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Corrigendum to "Production in advance versus production to order" [J. Econ. Behav. Organ. 54 (2004) 191-204]

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Recently, Montez and Schutz (2018) pointed out that restricting the firms' prices to $[c, p^m]$ changes the mixed-strategy equilibrium of the production in advance game for the case of large capacities in Tasnádi (2004) Section 4. Therefore, in Tasnádi (2004) it was erroneously stated that "... we may assume in the following without loss of generality that the set of prices equals $[c, p^m]$." Nevertheless, the analysis remains basically valid without this restriction because the results rely on $\widetilde{p}_\mu < p^m$, and Lemma 2 remains the only place where a reference to p^m is needed.

Dropping the superfluous assumption, requires replacing the occurrences of p^m with b in the following definitions:

- $S = [c, b] \times [0, k]$,
- for any price $p \in [c, b]$ we denote by $s_\mu(p) \subseteq [0, k]$ the set of those quantities $q \in [0, k]$ for which $(p, q) \in \text{supp}(\mu)$,
- $\mu^p(B) = \mu(B \times [0, k])$ for any Borel set $B \subseteq [c, b]$,
- $\widetilde{p}_\mu = \sup\{p \in [c, b] \mid \mu^p([c, p)) = 0\}$,
- $\widehat{p}_\mu = \inf\{p \in [c, b] \mid \mu^p((p, b]) = 0\}$, and
- $p^k = \sup\{p' \in [\widetilde{p}, b] \mid \forall p \in [\widetilde{p}, p') : s(p) = \{\min\{k, D(p)\}\} \text{ and } \mu^p(\{p\}) = 0\}$.

In addition, in the statements of Lemmas 2 and 4 as well as in the second paragraphs of the respective proofs "... there exists a price $p' \in (\widetilde{p}_\mu, p^m]$ such that ..." needs to be replaced with "... there exists a price $p' \in (\widetilde{p}_\mu, b]$ such that ..."

The end of the proof of Lemma 2 has to be modified by referring to strict concavity instead of montonicity because we are no longer restricting the strategy set to $[c, p^m]$. In particular, the last sentence in the proof of Lemma 2 has to replaced with "Clearly, $\lim_{\gamma \to \alpha+0} \pi_1((\gamma, D(\gamma)), \mu) = \pi_1((\alpha, D(\alpha)), \mu)$ since $\mu^p(\{\alpha\}) = 0$, and $\lim_{\gamma \to \beta-0} \pi_1((\gamma, D(\gamma)), \mu) = \pi_1((\beta, D(\beta)), \mu)$ since $\mu^p(\{\beta\}) = 0$. From $\alpha\mu^p((\alpha, \widehat{p}]) - c > 0$

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it follows that $\pi_1((\gamma, D(\gamma)), \mu)$ is strictly concave on $[\alpha, \beta]$ in γ because of Assumption 1 and therefore, $\pi_1((\gamma, D(\gamma)), \mu)$ has a unique maximizer in $[\alpha, \beta]$. We conclude that $s(\alpha) = \varnothing$ or $s(\beta) = \varnothing$, a contradiction."

Finally, though the statement "It can be verified that the definition of p^k and $p^k \mu^p([p^k, \widehat{p}]) - c > 0$ implies $\mu^p(\{p^k\}) > 0$." in the last paragraph of the proof of Lemma 3 remains true even after not limiting the strategy set to $[c, p^m]$, its proof does not remain that simple. Therefore, we are providing a proof of this statement in this corrigendum.

Suppose that $\mu^p(\{p^k\}) = 0$. Then there exists a $p^* \in (p^k, b]$ such that $p\mu^p((p, \widehat{p}]) - c > 0$ for all $p \in [p^k, p^*]$ by the continuity of $\mu^p((p, \widehat{p}])$ at p^k. Therefore, $\pi_1((p, q), \mu)$ is strictly increasing in q for any $p \in [p^k, p^*]$ since the two integrals in (4) are increasing in q and the third expression is strictly increasing in q, which in turn implies that either $s(p) = \varnothing$ or $s(p) = \{D(p)\}$ for any $p \in [p^k, p^*]$. If there exists a price $p^{**} \in (p^k, p^*]$ such that $s(p^{**}) = \{D(p^{**})\}$, then following the third and forth paragraphs of the proof of Lemma 2, we obtain that $\mu^p(\{p\}) = 0$ and $s(p) = \{D(p)\}$ for all $p \in [p^k, p^{**}]$, which contradicts the definition of p^k. Thus, we must have $\mu^p([p^k, p^*]) = 0$.

Let $\breve{p} = \inf\{p \in (p^*, \widehat{p}] \mid s(p) \neq \varnothing\}$. Assume that there is no atom at $\breve{p}$. Then

$$\breve{p}\mu^p((\breve{p}, \widehat{p}]) - c = \breve{p}\mu^p\left(\left(p^k, \widehat{p}\right]\right) - c > p^k \mu^p\left(\left(p^k, \widehat{p}\right]\right) - c > 0$$

and we must have either $s(\breve{p}) = \{D(\breve{p})\}$ or $s(\breve{p}) = \varnothing$ since the two integrals in (4) are increasing in q and the third expression is strictly increasing in q. In the former case following again the third and forth paragraphs of the proof of Lemma 2, we obtain a contradiction with the definition of p^k, while in the latter case (as it can be checked) the profits are continuous at $\breve{p}$ from which we can arrive to a contradiction by the strict concavity of $\pi_1((p, D(p)), \mu) = (p\mu^p((p, \widehat{p}]) - c) D(p)$ on $[p^k, \breve{p}]$.

Assume that there is an atom at $\breve{p}$ and pick a $\breve{q} \in s(\breve{p})$. Clearly, we must have $0 < \breve{q} < D(\breve{p})$, since otherwise we obtain a contradiction with the definition of p^k as it has been achieved already twice or with positive equilibrium profits (i.e. $\widetilde{p} > c$). Let $\delta = [p^k \mu^p(p^k, \widehat{p}] - c](D(\breve{p}) - \breve{q})$. Take an arbitrary sequence (p_n) such that $p^k < p_n < \breve{p}$ and $\lim_{n\to\infty} p_n = \breve{p}$. Then

$$\widetilde{\pi} = \pi_1((\breve{p}, \breve{q}), \mu) \leqslant \lim_{n\to\infty} \pi_1((p_n, \breve{q}), \mu) < \lim_{n\to\infty} \pi_1((p_n, D(p_n)), \mu) - \delta$$

by $p_n \mu^p((p_n, \widehat{p}]) - c > p^k \mu^p((p^k, \widehat{p}]) - c > 0$ for any n and the choice of δ; a contradiction.

We conclude that $\mu^p(\{p^k\}) > 0$ must be the case. The same statement also appears in the proof of Proposition 4 and can be established in an analogous way.

References

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