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FOREIGN TRADE IN MACROECONOMIC MODELS: PROGRAMMING VERSUS GENERAL EQUILIBRIUM

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Content

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1. Introduction

Multisectoral macroeconomic models fall roughly into three main classes: input-output (I-O) models, linear programming (LP) models, and general equilibrium (GE) models. In this paper we consider only models typical of the second and third classes, paying particular attention to the treatment of foreign trade, in these models.

The most important differences between the two modelling approaches examined here may be summarized as follows. The linear programming models contain mainly real (physical) variables; most of their relations take the form of inequalities (balances and special restrictions) and contain as a rule quite a few individual bounds on certain variables. Computable general equilibrium models, on the other hand, are specified in terms of both real and value (price and financial) variables, take the form of an equation system and include many nonlinear terms. The linear programming models optimize an overall objective (welfare) function, whereas in general equilibrium models distinguish various agents each optimization assume.

Despite these differences, computable general equilibrium models have many similarities to programming models. However, differences in the terminology used, conceptual and other difficulties have led to the impression that these two schools of macroeconomic modelling diverge rather than converge. LP models are tools designed for planning, whereas GE models for simulating the working of market economies. One of the authors (Zalai 1980, 1981) has demonstrated that – dispelling the neoclassical myth surrounding equilibrium models – computable general equilibrium models can be discussed in purely pragmatic terms, and they can be regarded as natural extensions of the programming models designed for planning.

This paper is concerned with the concepts of "equilibrium" and "optimum" in relation to export-import specification in macroeconomic models. In sections 2 and 3 we start by discussing the problem of overspecialization and possible methods of dealing with it in (linear) programming models as compared with computable general equilibrium models. The root of the problem is that most macroeconomic models adopt the common definition of small open economy, which implies that its terms of trade are dictated (fixed) by the world market. It can be easily shown (see, for example, Taylor 1975) that exogenously fixed terms of trade tend to produce overspecialized solutions in linear macroeconomic models, basically due to the constant ratios of substitution implied by the linearity of the model. Overspecialization manifests itself in the existence of only a small number of producing and exporting sectors and allow for little or no *intra*sectoral trade. Such overspecialized solutions cannot be defended on practical grounds. Thus, model builders must find ways to avoid such unrealistic solutions.

One can basically choose between two "pure" methods to prevent overspecialized solutions. One, used in linear programming models, is to introduce special (upper and/or lower) bounds on some important variables (e.g., sectoral output, export, import). The main criticism against this approach is that such bounds are rather arbitrarily chosen and influencing the solution. The other method, offered by computable general equilibrium models, is to use nonlinear export-import relationships, which imply diminishing returns. The main aim of this paper is to show that the difference between these two approaches can be viewed as the choice using rigid (fixed) or flexible (variable) bounds on certain variables. Ginsburgh and Waelbroeck (1981), for example, argued on that ground that it would be natural and useful to include flexible bounds, by using piece-wise linear relations in linear programming models, instead of using fixed bounds.

The paper provides also a basis for discussing a number of other points. For example, to argue that it is necessary to make clear distinction between export restrictions caused by supply and demand limitations in computable general equilibrium models, which is not always the case. A related issue is that export volume response to changes in relative prices is generally modelled by rather small export demand elasticities, which bring along unjustifiably large terms of trade effects. These problems call for a revision of common modelling practice in this field.

A related issue concerns the theoretical definition of small economies, which is incompatible with the assumption of less than perfectly elastic export demand. It is clearly inadequate to use this definition in applied models, since, due to market and product differentiation, even small countries face, as a rule, changing terms of trade as they change the volume of their exports. This has been realized by model builders and the use of less than perfectly elastic export demand as well as import demand functions is quite common. The theoretical justification is usually given as Armington's (1969) assumption of regional product differentiation.

Throughout of our discussion we will compare two modelling approaches used both in theoretical or applied macroeconomic policy analysis, the more traditional linear programming and the general equilibrium models. This gives rise to the issue of optimum tariffs. From the theoretical literature on international trade it is known that the pure competitive (laissez-faire) equilibrium is not (Pareto) optimal for an economy which faces less than perfectly elastic export demand[1] and optimum tariffs could be employed to produce optimal trade pattern in an otherwise competitive setting. This theoretical possibility is rightly neglected in the literature of computable (applied) general equilibrium models. The optimum tariffs, however, create a significant difference between the necessary conditions and the policy implications of the Pareto optimal and the laissez-faire equilibrium solution of the same resource allocation problem. The optimal solutions suggest rather severe import-export restrictions, whereas the laissez-faire solutions suggest a more open foreign trade policy. This problem will be briefly discussed in this paper too.

[1] See, for example, Dixit and Norman (1980). See also Srinivasan (1982) for a theoretical discussion of this separation in a different context.

2. PROGRAMMING MODELS WITH RIGID INDIVIDUAL BOUNDS

2.1. The Issue of Overspecialization

It is well known that development planning models based on linear programming tend to suggest overspecialization, simply because the linear nature of the model implies either perfect substitutability or perfect complementarity between commodities or factors of production. The most common means to prevent the model to extreme behaviour is to impose upper and/or lower bounds on different variables, particularly on production, export and import variables.

The use of individual bounds in linear programming planning models was not universally approved. One of the main criticisms is that they are ad hoc arbitrary restrictions, which can also distort the shadow-prices (see, for example, Taylor 1975, or Ginsburgh and Waelbroeck 1981). An alternative approach favoured by some model builders involves the introduction of more complicated nonlinear relationships into the model, perhaps in a piecewise linear fashion. We will come back to this possibility later.

The above criticism is, however, only partially justified. On the one hand, it is undoubtedly true that the individual constrains account for the inadequacy of the chosen model, reflecting our lack of knowledge and modelling ability. On the other hand, however, this problem, i.e., the arbitrariness of certain elements, is common to all economic models. In some models this is quite apparent, while in others it is partially hidden behind an elegant mathematical facade. Thus, for example, the use of nonlinear relationships (rather than individual bounds) to limit overspecialization can just be seen as introducing another type of arbitrariness into the model. Moreover, most of the individual bounds can be based on careful analysis of the underlying phenomena by experts; it is doubtful that this expertise could be replaced by some simple modelling device.

To avoid this argument becoming one-sided, we must make a brief mention of some points which will be discussed in more detail in later sections. It could be argued that the real choice is not between expert judgement and individual bounds, on the one hand, and nonlinear, econometrically estimated relationships, on the other. The parameters of the nonlinear forms in question could just as well be based on expert judgement as are the individual bounds in the other solution. Both solutions can provide equally realistic descriptions of the resource allocation problems analysed by the model.

In what follows it is argued that these nonlinear functions can be viewed as flexible bounds on certain variables. The main purpose of this and the next section is to demonstrate that most multisectoral computable general equilibrium models can be seen as programming models using flexible bounds. At the same time, through an illustrative example, some of the deficiencies of shadow-prices and post-optimization analysis in the case of linear models are also pointed to.

2.2. Rigid Bounds on Export in a Simple Linear Programming Macroeconomic Model

For the sake of simplicity an extremely stylized, textbook type of model will be used to open our discussion on the problem of overspecialization in linear models. Our attention is focused on the treatment of foreign trade. We assume that there is only one sector, whose net output

($\bar{Y}$) is given (determined by available resources). Intermediate use will be neglected. The emerging allocation problem is how to divide $\bar{Y}$ between domestic use (C_d) and export (Z), and how much to import, for the exported goods can be exchanged on the world market for import at given prices ($\hat{p}^{we}$, $\hat{p}^{wm}$). The imported commodity is assumed to be perfect substitute for the home commodity. The goal is to maximize the total amount ($C_d + C_m$) by means of foreign exchange.

Following the traditional linear programming approach, export (p^{we}) and import (p^{wm}) prices will be treated as (exogenously given) parameters in the model. Introducing M for the amount of imports purchased and C_m for the amount of imports used, our optimal resource allocation problem can be formulated in the following simple way:

LP I-II	Primal problem	Dual problem	
	$C_d, C_m, Z, M \geqq 0$	$p_d, p_m, v, \boxed{\tau_l,\ \tau_u} \geqq 0$	
(p_d)	$C_d + Z \leqq \bar{Y}$	$p_d \geqq 1$	(C_d)
(p_m)	$C_m - M \leqq 0$	$p_m \geqq 1$	(C_m)
(v)	$\hat{p}^{wm}\cdot M - \hat{p}^{we}\cdot Z \leqq 0$	$p_d \geqq v\cdot\hat{p}^{we} \boxed{+\ \tau_l - \tau_u}$	(Z)
$\boxed{(\tau_l,\ \tau_u)}$	$\boxed{\check{Z}_l \leqq Z \leqq \check{Z}_u}$	$p_m \leqq v\cdot\hat{p}^{wm}$	(M)
	$C_d + C_m \rightarrow \max!$	$p_d\cdot\bar{Y} \boxed{+\ \tau_u\cdot\check{Z}_u - \tau_l\cdot\check{Z}_l} \rightarrow \min!$	

where p_d, p_m, v and τ_l, τ_u are the dual variables associated with the constraints, i.e., the shadow-prices of domestic output, imports, foreign currency (shadow exchange rate), and of the individual lower and upper bounds on export, respectively.

In fact, two models are presented above, indicated by the broken line frames. *Model I* is defined by the variables and constraints other than those within frames, i.e., in which there are no individual bound prescribed for any variable. In the case of *model II* individual bounds ($\check{Z}_l \leqq Z \leqq \check{Z}_u$, where $\check{Z}_u < \bar{Y}$) constrain the volume of export. The solution of the above problems depends clearly on the relation of $\hat{p}^{we}$ and $\hat{p}^{wm}$, i.e., on the terms of trade.

In the case of *Model I*, if the terms of trade are favourable ($\hat{p}^{we} > \hat{p}^{wm}$), total available home product will be exported ($Z = \bar{Y}$), and only imported goods will be consumed ($C_d = 0$, $C_m = M = \hat{p}^{we}\cdot Z/\hat{p}^{wm}$). All constraints will be binding, and the optimal values of the dual variables will be $p_m = 1$, $v = 1/\hat{p}^{wm}$, $p_d = \hat{p}^{we}/\hat{p}^{wm}$. If the terms of trade are unfavourable ($\hat{p}^{we} < \hat{p}^{wm}$), then the optimal policy will be autarky, i.e., $C_d = \bar{Y}$, $C_m = M = Z = 0$. $p_d = 1$, $1/\hat{p}^{we} \geqq v \geqq 1/\hat{p}^{wm}$, $1 \leqq p_m \leqq v\cdot\hat{p}^{wm}$. In the case, when $\hat{p}^{we} = \hat{p}^{wm}$, any solution exhausting available resources is optimal.

In the case of *Model II* the individual bounds set on export prevent such extreme solution as in *Model I* (*everything or nothing*). All primal variables (C_d, C_m, Z, M) will be positive, thus all dual constraints complementing them will become equalities in the optimal solution. Depending on the terms of trade, the optimal volume of export will be either its upper bound ($Z = \check{Z}_u$, if $\hat{p}^{we} > \hat{p}^{wm}$) or lower bound ($Z = \check{Z}_l$, if $\hat{p}^{we} < \hat{p}^{wm}$). The case of $\hat{p}^{we} = \hat{p}^{wm}$ is a neutral case, any number between $\check{Z}_u$ and $\check{Z}_l$ is an optimal value for Z. Otherwise the solution is

$$Z = \check{Z}_u \text{ or } \check{Z}_l, \qquad C_d = \bar{Y} - Z, \qquad C_m = M = \hat{p}^{we} \cdot Z / \hat{p}^{wm};$$

$$p_d = p_m = 1, \qquad v = 1/\hat{p}^{wm}, \qquad \tau_l - \tau_u = 1 - \hat{p}^{we}/\hat{p}^{wm}, \quad \tau_l \cdot \tau_u = 0 \text{ (one of them is zero).}$$

As can be seen, in this simple model, the domestic prices of the domestically produced and imported commodity, which are assumed to be perfect substitutes, are equal, as they should be in perfect market equilibrium. The term τ_l or τ_u can be interpreted as a tax or subsidy on export, equalizing the income earned by the producer selling the home commodity on the domestic and foreign market. This is all in line with the working of a competitive market.

Introducing lower and upper bounds for Z forces thus its value stay within a "reasonable" region, and thereby constrains the values of the other variables too. One could introduce individual bounds on the volume of the import too, or on its ratio to domestic supply, as will be discussed soon.

One of the problems of using simply lower and upper bounds ($\check{Z}_l$, $\check{Z}_u$) to limit the volume of export is that within these limits its changes are not influenced by any economic variable. What is more, the export takes up, as a rule, one or the other extreme, arbitrarily fixed value. This is basically caused by the linearity of the model used. In a nonlinear model it would be possible to make the volume of export depend on foreign and/or domestic variables.

3. The Model with Flexible Bound Based on Export Demand

The analysis of such a model should not therefore stop here. The bounds set on export are estimated on certain estimated export price. If we changed $\hat{p}^{we}$, these bounds would change too. A decrease in the export price, for example, would increase the export absorption capacity. So, instead of rigid lower and upper bounds, one could introduce, by means of an export demand function, $Z_d(p^{we})$, a flexible upper bound, where the export price can change within certain limits itself. This would, however, turn our linear programming problem into a nonlinear one.

To keep the linear programming framework Srinivasan (1975) suggested to use piecewise linear functions. Another possibility would be to solve a series of linear programming, changing simultaneously the two parameters, $\check{Z}$ and $\hat{p}^{we}$. If the export constraint is binding, it indicates that relaxing the constraint, even decreasing simultaneously $\hat{p}^{we}$, would increase the value of the objective function. Thus, one could change, step by step, the value of parameters $\check{Z}$ and $\hat{p}^{we}$ and solve the problem again and again as long as the export constraint is binding.

In our simple model the logic of the primal and dual conditions of the linear programming problem offers an easy way to find where the above iteration would lead to. As one changes the $\check{Z}$ and $\hat{p}^{we}$ parameters in the LP model, the $Z \leqq \check{Z}$ constrain will be binding, i.e., $Z = Z_d(\hat{p}^{we})$ as long as the terms of trade is favourable (i.e., $\hat{p}^{we} > \hat{p}^{wm}$). Decreasing $\hat{p}^{we}$ increases $Z_d(\hat{p}^{we})$ and consequently the export will increase. The iteration would thus stop when one finds such a combination of the changing parameters $\hat{p}^{we}$ and $\check{Z}$, in which case $\hat{p}^{we} = p^{we}(\check{Z}) = \hat{p}^{wm}$, where $p^{we}(Z)$ is the inverse of the export demand function, $Z_d(p^{we})$.

Since all variables will be positive in such a case, which implies that all constraints will be fulfilled in the form of equality, the necessary conditions of such a solution can be rewritten in the form of the following nonlinear equation system, in which p^{we} is a variable too:

GEM I

$$(1)\quad C_d + Z = \bar{Y} \qquad\qquad (5)\quad p_d = 1$$

$$(2)\quad C_m - M = 0 \qquad\qquad (6)\quad p_m = 1$$

$$(3)\quad \hat{p}^{wm}\cdot M - p^{we}\cdot Z = 0 \qquad\qquad (7)\quad p_d = v\cdot p^{we}$$

$$(4)\quad Z = Z_d(p^{we}) \qquad\qquad (8)\quad p_m = v\cdot \hat{p}^{wm}$$

The eight equations (1)–(8) in eight variables (C_d, C_m, Z, M, p^{we}, p_d, p_m, v) can be reinterpreted as the necessary conditions for a pure competitive (Walrasian) general equilibrium in the above modelled economy, which consists of small households, all trading with the rest of the world. Observe that the exchange rate must assume such a value that equalizes the sales revenue or the purchasing cost of the same commodity on the domestic and world market, since $p_d = p_m = v\cdot p^{we} = v\cdot\hat{p}^{wm} = 1$. This is line with the assumption that we are dealing with a single commodity, which is not differentiated by its origin. In equilibrium the export price, p^{we} is in fact determined by the price of import, $\hat{p}^{wm}$ on the world market.

Necessary conditions of optimal solution or equilibrium containing export demand function of the above type cannot be derived directly from a programming model. To show that, observe first of all, that equation $Z = Z_d(p^{we})$ can be replaced by its inverse, by equation $p^{we} = p^{we}(Z)$. If one does that, a nonlinear programming problem (NLP I) can be formulated, whose primal conditions are the same as that of the LP model, except that the export price is no longer a parameter but a function of Z. The primal conditions of the resulting NLP I problem and the additional necessary (Kuhn–Tucker) complementary conditions of its (optimal) solution will be as follows:

NLP I	The primal problem	The Kuhn–Tucker conditions[2]	
	$C_d, C_m, Z, M \geqq 0$	$p_d, p_m, v \geqq 0$	
(p_d)	$C_d + Z \leqq \bar{Y}$	$p_d \geqq 1$	$(\partial L/\partial C_d)$
(p_m)	$C_m - M \leqq 0$	$p_m \geqq 1$	$(\partial L/\partial C_m)$
(v)	$\hat{p}^{wm}\cdot M - p^{we}(Z)\cdot Z \leqq 0$	$p_d \geqq (1 + 1/\varepsilon)\cdot v\cdot p^{we}(Z)$	$(\partial L/\partial Z)$
	$C_d + C_m \rightarrow \max!$	$p_m \leqq v\cdot p^{wm}$	$(\partial L/\partial M)$

where the two sets of inequalities must fulfil the usual complementary conditions, and p_d, p_m and v, are the Lagrange multipliers, associated with the given constraints (shadow-prices), as indicated in brackets.

From this it can be seen that the solution of NLP I will be different from that to which the solution of the above series of linear programming problems, in which parameters $\hat{p}^{we}$ and $\check{Z}$ were changing according to an assumed export demand function. This clearly shows up in the $(\partial L/\partial Z)$ dual condition by the $(1 + 1/\varepsilon)$ term, which indicates monopolistic price formation.

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[2] The inequalities of the dual conditions are derived by taking the partial derivatives of the Lagrangian function with respect to the original variables, indicated in brackets.

To make our discussion more transparent, we will use constant elasticity export demand curve in what follows, as customary in CGE models:

$$Z_d(p^{we}) = Z_{d0}\cdot\left(\frac{p^{we}}{\hat{p}^{we}}\right)^{\varepsilon} = z_d\cdot(p^{we})^{\varepsilon},$$

where ε (< -1) is the price elasticity of export demand, p^{we} is the price of the home produced good charged on the world market, $\hat{p}^{we}$ is the average world market price of the similar, but differentiated commodity set by the competitors and Z_{d0} is a constant multiplier (the export demand when $p^{we} = \hat{p}^{we}$) and $z_d = Z_{d0}\cdot(\hat{p}^{we})^{-\varepsilon}$.

This relationship can be derived as the solution of the cost minimizing problem the representative foreign buyer is facing, deciding how much should be bought from the given country at price p^{we} and from the rest of the world at price $\hat{p}^{we}$, considering the two types of export less than perfect substitutes. The conditional optimization problem takes the following form:

$$p^{we}\cdot Z_d + \hat{p}^{we}\cdot Z_r \to \min! \qquad\qquad \text{subject to: } Z(Z_d, Z_r) = Z_t,$$

where Z_r is the demand towards the rest of the world and Z_t the fixed total (composite) demand. $Z(Z_d, Z_r) = (\lambda_d\cdot Z_d^{-\kappa} + \lambda_r\cdot Z_r^{-\kappa})^{-1/\kappa}$, defining the composite export commodity, is CES function homogenous of degree 1, where $\kappa > -1$ is the parameter determining $\varepsilon > 0$ the elasticity of substitution between the two types of commodity, $\kappa = 1/\varepsilon - 1$.

The inverse of the above export demand function is

$$p^{we}(Z) = \left(\frac{Z}{Z_{d0}}\right)^{1/\varepsilon}\cdot\hat{p}^{we} = d_0\cdot Z^{1/\varepsilon},$$

where $d_0 = Z_{d0}^{-1/\varepsilon}\cdot\hat{p}^{we}$.

Rewriting the necessary conditions of optimum of the NLP I problem by using the export function $Z_d(p^{we})$, instaed of $p^{we}(Z)$, its inverse, one can rewrite the necessary conditions of the solution of NLP I as the following equation system:

GEM II	(1)	$C_d + Z = \bar{Y}$	(5)	$p_d = 1$
	(2)	$C_m - M = 0$	(6)	$p_m = 1$
	(3)	$\hat{p}^{wm}\cdot M - p^{we}\cdot Z = 0$	(7’)	$p_d = (1 + 1/\varepsilon)\cdot v\cdot p^{we}$
	(4)	$Z = Z_{d0}\cdot\left(\frac{p^{we}}{\hat{p}^{we}}\right)^{\varepsilon}$	(8)	$p_m = v\cdot\hat{p}^{wm}$

We arrived at almost the same eight equations (1)–(8) in the same eight variables (C_d, C_m, Z, M, p^{we}, p_d, p_m, v) as GEM I. The only difference is the term $(1 + 1/\varepsilon)$, which appears in the equation defining the relation between the domestic selling price to the export price, where ε is the price elasticity of export demand. This equation is in fact is the necessary condition of profit maximum, the marginal cost equals the marginal revenue associated with the demand curve, in the case of a firm with monopoly power. The monopoly price ($v \cdot p^{we}$), under normal conditions ($\varepsilon < -1$), will be higher than the cost (p_d). This relationship can be rewritten as

$$(1 + \pi) \cdot p_d = v \cdot p^{we},$$

where $\pi = -1/(1 + \varepsilon)$ (> 0) is the rate of the monopolist profit.

The above conditions can be interpreted as the necessary conditions of general equilibrium of an imperfect market economy, which consists of one household only, or many households but only one export-import monopolist company, trading with the rest of the world. The conditions of general equilibrium will also be the same in another type of imperfect market economy. This one consists of many small entrepreneurs, who cannot recognize the monopolistic position of their country. Therefore, the government charges $1/\varepsilon$ *ad valorem* tax (tariff) on their export. The introduction of such a tariff can make the small entrepreneurs behave collectively as a monopolist, thereby increasing the level of national welfare. This idea is known in international trade theory as optimal tariff (see, for example, Limão, 2008). We will come back to this problem later.

4. The Model with Flexible Bound Based on Export Supply Function

In most of the numerical general equilibrium models export demand is assumed to be less than perfectly, but not perfectly inelastic (*imperfectly elastic*), whereas the export supply is assumed to be perfectly elastic. The reason for introducing imperfectly elastic export demand in the programming approach was to substitute the rigid bounds on the volume of export, i.e., replace the $\check{Z}_l \leqq Z \leqq \check{Z}_u$ rigid constraints, with flexibly bounds in LP Model I.

Relying on neoclassical economic theory, the export demand function can be derived by assuming that foreign buyers treat the exported product as a close, but less than perfect substitutes of the same products offered by the competitors. This assumption implies that the modelled economy is not a small open economy, and it could increase the total welfare by exploiting its monopolistic position, as it was shown, in the case of the programming model.

The purpose of introducing imperfectly elastic export demand function into a general equilibrium model is to hinder large changes in export volume at the cost of bringing in terms of trade changes, which would be difficult to explain. The more one wants to restrain changes in the export volume, the larger will be the change in the terms of trade.

One may, therefore, prefer to maintain the assumption of a small open economy, that is, let the export prices be defined by the world market price ($\hat{p}^{we}$), and replace the $\check{Z}_l \leqq Z \leqq \check{Z}_u$ constraints, limiting changes in export volume, by a supply rather than in export demand function. The simplest solution would be to modify the necessary conditions of equilibrium GEM I the following way.

GEM III	(1)	$C_d + Z = \bar{Y}$	(5)	$p_d = 1$
	(2)	$C_m - M = 0$	(6)	$p_m = 1$
	(3a)	$\hat{p}^{wm} \cdot M - \hat{p}^{we} \cdot Z = 0$	(8)	$p_m = v \cdot \hat{p}^{wm}$

(4a) $$Z = Z_{s0} \cdot \left(\frac{p_d}{v \cdot \hat{p}^{we}} \right)^{\nu}$$

We have this time only seven equations, (1)–(6) and (8) with seven variables (C_d, C_m, Z, M, p_d, p_m, v). The world market price of export ($\hat{p}^{we}$) is constant in this model, as in the LP model, therefore, the balance of trade constraint (3a) changes compared to GEM I-II and the price of the product on domestic (p_d) and foreign markets ($v \cdot \hat{p}^{we}$) can vary from each other. The equation (7) in the previous models, which prescribes their equality, drops thus out, compensating for the loss of the p^{we} variable. Indtead of a demand function a constant elasticity export supply function, (4a) was introduced, where ν (< 0) is the price elasticity of export supply and Z_{s0} is a constant multiplier, the export supply when $p_d = v \cdot \hat{p}^{we}$. In CGE models it should be equal to Z_{d0}. This function is not derived from optimizing decision, it is just an econometrically estimated function. We will come back to this issue later.

5. Equilibrium of Imperfectly Elastic Export Supply and Demand

It is interesting to note that perfectly elastic export supply combined with imperfectly elastic export demand (the standard assumption) leads formally to the same export function as the opposite assumption, namely, imperfectly elastic supply with perfectly elastic demand. To show that, first observe that perfectly elastic export supply means that $p^{we} = p_d/v$. Substituting p_d/v for p^{we} in the export demand function yields:

$$Z = Z_{d0} \cdot \left(\frac{p^{we}}{\hat{p}^{we}} \right)^{\varepsilon} = Z_{d0} \cdot \left(\frac{p_d}{v \cdot \hat{p}^{we}} \right)^{\varepsilon}.$$

Thus, if both the export supply and demand are imperfectly elastic, one can convert their functions into the same form and combine them into an export supply-demand equilibrium function in the following way. First, from the demand function one gets:

$$p^{we} = z_d^{-1/\varepsilon} \cdot \hat{p}^{we} \cdot Z^{1/\varepsilon}.$$

Substituting this expression for p^{we} in the $Z = Z_{s0} \cdot \left(\frac{p_d}{v \cdot p^{we}} \right)^{\nu}$ export supply function (which is derived from equation (4a) of the GEM III by replacing the $\hat{p}^{we}$ by p^{we}) and solving the resulting equation for Z yields

$$Z = \left[\left(Z_{d0}^{\varepsilon} \cdot Z_{s0}^{\nu} \right) \left(\frac{p_d}{v \cdot \hat{p}^{we}} \right)^{\varepsilon \cdot \nu} \right]^{\frac{1}{\varepsilon + \nu}} = Z_{e0} \cdot \left(\frac{p_d}{v \cdot \hat{p}^{we}} \right)^{\eta},$$

where

$$Z_{e0} = (Z_{d0}^{\varepsilon} \cdot Z_{s0}^{\nu})^{1/(\varepsilon+\nu)},$$

$$\eta = \frac{\varepsilon \cdot \nu}{\varepsilon + \nu}.$$

Thus, the export demand, export supply and supply-demand equilibrium export functions, i.e, $Z = Z_{d0} \cdot \left(\dfrac{p_d}{v \cdot \hat{p}^{we}} \right)^{\varepsilon}$, $Z = Z_{s0} \cdot \left(\dfrac{p_d}{v \cdot p^{we}} \right)^{\nu}$, $Z = Z_{e0} \cdot \left(\dfrac{p_d}{v \cdot \hat{p}^{we}} \right)^{\eta}$,

have the same mathematical forms assuming constant price elasticity. This implies that it is difficult to tell which effect is, in fact, reflected and to what extent by an econometrically estimated function of that form.

Note also that the equilibrium specification is, in certain sense, an "average" of the pure supply and demand specifications, since the scaling parameter is the geometric average and the elasticity is half of the harmonic average of the corresponding "pure" parameters. It is interesting to see that the "equilibrium elasticity" is less than either the supply or the demand elasticity, and this may partially explain why empirical estimates of the export demand elasticity tend to be rather small, even for small economies.

One should add that econometric estimates of export functions are on the whole rather scarce and unreliable, and estimates of elasticities are especially sensitive to differences in samples, estimation techniques, and model specification[3]. This indicates that one has to choose with special care both the export specification and the size of parameters.

We have found thus that export functions determined on the basis of pure supply or pure demand or supply-demand equilibrium has the same algebraic form. Does this mean that it makes no difference which export specification is used in a general equilibrium model? Not at all! Their difference shows up in the unit export earning, i.e., in the current account balance. The income earned by exporting one unit (p^{we}) will be equal to p_d/v (endogenous) in the pure demand case and $\hat{p}^{we}$ (exogenous) in the case of pure supply. Expressing first p^{we} from the demand function, substituting next the supply term for Z into the resulting equation, and solving finally this new equation for p^{we} one gets the following relationship for the demand-supply equilibrium case:

$$p^{we} = \left[\frac{Z_{s0}}{Z_{d0}} \cdot \left(\frac{p_d}{v} \right)^{\nu} \cdot \hat{p}^{we^{\,\varepsilon}} \right]^{1/(\varepsilon+\nu)} .$$

If Z_{s0} and Z_{d0} are equal, the export price will be equal to the geometric average of the exogenous world market price of export ($\hat{p}^{we}$) and the foreign current equivalent of its domestic price (p_d/v).

The main characteristics of the different export specifications are summarized in Table 1. The table contains all possible pairs of supply-demand elasticity situations. Some of them are not really relevant, since the export functions are only discussed here as part of more complicated (multisectoral) models.

It should be perhaps pointed out, and this is important from a computational point of view, that the usual demand-specified general equilibrium model can easily be modified to allow for alternative export specifications. If either ε or ν decreases beyond a certain limit, our specification will reduce to the pure supply or demand case.

[3] See, for example, Houthakker and Magee (1969), Hickman and Lau (1973), Sato (1977), Goldstein and Khan (1978), Stone (1979), Browne (1982).

Table 1. The choice of elasticity of export demand and supply and its effect on the model specification

Supply Demand	Perfectly elastic $(\gamma = -\infty)$	Imperfectly elastic $(-\infty < \gamma < 0)$	Perfectly inelastic $(\gamma = 0)$
Perfectly elastic $(\varepsilon = -\infty, \;\; p^{\mathrm{we}} = \hat{p}^{\mathrm{we}})$	*No Bounds* $p_{\mathrm{d}} = v \cdot \hat{p}^{\mathrm{we}}$ $(0 \leqq Z \leqq \bar{Y})$	*Flexible Supply Bound* $p_{\mathrm{d}} = p_{\mathrm{m}} = v \cdot \hat{p}^{\mathrm{wm}}$ $Z = Z_{\mathrm{s0}} \cdot \left(\frac{p_d}{v \cdot \hat{p}^{\mathrm{we}}} \right)^{\gamma}$	*Rigid (Supply) Bounds* $p_{\mathrm{d}} \geqq v \cdot \hat{p}^{\mathrm{we}} + \tau_{\mathrm{l}} - \tau_{\mathrm{u}}$ $\check{Z}_{\mathrm{l}} \leqq Z \leqq \check{Z}_{\mathrm{u}}$
Imperfectly elastic $(-\infty < \varepsilon < 0)$	*Flexible Demand Bound* $p^{\mathrm{we}} = p_{\mathrm{d}} / v$ $Z = Z_{\mathrm{d0}} \cdot \left(\frac{p_d}{v \cdot \hat{p}^{\mathrm{we}}} \right)^{\varepsilon}$	*Supply-Demand Equilibrium* $p^{\mathrm{we}} = \left[\frac{Z_{\mathrm{s0}}}{Z_{\mathrm{d0}}} \cdot \left(\frac{p_{\mathrm{d}}}{v} \right)^{\nu} \cdot \hat{p}^{\mathrm{we}\,\varepsilon} \right]^{1/(\varepsilon+\nu)}$ $Z = \left[\left(Z_{\mathrm{d0}}^{\varepsilon} \cdot Z_{\mathrm{s0}}^{\nu} \right) \left(\frac{p_{\mathrm{d}}}{v \cdot \hat{p}^{\mathrm{we}}} \right)^{\varepsilon \cdot \nu} \right]^{\frac{1}{\varepsilon+\nu}}$	*Fixed Supply* $p^{\mathrm{we}} = \left(\frac{Z_{\mathrm{s0}}}{Z_{\mathrm{d0}}} \right)^{1/\varepsilon} \cdot \hat{p}^{\mathrm{we}}$ $Z = Z_{\mathrm{s}}$
Perfectly inelastic $(\varepsilon = 0)$	*Rigid (Demand) Bounds* $p_{\mathrm{d}} = v \cdot \hat{p}^{\mathrm{we}} + \tau_{\mathrm{l}} - \tau_{\mathrm{u}}$ $\check{Z}_{\mathrm{l}} \leqq Z \leqq \check{Z}_{\mathrm{u}}$	*Fixed Demand* $p^{\mathrm{we}} = \left(\frac{Z_{\mathrm{s0}}}{Z_{\mathrm{d0}}} \right)^{1/\nu} \cdot \frac{p_{\mathrm{d}}}{v}$ $Z = Z_{\mathrm{d}}$	*Both Fixed* (possible disequilibrium, no adjustment is feasible)

Figures 1 and 2, based on numerical simulations, summarize the main features of the alternative export specifications in geometrical form. Along the horizontal axis one can see the export volume (Z) in both cases. In Figure 1 the vertical axis represents the unit export price (p^{we}), whereas in Figure 2 the foreign currency equivalent of the domestic price (p_d/v).

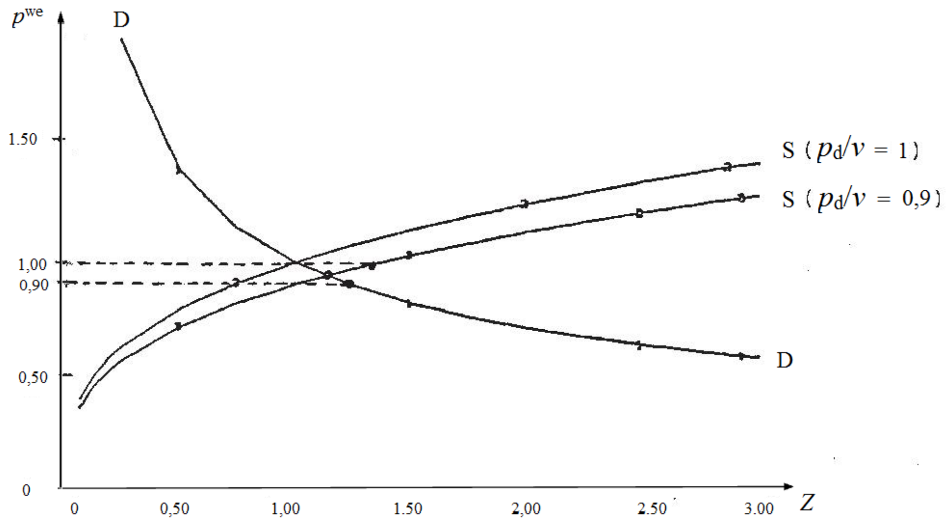

Figure 1. Export demand (D) and supply (S) as function of the export price (p^{we})

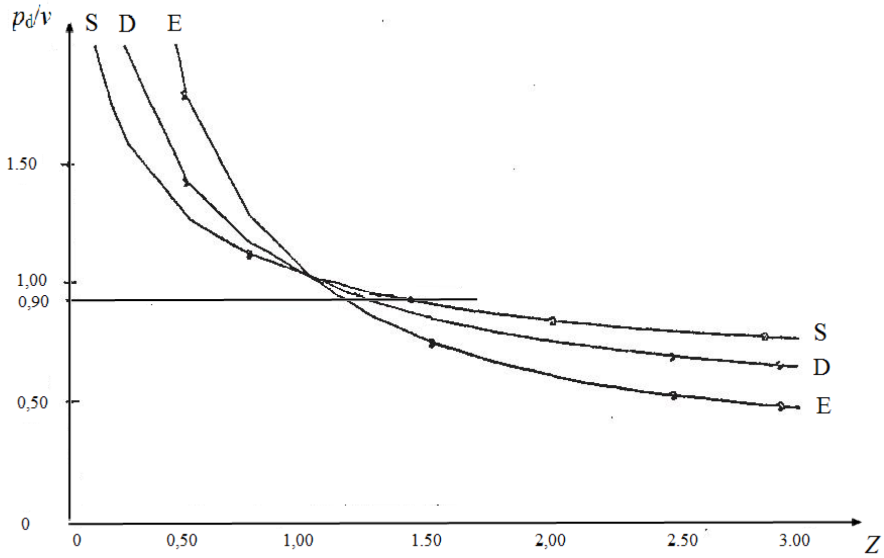

Figure 2. Export demand (D), supply (S) and equilibrium (E)

Figures 1 and 2, based on numerical simulations, summarize the main features of the alternative export specifications in geometrical form. Along the horizontal axis one can see the export volume (Z) in both cases. In Figure 1 the vertical axis represents the unit export price (p^{we}), whereas in Figure 2 the foreign currency equivalent of the domestic price (p_d/v).

The figures illustrate the impact of a 10 percent change in p_d/v on the volume of export in each case, which increased by 37, 23 and 13 percent in the supply, demand and equilibrium specifications, respectively. The elasticities of supply and demand are -3 and -2, respectively, and therefore the export elasticity in the equilibrium specification will be -1,2.

6. Export Supply Function Derived from Optimizing Decision

The necessary conditions of optimal solution or equilibrium containing an export supply function as above could be derived from a programming model or on the bases of neoclassical economic theory, i.e., assuming profit maximizing behaviour, if the same product sold on the domestic and on the foreign market were less than perfect subsitutes, i.e., differentiated commodities.

This can be built into the model by means of a production function extended to joint production. In this function, on the one hand, a transformation (disaggregating) function, $X(C_d, Z) = CAP$ shows what combinations of C_d and Z can be produced by distributing the given capacity CAP between the two sorts of output. The capacity (CAP) itself, on the other hand, provided by the available amount of production factors, say, labour (L) and capital (K), is expressed by a $CAP = F(L, K)$ production (aggregating) function. Such a production function will be thus defined as $X(C_d, Z) = F(L, K)$. In our model the capacity is assumed to be fixed ($\bar{Y}$), therefore, the production function is reduces to equation $X(C_d, Z) = \bar{Y}$.

Changing the composition of C_d and Z, the factors of production have to be reallocated, and consequently, their productivity, that is the effective volume of the total capacity would fall, as a rule. This phenomenon can be represented by transformation functions. The most commonly used Constant Elasticity of Transformation (CET) function has the following linearly homogenous (homogenous of degree one) form:

$$X(C_d, Z) = (a\cdot C_d^{\delta} + b\cdot Z^{\delta})^{1/\delta},$$

where a and b are the usual share parameters, $\delta > 1$ is the parameter determining $\nu < 0$, the elasticity of transformation between the two types of products: $\nu = 1/(1 - \delta)$. This will be the price elasticity of export supply. This is why we use the same symbol here as in the case of the simple (econometrically estimated) export supply function.

At given p_d and p_e, i.e., the price of the product sold on domestic market and the price of its export converted to domestic currency, the profit maximizing producers will choose such a combination of C_d and Z, which maximizes their total revenue ($p_d\cdot C_d + p_e\cdot Z$) subject to the capacity constraint $X(C_d, Z) = \bar{Y}$. From the necessary conditions of that maximum one can derive convenient forms, which could be used in a general equilibrium model. For example, the following variables and equations:

– the unit (CET average) price of the composite product, $X = X(C_d, Z)$:

$$p_a = (p_d\cdot C_d + p_e\cdot Z)/X,$$

– r_e and s_e, the optimal ratio of export to domestic supply (Z/C_d) and to total production (Z/X):

$$r_e = r_{e0}\cdot\left(\frac{p_d}{p_e}\right)^{\nu}, \qquad \text{and} \qquad s_e = s_{e0}\cdot\left(\frac{p_a}{p_e}\right)^{\nu},$$

– which implies the export supply functions of the following type:

$$Z = r_e \cdot C_d = r_{e0} \cdot \left(\frac{p_d}{p_e} \right)^{\nu} \cdot C_d \qquad \text{and} \qquad Z = s_e \cdot C_d = s_{e0} \cdot \left(\frac{p_a}{p_e} \right)^{\nu} \cdot X.$$

Introducing the nonlinear transformation function into the programming model will produce similar effect as the export demand function: it will constrain the shift in the export volume. Unlike in the case of the demand function, one can maintain the assumption of a small open economy, i.e., the export price is dictated by the world market ($\hat{p}^{we}$). Change in the export volume will not bring about unexplainable change in the terms of trade and the assumption of optimising behaviour will not result in optimal tariff.

Assuming optimising behaviour will thus lead to the following programming problem:

NLP II	The primal problem	The Kuhn–Tucker complementary conditions	
	$C_d, C_m, Z, M \geqq 0$	$p_a, p_m, v \geqq 0$	
(p_a)	$X(C_d, Z) \leqq \bar{Y}$	$p_a \cdot \partial X/\partial C_d \geqq 1$	$(\partial L/\partial C_d)$
(p_m)	$C_m - M \leqq 0$	$p_m \geqq 1$	$(\partial L/\partial C_m)$
(v)	$\hat{p}^{wm} \cdot M - \hat{p}^{we} \cdot Z \leqq 0$	$p_a \cdot \partial X/\partial Z \geqq v \cdot \hat{p}^{we}$	$(\partial L/\partial Z)$
	$C_d + C_m \to \max!$	$p_m \geqq v \cdot \hat{p}^{wm}$	$(\partial L/\partial M)$

Assuming again that in the solution all variables become positive, all conditions, defined in the form of weak inequalities, will be fulfilled as equations. Using auxiliary variables p_d and p_e to denote the domestic price of the goods supplied on the domestic and the export markets, respectively, we get the following chain of equations:

$$p_d = p_a \cdot \partial X/\partial C_d \; (= 1), \;\; p_e = p_a \cdot \partial X/\partial Z \; (= v \cdot \hat{p}^{we}), \qquad p_m = v \cdot \hat{p}^{wm} \; (= 1).$$

By virtue of Euler's theorem, one gets the following identities:

$$p_d \cdot C_d + p_e \cdot Z = p_a \cdot (\partial X/\partial C_d \cdot C_d + \partial X/\partial Z \cdot Z) = p_a \cdot \bar{Y}.$$

The necessary conditions of the optimal solution can be thus rewritten in the form of the following equation system,

GEM IV				
(1)	$X(C_d, Z) = \bar{Y}$	(5)	$p_d = 1$
(2)	$C_m - M = 0$	(6)	$p_m = 1$
(3a)	$\hat{p}^{wm} \cdot M - \hat{p}^{we} \cdot Z = 0$	(7a)	$p_e = v \cdot \hat{p}^{we}$
(4b)	$Z = s_{e0} \cdot \left(\frac{p_a}{p_e} \right)^{\nu} \cdot \bar{Y}$	(8)	$p_m = v \cdot \hat{p}^{wm}$
		(9)	$p_a = (p_d \cdot C_d + p_e \cdot Z)/\bar{Y}$

Compared to GEM II one can see that this equation system has nine variables instead of eight. There are two new variables, p_e and p_a (the product price became differentiated and their average price became a new variable) and p^{we} has been dropped (the export price is no longer variable). At the same time, the equation defining the average product price entered into the model. It should be noted that p_a could be defined also as a CET dual function of p_e and p_a alone, i.e., without using C_d, Z and $\bar{Y}$. Note also that the form of the export supply function matches the one used in GEM III, as long as the capacity, $\bar{Y}$ is fixed, since $s_{e0}\cdot\bar{Y}$ suits Z_{s0}.

7. Individual Bounds on Imports

A similar flexible bound approach can be used in case of the import as well, instead of individual rigid bounds. In our simple model it will be enough to constrain either the volume of export or import by individual bounds. In the case of import, the ratio of imported goods to domestic supply ($r_{md} = C_m/C_d$) is typically constrained. We introduce therefore only an upper ($\check{r}_u$) and lower ($\check{r}_l$) bound on the ratio of imported goods to domestic supply ($r_{md} = C_m/C_d$) into LP model III. Let us denote by τ_{lm} and τ_{um} the shadow-prices associated with the lower and upper constraint on import ratio, respectively. Modifying accordingly the LP problem one gets the following primal and dual problem.

LP III	Primal problem	Dual problem	
	$C_d, C_m, Z, M \geqq 0$	$p_d, p_m, v, \tau_{lm}, \tau_{um} \geqq 0$	
(p_d)	$C_d + Z \leqq \bar{Y}$	$p_d \geqq 1 - \tau_{lm}\cdot\check{r}_l + \tau_{um}\cdot\check{r}_u$	(C_d)
(p_m)	$C_m - M \leqq 0$	$p_m \geqq 1 + \tau_{lm} - \tau_{um}$	(C_m)
(v)	$\hat{p}^{wm}\cdot M - \hat{p}^{we}\cdot Z \leqq 0$	$p_d \geqq v\cdot\hat{p}^{we}$	(Z)
(τ_{lm}, τ_{um})	$\check{r}_l\cdot C_d \leqq C_m \leqq \check{r}_u\cdot C_d$	$p_m \leqq v\cdot\hat{p}^{wm}$	(M)
	$C_d + C_m \rightarrow \max!$	$p_d\cdot\bar{Y} \rightarrow \min!$	

Observe that if the lower limit on imports is binding (neglecting degenerate solutions), we will have $\tau_{lm} > 0$, $\tau_{um} = 0$ and $p_d = 1 - \tau_{lm}\cdot\check{r}_l < 1$, $p_m = 1 + \tau_{lm} > 1$. If the upper limit is binding then $\tau_{lm} = 0$, $\tau_{um} > 0$ and $p_d = 1 + \tau_{um}\cdot\check{r}_u > 1$, $p_m = 1 - \tau_{um} < 1$. Otherwise $p_d = p_m = 1$. This means, that if $p_d > p_m$ (the shadow-price of the commodity imported is smaller than that of the domestic), the volume of import will be as large as allowed for, and it will be the other way around, the import will be the minimum prescribed, if $p_d < p_m$.

The import ratio can be defined, thus, by the following function (see its graph in Figure 3):

$$r_{md} = r_m(p_d, p_m) = \begin{cases} \check{r}_l & \text{if } p_d/p_m < 1 \\ (\check{r}_l, \check{r}_u) & \text{if } p_d/p_m = 1 \\ \check{r}_u & \text{if } p_d/p_m > 1 \end{cases}$$

Observe that the same import restriction could be achieved by modifying the $C = C_d + C_m$ objective function. One could introduce in its place a piecewise linear objective function with indifference curves as illustrated in Figure 4. That would in effect restrict the import ratio by the same lower ($\check{r}_l$) and upper ($\check{r}_u$) bounds as before. Such an objective function can be viewed as a piecewise linear welfare or utility function, whose indifference curves consist of three different sections. Between the lines defined by $C_m = \check{r}_u \cdot C_d$ and $C_m = \check{r}_l \cdot C_d$, i.e., when $\check{r}_l \cdot C_d \leqq C_m \leqq \check{r}_u \cdot C_d$, the two types of the commodity are perfect substitutes, beyond it they behave as perfect complements.

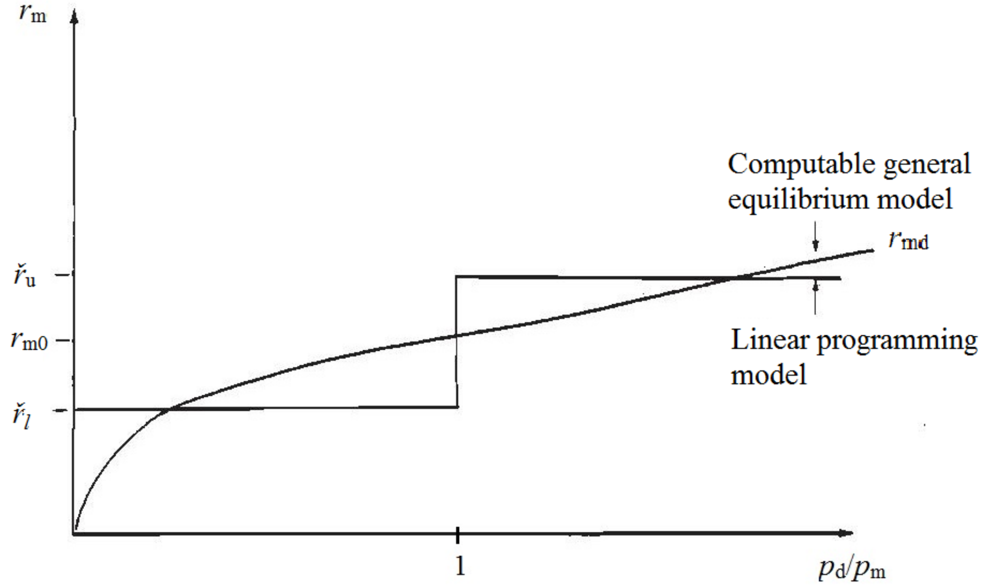

Figure 3. Smooth versus piecewise linear import ratio functions

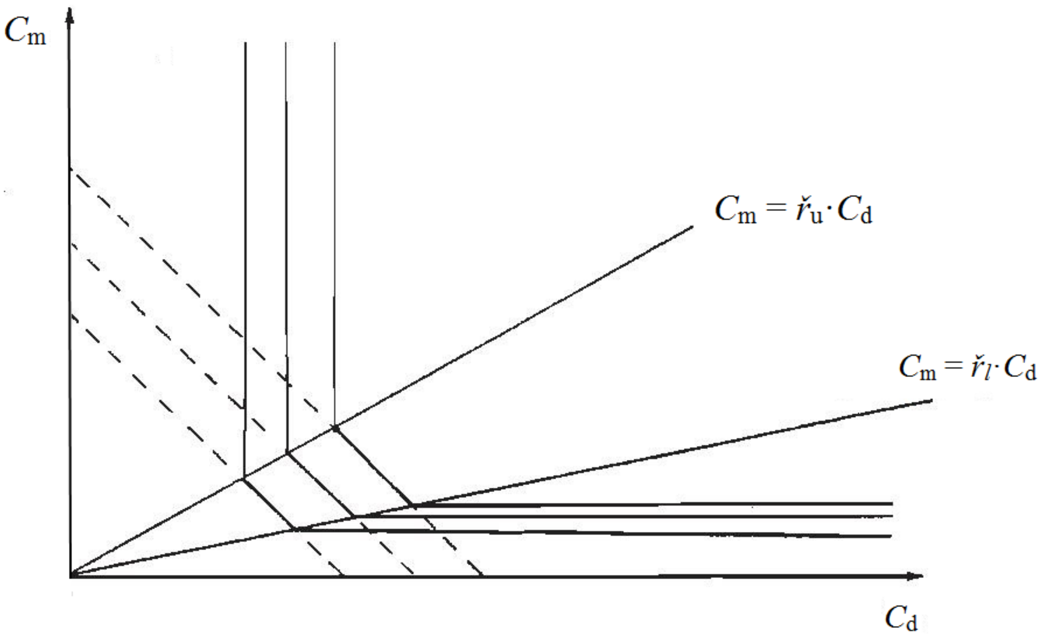

Figure 4. Import restriction built into the objective function

In the computable general equilibrium (CGE) models, it is usually assumed (the so-called Armington assumption) that the domestic and the imported variety of the same commodity are less than perfect substitutes, represented by a Constant Elasticity of Substitution (CES) utility (use value aggregation) function of the following form:

$$C = C(C_d, C_m) = (\alpha_d \cdot C_d^{-\beta} + \alpha_m \cdot C_m^{-\beta})^{-1/\beta},$$

where $\beta > -1$ is the parameter determining $\mu > 0$ the elasticity of substitution between the two types of commodity: $\mu = 1/(1 + \beta)$. The import ratio function (r_{md}) can be derived from maximizing the $C(C_d, C_m)$ aggregation function subject to cost constraint $p_d \cdot C_d + p_m \cdot C_m = 1$. The additional two constraints derived from the Lagrange function are as follows:

$$p_d = \partial C/\partial C_d \qquad (\partial L/\partial C_m), \qquad\qquad p_m = \partial C/\partial C_m \qquad (\partial L/\partial C_m)$$

From these necessary conditions one can derive the determination of the optimal ratio of the domestic and imported supply (C_m/C_d) in the form of a smooth function of their prices:

$$r_{md} = \frac{C_m}{C_d} = r_m(p_d, p_m) = r_{m0} \cdot \left(\frac{p_d}{p_m}\right)^{\mu}$$

(see its curve in Figure 3).

The difference in the treatment of import restrictions between linear programming and computable equilibrium models can be seen again as the difference between using *rigid or flexible* individual bounds. The relative-price-dependent import ratio implies a flexible individual bound on imports. The larger is the gap between the shadow-prices of the domestic and imported commodities the larger will be the deviation from the observed (or planned) import ratio (r_{m0}).

Smooth import ratio functions could be incorporated into an otherwise linear model, as mentioned above, using a piecewise linearization technique.[4] Thanks to availability of efficient programs solving nonlinear programming or computable general equilibrium models, it is more advantageous to transform the model into a nonlinear form.

Suppose we have a linear programming model with fixed individual bounds on both exports and import ratios. If we want to replace the fixed individual bounds by flexible ones, as described earlier, one should replace the objective function with a smooth preference function reflecting import limitations and introduce an export demand function as before. These changes yield the following nonlinear programming model.

NLP III	The primal problem	The Kuhn–Tucker complementary conditions	
	$C_d, C_m, Z, M \geqq 0$	$p_d, p_m, v \geqq 0$	
(p_d)	$C_d + Z \leqq \bar{Y}$	$p_d \geqq \partial C/\partial C_d$	$(\partial L/\partial C_d)$
(p_m)	$C_m - M \leqq 0$	$p_m \geqq \partial C/\partial C_m$	$(\partial L/\partial C_m)$
(v)	$\hat{p}^{wm} \cdot M - \hat{p}^{we} \cdot Z \leqq 0$	$p_d \geqq v \cdot \hat{p}^{we}$	$(\partial L/\partial Z)$
	$C(C_d, C_m) \rightarrow \max!$	$p_m \leqq v \cdot \hat{p}^{wm}$	$(\partial L/\partial M)$

[4] See, for example, Ginsburgh and Waelbroeck (1981) again, who give examples showing how piecewise linear (nonlinear) relationships can be introduced into linear programming models and outline some applications.

If all variables are positive, which implies that all constraints are fulfilled in the form of equality, the necessary conditions of optimum can be reformulated into the form of the following system of simultaneous equations (containing C_d, C_m, Z, M, p_d, p_m, v as variables).

GEM V

(1) $C_d + Z = \bar{Y}$

(2) $C_m - M = 0$

(3) $\hat{p}^{wm} \cdot M - \hat{p}^{we} \cdot Z = 0$

(4) $C_m = r_{m0} \cdot \left(\dfrac{p_d}{p_m} \right)^{\mu} \cdot C_d$

(5) $p_d = v \cdot \hat{p}^{we}$

(6) $p_m = v \cdot \hat{p}^{wm}$

(7) $p_d \cdot C_d + p_m \cdot C_m = 1$

These are again the same as the necessary conditions of general equilibrium.

8. Equilibrium *versus* Optimum: Optimal Tariff Revisited

It is worth taking a short detour and to show that by means of a slight modification of the NLP III model and making use of the parametric programming technique one can arrive at such a solution of the programming model, in which the necessary conditions of the optimal solution coincide with conditions if a perfect market equilibrium even in the case of downward sloping export demand function. The underlying idea is very simple. [5]

We will simplify the description of the NLP model by assuming that all variables will be positive, thus all weak inequalities will be fulfilled as equations in the optimal solution. Since M will be equal to C_m, one can reduce the model by omitting variable M and dual variable p_m as well the corresponding complemetarity dual condition. Our programming problem will have only three variables and two constraints This will allow us to illustrate together and compare the optimal tariff solution (the "planners optimum") and the perfect market equilibrium solution on Figures 5 and 6, making use of the $C_d = \bar{Y} - Z$ correspondence.

The modified model is as follows:

NLP IV	The primal problem	The Kuhn–Tucker conditions	
	$C_d, C_m, Z \geqq 0$	$p_d, p_m, v \geqq 0$	
(p_d)	$C_d + Z = \bar{Y}$	$p_d = 1$	$(\partial L/\partial C_d)$
(v)	$\hat{p}^{wm} \cdot C_m - [\varepsilon/(1+\varepsilon)] \cdot p^{we}(Z) \cdot Z = k$	$v \cdot \hat{p}^{wm} = 1$	$(\partial L/\partial C_m)$
	$C(C_d, C_m) \rightarrow \max!$	$p_d = v \cdot p^{we}(Z)$	$(\partial L/\partial Z)$

The model which results in the optimal tariff solution has been modified in such a way that its dual conditions will satisfy the pricing requirements of perfect market equilibrium. This is achieved simply by multiplying the export term in the foreign currency constraint by factor

[5] This idea was inspired by Lundgren (1982), who proposed such an algorithm for solving a special type of multisectoral equilibrium model, which incorporates non-smooth relationships.

$\varepsilon/(1+\varepsilon)$, the reciprocal of the optimal tariff term, in order to offset the "monopoly distortion" effect in the Kuhn–Tucker conditions. This change, however, alters the meaning of the foreign currency condition and this must be taken into account in the method of solution. This is achieved by varying the left-hand side (k) parametrically until the solution (C_m and Z, in particular) also satisfies the original current account, $\hat{p}^{wm}\cdot M - \hat{p}^{we}\cdot Z = 0$ condition.

Figure 5 sheds more light on the nature of the competitive equilibrium solution. The horizontal axis represents primarily the value of Z. However, since the difference between $\bar{Y}$ and Z yields C_d, whose value can be also represented along the horizontal axis. A vertical axis represents C_m in both cases As a result one can represent the indifference curves of $C(C_d, C_m)$, the balance of payment condition, as well as the similar second constraint of the programming problem all on the same figure.

The curve from 0 to $d = 0$ represents the export-import combinations which fulfil the current account requirement, where d is the balance of current account. Notice that the only difference between the latter and the second constraint in the programming model at $k = 0$ is that the export term is multiplied by the constant $\varepsilon/(1 + \varepsilon)$, which is, by assumption, greater than 1. Therefore, the curve from 0 to $k = 0$ is steeper than the current account curve. The curve $S\bar{Y}$ is the locus of the points, where the indifference curves of $C(C_d, C_m)$ is tangent to the curve of 0 to k at various values of k.

Observe, that the optimal solution of the programming problem at $k = 0$ clearly does not meet the current account requirement. If, however, we change k parametrically then the optimal solution will lie on the curve $S\bar{Y}$. The competitive equilibrium is there, where this latter curve intersects the current account curve, the curve from 0 to $d = 0$.

From Figure 5 it is also clear, and it is even more apparent in Figure 6, that the pure competitive equilibrium point cannot be the point of optimal solution at the same time. For, at the optimal solution point the indifference curve and the curve from 0 to k must be tangential to each other. In the competitive equilibrium case the current account curve and the curve from 0 to k*, which contains competitive equilibrium point, intersect each other. A small movement along the current account curve toward the origin would increase the value of the objective (welfare) function (see in Figure 6).

Observe the tangent line separating the indifference curve and to the transformed current account curve from 0 to k*at the equilibrium point is the consumers' budget line. This line passes through the origin (no foreign trade) as well, since the only source of income is the sale of domestic resources ($p_d\cdot\bar{Y}$). Observe, however, that this is not the case for the planners optimal solution, in which case part of the income is provided by the export tariffs.

9. SUMMING UP: NLP *VERSUS* CGE MODEL WITH FLEXIBLE BOUNDS

In the previous sections we have shown, case by case, how one can constrain the shift in export and import volume in macroeconomic models by means of flexible instead of rigid individual bounds as it is common in models of linear programming type. The basic idea was to use nonlinear relationships and thus assuming less than perfect substitutability between the commodities and production factors used or produced jointly, borrowing the well-known techniques of microeconomics.

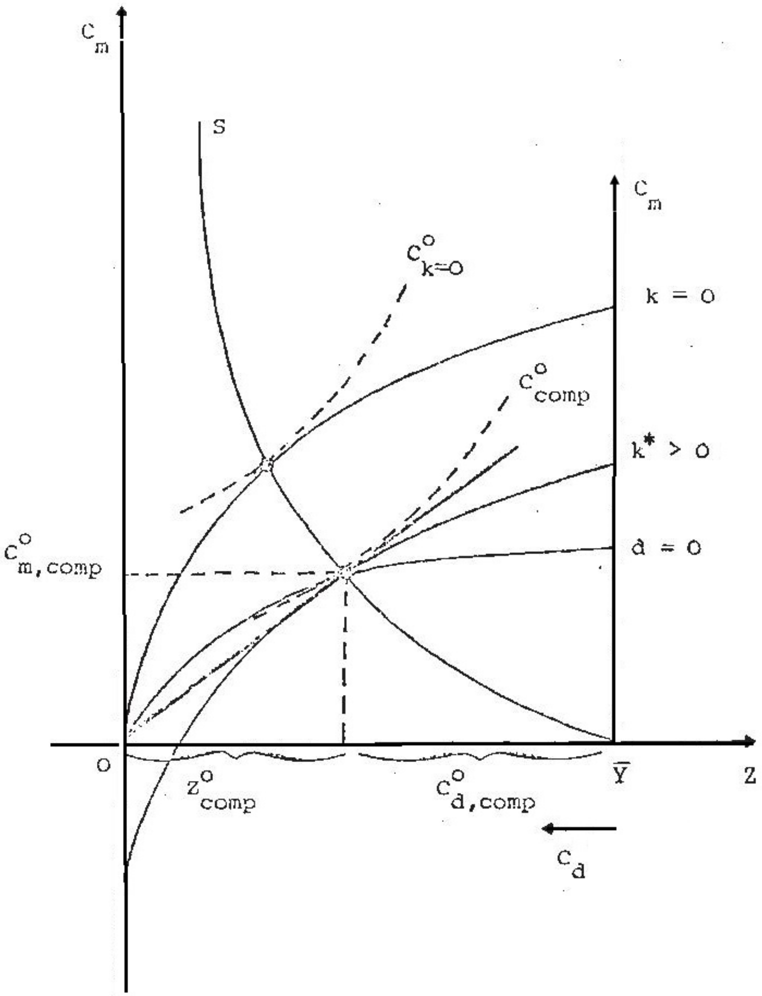

Figure 5. Finding the equilibrium solution by parametric programming.

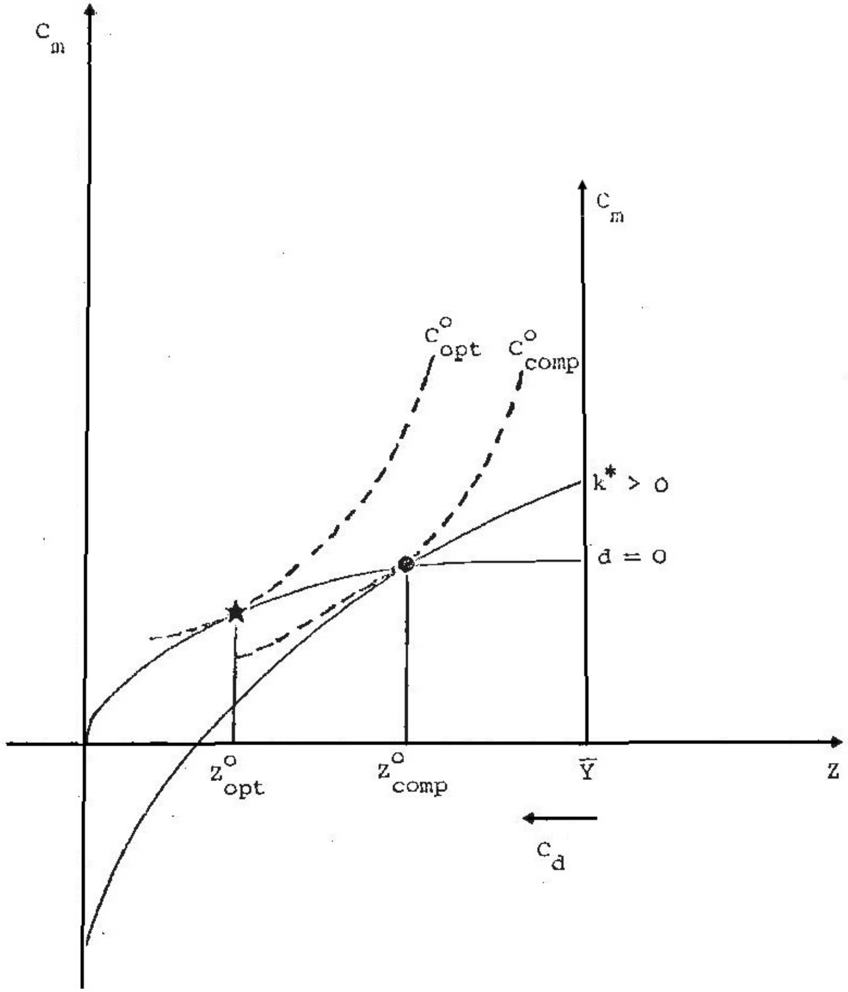

Figure 6. Planners optimum (★) and pure competitive equilibrium (●).

With examples based on a simple model, using macroeconomic aggregate indicators (e.g., production, consumption, export, import), it was also demonstrated that the necessary conditions of optimal and perfect market equilibrium resource allocation correspond with each other, as known from the theorems of welfare economics. In the case of using export demand functions, however, the programming model brings in an unwanted effect, the so-called optimal tariff phenomenon, whereas in a general equilibrium model unjustifiable changes in the terms of trade.

Hereby, combining the individually discussed cases into one model, we sum up our findings. The nonlinear programming model version of the resource allocation problem, incorporating all possibilities discussed, can be formulated as follows.

NLP V	The primal problem	The Kuhn–Tucker complementary conditions	
	$C_d, C_m, C, Z, M \geqq 0$	$p_d, p_m, p_{hm}, v \geqq 0$	
(p_a)	$X(C_d, Z) \leqq \bar{Y}$	$p_a \cdot \partial X/\partial C_d \geqq p_{hm} \cdot \partial C/\partial C_d$	$(\partial L/\partial C_d)$
(p_m)	$C_m - M \leqq 0$	$p_m \geqq p_{hm} \cdot \partial C/\partial C_m$	$(\partial L/\partial C_m)$
(p_{hm})	$C - C(C_d, C_m) \leqq 0$	$p_{hm} \geqq 1$	$(\partial L/\partial C)$
(v)	$\hat{p}^{wm} \cdot M - p^{we}(Z) \cdot Z \leqq 0$	$p_a \cdot \partial X/\partial Z \geqq (1 + 1/\varepsilon) \cdot v \cdot p^{we}(Z)$	$(\partial L/\partial Z)$
	$C \to \max!$	$p_m \leqq v \cdot \hat{p}^{wm}$	$(\partial L/\partial M)$

Assuming again that all variables will be positive in the solution, the first order necessary conditions of the optimal solution will be all fulfilled as equations. The necessary conditions of optimality can be thus reformulated as a system of equations, similar to those, which characterize general equilibrium of a market economy. In order to be able to use more familiar equivalent forms used in microeconomics, some auxiliary variables and additional equations will be introduced.

We have already done it above by introducing C to denote the aggregate volume of consumption in formulating the primal problem. $C(C_d, C_m)$ can be interpreted as a welfare function and C as the level of welfare in this model. Variable p_e will represent the price of export converted to domestic currency, as before: $p_e = (1 + 1/\varepsilon) \cdot v \cdot p^{we}(Z)$. Variable p_d denotes the price of the domestically produced commodity on the home market. Observe that p_{hm} is, in fact, the unit price of $C = C(C_d, C_m)$, the composite supply of commodities on the home market, whereas p_a is the unit price of the composite output, $X = X(C_d, Z)$, but X will not be introduced into the model as an additional variable. The prices of these composite commodities can be defined as the average of the prices of the components:

$$p_{hm} = (p_d \cdot C_d + p_m \cdot C_m)/C, \qquad p_a = (p_d \cdot C_d + p_e \cdot Z)/\bar{Y}.$$

The necessary conditions of optimality can be equivalently reformulated thus as follows.

Variables (altogether 11): Primal $C_d, C_m, C, Z, M,$ Dual $p_a, p_{hm}, p_d, p_m, p_e, v$

GEM VI	Primal		Dual
(1)	$X(C_d, Z) = \bar{Y}$	(5)	$p_m = v \cdot \hat{p}^{wm}$
(2)	$C_m = M$	(6)	$p_e = (1 + 1/\varepsilon) \cdot v \cdot p^{we}(Z)$
(3)	$C = C(C_d, C_m)$	(7)	$p_a = (p_d \cdot C_d + p_e \cdot Z)/\bar{Y}$
(4)	$\hat{p}^{wm} \cdot M - p^{we}(Z) \cdot Z = 0$	(8)	$p_{hm} = (p_d \cdot C_d + p_m \cdot C_m)/C$
		(9)	$p_d = p_{hm} \cdot \partial C/\partial C_d$
		(10)	$p_e = p_a \cdot \partial X/\partial Z$
		(11)	$p_{hm} = 1$

The equations speak for themselves. It seems still appropriate to add a few comments to them, which shed light on certain aspects that connect the necessary conditions of optimum to those of general equilibrium.

Equations (1), (7) and (10) are the necessary conditions C_d and Z have to fulfil in order to maximize the producer's revenue (and profit) at given p_d and p_e prices, and $\bar{Y}$ capacity. The conditional optimization problem of the producer takes the following form:

$$p_d \cdot C_d + p_e \cdot Z \to \max! \qquad \text{subject to: } X(C_d, Z) = \bar{Y},$$

where p_a is the Lagrange multiplier attached to the constraint. Here it is defined by equation (7) as the (average) sales price of the composite commodity $X = X(C_d, Z)$.

Equations (3), (8) and (9) are the necessary conditions for C_d and C_m to minimize the cost of achieving welfare level C (or purchase that amount of the composite good C) at prices p_d and p_e. The conditional optimization problem of the consumer takes the following form:

$$p_d \cdot C_d + p_m \cdot C_m \to \min! \qquad \text{subject to: } C(C_d, C_m) = C,$$

where p_{hm} is the Lagrange multiplier attached to the constraint. Here it is defined by equation (8) as the (average) price of the composite commodity $C = C(C_d, C_m)$.

Equations (5) and (6) are identities defining the auxiliary variables p_m and p_e.

Equation (11), $p_{hm} = 1$ sets the general price level, as usual in general equilibrium models. For, as we know, the equations defining the necessary conditions of equilibrium alone do not determine the general price level. The numeraire in this case is the composite commodity C, the maximand in the programming problem.

Note also that conditions (9) and (10) could be replaced by alternative necessary conditions. It can be shown, for example, that they could be replaced by the following derived, conditional import and export functions, as will be done in the general equilibrium model:

$$Z = r_{e0} \cdot \left(\frac{p_d}{p_e}\right)^{\nu} \cdot C_d, \qquad C_m = r_{m0} \cdot \left(\frac{p_d}{p_m}\right)^{\mu} \cdot C_d.$$

All these confirm that equation system GEM VI is equivalent to the condition of general equilibrium of a perfect market economy, except only for one alien condition: equation (6), the formation of the world market price of export, which contains the optimal tariff (tax). This can be however easily modified, changing it for $p_e = v \cdot p^{we}(Z)$.

10. Illustrative model simulations

We have calibrated the GEM VI model using the data of the CGE model presented in Zalai and Révész (2016). We assumed that the observed data represent a competitive general equilibrium, thus, we calibrated the model version in which the optimal tariff was absent. In order to be consistent with the benchmark data, the trade balance requirement, which was assumed to be zero in or models, has been replaced by the observed trade balance. The solution of the calibrated version has to reconstruct the benchmark data of the variables, i.e., their base values. The values of the calibrated parameters can be seen in Table 2, whereas Table 3 contains the base values of the variables (see column GEM0) and the simulation results, as well as the actual value of the export demand price elasticity and the elasticity of CET function, which may be different in various simulation runs.

Table 2. The values of the calibrated parameters

Notation	Parameter	Value
$\bar{Y}$	output (10^{12} HUF)	55.12
D_e	balance of trade (10^{12} HUF)	-2.13
z_d	scale parameter of export (10^{12} HUF)	20.37
ε	export demand price elasticity	-4
v	elasticity of CET function	-0.25
a	share par. of C_d in CET function	6.33
b	share par. of Z in CET function	53.61
μ	elasticity of CES function	0.5
α_d	share par. of C_d in CES function	0.43
α_m	share par. of M in CES function	0.12

The GEM VI model versions, in which the optimal tariff is omitted, will be simply referred to as GEM-Nr, where Nr = 0, 1, 2, … , 10. The simulations presented will be done, with one exception, with this version of GEM VI, assuming different values and combinations of the elasticity parameters, which affect the export demand or supply.

As it was shown, the version of GEM VI with optimal tariff is equivalent to the optimality conditions of NLP V, i.e., of the (Pareto-) optimal solution of the given resource allocation problem. This will be referred to simply as NLP in our simulation exercise. The calibrated parameters used in GEM0 provide the base parameters for the NLP models as well, whereas the optimal solution of NLP V (NLP0) serves the base values of the variables. In Table 3/A or B (see under GEM0, NLP0, % column heads) one can see the base values of the variables in both models, and their differences.

Table 3/A: The effect of 2% increase in world market import price level or in export demand (**bold**: absolute values, rest: percentage changes)

Notation	Indicator	Base values			Simulation results									
		GEM0	NLP0	diff. %	NLP-1	GEM-1	GEM-2	GEM-3	GEM-4	GEM-5	GEM-6	GEM-7	GEM-8	GEM-9
ν	elasticity of CET function	**-0.25**	**-0.25**	0.00	**-0.25**	**-0.25**	**-0.25**	**-0.25**	**-2.5**	**-2.5**	**-2.5**	**-5**	**-5**	**-5**
ε	export demand price elasticity	**-4**	**-4**	0.00	**-4**	**-4**	**-8**	**-1**	**-4**	**-8**	**-1**	**-4**	**-8**	**-1**
					2% increase in world market import price level ($\hat{p}^{wm}$)									
C_d	output sold at home*	**34.75**	**35.33**	1.67	-0.11	-0.12	-0.12	-0.14	-0.31	-0.30	-0.41	-0.34	-0.33	-0.47
Z	output exported*	**20.37**	**19.73**	-3.14	0.23	0.21	0.21	0.23	0.53	0.51	0.71	0.59	0.56	0.80
M	imported products*	**18.54**	**18.06**	-2.59	-1.78	-1.79	-1.77	-1.96	-1.53	-1.48	-1.96	-1.49	-1.43	-1.96
C	total domestic use* (welfare)	**53.29**	**53.37**	0.14	-0.71	-0.71	-0.70	-0.78	-0.74	-0.71	-0.96	-0.74	-0.72	-0.99
p_d	domestic price of output#	**1**	**0.97**	-3.00	-1.21	-1.17	-1.15	-1.28	-0.85	-0.82	-1.09	-0.80	-0.77	-1.05
p_m	domestic price of import#	**1**	**1.06**	6.00	2.18	2.21	2.18	2.43	1.61	1.55	2.06	1.52	1.46	1.99
p_a	average price of output#	**1**	**0.91**	-9.00	-0.79	-0.68	-0.67	-0.74	-0.73	-0.70	-0.92	-0.74	-0.71	-0.96
p_e	domestic price of export#	**1**	**0.80**	-20.00	0.12	0.16	0.15	0.18	-0.52	-0.50	-0.65	-0.62	-0.60	-0.80
p^{we}	world market price of export #	**1**	**1.01**	1.00	-0.06	-0.05	-0.03	-0.23	-0.13	-0.06	-0.70	-0.15	-0.07	-0.79
v	exchange rate#	**1**	**1.06**	6.00	0.18	0.21	0.18	0.42	-0.38	-0.44	0.06	-0.47	-0.53	-0.01
	* 10^{12} HUF #index				*2% increase in the volume (scale parameter) of export demand* (Z_{d0})									
C_d	output sold at home*	**34.75**	**35.33**	1.67	0.03	0.04	0.02	0.16	0.09	0.05	0.49	0.10	0.05	0.56
Z	output exported*	**20.37**	**19.73**	-3.14	-0.07	-0.06	-0.03	-0.28	-0.16	-0.08	-0.84	-0.17	-0.08	-0.95
M	imported products*	**18.54**	**18.06**	-2.59	0.49	0.49	0.24	2.20	0.41	0.20	2.20	0.40	0.19	2.20
C	total domestic use* (welfare)	**53.29**	**53.37**	0.15	0.19	0.20	0.10	0.86	0.20	0.10	1.08	0.21	0.10	1.12
p_d	domestic price of output#	**1**	**0.97**	-3.00	0.33	0.32	0.16	1.40	0.22	0.11	1.17	0.21	0.10	1.13
p_m	domestic price of import#	**1**	**1.06**	6.00	-0.59	-0.59	-0.29	-2.60	-0.41	-0.20	-2.18	-0.39	-0.19	-2.09
p_a	average price of output#	**1**	**0.91**	-9.00	0.20	0.17	0.08	0.75	0.18	0.09	0.97	0.19	0.09	1.01
p_e	domestic price of export#	**1**	**0.80**	-20.00	-0.07	-0.08	-0.04	-0.37	0.12	0.06	0.63	0.15	0.07	0.82
p^{we}	world market price of export #	**1**	**1.01**	1.00	0.51	0.51	0.25	2.29	0.54	0.26	2.87	0.54	0.26	2.98
v	exchange rate#	**1**	**1.06**	6.00	-0.59	-0.59	-0.29	-2.60	-0.41	-0.20	-2.18	-0.39	-0.19	-2.09

Table 3/B: The effect of 2% increase in world market import price level or in export demand (**bold**: absolute values. rest: percentage changes)

Notation	Indicator	Base values			Simulation results									
		GEM0	NLP0	diff. %	NLP-1	GEM-1	GEM-4	GEM-7	GEM-2	GEM-5	GEM-8	GEM-3	GEM-6	GEM-9
ν	elasticity of CET function	**-0.25**	**-0.25**		**-0.25**	**-0.25**	**-2.5**	**-5**	**-0.25**	**-2.5**	**-5**	**-0.25**	**-2.5**	**-5**
ε	export demand price elasticity	**-4**	**-4**		**-4**	**-4**	**-4**	**-4**	**-8**	**-8**	**-8**	**-1**	**-1**	**-1**
					2% increase in world market import price level ($\hat{p}^{wm}$)									
C_d	output sold at home*	**34.75**	**35.33**	1.67	-0.11	-0.12	-0.31	-0.34	-0.12	-0.30	-0.33	-0.14	-0.41	-0.47
Z	output exported*	**20.37**	**19.73**	-3.14	0.23	0.21	0.53	0.59	0.21	0.51	0.56	0.23	0.71	0.80
M	imported products*	**18.54**	**18.06**	-2.59	-1.78	-1.79	-1.53	-1.49	-1.77	-1.48	-1.43	-1.96	-1.96	-1.96
C	total domestic use* (welfare)	**53.29**	**53.37**	0.14	-0.71	-0.71	-0.74	-0.74	-0.70	-0.71	-0.72	-0.78	-0.96	-0.99
p_d	domestic price of output#	**1**	**0.97**	-3.00	-1.21	-1.17	-0.85	-0.80	-1.15	-0.82	-0.77	-1.28	-1.09	-1.05
p_m	domestic price of import#	**1**	**1.06**	6.00	2.18	2.21	1.61	1.52	2.18	1.55	1.46	2.43	2.06	1.99
p_a	average price of output#	**1**	**0.91**	-9.00	-0.79	-0.68	-0.73	-0.74	-0.67	-0.70	-0.71	-0.74	-0.92	-0.96
p_e	domestic price of export#	**1**	**0.80**	-20.00	0.12	0.16	-0.52	-0.62	0.15	-0.50	-0.60	0.18	-0.65	-0.80
p^{we}	world market price of export #	**1**	**1.01**	1.00	-0.06	-0.05	-0.13	-0.15	-0.03	-0.06	-0.07	-0.23	-0.70	-0.79
v	exchange rate#	**1**	**1.06**	6.00	0.18	0.21	-0.38	-0.47	0.18	-0.44	-0.53	0.42	0.06	-0.01
	* 10^{12} HUF #index				*2% increase in the volume (scale parameter) of export demand (Z_{d0})*									
C_d	output sold at home*	**34.75**	**35.33**	1.67	0.03	0.04	0.09	0.10	0.02	0.05	0.05	0.16	0.49	0.56
Z	output exported*	**20.37**	**19.73**	-3.14	-0.07	-0.06	-0.16	-0.17	-0.03	-0.08	-0.08	-0.28	-0.84	-0.95
M	imported products*	**18.54**	**18.06**	-2.59	0.49	0.49	0.41	0.40	0.24	0.20	0.19	2.20	2.20	2.20
C	total domestic use* (welfare)	**53.29**	**53.37**	0.15	0.19	0.20	0.20	0.21	0.10	0.10	0.10	0.86	1.08	1.12
p_d	domestic price of output#	**1**	**0.97**	-3.00	0.33	0.32	0.22	0.21	0.16	0.11	0.10	1.40	1.17	1.13
p_m	domestic price of import#	**1**	**1.06**	6.00	-0.59	-0.59	-0.41	-0.39	-0.29	-0.20	-0.19	-2.60	-2.18	-2.09
p_a	average price of output#	**1**	**0.91**	-9.00	0.20	0.17	0.18	0.19	0.08	0.09	0.09	0.75	0.97	1.01
p_e	domestic price of export#	**1**	**0.80**	-20.00	-0.07	-0.08	0.12	0.15	-0.04	0.06	0.07	-0.37	0.63	0.82
p^{we}	world market price of export #	**1**	**1.01**	1.00	0.51	0.51	0.54	0.54	0.25	0.26	0.26	2.29	2.87	2.98
v	exchange rate#	**1**	**1.06**	6.00	-0.59	-0.59	-0.41	-0.39	-0.29	-0.20	-0.19	-2.60	-2.18	-2.09

As can be expected, due to the optimal tariff (25% in the base case), the export volume is smaller, therefore – because of the trade balance condition – the import volume diminishes too, and domestic sale increases. The positive effect of the optimal tariff shows up in the (slightly) bigger volume of the total domestic use, which can be interpreted as a measure of welfare level in this stylised model. The domestic price of export decreases by 20%, which is partly counterbalanced by the 6% increase of the exchange rate, which makes the domestic price of import increase too. The corresponding changes in the other prices follow logically the above changes. (The price level was set by the average price level of the commodity supply on the home market, $p_{\mathrm{hm}} = 1$.)

The next two columns in Table 3 (see under column heads NLP-1 and GEM-1) show the results of four comparative static simulations. In the upper part of the table the effects of a negative shock is illustrated, those of 2% increase in the world market price of import. In the lower part of the table the effects of a positive shock, 2% growth of the scale parameter of export demand function, i.e., in the volume of export demand can be seen. These two shocks were analysed with both the NLP and GEM model (NLP-1 and GEM-1).

The effect of the increase in world market price of import shows up directly in its increasing (relative) domestic price level and in its decreasing volume, as well as in rising exchange rate, reflecting the increased scarcity of the foreign exchange. Because of the higher import price and the fixed balance of trade, the volume of export must increase. Subsequently, the world market price of export will decrease. However, its domestic value must exceed the price of the output on the domestic market, which is secured by the increasing exchange rate. The level of welfare, i.e., the volume of the total domestic use will obviously decrease, and the other variables adjust adequately to the above changes.

In the case of the positive shock, the increasing export demand makes foreign exchange less scarce, and consequently, the rate of foreign exchange diminishes. This affects positively the import, its volume increases, and – due to the fixed balance of trade, the volume of export must decrease and its world market price will thus increase. All these require the domestic price of export to decrease compared to the price of the output on domestic market. (In fact it decreases relative to p_{hm} too). The total volume of domestic use, i.e., the welfare level increases as a result of the positive shock.

In the subsequent simulations from GEM-1 to GEM-9 only the CGE model was used. In these runs the same negative and positive shocks were assumed as before, but the assumed value of the price elasticity of the export supply (the elasticity parameter in the CET function) and the export demand (the elasticity parameter in the CES function), and their combination were varied in the different runs.

These simulations illustrate thus the sensitivity of the solutions with respect to the price elasticities of the export supply (ν) and demand (ε). Table 3/A and B contain the same simulation results but in different order, as can be seen: in Table 3/A sorted by the value of ν, while in Table 3/B sorted by the value of ε.

In simulations 1-3 in Table 3/A the value of ν was set to –0.25, assuming relatively high adjustment flexibility in the domestic and export composition of the output, as in Zalai and

Révész (2016). In the simulations 4-6 and 7-9 it was set to –2.5 and –5, respectively, which imply lower flexibility. These values were used in a similar simulation exercise in Zalai (1982). Within these groups of three simulations the price elasticity of export demand, ε was set first to –4 as in Zalai and Révész (2016), which indicates a relatively flexibility, next to –8, an even higher value, and third to –1.

Note that –1 is already an extreme borderline. At this value the export revenue remains the same as the export volume changes in any direction, and as this elasticity exceeds –1 the export revenue decreases as its volume increases. From this it follows that the size of the elasticity of export demand should in normal case be smaller than –1 (larger than 3 in absolute value).

Table 3/A makes more transparent the effect of the change in elasticity of export demand (ε), whereas Table 3/B the effect of the change in the export supply (ν). Summarizing the above elasticity settings we can see that they form all possible pairs (Cartesian product) of the 3x3 different values of the supply and demand elasticities. In each simulation we recalibrated the models with the corresponding above elasticities.

Note, that from the $p^{\mathrm{we}}(Z) = d_0 \cdot Z^{1/\varepsilon}$, $d_0 = Z_{\mathrm{d0}}^{-1/\varepsilon} \cdot \hat{p}^{\mathrm{we}}$ relationship (see further above) one can see that whatever is the export volume, it can be sold at -1/ε higher price (p^{we}) than before (i.e. than in the corresponding simulations without this assumption). Therefore – as opposed to the import price scenarios – the terms of trade effect depends on the assumed magnitude of the export demand price elasticity. Note that 2% increase of the scale parameter implies 2% increase in the export price only if the export demand price elasticity is –1. In other words, only in this case will it produce similar magnitude change in the welfare (C) as 2 increase in the import price, but of course in the opposite direction (since increasing import price deteriorates the terms of trade, while increasing export price improves it). These can be seen by comparing the results of simulations 3, 6 and 9 in the two shocks (-0.78; -0.96; -0.99 versus 0.86; 1.08; 1.12).

Note that in the case of the NLP-1 and GEM-1 simulations the percentage changes in the variables are practically the same despite the differences in their base values. That is, the comparative static analyses produced basically the same results both with the GEM equilibrium and the NLP programming models. This seems to be not so surprising, since the only difference between the two models is caused by the optimal tariff, the necessary conditions of the optimal and the competitive equilibrium solution are otherwise equivalent. In the case of a full-fledged CGE model, containing taxes, subsidies and complex income distribution schemes, one can also see close similarity between the direction and size of changes in the main macroeconomic indicators comparing the comparative static simulations done with a full-fledged CGE, containing taxes, subsidies and complex income distribution schemes, and an optimal resource allocation model without the latter components otherwise based on the same data (more about this see Zalai, 2012). This demonstrates the robustness of the 'identity-centered models' (the central model is based on the identities for each sector, see Almon, 1995).

In this simple model, however, especially since the production is fixed, the size of the elasticity parameter of the CET function (ν) plays important role and can make larger differences between some macroeconomic indicators gained from the GEM and the NLP model. For this elasticity parameter determines how far the ratio of export and domestic supply can depart from the base case.

Table 4: The effect of the size of the CET elasticity parameter on some indicators gained from the GEM and the NLP model

Notation	Indicator	GEM0	NLP0	GEM-1	NLP-1	GEM-10	NLP-10
ν	elasticity of CET function	**–**	**–0.25**	**–0.25**	**–0.25**	**–2**	**–2**
				2% increase in import price level			
Z	output exported	**20.37**	**19.73**	0.21	0.23	0.51	-3.87
M	imported products	**18.54**	**18.06**	-1.79	-1.78	-1.55	-5.11
C	total domestic use (welfare)	**53.29**	**53.37**	-0.71	-0.71	-0.74	-0.53
v	exchange rate	**1**	**1.06**	0.21	0.18	-0.34	7.74
				2% increase in export demand			
Z	output exported	**20.37**	**19.73**	-0.06	-0.07	-0.15	-4.55
M	imported products	**18.54**	**18.06**	0.49	0.49	0.42	-3.25
C	total domestic use (welfare)	**53.29**	**53.37**	0.20	0.19	0.20	0.41
v	exchange rate	**1**	**1.06**	-0.59	-0.59	-0.43	7.72

Table 4 illustrates this effect comparing the results gained in the first simulation from the GEM and the NLP model at two different values of ν, -0.25 and 2, at fixed export demand price elasticity ($\varepsilon = -4$), see GEM-10 and the NLP-10. One can see the full simulation results in the case of ν = -0.25 in the two versions of Table 3. When $\nu = -0.25$ (a case close to fixed ratio), the change in export supply is constrained into a very narrow range, therefore the optimal tariff effect remains quite limited and the results gained from the GEM and the NLP model are close to each other. When $\nu = -2$, export supply can adjust much more flexibly to the changes in both models. The results of the GEM model are already close to those gained at $\nu = -2.5$ or –5 (see GEM-4 in Tables 3). Note also that they do not change much further, when ν reaches –5. See the percentage changes of some variables at $\nu = -2.5$ and –5 (the latter in brackets) below.

	Z	M	C	v	(Z	M	C	v)
increase in import price:	0.53;	-1.53;	-0.74;	-0.38;	(0.59;	-1.49;	-0.74;	-0.47);
increase in export demand:	-0.16;	0.41;	0.20;	-0.41;	(-0.17;	0.40;	0.21;	-0.39).

The results of the NLP model differ quite significantly from those the GEM model, due to the effect of the optimal tariff. What is interesting to see is that differences between the GEM and the NLP model results are almost the same in the case of all three scenarios (the base case, the 2% increase in import price and export demand). The difference in the welfare level resulted in the NLP model is 0.14-0.15% higher than in the GEM model in the case of $\nu = -0.25$ (see in Table 3), whereas it is 0.35-0.36% higher in the case of $\nu = -2$ (not shown in the tables).

It is also interesting to observe that in the NLP model the foreign trade flows do not change always monotonously with ν. While in the import price increase scenario the export grows at small elasticity values and decreases considerably at higher values, and the opposite applies to the import: in the case of increasing export demand import increases at small ν values and decreases at higher ones.

The main observations about the simulation results can be summarized in the following:

a) In the first series of the CGE-model simulations the 2% increase in the world import prices resulted in 1.43 – 1.96% decrease in the import demand. This is the combined effect of the rather small (–0.5) import price elasticity and the 0.7 – 1% fall in the consumption. The fall of the consumption (the objective in the NLP V model) is the necessary effect of the assumed deterioration of the terms of trade. In absolute terms, the 370 Bln HUF terms of trade loss resulted in a 370 – 530 Bln HUF reduction of the consumption. Higher reduction occurred when the export demand price elasticity was set to 1. In other words, to maintain the original trade balance requires larger export (note that the 1.43 – 1.96% decrease in the import demand combined with the 2% world price increase and thus the cost of import increases, although in the 1.96 case only slightly), but to sell this higher amount requires lowering the offer price by a magnitude consistent with the $-1/\varepsilon$ elasticity. Clearly, this necessary price cut is the highest when $\varepsilon = -1$. In that case the export revenue does not increase and the somewhat higher export volume is a vain effort, which is due only to the real devaluation of the currency, more precisely, to the favourable changes in the relative price term of the export supply function. Also note that the *real* exchange rate of the foreign currency, computed as the ratio of the nominal exchange rate and the price of the domestic product on the home market, increases.
b) In the second series of the CGE-model simulations the 2% increase in the scale parameter of the export demand improves the terms of trade. Therefore the quantity of the import could increase, induced mostly by the lower foreign exchange rate. Similarly exports could be cut, most significantly in the $\varepsilon = -1$ case, when it did not cause revenue loss. As a consequence, higher part of the (fixed) output could be sold on the home market. Since both the domestic and import component of the domestic use (i.e. consumption) increased, the aggregate level of consumption increased as well by 0.2 – 1.12%. Not surprisingly, the highest increases showed up in those simulations where the $\varepsilon = -1$ assumption was applied.
c) Only simulation GEM-1 is directly comparable with the similar simulation based on a one-sector model with neoclassical closure, fixed trade balance and 2% import price increase (see Table 4 in Zalai and Révész, 2016). Comparable the corresponding import, export, domestic use, exchange rate etc. figures one can see that the results are exactly the same, despite the fact that the one-sector model of the referred article is more elaborated from the point of view of the production function (factors are distinguished), final demand (investment and government consumption are also separated out) and income distribution (income tax, savings rate). This demonstrates that in such simple one-sector models the neoclassical closure affects only the allocation of the income among the components of the final demand without changing the foreign trade flows.

11. Conclusion

We will leave the further analysis for the reader. Here we can just summarize our exercise by pointing out the following:

- the simulation results proved to be rather sensitive to the export supply- and demand elasticities
- their effect depends very much on each other (they act in synergy or may block each other)
- nevertheless the direction of the changes are mostly rather independent of the chosen elasticity values, provided they are in the reasonable range
- the models performed well, according to the expectations based on the above methodological discussion of the matter

Given the above conclusions and the fact that the model is programmed in a rather transparent way in GAMS (see the NLP-CGE-models.gms main segment and its ResultNLPCGE.gms auxiliary file and the CGENLPruns.xls output-file) the model is suitable both for testing various parameter estimations for more complex models and for being a practical tool of university teaching of macroeconomic model building.

REFERENCES

Almon, C. (1995) Identity-centered Modeling in the Accountant of SNA Based Model, INFORUM Working Paper, Series 95, University of Maryland, on-line access: http://inforumweb.umd.edu/Workpapr.html.

Armington, P. (1969) A Theory of Demand for Products Distinguished by Place of Production. IMF Staff Papers 16, pp, 159-178.

Blitzer, C-R., P.C. Clark and L. Taylor, eds (1975) Economy-Wide Models and Development Planning. Oxford: Oxford University Press.

Browne, F.X. (1982) Modelling Export Prices and Quantities in a Small Open Economy. The Review of Economics and Statistics, Vol. LXIU, No. 2.

Dixit, A., and V.D. Norman (1980) Theory of International Trade. Cambridge: Cambridge University Press.

Ginsburgh, V. and J. Waelbroeck (1981) Activity Analysis and General Equilibrium Modelling. North-Holland Publishing Company, Amsterdam.

Goldstein, M. and M.S'. Khan (1978) The Supply and Demand for Exports: A Simultaneous Approach. The Review of Economics and Statistics. Vol. LX., No. 2.

Hickman, 3. G. and L.J. Lau (1973) Elasticities of Substitution and Export Demand in a World Trade Model. European Economic Review. Vol. 4, No. 12.

Houthakker, H.S. and S.P. Magee (1969) Income and Price Elasticities in World Trade. The Review of Economics and Statistics. Vol. LI, No. 2.

Limão, N. "Optimal tariffs." The New Palgrave Dictionary of Economics. Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan, 2008.

Lundgren, S. (1982) A Mevhod for Integrating Activity Analysis Submodels with Neoclassical General Equilibrium Models. WP-82-44. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Sato, K. (1977) The Demand Function for Industrial Exports: A Cross-Country Analysis. The Review of Economics and Statistics, Vol. LIX, No. 4.

Srinivasan, T.N. (1975) The Foreign Sector in Planning Models. In (2), pp. 155-176.

Srinivasan, T.N. (1982) International Factor Movements, Commodity Trade and Commercial Policy. Discussion Paper, New Haven: Ȳale University, Economic Growth Center.

Stone, J.A. (1979) Price Elasticities of Demand for Imports and Exports. Industry Estimates for the U.S., the E.E.C. and Japan. The Review of Economics and Statistics, Vol. LXI, No. 2.

Taylor, L. (1975) Theoretical Foundations and Technical Implications. In (2), pp. 33-110,

Zalai, E. (1982) Foreign Trade in Macroeconomic Models: Equilibrium, Optimum, and Tariffs. WP-82-132, Laxenburg, Austria: International Institute for Applied Systems Analysis.

Zalai Ernő (2012) Matematikai közgazdaságtan II.: Többszektoros modellek és makrogazdasági elemzések (Mathematical Economics II: Multisectoral Models and Macroeconomic Analyses). Budapest: Akadémiai Kiadó

Zalai, E. and Révész, T. (2016) The issue of macroeconomic closure revisited and extended. Acta Oeconomica, 2016, vol. 66, No. 1