the external shock is created by change only in one exogenous variable. In policy simulations made with CGE models, one

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[9] The simulation results are rather sensitive to the size of the elasticity, especially in the case of fixed balance of trade.

designs normally a scenario, in which forecasted changes in all important exogenous variables are taken into account and harmonised with each other.

5.1. The effect of a 5% increase in the government expenditure under various closures

We have chosen six closure alternatives, neo-classical (I.), Johansen (III.), Keynes (II.), neo-Keynesian II. (V.), structuralist I. and II. (VI-VII.) for illustrative exercises. We have first simulated the effect of an internal demand shock, assuming 5% increase in the government expenditure. We have estimated the results both in the case of fixed real exchange rate and fixed balance of trade. The results are shown in Table 3.

We will analyse in details only the *case of fixed real exchange rate*, since in the case of fixed balance of trade the real effects will be very similar, although somewhat sharpened. Savings will behave, of course, differently in the two cases. In all but the first two simulations (closures) foreign savings decreases in the first case and – only in domestic currency - increases in the second. Private saving increases also faster (due to the larger increase of employment) and government saving increases therefore slower.

In the *neo-classical* and the *Johansen closure* total output and net output for final use, the wage and profit rate, all prices, foreign trade and foreign saving remain the same as in the base. The increase in government expenditure affects therefore only the distribution of the net output. In the case of the *neo-classical closure* the increase of the government expenditure takes place at the cost of investment and government saving. In the *Johansen closure* government expenditure increases at the expense of consumption, which is enforced by the higher tax rate, which allows at the same time to increase government saving, making up for the decreasing private saving (which is assumed to decrease proportionately with consumption).

In the *Keynesian closure L* becomes variable instead of *I*, available labour does not constrain the expansion of production. The increase in the government expenditure creates a multiplier effect: output grows by 2.6% and employment by 4.5%. The marginal product of labour, consequently the wage rate decreases by 4%, which in turn increases the rate of return on capital by almost 5%. Despite the falling wage rate, total private income and consumption increases (1.9%), multiplying the effect of the autonomous growth of final demand.

With increasing output comes increasing import (2.3%), which has to be compensated by growing export (2.6%), reinforcing also the multiplier effect. Increasing export reduces slightly the price level of export (by 0.7% leading to a terms of trade loss equivalent to 2% of total savings) and increases the nominal exchange rate (0.4%). As a result of the latter, the domestic output price diminishes too (-0.2%). Since investment is fixed, savings remain the same, but its composition changes. Government's saving decreases by about 17%, compensated by 2.4% increase in private and 0.5% increase in foreign saving.

Table 3: The effect of 5% increase in the government expenditure (percentage changes, base values in trillion HUF or ratios)

5% increase in G 1-sector macro model		Base values	fixed real exchange rate						fixed trade balance					
			neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.	neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.
L	level of employment[1]	4.04	0	0	4.52	5.70	4.44	8.10	0	0	4.85	6.26	4.77	9.31
X	output	55.12	0	0	2.64	3.32	2.60	4.68	0	0	2.84	3.64	2.79	5.35
X^h	output for domestic use	34.75	0	0	2.64	3.32	2.60	4.68	0	0	2.82	3.62	2.77	5.32
Z	export	20.37	0	0	2.64	3.32	2.60	4.68	0	0	2.86	3.67	2.81	5.40
M	import	18.54	0	0	2.31	2.90	2.27	4.08	0	0	2.39	3.06	2.34	4.48
X^{hm}	domestic supply	53.29	0	0	2.53	3.17	2.49	4.47	0	0	2.67	3.42	2.62	5.03
C	private consumption	10.85	0	-2.84	1.85	3.05	2.38	3.65	0	-2.84	1.99	3.34	2.56	4.17
I	investment	4.58	-6.74	0	0	0	-1.45	4.26	-6.74	0	0	0	-1.56	4.87
w	real wage rate[2]	3.458	0	0	-4.06	0	0	-4.43	0	0	-4.36	0	0	-5.04
$\pi\,(q)$	rate of return on capital	0.047	0	0	4.80	-1.90	-1.45	4.26	0	0	5.16	-2.10	-1.56	4.87
v	nominal exchange rate	1.00	0	0	0.43	0.53	0.42	0.75	0	0	0.56	0.72	0.55	1.05
v_r	real exchange rate	1,00	0	0	0	0	0	0	0	0	0,15	0,19	0,15	0,29
$v\cdot p^{we}$	domestic export price	1.00	0	0	-0.23	-0.28	-0.22	-0.40	0	0	-0.15	-0.19	-0.15	-0.27
p^h	domestic output price	1.00	0	0	-0.23	-0.28	-0.22	-0.40	0	0	-0.30	-0.38	-0.29	-0.56
p^a	average price of output	1.00	0	0	-0.23	-0.28	-0.22	-0.40	0	0	-0.24	-0.31	-0.24	-0.45
S^p	private saving	7.91	0	-2.84	2.44	2.06	1.62	3.77	0	-2.84	2.62	2.26	1.74	4.31
S^g	government saving	-1.20	25.71	-18.74	16.95	14.72	17.09	10.25	25.71	-18.74	16.26	13.61	16.43	7.96
$v\cdot D_e$	foreign saving (in HUF)	-2.13	0	0	-0.50	-0.64	-0.49	-0.94	0	0	0.56	0.72	0.55	1.05
α_w	wage/marginal product	1.00	0	0	0	5.36	4.17	2.89	0	0	0	5.89	4.47	3.33
τ	tax rate	0.20	0	11.42	0	0	0	0	0	11.42	0	0	0	0
p^{we}	foreign export price	1.00	0	0	-0.65	-0.81	-0.64	-1.14	0	0	-0.70	-0.90	-0.69	-1.31
D_e	foreign trade deficit	-2.13	0	0	-0.92	-1.17	-0.91	-1.67	0	0	0.00	0.00	0.00	0.00
	domestic savings	6.71	-4.60	0	-0.16	-0.20	-1.15	2.61	-4.60	0	0.18	0.23	-0.89	3.66
	terms of trade loss/GDP	0	0	0	-0.57	-0.71	-0.56	-0.99	0	0	-0.59	-0.75	-0.58	-1.10

[1] million persons   [2] million HUF/year/person

In the *neo-Keynesian closures* the wage rate is no longer determined by the marginal product of labour (α_w becomes endogenous instead of I). In its first version, discussed by Sen, L remains fixed, as in the neoclassical closure, in the second, it is let vary at the expense of fixing the real wage rate.[10] We present only the results of this second, *neo-Keynesian closure II*, in which all other aspects are the same as in the Keynesian closure. As a result of fixed real wage rate, total wage fund grows at the same rate as the level of employment (5.7%), exceeding the growth rate of the output and the value added ($p^a \cdot X - p^{hm} \cdot A \cdot X$), which grow only by about 3%. As a result, the rate of return on capital decreases (by 2%). To make up for the lost saving the level of employment and output has to expand more than in the case of the Keynesian closure. The rest of the changes are similar to those experienced in the case of Keynesian closure.

The *structuralist closures* depart more drastically from the Keynesian by using markup pricing and incorporating an accelerator effect in addition to the multiplier effect, because investment depends on the rate of return on capital. We used a simple investment function of the following form:

$$I = I^0 \left(\frac{\pi}{\pi^0} \right)^\delta ,$$

where we have chosen I^0 and π^0 to be equal to the base values of the investment and the rate of return on capital, and δ, the elasticity parameter 1. Since K is fixed, all these mean that investment will change in proportion to capital income.

In *structuralist closure I.* the (real) wage level is fixed, and the profit markup is free to adjust. The profit markup decreases by 4.5%, the rate of return on capital and investment both by 1.5% ($\delta = 1!$). One can observe thus a reverse accelerator effect in this case, which slows down the growth, compared to the neo-Keynesian closure.

In *structuralist closure II.* the rate of the profit markup is fixed instead of the real wage. The real wage decreases by 4.4%, the rate of return on capital as well as the investment level grows by 4.26%, adding an accelerator effect to the multiplier effect. As a result, employment increases by 8.1%, output 4.7% and consumption by 3.7%, indicating thus quite an economic boom.

In Table 3 one can also see how the simulation results are modified by *fixing the balance of trade* instead of the real exchange rate. This makes foreign currency scarcer, than in the previous simulations, therefore, the domestic currency devaluates, increasing the domestic value of the fixed foreign surplus, i.e., decreasing foreign saving. In the case of the *neo-classical* and *Johansen closure*, in which foreign saving remains unchanged, the results do not change either. In the case of the *other closures* the decreasing government and foreign saving can be made up only by growing domestic income. Therefore, the economy must grow at faster rate than in the case of fixed real exchange rate, resulting in larger terms of trade losses as well.

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[10] This scenario assumption is somewhat unrealistic, since real wage should decrease to some extent to enable employment to increase. But we stick to the rule to change only one assumption at one time.

5.2. The effect of a 2% increase in world market import prices under various closures

In the second series of simulations we analysed the likely effect of an external shock, represented by 2% increase in the world market import prices. As can be seen in Table 4, the differences between the results obtained in the case of *fixed real exchange rate* and *fixed balance of trade* become larger. *At fixed real exchange rate* the trade balance deteriorates, by about 9% in all closures, which must be compensated by 1.34-1.48% devaluation, if the balance of trade is fixed. This leads to larger changes in relative prices and consequently in all volume and income variables as well.

In the case of *fixed real exchange rate* the increasing net foreign saving (resulting from smaller trade deficit) makes up for a large part of the lost income available for domestic use, therefore the changes in final demand are less drastic than in the case of fixed balance of trade. In the case of the *neo-classical and Johansen* closures the GDP (effected by terms of trade changes too) drops by about 1.6% and both the wage rate and the rate of the return on capital decreases, at roughly the same per cent.

In the *Keynesian* and *neo-Keynesian closures* the results of the neo-classical closure are almost reproduced in the case of *fixed real exchange rate*. This is simply due to the fact that the relative prices on the domestic and foreign markets, and consequently, the domestic and foreign supply/demand structures change only slightly. Since government expenditure and investment are fixed, the level of consumption drops as a consequence of diminishing GDP. The case of *fixed balance of trade* is quite different. The growing real exchange rate increases the share of export and decreases that of import, and the resulting increase of the net export in final demand enhances the multiplier effect. The higher level of employment (3.3% and 4.7%) and consumption strengthens further the multiplier effect, especially in the case of the neo-Keynesian closure. Increasing export volume is accompanied by falling export price, which increases the terms of trade loss in both Keynesian closures.

In the case of the *structuralist closures*, the investment function brings in a negative accelerator effect, when the rate of return on capital drops. This happens in all cases, except for one, the case of *structuralist II., with fixed trade balance*. There the rate of return on capital and investment increases by close to 2%, creating quite a boom.

5.3. The effect of the less than perfectly elastic export demand

The assumption of less than perfectly elastic export demand function, which is quite common in CGE models, deserves a brief comment at the end. We used the following form:

$$p^{\text{we}}(Z) = a_z \cdot Z^{-1/\varepsilon}$$

where ε is the price elasticity of export demand, and a_z a constant shift parameter, which represents the export price level of the competing countries.

The price elasticity of demand was set at a reasonable large value of 4 (i.e. -4) in our scenarios, which means that 1 per cent increase in export volume will be accompanied by 0.25 per cent decrease in export price. In most CGE models somewhat smaller values are used, which yield larger price effects. (Sometimes one can even see elasticity smaller than 1, which means that a reduction in export volume will increase export revenue!!)

Table 4: The effect of 2% increase in world market import prices (percentage changes, base values in trillion HUF or ratios)

2% increase in p^{wm} 5-sector CGE model	Base values	fixed real exchange rate						fixed trade balance					
		neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.	neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.
L level of employment[1]	4.04	0	0	0.09	0.54	-2.19	-0.89	0	0	3.27	4.71	0.73	4.99
X output	55.12	0	0	0.05	0.32	-1.32	-0.53	0	0	1.92	2.76	0.43	2.92
X^h output for domestic use	34.75	0	0	0.05	0.32	-1.32	-0.53	0	0	1.79	2.62	0.31	2.78
Z export	20.37	0	0	0.05	0.32	-1.32	-0.53	0	0	2.15	2.99	0.64	3.16
M import	18.54	-0.99	-0.99	-0.94	-0.71	-2.13	-1.45	-1.79	-1.79	-0.19	0.49	-1.43	0.63
X^{hm} domestic supply	53.29	-0.34	-0.34	-0.30	-0.04	-1.60	-0.85	-0.71	-0.71	1.09	1.87	-0.31	2.02
C private consumption	10.85	-1.62	-1.68	-1.58	-1.11	-2.59	-2.11	-1.60	-3.49	-0.28	1.11	-0.99	0.55
I investment	4.58	-0.13	0	0	0	-3.33	-1.17	-4.48	0	0	0	-4.20	1.96
w real wage rate[2]	3.458	-1.62	-1.62	-1.70	0	0	-1.75	-1.60	-1.60	-4.52	0	0	-4.96
$\pi\,(q)$ rate of return on capital	0.047	-1.62	-1.62	-1.53	-4.19	-3.33	-1.17	-1.60	-1.60	1.81	-5.62	-4.20	1.96
v nominal exchange rate	1.00	-0.69	-0.69	-0.68	-0.64	-0.90	-0.77	0.21	0.21	0.58	0.74	0.29	0.77
v_r real exchange rate	1.00	0	0	0	0	0	0	1.34	1.34	1.43	1.47	1.36	1.48
$v \cdot p^{we}$ domestic export price	1.00	-0.69	-0.69	-0.69	-0.72	-0.57	-0.64	0.15	0.15	0.04	0.00	0.13	-0.01
p^h domestic output price	1.00	-0.69	-0.69	-0.69	-0.72	-0.57	-0.64	-1.17	-1.17	-1.37	-1.45	-1.21	-1.47
p^a average price of output	1.00	-0.69	-0.69	-0.69	-0.72	-0.57	-0.64	-0.68	-0.68	-0.85	-0.92	-0.72	-0.93
S^p private saving	7.91	-1.62	-1.68	-1.57	-1.72	-2.73	-1.93	-1.60	-3.49	0.14	-0.23	-1.63	0.83
S^g government saving	-1.20	6.31	5.45	6.14	5.25	10.64	8.05	6.17	-23.36	-0.13	-2.85	4.75	-3.38
$v \cdot D_e$ foreign saving (in HUF)	-2.13	-9.29	-9.29	-9.30	-9.36	-9.00	-9.17	0.21	0.21	0.58	0.74	0.29	0.77
α_w wage/marginal product	1.00	0	0	0	2.2	-0.4	-1.0	0	0	0	6.1	2.3	1.1
τ tax rate	0.20	0	0.22	0	0	0	0	0	7.71	0	0	0	0
p^{we} foreign export price	1.00	0	0	-0.01	-0.08	0.33	0.13	0	0	-0.53	-0.73	-0.16	-0.77
D_e foreign trade deficit	-2.13	-8.66	-8.66	-8.68	-8.78	-8.17	-8.46	0.00	0.00	0.00	0.00	0.00	0.00
domestic savings	6.71	-3.04	-2.95	-2.95	-2.97	-5.13	-3.71	-2.99	0.07	0.18	0.23	-2.78	1.58
terms of trade loss/GDP	0	-1.58	-1.58	-1.59	-1.65	-1.29	-1.47	-1.62	-1.62	-2.03	-2.20	-1.71	-2.23

[1] million persons [2] million HUF/year/person

The simulation results are rather sensitive to the size of the elasticity, especially in the case of fixed balance of trade. For example, in the case of the neo-Keynesian closure, if the size of elasticity decreases, in absolute term, say from 4 to 2, the nominal exchange rate will increase by more than 3.5% (instead of 0.8%), the terms of trade loss four times, export will grow by 7.5% (instead of 3.9%) and its foreign price will decrease by 3.5% (instead of 0.9%).

6. Macro closure options in multi-sectoral models

We turn now our attention to the issue of closure in multi-sectoral CGE models. We will first review their typical variables and equations of a still stylised multi-sectoral CGE model. Then we present a more sophisticated and more practical concrete version of this, developed for the analysis of various policy issues in Hungary. Finally, with this concrete fully-fledged multi-sectoral CGE model we replicate the simulation runs made with the one-sector model.

6.1. The structure of a stylised multi-sectoral CGE model

The following brief overview of the main components of a stylized CGE model will hopefully suffice for the reader, on the bases of the Walrasian and the aggregate macroeconomic model presented above.

In the following brief overview of the model, for the sake of simplicity, we will not present the various derived demand or supply functions in their concrete parametric form. On the basis of the aggregate macroeconomic model presented earlier the reader will be able to reconstruct them.

As will be seen, the equations defining the equilibrium conditions of the supply and pricing side of the model will be practically the same as in the case of the one-sector model, except that we will use their dual and mixed forms, as described earlier, in defining the necessary conditions of equilibrium.

With each equation a variable will be associated. In this way one can easily check if the model is well defined or not, i.e., the number of equations and variables are equal or not. In most cases the variable associated with the equation is the one standing on its left hand side. Therefore, it will not be indicated explicitly. If it is another variable, it will be attached to the equation numbers appearing on their right hand side. For example, (S-01: k_j) in the case of the first equation.

It is obvious that in a multi-sectoral model one has to distinguish between commodities and prices according to their sectoral origin. We will use i and/or j indexes for different sectors, depending on whether we refer to sectoral commodity or activity, For example, x_j, x_j^h, z_j, x_i^{hm}, m_i, p_i^{hm}, p_i^h, p_i^{wm}, p_i^{we} and so on. As can be seen, the equations defining the equilibrium conditions of the supply and pricing side of the model will be the same as in the case of the one-sector model, except that we will use dual and mixed forms, as was indicated earlier, in describing the necessary conditions of equilibrium.

$1 = f_j(l_j, k_j)$	sectoral production function	(S-01: k_j)
$l_j = l_j(w_j/\alpha, q_j)$	labour demand coefficient	(S-02)
$p_j^a = \Sigma_i p_i^{hm} \cdot a_{ij} + w_j \cdot l_j + q_j \cdot k_j + p_j^a \cdot \tau_j^t$	price-cost identity	(S-03)

$x_j = x_j(x_j^{\text{h}}, z_j)$	composition of the sectoral outputs	(S-04)
$z_j = r_j^{\text{eh}}(p_j^{\text{h}}, p_j^{\text{e}}) \cdot x_j^{\text{h}}$	exports and supply on home markets	(S-05)
$p_j^{\text{a}} = p_j^{\text{h}} \cdot (x_j^{\text{h}}/x_j) + p_j^{\text{e}} \cdot (z_j/x_j)$	(average) producers' prices	(S-06: p_j^{h})
$x_i^{\text{hm}} = x_i^{\text{hm}}(x_i^{\text{h}}, m_i)$	composition of domestic supply	(S-07: x_j^{h})
$m_i = r_i^{\text{mh}}(p_i^{\text{h}}, p_i^{\text{m}}) \cdot x_i^{\text{h}}$	imports and supply on home markets	(S-08)
$p_i^{\text{hm}} = p_i^{\text{h}} \cdot (x_i^{\text{h}}/x_i^{\text{hm}}) + p_i^{\text{m}} \cdot (m_i/x_i^{\text{hm}})$	(average) domestic users' prices	(S-09)

where:

$l_j(w_j/\alpha_{\text{w}}, q_j)$, $r_j^{\text{eh}}(p_j^{\text{h}}, p_j^{\text{e}})$ and $r_i^{\text{mh}}(p_i^{\text{h}}, p_i^{\text{m}})$ are all dual forms,

derived from assumed optimizing behaviour (used to determine labour demand, the ratios of exports and imports to domestic supply). The auxiliary (sectoral) cost and price variables are as follows:

$w_j = (1+\tau_j^{\text{w}}) \cdot w \cdot d_j^{\text{w}}$	cost of labour	(S-10)
$q_j = p_j^{\text{b}} \cdot (r_j^{\text{d}} + \pi d_j^{\pi})$	Walras's cost of capital	(S-11)

where

$p_j^{\text{b}} = \Sigma_i p_i^{\text{hm}} \cdot b_{ij}$	price index of the capital goods	(S-12)

where the b_{ij} parameters are the investment coefficients,

$p_i^{\text{c}} = (1+\tau_i^{\text{c}}) \cdot p_i^{\text{hm}}$	consumers' prices	(S-13)
$p_j^{\text{e}} = (1+\tau_j^{\text{e}}) \cdot v \cdot p_j^{\text{we}}(z_j)$	domestic price of exports	(S-14)
$p_i^{\text{m}} = (1+\tau_i^{\text{m}}) \cdot v \cdot p_i^{\text{wm}}$	domestic price of imports	(S-15)

As can be seen, we took into considerations various net taxes/subsidies (τ_j^{w}, τ_j^{e}, τ_i^{m}, τ_i^{c} *ad valorem* rates), as well as the observed differences in sectoral wage levels and rates of return on capital (d_j^{w}, d_j^{π}), as common in CGE models. Observe also that the p_j^{a} producers' prices include also net direct taxes ($p_j^{\text{a}} \cdot \tau_j^{\text{t}}$), which will be added to the operating surplus, when we compare the simulation results of the simple one-sector and complex five-sector model.

For the sake of simplicity we will assume that both labour and capital are homogenous, i.e., mobile across sectors as any other factor of production. The aggregate demand is thus the algebraic sum of the sectoral demands:

$\Sigma_j l_j \cdot x_j = L$	total use of labour	(S-16: L)
$\Sigma_j k_j \cdot x_j = K$	total capital demand	(S-17: π)

Capital supply will be assumed to be given exogenously in all closures, thus, (S-17) is the condition of equilibrium on capital market. The same holds, *mutatis mutandis*, in the case of labour too, if its supply is fixed. If it can vary, then (S-16) defines simply the labour demand.

As one can see, equations (S-01) - (S-10), supplemented with equations (S-11) - (S-17), are simply the multisectoral equivalents of the equations (1)-(9) of the fully aggregated, one-sector macroeconomic model analysed above. We will now define the equations equivalent to (10)-(14) in the one-sector model.

First, appropriate supplementary variables and equations are introduced to define consumption and investment, as parts of final demand. As in most CGE models, consumer demand is represented by a linear expenditure system (LES), introduced by Stone (1954), which can be expressed as:

$$c_i = C_i^{\text{c}} + s_i^{\text{cv}}(p_1^{\text{c}}, p_2^{\text{c}}, \dots , p_n^{\text{c}}) \cdot C^{\text{v}} \qquad \text{consumers' demand (LES)} \qquad (\text{S-18})$$

where the s_i^{cv} functions define the expenditure minimizing shares of commodities in variable consumption, whose level is C^{v} and C_i^{c} is the 'committed' part of consumer demand.

In our static model in the non-structuralist closures the sectoral investment levels are determined by assuming fixed sectoral investment shares (s_j^{a}):

$$I_j = s_j^{\text{a}} \cdot I \qquad \text{sectoral investment levels,} \qquad (\text{S-19})$$

In the structuralist closures the sectoral investment functions (replacing S-19) will be similar to those used in the one-sector model:

$$I_j = I_j(q_j) = I_j^0 \cdot \{q_j/q_j^0 \cdot (k_j/k_j^0)\}^{\delta_j} \qquad \text{sectoral investment functions,} \qquad (\text{S-19'})$$

Of course, in this case the total investment level (I) drops out of the model.

The equations representing income (re)distribution and the final demand side of the model will be as follows:

$$x_i^{\text{hm}} = \Sigma_j\, a_{ij} \cdot x_j + c_i + s_i^{\text{g}} \cdot G + \Sigma_j\, b_{ij} \cdot I_j \qquad \text{equilibrium on commodity markets} \qquad (\text{S-20})$$

The balance of trade (D_{e}) is defined in the multisectoral model by the following equation:

$$\Sigma_i\, (p_i^{\text{wm}} \cdot m_i - p_i^{\text{we}}(z_i) \cdot z_i) = D_{\text{e}} \qquad \text{foreign trade balance} \qquad (\text{S-21: } D_{\text{e}})$$

The budget balances of the other (domestic) agents are formulated as follows:

$$S^{\text{rw}} = v \cdot D_{\text{e}} + tr^{\text{sw}}(\cdot) \qquad \text{current account (foreign net savings)} \qquad (\text{S-22})$$

$$\Sigma_i\, p_i^{\text{hm}} \cdot s_i^{\text{g}} \cdot G + S^{\text{g}} = \Sigma_j\, \{\tau_j^{\text{w}} \cdot w \cdot d_j^{\text{w}} \cdot l_j \cdot x_j + p_j^{\text{a}} \cdot x_j \cdot \tau_j^{\text{t}}\} + \Sigma_i\, \tau_i^{\text{c}} \cdot p_i^{\text{hm}} \cdot c_i + \Sigma_i\, \{\tau_i^{\text{m}} \cdot v \cdot p_i^{\text{wm}} \cdot m_i -$$
$$- \tau_i^{\text{e}} \cdot v \cdot p_i^{\text{we}}(z_i) \cdot z_i\} + tr^{\text{g}}(\cdot) \qquad \text{government's budget} \qquad (\text{S-23: } S^{\text{g}})$$

$$p_j^{\text{b}} \cdot I_j + S_j^{\text{s}} = q_j \cdot k_j \cdot x_j + tr_j^{\text{s}}(\cdot) \qquad \text{sectoral budgets} \qquad (\text{S-24: } S_j^{\text{s}})$$

$$\Sigma_i\, p_i^{\text{c}} \cdot c_i + S^{\text{h}} = \Sigma_j\, w \cdot d_j^{\text{w}} \cdot l_j \cdot x_j + tr^{\text{h}}(\cdot) \qquad \text{households' budget} \qquad (\text{S-25: } S^{\text{h}})$$

The first equation defines the equilibrium condition on the commodity markets at home. Change in stocks is, for the sake of simplicity, assumed to be part of investment demand, defined as $\Sigma_j\, b_{ij} \cdot I_j$.

The budget identities are thus somewhat different but equivalent to their counterparts in the one-sector model. Observe, for example, that net foreign saving (S^{rw}) is determined by the current account, not by the balance of foreign trade (net income from abroad) alone, since net current transfers, tr^{w} is also taken into account. In addition to households and government, the production sectors appear also as economic actors in the model. They represent the enterprise managers, entrepreneurs and rentiers, who make production and investment decisions as well as distribute the profits. As a result, unlike in the one-sector model, profit income does not go directly to the households, but through income redistribution.

Each actor (domestic and foreign) disposes with a given portion of GDP, which forms their initial (gross or net) income position. Households get hold of gross wages, the sectors

retain their gross operating surplus (amortization plus profit), whereas the government collects taxes and pays subsidies, foreign net income is formed by the balance of trade. The initial incomes are then transferred from one group to another through different channels. In the CGE models all sorts of transfer payments are taken into account one by one. (Typical examples of transfers include income taxes, pension contributions, property taxes, investment taxes, social benefits and the likes.) Their net effects on the initial incomes of the agents are denoted by the tr^w, tr^g, tr_j^s and tr_k^h variables. The value of these agent specific net transfers can be either positive or negative, but their total sum has to be always zero (one transfers some amount to another actor). The disposable income of the household groups formed in this way can be seen on the right hand sides of the budget equations.

The households and the government make the decisions concerning private and public consumption expenditures and net financial savings (see them on the left hand sides of the budget equations), the sum of which must be equal to their disposable income, as stated by the budget constraints. The sum total of net financial savings ($S^h + S^g + \Sigma_j S_j^s + S^{rw}$) has to be zero, which secures that total private, public and foreign savings plus the retained earnings by the sectors will be equal to investments. It can be also shown that equation system (S-01) - (S-25) fulfils the requirement of Walras law.

Equation (S-26) establishes relationship between households' savings (S^h) and disposable income by means of a saving rate (σ_w):

$$S^h = \sigma_w \cdot \{\Sigma_j w \cdot d_j^w \cdot l_j \cdot x_j + tr^h(\cdot)\} \qquad \text{households' total savings} \qquad \text{(S-26: } C^v\text{)}$$

In the one-sector macroeconomic model we distinguished saving rates from wages and profits. This latter relationship, used in CGE models, is a more realistic way to represent the saving behaviour, since most households receive wages as well as profit shares or rents. Saving rates differ across income groups rather than depend on the source of income.

In the multisectoral model the real exchange rate will be defined as

$$v_r = v \cdot \Sigma_i p_i^{we}(z_i) \cdot z_i / \Sigma_i p_i^h \cdot z_i \qquad \text{real exchange rate} \qquad \text{(S-27)}$$

Finally, for practical reasons we will use the following

$$p_c = \Sigma_i p_i^c \cdot c_i / \Sigma_i p_i^{c0} \cdot c_i \qquad \text{consumers' price index,} \qquad \text{(S-28)}$$

which will be set to unity, in a way similar to $p^{hm} = 1$ in the one-sector model:

$$p_c = 1 \qquad \text{(S-29: } v\text{)}$$

Setting the price level in this way means that variable v becomes the consumer purchasing power of the foreign currency. Setting the price level in this way will mean that the w will be the real wage rate, the same way as in the one-sector model in the case of $p^{hm} = 1$.

Similarly, we introduce the average investment price index as

$$p^b = (\Sigma_i p_i^{hm} \cdot \Sigma_j b_{ij} \cdot I_j) / \Sigma_j I_j \qquad \text{price index of the capital goods} \qquad \text{(S-30)}$$

The above equation system (S-01)-(S-28) is an equivalent variant of equations (1)-(15) in the one-sector model, except that we have not yet defined the tax level, which will adjust in the Johansen closure as τ did in the one-sector model. This additional tax level variable will be denoted by α_τ, and it will be added to the model. It is a potentially variable multiplier attached to $(1+\tau_j^w)$ and $(1+\tau_i^c)$ factors in the definition of disposable income.

In order to implement the structural closures too, the model has to be modified further. Equation (S-19) is replaced by (S-19'), containing sectoral investment functions and total investment, I is dropped as a variable. New variables and equations have to be the introduced, as in the case of the one-sector model. First of all, the potentially variable profit mark-up rates ($\alpha_\pi \cdot c_j^\pi$) have to be introduced, where α_π is the general level of the profit mark-up (equal to 1 in the base) and the c_j^π parameters denote their sectoral differences (their values are set at their observed base level). Parallel to that, gross operating surplus ($q_j \cdot k_j$) has to be accordingly redefined and replaced in equations (S-03) and (S-24) as $p_j^b \cdot r_j^d \cdot k_j + \alpha_\pi \cdot p_j^a \cdot c_j^\pi$, where the first term represents the amortization and the second the profit (net operating surplus).

Next, the relationship between the general level of the net rate of return (π) and the profit mark-up (α_π) has to be specified, as it was done by equation (16) in the one-sector model. In the multisectoral model the following two equations will do the same:

$$\pi d_j^\pi \cdot p_j^b \cdot k_j = \alpha_\pi \cdot p_j^a \cdot c_j^\pi \qquad \text{sectoral rate of return differences} \qquad \text{(S-31: } d_j^\pi\text{)}$$

$$\Sigma_i d_i^\pi = \Sigma_i d_i^{\pi 0} \qquad \text{normalization of the rates of return} \qquad \text{(S-32: } \alpha_\pi\text{)}$$

In closures in which total investment is exogenous no more equations are needed. However, when total investment is endogenous so that the sectoral investment are determined by (S-19) we still need an equation to determine the aggregate investment behaviour. This can be formulated in a similar manner to what we used in the one-sector model:

$$I = I(\pi) = I^0 \cdot (\pi/\pi^0)^\delta \qquad \text{investment function} \qquad \text{(S-33)}$$

which can represent fixed investment level as well ($\delta_j = 0$).

6.2. Closure options in the stylised CGE models

The equation system (S-01)-(S-30) is thus the multisectoral counterpart of equations (1)-(15) in the case of the one-sector model. It contains $19n+11$ equations and the following, altogether $19n+8$ variables will be endogenous in all closures in our simulation:

$k_j, l_j, p_j^a, x_j, z_i, p_j^h, x_i^h, m_i, p_i^{hm}, w_j, q_j, p_j^b, p_i^c, p_j^e, p_i^m, \pi, c_i, I_j, x_i^{hm}, S^{rw}, S^g, S_j^s$ and S^h, C^v, p_c, v and p^b.

This means that 3 further (endogenous) variables are still to be chosen to make the system well determined.

σ_w will be exogenous in all closures, as indicated above. G will be also exogenous as in the one-sector model. The additional endogenous variables can be chosen thus from among

I, L, w, α_w, v_r or D_e, α_τ

in the case of the neoclassical, Johansen, Keynesian and the neo-Keynesian closures.

In the case of the structuralist closures d_j^π is added to the list of the always endogenous variables, and α_π is added to the former list of the potential endogenous variables. Since I is at the same time dropped from the model (or, alternatively, it becomes an always endogenous variable, associated with identity $I = \Sigma_j I_j$) the number of potential endogenous variables remains seven. The degree of freedom increases by one. Four more endogenous variables have to be chosen out of the seven candidate variables and the remaining three fixed, in order to close the model. So, the closure options are the same as in the case of the one-sector model

(see in Table 1), the only differences would be in the notation: τ and c^{π} is replaced by α_{τ} and α_{π}.

6.3. The equations of the disaggregated CGE model used in our simulations

To make the model as practical as possible, household sector is disaggregated into three groups (income terciles), and three main areas of domestic use (private consumption, investment and other areas) are also distinguished, assuming different elasticity parameters determining their import-domestic composition. In addition to disaggregation we use, for practical reasons, somewhat different forms of specification for various functions describing substitution possibilities than in the one-sector model. The detailed representation of the complex tax- and income redistribution system also requires a model somewhat different from the stylised model. The export supply functions used in this model are also different from the one used in the one-sector model. They may depend, apart from the prices, on the level of output too.

Further on, following the example of recent CGE-models designed for policy analysis, nested CES production functions are used in our model too, in which *energy appears also as a production factor* substitutable with labour and capital. As a result, wage induced shifts will be smaller than in the one-sector model. Substitution between various types of energy inputs will be also allowed for in the production function.

In the following equation list of the model we preserve, as much as it was possible, the logic and sequence of the equations of the stylised model. Therefore, the first block contains the production functions and the derived factor demand equations:

$$1 = f_j(l_j, k_j, e_j) \qquad \text{production functions (unit isoquants)} \qquad \text{(M-01: } k_j)$$

$$l_j = l_j(w_j/\alpha_{\mathrm{w}}, q_j, p_j^{\mathrm{en}}) \qquad \text{labour input coefficient} \qquad \text{(M-02)}$$

$$e_j = e_j(w_j/\alpha_{\mathrm{w}}, q_j, p_j^{\mathrm{en}}) \qquad \text{aggregate energy input coefficient} \qquad \text{(M-03)}$$

$$a_{ij} = a_{ij}(p_{s+1,j}^{\mathrm{hmut}}, p_{s+2,j}^{\mathrm{hmut}}, \ldots, p_{nj}^{\mathrm{hmut}}, e_j), \ i \in \mathrm{EN} \quad \text{variable input coefficients} \qquad \text{(M-04)}$$

$$p_{ij}^{\mathrm{hmut}} = p_i^{\mathrm{ohm}} \cdot (1 + \tau_{ij}^{\mathrm{fu}}), \ i \in \mathrm{EN} \qquad \text{user's price of energy in production} \qquad \text{(M-05)}$$

$$p_j^{\mathrm{en}} = \sum_{i=s+1}^{n} p_{ij}^{\mathrm{hmut}} \cdot a_{ij}/e_j \qquad \text{user price (cost) of aggregate energy,} \qquad \text{(M-06)}$$

where $\mathrm{EN} = \{s+1, s+2, \ldots, n\}$ is the index set of energy products.

The following three equations define the composite sectoral outputs:

$$x_j = x_j(x_j^{\mathrm{h}}, z_j) \qquad \text{composition of the sectoral outputs} \qquad \text{(M-07)}$$

$$z_j = r_j^{\mathrm{eh}}(p_j^{\mathrm{h}}, p_j^{\mathrm{e}}) \cdot x_j^{\mathrm{h}} \qquad \text{exports supply} \qquad \text{(M-08)}$$

$$p_j^{\mathrm{a}} = p_j^{\mathrm{h}} \cdot (x_j^{\mathrm{h}}/x_j) + p_j^{\mathrm{e}} \cdot (z_j/x_j) \qquad \text{(average) producers' prices} \qquad \text{(M-09: } p_j^{\mathrm{h}})$$

By splitting the market of each product into the mentioned three main areas of use, the total supply of domestic and import products on the domestic market is defined as

$$x_i^{\mathrm{h}} = x_i^{\mathrm{oh}} + x_i^{\mathrm{ch}} + x_i^{\mathrm{bh}} \qquad \text{decomposition of the domestic supply} \qquad \text{(M-10)}$$

$$m_i = m_i^{\mathrm{o}} + m_i^{\mathrm{c}} + m_i^{\mathrm{b}} \qquad \text{decomposition of the import supply} \qquad \text{(M-11)}$$

The multisectoral and use-specific equivalents of equations (7)-(9) are as follows:

$$x_i^{\mathrm{rhm}} = x_i^{\mathrm{rhm}}(x_i^{\mathrm{rh}}, m_i^{\mathrm{r}}), \qquad \text{composite supply by market segments} \qquad \text{(M-12)}$$

$$m_i^r = r_i^{r\text{mh}}(p_i^{\text{h}}, p_i^{\text{m}}) \cdot x_i^{r\text{h}} \qquad\qquad \text{imports supply by market segments} \qquad\qquad \text{(M-13)}$$

$$p_i^{r\text{hm}} = (p_i^{\text{h}} \cdot x_i^{r\text{h}} + p_i^{\text{m}} \cdot m_i^r)/x_i^{r\text{hm}} \qquad\qquad \text{average domestic users' prices,} \qquad\qquad \text{(M-14)}$$

where $r = \text{o, c, b}$ (c consumption, b investment, o other areas of use).

In the above equations $l_j(w_j/\alpha_{\text{w}}, q_j, p_j^{\text{en}})$, $x_j(x_j^{\text{h}}, z_j)$, $r_j^{\text{eh}}(p_j^{\text{h}}, p_j^{\text{e}})$, $x_i^{r\text{hm}}(x_i^{r\text{h}}, m_i^r)$ and $r_i^{r\text{mh}}(p_i^{\text{h}}, p_i^{\text{m}})$ are all dual forms, derived from assumed optimizing behaviour.

The auxiliary (sectoral) cost and price variables are as follows:

$$w_j = (1+\tau_j^{\text{w}}) \cdot w \cdot d_j^{\text{w}} \qquad\qquad \text{cost of labour} \qquad\qquad \text{(M-15)}$$

$$q_j = p_j^{\text{b}} \cdot (r_j^{\text{a}} + \pi d_j^{\pi}) \qquad\qquad \text{Walras's cost of capital,} \qquad\qquad \text{(M-16)}$$

where

$$p_j^{\text{b}} = \Sigma_i p_i^{\text{bhm}} \cdot b_{ij} \qquad\qquad \text{price indices of the capital goods} \qquad\qquad \text{(M-17)}$$

$$p_j^{\text{e}} = (1+\tau_j^{\text{e}}) \cdot v \cdot p_j^{\text{we}}(z_j) \qquad\qquad \text{domestic price of exports} \qquad\qquad \text{(M-18)}$$

$$p_j^{\text{a}} = \Sigma_{i=1}^{s} p_i^{\text{ohm}} \cdot a_{ij} + \Sigma_{i=s+1}^{n} p_{ij}^{\text{hmut}} \cdot a_{ij} + w_j \cdot l_j + q_j \cdot k_j + p_j^{\text{a}} \cdot \tau_j^{\text{t}}, \quad \text{price-cost identity} \qquad \text{(M-19)}$$

$$p_i^{\text{m}} = (1+\tau_i^{\text{m}}) \cdot v \cdot p_i^{\text{wm}} \qquad\qquad \text{domestic price of imports} \qquad\qquad \text{(M-20)}$$

The aggregate demand for labour and capital is again simply the algebraic sum of the sectoral demands:

$$\Sigma_j l_j \cdot x_j = L \qquad\qquad \text{total use of labour} \qquad\qquad \text{(M-21: } L\text{)}$$

$$\Sigma_j k_j \cdot x_j = K \qquad\qquad \text{total capital demand} \qquad\qquad \text{(M-22: } \pi\text{)}$$

As can be seen, equations (M01)-(M14), supplemented with equations (M15)-(M22), are simply the multisectoral equivalents of equations (S-1)-(S-17) of the stylised CGE model.

As in this disaggregated CGE-model consumer demand is represented by a linear expenditure system (LES):

$$y_{ik}^{\text{c}} = y_{ik}^{\text{e}} + s_{ik}^{\text{cv}}(p_1^{\text{c}}, p_2^{\text{c}}, \dots, p_n^{\text{c}}) \cdot C_k^{\text{v}} \qquad\qquad \text{consumers' demand (LES),} \qquad\qquad \text{(M-23)}$$

where consumer's prices contain taxes and subsidies defined by ad valorem consumption tax rates (τ_i^{c}) in addition to their seller's basic prices:

$$p_i^{\text{c}} = (1+\tau_i^{\text{c}}) \cdot p_i^{\text{chm}} \qquad\qquad \text{consumers' prices} \qquad\qquad \text{(M-24)}$$

The s_{ik}^{cv} functions define the expenditure minimizing shares of commodities in variable consumption, whose level is C_k^{v} and y_{ik}^{e} is the fixed ('committed') part of consumer demand. The total consumption of the households by sectors of origin (c_i) can be computed by summing over the consumption of the individual household groups:

$$c_i = \Sigma_k y_{ik}^{\text{c}} \qquad\qquad \text{(M-25)}$$

In the equilibrium the composite supply of consumer goods should be equal to the demand for them, where the demand is the sum of the consumption of the inbound tourists (y_i^{ct}) and the households (c_i):

$$x_i^{\text{chm}} = c_i + y_i^{\text{ct}} \qquad\qquad \text{(M-26: } x_i^{\text{ch}}\text{)}$$

Similarly, the composite supply of investment goods should be equal, in equilibrium, to the total investment demand:

$$x_i^{\text{bhm}} = \Sigma_j b_{ij} \cdot I_j, \qquad\qquad \text{(M-27: } x_i^{\text{bh}}\text{)}$$

In the non-structuralist closures the sectoral investment levels are determined again by assuming fixed sectoral investment shares (s_j^{a}):

$$I_j = s_j^{\text{a}} \cdot I \qquad\qquad \text{sectoral investment levels,} \qquad\qquad \text{(M-28)}$$

while in the structuralist closures the sectoral investment functions will be the same as in (S-19'):

$$I_j = I_j(q_j) = I_j^0 \cdot \{q_j/q_j^0 \cdot (k_j/k_j^0)\}^{\delta_j} \qquad\qquad \text{sectoral investment functions,} \qquad\qquad \text{(M-28')}$$

which can represent fixed investment level as well ($\delta_j = 0$). Of course, in this case the total investment level (I) drops out of the model.

The equilibrium condition on other market areas is as follows:

$$x_i^{\text{ohm}} = \Sigma_j \, a_{ij} \cdot x_j + s_i^{\text{g}} \cdot G + y_i^{\text{s}} \qquad\qquad (i = 1, 2, \ldots, n), \qquad\qquad \text{(M-29: } x_i^{\text{oh}}\text{)}$$

where s_i^{g} represents the commodity-structure of the government consumption and y_i^{s} stands for inventory accumulation.

The balance of trade (D_{e}) is defined in the multisectoral model by the following equation:

$$\Sigma_i \, (p_i^{\text{wm}} \cdot m_i - p_i^{\text{we}}(z_i) \cdot z_i - p_i^{\text{c}} \cdot y_i^{\text{ct}}/v) = D_{\text{e}} \qquad\qquad \text{foreign trade balance (deficit)} \qquad\qquad \text{(M-30: } D_{\text{e}}\text{)}$$

Note that fixing the gross trade balance (i.e., the trade deficit without the $-p_i^{\text{c}} \cdot y_i^{\text{ct}}/v$ term) turns foreign currency reserves into a resource constraint similar to fixed labour and capital.

The budget balances of the other (domestic) agents are formulated as follows:

$$S^{rw} = v \cdot D_{\text{e}} + tr^{\text{w}} \qquad\qquad \text{current account (foreign net savings)} \qquad\qquad \text{(M-31)}$$

$$\Sigma_i \, p_i^{\text{ohm}} \cdot s_i^{\text{g}} \cdot G + S^{\text{g}} = \Sigma_j \, \{ \tau_j^{\text{w}} \cdot w \cdot d_j^{\text{w}} \cdot l_j + p_j^{\text{a}} \cdot \tau_j^{\text{t}} \} \cdot x_j + \Sigma_i \, \{ \tau_i^{\text{c}} \cdot p_i^{\text{chm}} \cdot x_i^{\text{chm}} + \tau_i^{\text{m}} \cdot v \cdot p_i^{\text{wm}} \cdot m_i -$$
$$- \tau_i^{\text{e}} \cdot v \cdot p_i^{\text{we}}(z_i) \cdot z_i + \Sigma_j \, \tau_{ij}^{\text{fu}} \cdot p_i^{\text{ohm}} \cdot a_{ij} \cdot x_j \} + tr^{\text{g}} \qquad \text{government's budget} \qquad\qquad \text{(M-32: } S^{\text{g}}\text{)}$$

$$p_j^{\text{b}} \cdot I_j + \Sigma_i \, s_{ij}^{\text{s}} \cdot p_i^{\text{ohm}} \cdot y_i^{\text{s}} + S_j^{\text{s}} = q_j \cdot k_j \cdot x_j + tr_j^{\text{s}} \qquad\qquad \text{sectoral budgets} \qquad\qquad \text{(M-33: } S_j^{\text{s}}\text{)}$$

$$\Sigma_i \, p_i^{\text{c}} \cdot y_{ik}^{\text{c}} + S_k^{\text{h}} = \alpha_{kj}^{\text{w}} \Sigma_j \, w \cdot d_j^{\text{w}} \cdot l_j \cdot x_j + tr_k^{\text{h}} \qquad\qquad \text{household groups' budget,} \qquad\qquad \text{(M-34: } S_k^{\text{h}}\text{)}$$

where the α_{kj}^{w} parameters show the individual hoseholds relative share of wage incomes.

The components of the net transfers are defined in the following ways:

$$tr^{\text{w}} = v \cdot T^{\text{w}}, \qquad\qquad \text{(M-35)}$$

where T^{w} is the net transfer income of the foreign sector assumed to be exogenous in foreign currency.

$$tr^{\text{g}} = \Sigma_k \{ (\tau_k^{\pi} + \tau_k^{\text{s}}) \cdot w \cdot \Sigma_j \, \alpha_{kj}^{\text{w}} \cdot d_j^{\text{w}} \cdot l_j + J_k^{\text{d}} \cdot \alpha_{\tau} \} - p_{\text{c}} \cdot (t_{\text{p}} + t_t) + \Sigma_j \, (J_j^{\pi} + p_{\text{c}} \cdot T_j) - v \cdot T^{\text{w}}, \qquad\qquad \text{(M-36)}$$

where

- τ_k^{π} and τ_k^{s} are the personal income tax rates and employees social security contribution rates,

- t_{p} and t_t are the households' cash and in-kind benefits in real terms (all routed through the government),

- T_j is the net other transfer expenditure of the sectors (also expressed in real terms and assumed to go through the government).

The J_j^{π} profit tax by sectors and the J_k^{d} disposable income of the households are defined as follows:

$$J_j^{\pi} = p_j^{\text{b}} \cdot J_j^{\pi 0} \qquad\qquad \text{profit tax by sectors,} \qquad\qquad \text{(M-37)}$$

where $J_j^{\pi 0}$ are the sector-specific level of the real value of the profit taxes.

$$J_k^{d} = (1 - \tau_k^{\pi} - \tau_k^{s}) \cdot w \cdot \Sigma_j \alpha_{kj}^{w} \cdot d_j^{w} \cdot l_j + p_c \cdot \beta_k \cdot t_p \quad \text{disposable income of the households,} \quad \text{(M-38)}$$

where β_k coefficients show the relative shares of the individual strata in total cash-benefits.

Using these auxiliary variables the net transfers of the sectors and households can be formulated as

$$tr_j^{s} = -J_j^{\pi} - p_c \cdot T_j + r_j p_l^{b} \cdot \Sigma_k B_k^{l}, \quad \text{(M-39)}$$

where r_j represents the housing sector (as the dummy for), l denotes its index and B_k^{l} is the households housing investment expenditure (accounted as transfer to the housing sector) in real terms.

$$tr_k^{h} = -(\tau_k^{\pi} + \tau_k^{s}) \cdot w \cdot \Sigma_j \alpha_{kj}^{w} \cdot d_j^{w} \cdot l_j + p_c \cdot \beta_k \cdot t_p + p_c \cdot \varphi_k \cdot t_t - p_l^{b} \cdot B_k^{l} - J_k^{d} \cdot \alpha_\tau, \quad \text{(M-40)}$$

where φ_k coefficients show the relative shares of the individual strata in total in-kind benefits and α_τ is the Johansen-type tax rate (its default value being 0), which adjusts in the Johansen closure as τ did in the one-sector model, and its tax base is the disposable income of the households.

The sum total of net financial savings ($\Sigma_k S_k^{h} + S^{g} + \Sigma_j S_j^{s} + S^{rw}$) has to be zero, which secures that total private, public and foreign savings plus the retained earnings by the sectors will be equal to investments. It can be shown that equation system (M-01)-(M-34) fulfils the requirement of Walras law.

To take into account the different behaviour of different types of households, and the fact that changes in the tax and social security systems have varying effects on them, different household groups are defined in many CGE models. The GGE model, which was used in our simulations, classifies them into three groups too. Equation (M-41) establishes relationship between households' savings (S^{h}) and disposable income by means of a saving rate (δ_k):

$$S_k^{h} = \delta_k \cdot J_k^{d} \cdot (1 - \alpha_\tau) \qquad \text{households' total savings} \qquad \text{(M-41: } C_k^{v})$$

In the multisectoral model the real exchange rate will be defined as

$$v_r = v \cdot \Sigma_i p_i^{we}(z_i) \cdot z_i / \Sigma_i p_i^{h} \cdot z_i \qquad \text{real exchange rate} \qquad \text{(M-42)}$$

Observe that the equation system (M-01)-(M-42) is equivalent to equations (1)-(15) of the one-sector model.

Finally, the price level in running simulations with the CGE model will be fixed by the following equation:

$$p_c = \Sigma_i p_i^{c} \cdot c_i / \Sigma_i p_i^{c0} \cdot c_i \qquad \text{consumers' price index,} \qquad \text{(M-43)}$$

which will be set to unity, in a way similar to $p^{hm} = 1$ in the one-sector model:

$$p_c = 1 \qquad \text{(M-44: } v)$$

Setting the price level in this way means that variable v becomes the consumer purchasing power of the foreign currency.

Similarly, we introduce the average investment price index as

$$p^{b} = (\Sigma_i p_i^{bhm} \cdot \Sigma_j b_{ij} \cdot I_j) / \Sigma_j I_j \qquad \text{price index of the capital goods} \qquad \text{(M-45)}$$

With the model defined by the above equations one can easily and quite closely replicate the simulations done with the one-sector model under the neo-classical, Johansen, Keynesian and neo-Keynesian closure rules. In order to implement the *structural closures*, the model has to be modified. Equation (M-28) is replaced by (M-28'), containing sectoral investment functions and total investment, I is dropped as a variable. New variables, equations and parameters have to be introduced again (α_π, d_j^π variables and the c_j^π parameters) as in the case of the one-sector model or in the stylised CGE-model. The unit gross operating surplus ($q_j \cdot k_j$) has to be accordingly redefined and replaced in equations (M-19) and (M-34) as $p_j^b \cdot r_j^a \cdot k_j + \alpha_\pi \cdot p_j^a \cdot c_j^\pi$, where the first term represents the amortization and the second the profit (net operating surplus).

As in the one-sector model and the stylised CGE-model the relationships between the general level of the net rate of return (π) and the profit mark-up (α_π) are the following:

$$\pi \, d_j^\pi \cdot p_j^b \cdot k_j = \alpha_\pi \cdot p_j^a \cdot c_j^\pi \qquad\qquad \text{sectoral rate of return differences} \qquad \text{(M-46: } d_j^\pi)$$

$$\sum_i d_i^\pi = \sum_i d_i^{\pi 0} \qquad\qquad \text{normalization of the rates of return} \qquad \text{(M-47: } \alpha_\pi)$$

6.4. Closure options in the applied CGE models

The equation system (M-01)-(M-45) is thus the multisectoral counterpart of equations (1)-(15) in the case of the one-sector model. In all multisectoral simulations G will be *exogenous*, whereas all variables associated with the equations (M-01)-(M-45), except for L, D_e and v_r, which are associated to (M-21), (M-30) and (M-42) respectively, will be *endogenous*.(i.e., k_j, l_j, e_j, a_{ij}, p_{ij}^{hmut}, p_j^{en}, x_j, z_j, p_j^h, x_i^h, m_i, x_i^{rhm}, m_i^r, p_i^{rhm}, w_j, q_j, p_j^b, p_j^e, p_j^a, p_i^m, π, y_{ik}^c, p_i^c, c_i, x_i^{ch}, x_i^{bh}, I_j, x_i^{oh}, S^{rw}, S^g, S_j^s, S_k^h, tr^w, tr^g, J_j^π, J_k^d, tr_j^s, tr_k^h, C_k^v, p_c, v and p^b will be endogenous). The remaining degree of freedom is thus 3. In addition to L, D_e and v_r, variables I, w, α_w and α_τ, which were not associated with any equation, make up thus seven macro and auxiliary variables from among which three further endogenous variables can be chosen, in order to make the system well determined in the case of the neo-classical, Johansen, Keynesian and the neo-Keynesian closures.

In the case of the structuralist closures d_j^π is added to the list of the always endogenous variables, and α_π, associated with (M-47), is added to the former list of the potential endogenous variables. Since I is at the same time dropped from the model (or, alternatively, it becomes an always endogenous variable, associated with identity $I = \sum_j I_j$) the number of potential endogenous variables remains seven. The degree of freedom increases by one. Four more endogenous variables have to be chosen out of the seven candidate variables and the remaining three fixed, in order to close the model. So, the closure options are the same as in the case of the one-sector model (see in Table 1), the only differences would be in the notation: τ and c^π is replaced by α_τ and α_π.

7. Replicated simulations with the applied CGE model

We have thus repeated the simulations with a five-sector[11], three-household CGE model, which was calibrated on the bases of the same data set, as the one-sector model. We will refer

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[11] The five aggregate sectors are as follows: raw materials (including the energy sectors), manufacturing industry (without food industry), food and agriculture, material services and non-material services.

to our numerical model as CGE-mini. As demonstrated in the previous section, the structure of the CGE-mini is in some aspects different from the stylized one-sector model presented above.

Because of these and other differences, it was not possible to reproduce exactly the benchmark values of the macroeconomic indicators of the one-sector model either. What impedes further the comparison of the results gained from the one-sector macro model and the five-sector CGE model is that the macro variables are sectoral aggregates, which alone can be a serious distorting factor. Due to these difficulties, the reproduction of the scenarios of the one-sector model with CGE-mini required occasionally in-depth considerations too.

7.1. Main characteristics of the simulation results

The results of the simulations can be seen in Tables 5 and 6. Notice that in the CGE model the export, unlike in the one-sector model, does not contain the turnover of tourist revenues, it is part of private consumption. Therefore, the volumes of Z and consequently X^h and X^{hm} differ from their counterparts in the one-sector model. Note also, that when gross trade deficit is fixed, the trade deficit (D_e) might still vary slightly due to the changing foreign currency value of the fixed consumption of the inbound tourists.

From Table 5 and 6 the reader can follow and analyse the results, most of which can be expected from the applied theoretical model. It suffices to comment only briefly on the observable differences between the results obtained from one- and the five-sector model. In the case of *5% increase in G*, the level of employment and the aggregate volume of production changes into the same direction as in the one-sector model, but more moderately in the five-sector model. The same applies to the aggregate volume of exports and imports, as well as the general wage and profit rates, which change even more moderately.

In the five-sector CGE-model, which links together domestic and export supply by CET transformation functions, exports move practically in proportion with domestic demand. It takes also into account that government consumption consists basically of non-tradable goods. Therefore, its change generates smaller exports and further repercussions.

In the case of *2% increase in import prices*, one can observe more significant differences between the results obtained by the two models. It is understandable, because changes in the import prices affect the input structures and consumption patterns too. The neo-Keynesian closures produce surprisingly more drastic changes than the one-sector model, some variables (imports, consumption, domestic demand, the exchange rate and the government saving) move even in opposite direction than in the one-sector model. The structuralist closures (the first in the case of fixed trade balance, the second in the case of fixed real exchange rate) also provide results qualitatively rather different from those obtained in the one-sector model. Concretely, in the first structuralist closure with fixed trade balance the employment increases by 0.7 per cent in the one-sector model while in the applied CGE-model it decreases by 1 per cent. In the latter, along with the decreasing employment the output and the private savings also turn into decrease, as opposed to the one sector model, where these categories they increased. The decrease of the private saving is partly due to the lower decrease of the household consumption.

Table 5: The effect of 5% increase in the government expenditure (percentage changes, base values in trillion HUF or ratios)

5% increase in G 1-sector macro model		Base values	fixed real exchange rate						fixed trade balance					
			neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.	neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.
L	level of employment[1]	4.04	0	0	2,26	4,94	3,23	4,81	0	0	2,63	4,73	3,31	4,89
X	output	55.12	-0,48	-0,30	0,88	2,12	1,30	2,20	-0,43	-0,27	1,14	2,01	1,35	2,25
X^{h}	output for domestic use	35.69	-0,05	-0,09	1,20	2,29	1,54	2,39	-0,10	-0,11	1,36	2,21	1,56	2,43
Z	export	19.43	-1,27	-0,69	0,30	1,80	0,86	1,86	-1,04	-0,57	0,73	1,63	0,95	1,93
M	import	18.54	-0,79	-0,46	0,42	1,37	0,73	1,49	-0,83	-0,45	0,58	1,29	0,75	1,53
X^{hm}	domestic supply	54.23	-0,30	-0,21	0,93	1,98	1,26	2,08	-0,35	-0,23	1,10	1,90	1,29	2,12
C	private consumption	10.85	-0,06	-2,45	0,05	2,66	1,77	1,76	-0,08	-2,64	0,04	2,55	1,80	1,78
I	investment	4.58	-4,96	0	0	0	-1,63	1,86	-5,94	0	0	0	-1,75	1,89
w	real wage rate[2]	3.459	-0,07	-0,03	-2,11	0,05	0,10	-1,43	-0,12	-0,05	-2,49	0,05	0,10	-1,47
$\pi\,(q)$	rate of return on capital	0.046	0,53	0,41	2,83	-1,01	-0,47	1,28	0,54	0,41	3,22	-0,94	-0,50	1,31
v	nominal exchange rate	1.00	-0,40	-0,10	0,86	0,45	0,07	1,05	-0,13	0,03	1,29	0,37	0,13	1,11
v_{r}	real exchange rate	1.00	0	0	0	0	0	0	0,42	0,16	0,34	-0,08	0,08	0,04
$v \cdot p^{we}$	domestic export price	1.00	-0,09	0,07	0,78	0,01	-0,14	0,59	0,13	0,17	1,10	-0,04	-0,10	0,63
p^{h}	domestic output price	1.00	0,11	0,06	-0,12	-0,14	-0,03	-0,20	0,03	0,03	-0,22	-0,12	-0,05	-0,22
p^{a}	average price of output	1.00	0,04	0,07	0,20	-0,09	-0,07	0,07	0,07	0,08	0,24	-0,09	-0,07	0,08
S^{p}	private saving	7.91	0,49	-0,25	3,39	-0,57	-0,20	1,99	0,56	-0,28	3,92	-0,53	-0,21	2,04
S^{g}	government saving	-1.20	25,34	-0,16	23,69	-5,05	5,23	4,93	25,96	-1,83	23,94	-3,91	4,87	4,73
$v \cdot D_{e}$	foreign saving (in HUF)	-2.13	-2,06	-0,84	-1,29	0,69	-0,33	0,23	-0,07	0,01	0,49	0,18	0,08	0,47
α_{w}	wage/marginal product	1.00	0,00	0,00	0,00	11,85	7,66	7,34	0	0	0	11,33	7,84	7,43
τ	tax rate	0.00	0	7,05	0	0	0	0	0	7,55	0	0	0	0
p^{we}	foreign export price	1.007	0,32	0,17	-0,07	-0,45	-0,21	-0,46	0,26	0,14	-0,18	-0,40	-0,24	-0,48
D_{e}	foreign trade deficit	-2.13	-1,66	-0,74	-2,12	0,23	-0,40	-0,82	0,06	-0,02	-0,78	-0,19	-0,05	-0,64
	domestic savings	6.71	-4,10	-0,26	-0,35	0,26	-1,21	1,45	-4,13	0,01	0,23	0,10	-1,15	1,55
	terms of trade loss/GDP	0	0,26	0,14	-0,06	-0,37	-0,18	-0,38	0,22	0,12	-0,22	-0,35	-0,20	-0,45

[1] million persons   [2] million HUF/year/person

Table 6: The effect of 2% increase in world market import prices (percentage changes, base values in trillion HUF or ratios)

2% increase in p^{wm} 5-sector CGE model		Base values	fixed real exchange rate						fixed trade balance					
			neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.	neo-classical	Johansen	Keynes	neo-Keynes	structu-ralist I.	structu-ralist II.
L	level of employment[1]	4.04	0	0	0,75	3,63	-2,75	0,25	0	0	2,85	6,69	-1,04	3,73
X	output	55.12	-0,39	-0,33	0,07	1,39	-2,02	-0,24	-0,18	0	1,52	3,05	-0,86	1,90
X^h	output for domestic use	34.75	-0,37	-0,38	0,05	1,22	-1,87	-0,19	-0,61	-0,62	0,96	2,44	-1,25	1,38
Z	export	20.37	-0,42	-0,23	0,10	1,72	-2,31	-0,33	0,61	1,15	2,55	4,16	-0,16	2,87
M	import	18.54	-1,31	-1,20	-0,91	0,10	-2,73	-1,21	-1,49	-1,07	0,01	1,25	-2,09	0,26
X^{hm}	domestic supply	53.29	-0,69	-0,66	-0,28	0,83	-2,16	-0,54	-0,92	-0,78	0,62	2,03	-1,54	0,98
C	private consumption	10.85	-0,90	-1,69	-0,87	1,92	-1,40	-1,44	-1,02	-3,73	-0,90	3,52	-0,52	-0,60
I	investment	4.58	-1,63	0	0	0	-6,87	0,05	-6,21	0	0	0	-9,55	1,37
w	real wage rate[2]	3.458	-1,74	-1,73	-2,42	-0,09	0,09	-3,00	-1,99	-1,92	-4,51	-0,19	0,02	-4,80
$\pi(q)$	rate of return on capital	0.047	-1,67	-1,71	-0,92	-5,02	-3,49	-0,05	-1,65	-1,79	1,19	-6,13	-4,09	1,38
v	nominal exchange rate	1.00	0,76	0,86	1,18	0,73	-0,55	1,46	2,04	2,23	3,63	1,94	0,93	4,07
v_r	real exchange rate	1.00	0	0	0	0	0	0	1,93	1,66	1,86	1,11	1,88	1,77
$v \cdot p^{we}$	domestic export price	1.00	0,86	0,92	1,16	0,30	0,03	1,55	1,89	1,94	2,98	0,90	0,97	3,34
p^h	domestic output price	1.00	-0,71	-0,73	-0,79	-0,81	-0,46	-0,82	-1,08	-1,08	-1,35	-1,16	-0,90	-1,43
p^a	average price of output	1.00	-0,16	-0,15	-0,10	-0,41	-0,29	0,01	-0,03	-0,01	0,19	-0,43	-0,23	0,27
S^p	private saving	7.91	-1,89	-2,13	-0,94	-5,17	-4,33	0,01	-1,59	-2,46	2,00	-5,86	-4,51	2,38
S^g	government saving	-1.20	7,37	-1,01	6,85	-23,81	13,32	13,02	10,27	-18,94	8,32	-40,71	5,05	4,97
$v \cdot D_e$	foreign saving (in HUF)	-2.13	-8,52	-8,13	-8,33	-6,23	-9,43	-8,56	0,78	0,88	1,42	0,83	0,36	1,61
α_w	wage/marginal product	1.00	0	0	0	13,03	-1,71	-2,34	0	0	0	21,2	2,5	1,3
α_τ	additional income tax rate	0.00	0	2,49	0	0	0	0	0	8,22	0	0	0	0
p^{we}	foreign export price	1.00	0,11	0,06	-0,02	-0,43	0,59	0,08	-0,15	-0,29	-0,63	-1,02	0,04	-0,70
D_e	foreign trade deficit	-2.13	-9,20	-8,92	-9,40	-6,91	-8,93	-9,87	-1,24	-1,33	-2,13	-1,09	-0,57	-2,36
	domestic savings	6.71	-3,59	-2,34	-2,38	-1,73	-7,58	-2,39	-3,78	0,58	0,83	0,57	-6,27	1,91
	terms of trade loss/GDP	0	-1,48	-1,52	-1,58	-1,92	-1,09	-1,49	-1,79	-1,91	-2,24	-2,50	-1,58	-2,31

[1] million persons   [2] million HUF/year/person

In the second structuralist closure with fixed exchange rate the employment decreases by 0.9 per cent in the one-sector model while in the applied CGE-model it increases by 1/4 per cent. In the latter the rate of return and the investment practically remains at the base level, while in the one-sector model it decreased by 1.2 per cent.

These results show clearly, that in multi-sectoral models the effects are difficult to trace back even in such theoretically transparent model and in such simple simulation scenarios. One has to look at the details and bear in mind all those equations in which the changing parameters have significant direct or indirect role.

7.2. The effect of differentiating the changes in parameters across sectors

It is also important to note, that one could differentiate the expected changes in parameters across sectors in a multi-sectoral model, which would produce even more different results for otherwise similar scenarios than the one-sector model. For example, if one assumed instead of the general 2% increase in the import prices that only the price of the raw materials increased by 8.8 %, which would generate the same 2% increase in the aggregate import price index, then the structural effects would be more pronounced. For example, in the neo-Keynesian closure, in the case of fixed trade balance, the sectoral imports would change the way as shown in Table 7.

Table 7. Change in import demand by sector, %

Scenario \ Sector code	Row materials	Manufac-turing	Food and agriculture	Material services	Non-mat. services	Total
Overall 2% price increase	-3.14	3.25	1.62	0.32	-1.01	1.25
8.8 % in raw material prices	-5.68	2.91	2.56	1.33	0.17	0.66
Difference	-2.54	-0.34	0.94	1.01	1.18	-0.59

One can see that concentrating the assumed import price changes to one sector resulted in only half as large increase in total import, for which only two sectors were responsible (understandably the row materials and less intuitively the manufacturing products).

We should warn the reader that our necessarily limited number of simulations served only demonstrative purposes. Only one exogenous variable was assumed to change in each of them. In other words, we did not attempt to formulate changes consistent (both theoretically and empirically) in all important exogenous categories as it should be done in a more realistic scenario package. Our aim was only to replicate the simulations done with the one-sector model.

8. Concluding remarks

In this paper the problem of macro closure was revisited, a problem that arises in one-period general equilibrium models if they are based strictly on neo-classical assumptions. Following and extending the earlier literature on the subject, alternative "closure rules" were discussed, and these can be used to allow for autonomous investment decisions and other more realistic adjustment mechanisms in such models.

In a one-sector model, it was first demonstrated how the effects of exogenous shocks depend on the adjustment mechanisms one assumes to be operating. Running similar simulations with a complex CGE model, based on a 2010 Hungarian database compiled by us, we demonstrated the robustness of the model framework, i.e., that applying the same closure rule, one will get roughly the same results for the macroeconomic aggregates from the one-sector and multi-sector CGE model. The results gained from assuming different closures will, however, yield characteristically different results.

We have illustrated the limitations of the simple macroeconomic model, and its inability to project the changes in key macro variables well enough. A CGE model provides not only more realistic predictions, but also clearer explanations of the mechanisms that lie behind the projected changes. In particular, we pointed out that a multi-sectoral model may use more elaborate technology, income distribution and demand functions, and take into account different exogenous changes affecting various sectors. All these features warrant different and more trustable results from a multisectoral than an aggregate macroeconomic model.

The lack of sufficiently reliable economic theories prevents model builders from formulating enough consistency criteria (equations) to match all the potential variables of the model. This is why some have to be set exogenously. The emerging dilemma, concerning which variables should be exogenous and/or endogenous, can be perceived as the generalization of the classical closure problem. The given policy problem being analysed not only limits the choice of macro closure, but also suggests what particular closure rules could or should be chosen. When doubts remain, the modeller should test the model by choosing alternative closure options, on the basis of which she/he can formulate alternative scenarios, in which the exogenous variables are better harmonized.

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