

A Price-Setting Game with a Nonatomic Fringe

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Abstract: This paper extends the Bertrand-Edgeworth price-setting game with finitely many firms to a game with infinitely many firms. Taking a market with one significant firm and a nonatomic fringe, we present a micro-foundation of dominant-firm price leadership.

Keywords: Bertrand-Edgeworth; Dominant firm; Price leadership

JEL classification: D43; L13

1 Introduction

In the following we shall consider a homogenous good market with one significant firm and a nonatomic fringe containing many infinitesimal firms. Our model may be regarded as an extension of the Bertrand-Edgeworth game in which there are finitely many firms to a game with infinitely many firms.

Mixed measure theoretic models have been considered, for instance, by Gabszewicz and Mertens (1971), Shitovitz (1973), and Okuno, Postlewaite, and Roberts (1980) in a general equilibrium framework. Sadanand and Sadanand (1996) used in their analysis a partial equilibrium model containing a dominant firm and a nonatomic competitive fringe in order to investigate the timing of quantity-setting oligopoly games. Our model may be considered as the price-setting counterpart of their model.

In our analysis we will assume that the large firm is the exogenously specified first mover while the small firms follow simultaneously. We will show that our model gives a game theoretic foundation of dominant-firm price leadership. In the dominant-firm price leadership model introduced by Forchheimer (see Scherer and Ross, 1990) there is one large firm and many small firms. Furthermore, the large firm is able to set the price on the market and the

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firms in the competitive fringe act as price takers. Therefore, the large producer sets a price by maximizing profit subject to its residual demand curve. The large firm's residual demand curve can be obtained as the horizontal difference of the demand curve and the aggregate supply curve of the competitive fringe. The problem is that this model is not based on the firms' individual profit maximizing behavior since it does not explain the large firm's price-setting behavior nor why small firms act as price takers. A game theoretic foundation of price leadership on a duopolistic market was given by Deneckere and Kovenock (1992). They extended the capacity constrained Bertrand-Edgeworth duopoly game to a two-stage game with the timing of price decision in stage one. While they assumed constant average costs up to some capacity levels, in our analysis we shall assume strictly convex cost functions.

2 The model

We denote the set of producers by Ω . Let us denote by $\omega_d \in \Omega$ the dominant firm and by $\Omega_c := \Omega \setminus \{\omega_d\}$ the competitive fringe. There is a σ -algebra \mathcal{A} given above the set of producers such that $\{\omega_d\} \in \mathcal{A}$. We suppose that there is a finite measure μ given above the set of producers such that ω_d is its only μ -atom with $\mu(\{\omega_d\}) = 1$ and the restriction of μ to Ω_c is nonatomic.

Let us denote by $P := [0, \bar{p}]$ the set of possible prices. We denote by $s(p, \omega)$ the supply of producer $\omega \in \Omega$ at price level p .

Assumption 2.1. We assume that s is a bounded and \mathcal{A} measurable function for all $p \in P$. Furthermore, we assume that s is differentiable for all ω with respect to p , that s is integrable for all $p \in P$ with respect to μ , and that $s(0, \omega) = 0$, $\frac{\partial s(p, \omega)}{\partial p} > 0$ for all ω .

The supply function $s(\cdot, \omega)$ of firm $\omega \in \Omega$ can be obtained from its cost function $c(\cdot, \omega)$. Suppose that $c(0, \omega) = 0$ for all $\omega \in \Omega$. Then in the opposite direction we can reconstruct the cost functions from the supply functions since $s(\cdot, \omega)$ is invertible by Assumption 2.1. Hence, we have $\frac{\partial c}{\partial q}(q, \omega) = p$ if and only if $s(p, \omega) = q$.

The supply of producers $A \in \mathcal{A}$ is given at price level p by $S(p, A) := \int_A s(p, \omega) d\mu(\omega)$. Assumption 2.1 assures that function $S(p, A)$ is differentiable with respect to p and that $\frac{\partial S}{\partial p}(p, A) = \int_A \frac{\partial s}{\partial p}(p, \omega) d\mu(\omega)$. Hence, it follows that $\frac{\partial S}{\partial p}(p, A) > 0$ for any set A with positive measure, which means that $S(p, A)$ is strictly increasing in p .

We will model dominant-firm price leadership by a price-setting game. The price actions of the producers' are given by a measurable function \mathbf{p} :

$\Omega \rightarrow P$ that we will call from now on a *price profile*. Let us denote by \mathcal{P} the set of price profiles.

Lemma 2.2. *For any price profile \mathbf{p} function $f(\omega) := s(\mathbf{p}(\omega), \omega)$ is measurable and integrable with respect to μ .*

Proof. We pick an arbitrary set $A \in \mathcal{A}$ of producers and an arbitrary price profile $\mathbf{p} \in \mathcal{P}$. We will construct an increasing sequence $(f_n)_{n \in \mathbb{N}}$ of type $\Omega \rightarrow P$ measurable functions that converges pointwise to f . Let $p_i^n = i\bar{p}/2^n$, where $i = 0, 1, \dots, 2^n$; let $A_i^n = \{\omega \in A \mid p_{i-1}^n \leq \mathbf{p}(\omega) < p_i^n\}$, where $i = 1, \dots, 2^n - 1$; and let $A_i^n = \{\omega \in A \mid p_{i-1}^n \leq \mathbf{p}(\omega) \leq p_i^n\}$, where $i = 2^n$. Define the functions $f_n = \sum_{i=1}^{2^n} \mathbf{1}_{A_i^n} s(p_{i-1}^n, \cdot)$ and $g_n = \sum_{i=1}^{2^n} \mathbf{1}_{A_i^n} s(p_i^n, \cdot)$, where $\mathbf{1}_A$ denotes the characteristic function of set A . Obviously, $f_n \leq f \leq g_n$, $\int_A f_n(\omega) d\mu(\omega) = \sum_{i=1}^{2^n} S(p_{i-1}^n, A_i^n)$, and $\int_A g_n(\omega) d\mu(\omega) = \sum_{i=1}^{2^n} S(p_i^n, A_i^n)$ for any $n \in \mathbb{N}$.

For any $\epsilon > 0$ there exists a $\delta > 0$ such that

$$|s(\mathbf{p}(\omega), \omega) - s(p', \omega)| < \epsilon, \text{ if } |\mathbf{p}(\omega) - p'| < \delta,$$

because $s(\cdot, \omega)$ is continuous for any $\omega \in \Omega$. We can choose a sufficiently large number $n \in \mathbb{N}$ so that $|\mathbf{p}(\omega) - p_i^n| < \delta$ for some $i = 0, 1, \dots, 2^n - 1$. Hence,

$$f(\omega) - f_n(\omega) = s(\mathbf{p}(\omega), \omega) - \sum_{i=1}^{2^n} \mathbf{1}_{A_i^n}(\omega) s(p_{i-1}^n, \omega) < \epsilon.$$

Therefore, it follows that the sequence $(f_n)_{n \in \mathbb{N}}$ converges increasingly pointwise to function f and thus f is measurable. Now, we can conclude by Lebesgue's monotone convergence theorem that $\int_A f(\omega) d\mu(\omega)$ exists and that

$$\int_A f(\omega) d\mu(\omega) = \lim_{n \rightarrow \infty} \int_A f_n(\omega) d\mu(\omega) = \lim_{n \rightarrow \infty} \sum_{i=1}^{2^n} S(p_{i-1}^n, A_i^n).$$

Furthermore, $\int_A f(\omega) d\mu(\omega)$ has to be finite since $f \leq g_n$ and $\int_A g_n(\omega) d\mu(\omega)$ is finite for any $n \in \mathbb{N}$. \square

The supply of producers $A \in \mathcal{A}$ at price profile $\mathbf{p} \in \mathcal{P}$ can be defined by $\widehat{S}(\mathbf{p}, A) := \int_A s(\mathbf{p}(\omega), \omega) d\mu(\omega)$ because of Lemma 2.2. The consumers side is given by the demand function D .

Assumption 2.3. We assume that the demand curve is a continuously differentiable, decreasing function and intersects both axis. Formally $D \in \mathcal{C}^1$, $D' < 0$, $D(\bar{p}) = 0$ and $D(p) > 0$ for all $p \in [0, \bar{p})$.

We denote by $B(\mathbf{p}, \omega)$ the set of those producers setting prices below producer $\omega \in \Omega$, and by $C(\mathbf{p}, \omega)$ the set of those producers in the fringe setting the same price as producer $\omega \in \Omega$. Formally, $B(\mathbf{p}, \omega) := \{\omega' \in \Omega \mid \mathbf{p}(\omega') < \mathbf{p}(\omega)\}$ and $C(\mathbf{p}, \omega) := \{\omega' \in \Omega_c \mid \mathbf{p}(\omega') = \mathbf{p}(\omega)\}$. Assuming efficient rationing of consumers, we define the demand served by the firms in the following manner:

$$\Delta(\mathbf{p}, \omega) := \begin{cases} s(\mathbf{p}(\omega), \omega) \min \left\{ 1, \frac{D(\mathbf{p}(\omega)) - \widehat{S}(\mathbf{p}, B(\mathbf{p}, \omega))}{\widehat{S}(\mathbf{p}, C(\mathbf{p}, \omega))} \right\}, & \text{if } \widehat{S}(\mathbf{p}, C(\mathbf{p}, \omega)) > 0 \text{ and} \\ & D(\mathbf{p}(\omega)) \geq \widehat{S}(\mathbf{p}, B(\mathbf{p}, \omega)); \\ s(\mathbf{p}(\omega), \omega), & \text{if } \widehat{S}(\mathbf{p}, C(\mathbf{p}, \omega)) = 0 \text{ and} \\ 0, & \text{if } D(\mathbf{p}(\omega)) < \widehat{S}(\mathbf{p}, B(\mathbf{p}, \omega)); \end{cases} \quad (1)$$

for any firm $\omega \in \Omega_c$ and

$$\Delta(\mathbf{p}, \omega_d) := \min \left\{ s(\mathbf{p}(\omega_d), \omega_d), \left(D(\mathbf{p}(\omega_d)) - \widehat{S}(\mathbf{p}, B(\mathbf{p}, \omega_d) \cup C(\mathbf{p}, \omega_d)) \right)^+ \right\} \quad (2)$$

for the dominant firm ω_d . The definitions (1) and (2) assume that the dominant firm serves the consumers at a given price level after the competitive fringe has already sold its supply. However, this assumption is not necessary, but we impose it only for the purely technical reason of avoiding the need to have a competitive fringe setting their prices arbitrarily close to, but below, the dominant firm's price. This simplification has already been applied by Deneckere and Kovenock (1992) in their analysis. Now, we are ready to define the profit function of firm $\omega \in \Omega$ as $\pi(\mathbf{p}, \omega) := \mathbf{p}(\omega) \Delta(\mathbf{p}, \omega) - c(\Delta(\mathbf{p}, \omega), \omega)$.

Our next assumption concerns the timing of the game in that we suppose that firm ω_d is the exogenously given first mover.

Assumption 2.4. We suppose that firm ω_d set its price first and that the firms in the fringe set their prices simultaneously already in the knowledge of the price set by the dominant firm.

For any price profile \mathbf{p} and any firm $\omega \in \Omega$ we denote by $\mathbf{p}_{-\omega}$ the restriction of \mathbf{p} to the set $\Omega \setminus \{\omega\}$, that is $\mathbf{p}_{-\omega}$ contains the price decisions of firm ω 's rivals. We will also write $(\mathbf{p}(\omega), \mathbf{p}_{-\omega})$ for \mathbf{p} .

Definition 2.5. Let the dominant firms's action be $p_d \in P$. We call the price profile \mathbf{p}^* with $\mathbf{p}^*(\omega_d) = p_d$ a Nash equilibrium of stage two, if

$$\pi((\mathbf{p}^*(\omega), \mathbf{p}_{-\omega}^*), \omega) \geq \pi((p, \mathbf{p}_{-\omega}^*), \omega)$$

for any firm $\omega \in \Omega_c$ and for any price $p \in P$.

The following behavior is called the dominant-firm price-setting behavior: firm ω_d is maximizing profit subject to its residual demand function $D_d^r(p) :=$

$(D(p) - S(p, \Omega_c))^+$. The existence of a price $p_d^* \in P$ maximizing the residual profit function $\pi_d^r(p) := pD_d^r(p) - c(D_d^r(p), \omega_d)$ on set P is guaranteed by the continuity of the residual profit function. We want to show that in a subgame perfect Nash equilibrium of the price-setting game the dominant firm and all other firms set price p_d^* .

Proposition 2.6. *The extensive game $\langle \Omega, \mathcal{P}, \pi \rangle$ has a unique subgame perfect Nash equilibrium under Assumptions 2.1, 2.3, and 2.4. The equilibrium price profile is given by $\mathbf{p}^*(\omega) = p_d^*$ for all ω in Ω .*

Proof. Let the dominant-firm's action be any $p_d \in P$. We have to distinguish between three cases: (i) $S(p_d, \Omega_c) < D(p_d) - s(p_d, \omega_d)$, (ii) $D(p_d) - s(p_d, \omega_d) \leq S(p_d, \Omega_c) \leq D(p_d)$, and (iii) $D(p_d) < S(p_d, \Omega_c)$. Let p_l be the price level for which $S(p, \Omega_c) = D(p) - s(p, \omega_d)$ holds and let p_u be the price level for which $D(p) = S(p, \Omega_c)$. Such prices p_l and p_u exist uniquely because of Assumptions 2.1 and 2.3.

In case of (i) we have $p_l > p_d$ since $S(\cdot, \Omega_c)$ is a strictly increasing continuous function, and D is a decreasing continuous function. Firm $\omega \in \Omega_c$ will not set its price below p_l , because at price p_l it can sell its entire supply independently from its rivals' actions and its profit function by Assumption 2.1 increases on the interval $[0, p_l]$. Therefore, it follows that at price p_d the dominant firm will sell $s(p_d, \omega_d)$ amount of product.

In case of (ii) any firm $\omega \in \Omega_c$ will not set its price below p_d because at price p_d a firm in the fringe can sell its entire supply. Let us denote by \mathbf{p} the price profile for that $\mathbf{p}(\omega) = p_d$ for all $\omega \in \Omega$. Furthermore, suppose that there is a Nash equilibrium price profile \mathbf{p}' of the subgame so that $\mathbf{p}'(\omega_d) = p_d$, $\mathbf{p}'(\omega) \geq p_d$ for all $\omega \in \Omega_c$ and $\mathbf{p}' > \mathbf{p}$ above a set with positive measure. Let $A \in \Omega_c \cap \mathcal{A}$ be the set of those producers in the fringe for which $\mathbf{p}'(\omega) > p_d$. Denote by U the set of those producers in A that cannot sell their entire supply, i.e. $U := \{\omega \in A \mid \Delta(\mathbf{p}', \omega) < s(\mathbf{p}'(\omega), \omega)\}$. Let $A_p := \{\omega \in \Omega \mid \mathbf{p}'(\omega) \leq p\}$, $F := \left\{ p \in P \mid \widehat{S}(\mathbf{p}', A_p) \leq D(p) \right\}$, and $p_s := \sup F$. Of course, $p_s \geq p_d$ because $\widehat{S}(\mathbf{p}', A_p) = 0$ for any $p < p_d$. It can be verified that if $p_s = p_d$, then any firm $\omega \in A$ will not sell anything at all. Thus, \mathbf{p}' cannot be a Nash equilibrium of the subgame. Therefore, in what follows we suppose that $p_s > p_d$. Since A_p increases as p increases it follows that either $F = [0, p_s]$ or $F = [0, p_s)$.

First, suppose that $F = [0, p_s]$. Then $p_s < \bar{p}$ because otherwise $\widehat{S}(\mathbf{p}', A_{\bar{p}}) = \widehat{S}(\mathbf{p}', \Omega) \leq D(\bar{p})$ would follow, which is in contradiction to $\widehat{S}(\mathbf{p}', \Omega) > S(p_d, \Omega) \geq D(p_d) \geq D(\bar{p})$ because of the properties of \widehat{S} and D . $\Omega \setminus U = \{\omega_d\} \cup \{\omega \in \Omega_c \mid \Delta(\mathbf{p}, \omega) = s(\mathbf{p}(\omega), \omega)\}$ by the definition of set U . We claim that $\Omega \setminus U = A_{p_s}$. Clearly, ω_d is contained in both sets. If

$\omega \in A_{p_s} \setminus \{\omega_d\}$, then $\widehat{S}(\mathbf{p}', A_{\mathbf{p}'(\omega)}) \leq D(\mathbf{p}'(\omega))$ because of $\mathbf{p}'(\omega) \in F$. Thus $\omega \in \Omega \setminus U$ since $B(\mathbf{p}', \omega) \cup C(\mathbf{p}', \omega) = A_{\mathbf{p}'(\omega)}$. Hence, $\Omega \setminus U \supset A_{p_s}$. If $\omega \in \Omega_c \setminus U$, then $D(\mathbf{p}'(\omega)) \geq \widehat{S}(\mathbf{p}', B(\mathbf{p}', \omega)) + \widehat{S}(\mathbf{p}', C(\mathbf{p}', \omega)) = \widehat{S}(\mathbf{p}', A_{\mathbf{p}'(\omega)})$ regarding the definition of Δ (equation (1)). This means that $\mathbf{p}'(\omega) \in F$, which implies that $\mathbf{p}'(\omega) \leq p_s$. Thus, $\omega \in A_{p_s}$ and therefore $\Omega \setminus U \subset A_{p_s}$. It follows that $U = \{\omega \in \Omega \mid \mathbf{p}'(\omega) > p_s\} \in \mathcal{A}$. For any price $p \in (p_s, \bar{p}]$ we have $\widehat{S}(\mathbf{p}', A_p) > D(p)$; thus, we obtain $\widehat{S}(\mathbf{p}', A_p \setminus A_{p_s}) > 0$. But this implies that $\widehat{S}(\mathbf{p}', U) > 0$, which in turn implies $\mu(U) > 0$. Any producer $\omega \in U$ cannot sell anything at all because $\widehat{S}(\mathbf{p}', B(\mathbf{p}', \omega)) \geq \widehat{S}(\mathbf{p}', A_{\bar{p}}) > D(\bar{p}) > D(\mathbf{p}'(\omega))$ for any $\bar{p} \in (p_s, \mathbf{p}'(\omega))$. Hence, any producer in set U will prefer price p_d to $\mathbf{p}'(\omega)$, and therefore \mathbf{p}' cannot be a Nash equilibrium profile.

Second, suppose that $F = [0, p_s)$. We claim that

$$E := \{\omega \in \Omega \mid \mathbf{p}'(\omega) < p_s\} = \Omega \setminus U.$$

Clearly, ω_d is contained in both sets. If $\omega \notin E$, then $\mathbf{p}'(\omega) \notin F$ and therefore $\widehat{S}(\mathbf{p}', A_{\mathbf{p}'(\omega)}) = \widehat{S}(\mathbf{p}', B(\mathbf{p}', \omega)) + \widehat{S}(\mathbf{p}', C(\mathbf{p}', \omega)) > D(\mathbf{p}'(\omega))$. Thus, $\omega \in U$ and $E \supset \Omega \setminus U$. If $\omega \in U$, then regarding the definition of Δ (equation (1)) we obtain $\widehat{S}(\mathbf{p}', A_{\mathbf{p}'(\omega)}) > D(\mathbf{p}'(\omega))$, which implies that $\mathbf{p}'(\omega) \notin F$. Therefore, $\omega \notin E$ and $\Omega \setminus U = E \in \mathcal{A}$. Let $V := \{\omega \in \Omega \mid \mathbf{p}'(\omega) = p_s\} \in \mathcal{A}$. Set V has positive measure since $\widehat{S}(\mathbf{p}', \Omega \setminus U) \leq D(p_s) < \widehat{S}(\mathbf{p}', A_{p_s}) = \widehat{S}(\mathbf{p}', V \cup (\Omega \setminus U))$. Hence, producers setting price p_s can sell their supply only partly and producers setting prices above p_s cannot sell anything at all. Therefore, any producers from set V can sell their whole supply by setting a price slightly below p_s and therefore \mathbf{p}' cannot be a Nash equilibrium profile.

Furthermore, if $\mathbf{p}' = \mathbf{p}$ almost everywhere, then those producers that are setting a price above p_d will not sell anything since the restriction of μ to Ω_c is nonatomic. Hence, the only Nash equilibrium candidate is profile \mathbf{p} . Profile \mathbf{p} is a Nash equilibrium because if any producer raises its price unilaterally above p_d , then the demand it faces will be zero because the restriction of μ to Ω_c is nonatomic. We can conclude that at price p_d the dominant firm will sell $D(p_d) - S(p_d, \Omega_c)$ amount of product.

In case of (iii) any firm $\omega \in \Omega_c$ will not set its price below p_u ($p_u < p_d$) because setting prices below p_u yields less profits, since at price p_u a firm in the fringe can sell its entire supply. Let us denote by \mathbf{p}'' the price profile for that $\mathbf{p}''(\omega_d) = p_d$ and $\mathbf{p}''(\omega) = p_u$ for all $\omega \in \Omega_c$. We can proceed in an analogous way to case (ii) in order to establish that \mathbf{p}'' is a unique Nash equilibrium of the subgame. Therefore, it follows that at price p_d the dominant firm faces no demand at all. This implies that the dominant firm will not set a price above p_u .

We already know that $p_d \leq p_u$. If the dominant firm set its price p below p_l then $\pi_d^r(p) = ps(p, \omega_d) - c(s(p, \omega_d), \omega_d)$. Since

$$\frac{d\pi_d^r}{dp}(p) = p \frac{\partial s}{\partial p}(p, \omega_d) + s(p, \omega_d) - \frac{\partial c}{\partial q}(s(p, \omega_d), \omega_d) \frac{\partial s}{\partial p}(p, \omega_d) = s(p, \omega_d) > 0$$

the residual profit function $\pi_d^r(p)$ is strictly increasing on the interval $(0, p_l)$. Therefore, $p_d \geq p_l$, which means that the dominant firm set its price in the region corresponding to case (ii). Thus, the dominant firm will set its price to p_d^* in order to maximize its profits and the firms in the fringe will also set their price to p_d^* . This completes the proof of the proposition. \square

In order to present a complete model of dominant-firm price leadership we should resolve Assumption 2.4. Following Deneckere and Kovenock (1992) we shall investigate the outcome of the simultaneous-move price-setting game. Thereafter we have to compare the firm's profits in the simultaneous-move game to the profits in the extensive game. In case of the simultaneous-move game there is a lack of equilibrium in pure strategies because the large firm will have an incentive to slightly undercut price p_d^* and therefore it will trigger a price war. The competitive price will not be an equilibrium either since the large firm will prefer price p_d^* to it. Hence, one has to consider equilibrium in mixed strategies. The crucial point is that we cannot even guarantee the existence of a mixed strategy equilibrium. The main existence theorems for games with discontinuous payoffs given by Dasgupta and Maskin (1986), Simon (1987) and the recent one by Reny (1999) can be applied only in the case of finitely many firms.

3 Concluding remarks

The price-setting game with a continuum of firms, as presented in Section 2, may also be formulated for the case of more than one large firm, but the equilibrium behavior of the model would be quite different.

The random rationing rule for the Bertrand-Edgeworth game with a continuum of firms may be specified by the following residual demand function:

$$D^r(p) := D(p) \left(1 - \int_{\{\omega' \in \Omega \mid \mathbf{p}(\omega') < p\}} \frac{s(\mathbf{p}(\omega), \omega)}{D(\mathbf{p}(\omega))} d\mu(\omega) \right)^+.$$

Our proposition remains valid in the case of random rationing because it can be shown that in the relevant price region of the large firm the small firms will follow the price set by the large firm. Therefore, the large firm has to maximize the following residual demand function:

$$D_d^r(p) = D(p) (1 - S_c(p) / D(p)) = D(p) - S_c(p).$$

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