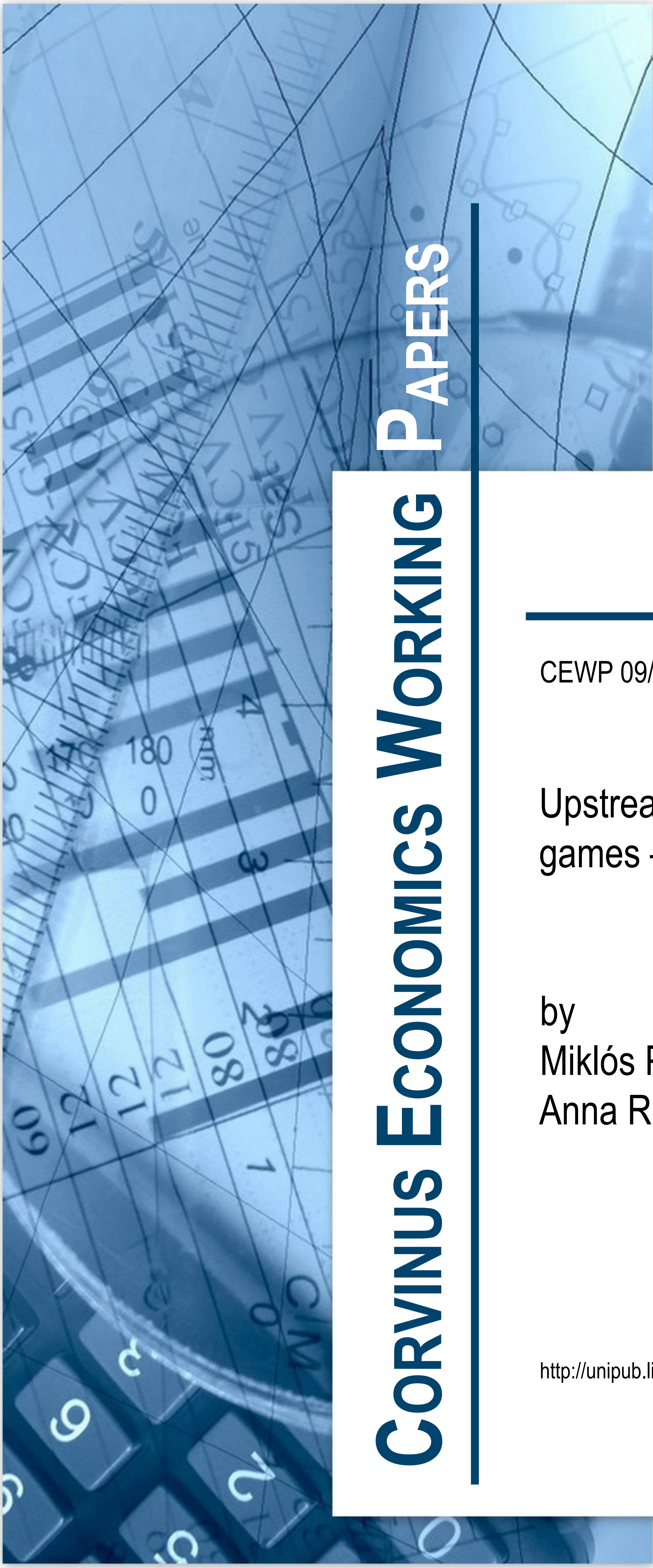

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Upstream responsibility games – the non-tree case

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Upstream responsibility games – the non-tree case*

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Abstract

In this paper the problem of sharing the cost of emission in supply chains is considered. (Gopalakrishnan et al, 2017) focus on allocation problems that can be described by rooted trees, called cost-tree problems, and on the induced transferable utility cooperative games, called upstream responsibility games. This paper generalizes the formal notion of upstream responsibility games to a non-tree model, and provides two (primal and dual) characterizations of the class of these games. Axiomatizations of the Shapley value under both characterizations are also provided.

This is a followup paper of Radványi (2018); Pintér and Radványi (2019).

Keywords: Upstream responsibility games; Cost sharing; Emission; Supply chain; Shapley value; Axiomatization of the Shapley value.

JEL Classification: C71.

1 Introduction

In this paper we consider cost sharing problems related to supply chains. We assign transferable utility (TU) cooperative games (henceforth games) to

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these cost sharing problems. Specializing the problem, we consider energy supply chains with a motivated dominant leader, who has the power to assign the suppliers responsibilities for both direct and indirect emissions. The induced games are called *upstream responsibility games* (Gopalakrishnan et al, 2017).

For an example consider a supply chain where we look at the responsibility allocation of greenhouse gas (GHG) emission among the firms in the chain. One of the main questions is how to share the costs related to the emission among the firms. The supply chain and the related firms (or any other actors) in Gopalakrishnan et al (2017) are represented by a rooted tree.

The root of the tree represents the end product produced by the supply chain. The root is connected to only one node which is the leader of the chain. Each further node represents one firm, and the edges of the rooted tree represent the producing process among the firms with the related emissions. The goal is to share the responsibility of the emission while embodying the principle of upstream emission responsibility.

In this paper we rely on the TU-game model of Gopalakrishnan et al (2017), called GHG Responsibility-Emissions and Environment (GREEN) game. Gopalakrishnan et al use the Shapley value (Shapley, 1953) as an allocation method, consider some pollution related properties that an emission allocation rule should meet, and provide several axiomatizations as well. In contrast we generalize the model of the upstream responsibility games to a non-tree approach, which model relies on the responsibility matrix of the emission of pollutants. We offer two characterizations of the generalized model, using the unanimity games on the one hand and the duals of the unanimity games on the other hand. In both cases we provide different axiomatizations of the Shapley value as well.

The setup of this paper is as follows: in the following section we introduce the notions and notations, then we define the upstream responsibility game in the special case where the supply chain can be represented by a rooted tree. In Section 3 we generalize our model behind the supply chain, that is, we characterize the class of upstream responsibility games where a graph is no longer needed to be a rooted tree, instead we use the so called responsibility matrix which allows for non-tree structures too. In Section 4 we introduce the axiomatizations we use for the Shapley value on the class of (generalized) upstream responsibility games.

2 Upstream Responsibility Games

2.1 Preliminaries

Notions, notations: $\#N$ is for the cardinality of set N, and 2^N denotes the class of all subsets of N. $A \subset B$ means $A \subseteq B$, but $A \neq B$. $A \uplus B$ stands for the union of disjoint sets A and B.

A *graph* is a pair $G = (V, E)$, where the elements of V are called *vertices* or nodes, and E stands for the ordered pairs of vertices, called *edges* or arcs. A *rooted tree* is a graph in which any two vertices are connected by exactly one path, and one vertex has been designated the root denoted as node root. In the case of a supply chain consisting several entities, which are cooperating in the production of a final product, the manufacturing process can be modeled with a directed rooted tree. The set of nodes V represents the entities, henceforth players, and a directed arc emanating from node i towards root represents the activity by player i contributing to the manufacturing of the final product. We assume that only one node emanates arc enters the root, this emanates from node 1 which node represents the end consumer.

The tree-order is the partial ordering on the vertices of a rooted tree with $i \leq j$, if the unique path from j to the root passes through i. The *chain* is a rooted tree such that any vertices $i, j \in V$, $i \leq j$ or $j \leq i$. That is, a chain is a rooted tree with only one "branch". For any pair $e \in E$, $e = \overline{ij}$ means e is an edge between vertices $i, j \in V$ such that $i \leq j$. For each $i \in V$, let $S_i(G) = \{j \in V : j \geq i\}$, that is, for any $i \in V$, $i \in S_i(G)$. For each $i \in V$ let $P_i(G) = \{j \in V : j \leq i\}$, that is, for any $i \in V$, $i \in P_i(G)$. Moreover, for any $V' \subseteq V$, let $(P_{V'}(G), E_{V'})$ be the sub-rooted-tree of (V, E), where $P_{V'}(G) = \cup_{i \in V'} P_i(G)$ and $E_{V'} = \{\overline{ij} \in E : i, j \in P_{V'}(G)\}$.

Let $c \colon E \to \mathbb{R}_+$, c and (G, c) are called *cost function* and *cost-tree* respectively. An interpretation of cost tree (G, c) might be as follows: there is a given product which is produced by a supply chain. $V = N \cup \{\text{root}\}$, N denotes the set of all entities of the supply process. We assume that only one arc emanates form the root and this emanates from the node 1. Node 1, the only node from which an arc goes to the root denotes the end-consumer, and the leaf nodes represents the most upstream members (that is, firms, etc.) of the supply chain. Each edge $e \in E$ is associated with a process in the supply chain emitting a pollution c(e). Let e_i denote the unique edge in the tree T emanating from node i (in the direction of the root). In this case $c(e_i)$ represents the direct pollution associated with e_i, the directly created pollution by node (firm) i. Besides i also can be responsible for the pollution of other processes in the chain. For each node i, $\mathcal{E}_i$ denotes the set of edges whose associated pollution is the direct or indirect responsibility of node i.

Let $N \neq \emptyset$, $\#N < \infty$, and $v : 2^N \to \mathbb{R}$ be a function such that $v(\emptyset) = 0$. Then N, v are called set of players, and *transferable utility cooperative game* (henceforth game) respectively. The class of games with player set N is denoted by $\mathcal{G}^N$.

A game $v \in \mathcal{G}^N$ is *convex*, if for all $S, T \subseteq N$, $v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$, moreover, it is *concave*, if for all $S, T \subseteq N$, $v(S) + v(T) \geq v(S \cup T) + v(S \cap T)$.

The *dual* of a game $v \in \mathcal{G}^N$ is a game $\bar{v} \in \mathcal{G}^N$ such that for all $S \subseteq N$, $\bar{v}(S) = v(N) - v(N \setminus S)$. It is well known that the dual of a convex game is a concave game and vice versa.

Let $v \in \mathcal{G}^N$ and $i \in N$, and $v'_i(S) = v(S \cup \{i\}) - v(S)$, where $S \subseteq N$. The vector v'_i is called player i's *marginal contribution function* in game v. Alternatively, $v'_i(S)$ is player i's marginal contribution to coalition S in game v.

Let $v \in \mathcal{G}^N$, the players $i, j \in N$ are *equivalent* in game v, $i \sim^v j$, if for all $S \subseteq N$ such that $i, j \notin S$, $v'_i(S) = v'_j(S)$.

Let N and $T \in 2^N \setminus \emptyset$, and for all $S \subseteq N$, let

$$u_T(S) = \begin{cases} 1, & \text{if } T \subseteq S, \\ 0 & \text{otherwise.} \end{cases}$$

The game u_T is called *unanimity game* on coalition T.

In this paper we also use the duals of the unanimity games. For any $T \in 2^N \setminus \emptyset$ and for all $S \subseteq N$,

$$\bar{u}_T(S) = \begin{cases} 1, & \text{if } T \cap S \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that every unanimity game is convex, and the duals of the unanimity games are concave.

Henceforth, we assume that in the considered cost-tree problems there are at least two players, that is, $\#V \geq 3$ and $\#N \geq 2$.

2.2 The TU-game model for trees

In the following we introduce the notion of *upstream responsibility games* (URG) (Gopalakrishnan et al, 2017). Let (G, c) be a cost-tree, representing a supply chain, and N be the set of the members of the chain, henceforth players (the vertices but the root). We denote by a_j the pollution associated with arc j. Let $c(\{i\})$ denote the total pollution emission that player i is directly or indirectly responsible for. $\mathcal{E}_i$ represents the set of edges whose associated pollution is the direct or indirect responsibility of player i, $c(\{i\}) =$

$a(\mathcal{E}_i) = \sum_{j \in \mathcal{E}_i} a_j$. For every $S \subseteq N$ let $\mathcal{E}_S$ denote the collection of edges whose associated pollution is the direct or indirect responsibility of the players in S, thus $\mathcal{E}_S = \cup_{i \in S} \mathcal{E}_i$ and the pollution which S is directly or indirectly responsible for is as follows $c(S) = a(\mathcal{E}_S) = \sum_{j \in \mathcal{E}_S} a_j$.

Definition 1 (Upstream Responsibility Game)**.** *For any cost-tree* (G, c)*, let* $N = V \setminus \{root\}$ *be the player set, and for any coalition* S *(the empty sum is* 0*) let the upstream responsibility game be defined as follows*

$$v_{(G,c)}(S) = \sum_{j \in \mathcal{E}_S} a_j \, .$$

Let $\mathcal{G}_G$ *denote the subclass of upstream responsibility games induced by cost-tree problems on a (specific) rooted tree* G*.*

The next example is an illustration of the above definition.

Example 2. Consider the cost-tree in Figure 1, where the rooted tree $G = (V, E)$ is as follows, $V = \{\text{root}, 1, 2, 3\}$, $A = \{\overline{\text{root}1}, \overline{12}, \overline{13}\}$, and the cost (pollution) function c is defined as $c(\overline{\text{root}1}) = a_1 = 2$, $c(\overline{12}) = a_2 = 4$ and $c(\overline{13}) = a_3 = 1$. $\mathcal{E}_1 = \{e_1, e_2, e_3\}$, $\mathcal{E}_2 = \{e_2\}$, $\mathcal{E}_3 = \{e_3\}$.

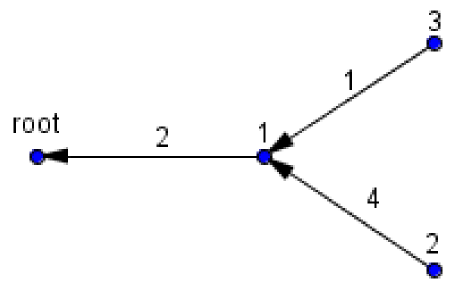

Figure 1: The cost-tree (G, c)

The upstream responsibility game $v_{(G,c)} = (0, 7, 4, 1, 7, 7, 5, 7)$, that is, $v_{(G,c)}(\emptyset) = 0$, $v_{(G,c)}(\{1\}) = 7$, $v_{(G,c)}(\{2\}) = 4$, $v_{(G,c)}(\{3\}) = 1$, $v_{(G,c)}(\{1,2\}) = 7$, $v_{(G,c)}(\{1,3\}) = 7$, $v_{(G,c)}(\{2,3\}) = 5$ and $v_{(G,c)}(N) = 7$. (In this example we assume that for any rooted tree G the associated pollutions sets are as follows $\mathcal{E}_i = P_i(G)$, $i \in V \setminus \{\text{root}\}$.)

In the following section we generalize the model of the upstream responsibility games, in the sense that the game depends only on the responsibility matrix meaning the graph structure is not necessarily a tree. That is we only need to know which player is responsible for which process in the supply chain, and no graph is needed to describe the connection, so in this case we get a more generalized model.

3 Characterization of the class of upstream responsibility games

This section is devoted to defining and analyzing the class of upstream responsibility games. First we define the notion of upstream responsibility game, then in two subsections we give two characterizations of this class, one by the unanimity games, and one by the duals of the unanimity games.

Assume that the class of processes M and the responsibility matrix B, which is an $|N| \times |M|$ binary matrix, $b_{i,j} = 1$ means Player i is responsible for process j and $b_{i,j} = 0$ means Player i is not responsible for process j, are given. Each process $k \in M$ involves footprint or cost of the activities the process defines, it is denoted by a non-negative scalar f_k. Then we can define a TU-game, an upstream responsibility game as follows:

Definition 3. *Let $F \coloneqq \{f_k\}_{k \in M}$, $f_k \geq 0$, $k \in M$, B $|N| \times |M|$ binary matrix are given. Then for each $S \subseteq N$ let*

$$v_{B,F}(S) \coloneqq \sum_{f \in \cup_{i \in S} \mathcal{P}_i} f,$$

where $\mathcal{P}_i \coloneqq \{f_k \in F : b_{i,k} = 1\}$.

Notice that $v_{B,F} \in \mathcal{G}^N$.

Let $\mathcal{G}^N_{UR}$ denote the class of upstream responsibility games with players set N.

In words, the value of a coalition is the sum of the footprints the players from the coalition are responsible for. We assume that at least one player is responsible for each process. Notice that the upstream responsibility games are cost games, the higher the value of a coalition the worse the position of the coalition (it involves more cost).

3.1 The primal characterization of the class of upstream responsibility games

Next we give a characterization of the upstream responsibility games. This characterization is based on the unanimity games.

Corollary 4. *For any upstream responsibility game $v \in \mathcal{G}^N$ it holds that*

$$v = \sum_{T \subseteq N} (-1)^{|T|+1} \left(\sum_{f \in \cap_{i \in T} \mathcal{P}_i} f \right) u_T. \tag{1}$$

Formula (1) is very intuitive, this practically is a sieve formula, and its validity comes directly from Definition 3.

Next we give the class of upstream responsibility games.

Lemma 5. *Let $v \in \mathcal{G}^N$ be a game, and $v = \sum_{T \in N} \alpha_T u_T$. Then v is an upstream responsibility game if and only if*

1. *$T \subseteq S$ implies $|\alpha_T| \geq |\alpha_S|$,*
2. *$\alpha_T > 0$ if and only if $|T|$ is even,*
3. *$\alpha_T < 0$ if and only if $|T|$ is odd.*

Proof. It is the direct corollary of Formula (1). □

Lemma 5 also says that any subgame of an upstream responsibililty game is an upstream responsibility game.

In the following proposition we show that every upstream responsibility game is totally balanced, meaning it is always possible to allocate the total footprint, the total cost of the supply chain, among the players in a stable way.

Proposition 6. *Every upstream responsibility game is totally balanced, but there exist non-negative totally balanced games which are not upstream responsibility games.*

Proof. Notice that because of Lemma 5 it is enough to consider a general upstream responsibility game.

Take an upstream responsibility game $v_{B,F} \in \mathcal{G}^N$, and for each $i \in N$ let

$$x_i := \sum_{\substack{T \subseteq N \\ i \in T}} \frac{\sum\limits_{f \in (\cap_{j \in T} \mathcal{P}_j) \setminus (\cup_{j \in N \setminus T} \mathcal{P}_j)} f}{|T|}. \tag{2}$$

Let $S \subseteq N$, then

$$\sum_{i\in S} x_i = \sum_{i\in S} \sum_{\substack{T\subseteq N\\ i\in T}} \frac{\sum\limits_{f\in(\cap_{j\in T}\mathcal{P}_j)\setminus(\cup_{j\in N\setminus T}\mathcal{P}_j)} f}{|T|}$$
$$\leq \sum_{i\in S}\sum_{f\in\mathcal{P}_i} f = \sum_{f\in\cup_{i\in S}\mathcal{P}_i} f = v_{B,F}(S).$$

Let $N=\{1,2\}$, $v\in\mathcal{G}^N$ be defined as $v=u_{\{1\}}-u_N$. It is easy to check that v is totally balanced, and by Lemma 5 $v\notin\mathcal{G}^N_{UR}$, that is, v is not an upstream responsibility game. □

Finally, by Lemma 5 we have that the class of upstream responsibility games is a convex cone. It is worth noticing that when we consider $\mathcal{G}^N_{UR}$ then the set of process M, the footprints F and the responsibility matrix B are not fixed, those can vary.

Corollary 7. *The class of upstream responsibility games is a convex cone.*

3.2 The dual characterization of the class of upstream responsibility games

Next we give another characterization of the upstream responsibility games. This characterization is based on the duals of the unanimity games. (For the special case on trees see the results of Radványi (2018).)

Corollary 8. *For any upstream responsibility game $v\in\mathcal{G}^N$ it holds that*

$$v = \sum_{\substack{k\in M\\ T:=\{i\in N\colon b_{i,k}=1\}}} f_k \bar{u}_T. \tag{3}$$

The formula (1) is very expressive, it says that each coalition must cover the cost its subgroups are responsible for exclusively. Since M, B and F can vary widely we have the following characterization of the class of upstream responsibility games:

Proposition 9. *The class of upstream responsibility games $\mathcal{G}^N_{UR}$ is the convex cone spanned by the duals of the unanimity games.*

Proof. It is the direct corollary of Corollary 8. □

Again, Proposition 8 says also that any subgame of an upstream responsibility game is an upstream responsibility game. Moreover, since the core of any non-negative weighted sum of the duals of the unanimity games are not empty, we have that

Proposition 10. *The class of upstream responsibility games is a proper subset of the class of totally balanced games.*

4 Axiomatization of the Shapley value

In this section we consider various characterizations of the Shapley value on the class of upstream responsibility games. We split this section into two subsections. In the first one we consider two axiomatizations which are based on the characterization of upstream responsibility game by Corollary 4. In the second one, very similarly, we consider an axiomatization which is based on the characterization of upstream responsibility game by Proposition 8. For an analogy to the case of trees see Pintér and Radványi (2019).

4.1 The Shapley value and the core

First let us define the two following solution concepts which provide solutions for sharing the cost of emissions.

A *solution* on set $A \subseteq \mathcal{G}^N$ ψ is a set-valued mapping $\psi : A \twoheadrightarrow \mathbb{R}^N$, that is, a solution assign a set of allocations to each game. In the following, we define two solutions.

Let $v \in \mathcal{G}^N$ and

$$p^i_{Sh}(S) = \begin{cases} \dfrac{\#S!(\#(N \setminus S) - 1)!}{\#N!}\,, & \text{if } i \notin S, \\ 0 & \text{otherwise.} \end{cases}$$

Then $\phi_i(v)$ the Shapley value (Shapley, 1953) of player i in game v is the p^i_{Sh} expected value of v'_i. In other words

$$\phi_i(v) = \sum_{S \subseteq N} v'_i(S)\; p^i_{Sh}(S)\,. \tag{4}$$

Furthermore, let ϕ denote the Shapley value.

It is obvious from its definition that the Shapley solution is a single valued solution, a single valued solution is called value.

Next, we introduce an other solution, the core (Shapley, 1955; Gillies, 1959). Let $v \in \mathcal{G}^N_{UR}$ be an upstream responsibility game. Then the core of upstream responsibility game v is defined as follows

$$\text{core }(v) = \left\{ x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N), \text{ and for any } S \subseteq N, \sum_{i \in S} x_i \leq v(S) \right\}.$$

The core consists of the stable allocations of the value of the grand coalition, that is, any allocation of the core is such that the allocated cost is the total cost ($\sum_{i \in N} x_i = v(N)$) and no coalition has incentive to deviate from the allocation scheme.

A game is balanced if its core is not empty, moreover it is totally balanced if the core of any of its subgames is not empty.

4.2 Axiomatizations based on the primal representation

First we introduce the axioms we use in this subsection to characterize the Shapley value.

Definition 11. *A value ψ on the class of games $A \subseteq \mathcal{G}^N$ meets*

- *PO (Pareto Optimality or Efficiency), if $\sum_{i \in N} \psi_i(v) = v(N)$, $v \in A$,*
- *ETP (Equal Treatment Property), if for each pair $i, j \in N$ such that $i \sim^v j$ it holds that $\psi_i(v) = \psi_j(v)$,*
- *NN (Non-negativity), if for all $v \in A$ such that $v \geq 0$ it holds that $\psi(v) \geq 0$,*
- *CSE (Coalitional Strategic Equivalence), if for all $v \in A$, $\alpha > 0$ and $T \in \mathcal{N}$ such that $v + \alpha u_T \in A$ it holds that $\psi_i(v + \alpha u_T) = \psi_i(v)$, $i \in N \setminus T$.*
- *WFP (Weak Fairness Property), if for all $v \in A$, $\alpha > 0$, $T \in \mathcal{N}$ such that $v + \alpha u_T \in A$ for all $i, j \in T$ it holds that $\psi_i(v + \alpha u_T) - \psi_i(v) = \psi_j(v + \alpha u_T) - \psi_j(v)$.*

The first three axioms (PO, ETP and NN) are considered from the beginning of the axiomatizations of the Shapley value. The axiom CSE was introduced by Chun (1989). We differ here from Chun's definition but the difference does not deserve to look into the details. The last axiom (WFP) is a new axiom, it is a weakening of the axiom FP (Fairness Property) by van den Brink (2001). The difference between WFP and FP is that FP is applied to cases where instead of αu_T can be any game, and the considered two players are equivalent in the game. It is clear that WFP is a real weakening of FP, but it is very similar in spirit to FP.

Our first characterization theorem is the refinement of Young (1985) and Chun (1989).

Theorem 12. *A value on the class of upstream responsibility games $\mathcal{G}_{UR}^N$ meets the axioms PO, ETP, CSE and WFP if and only if it is the Shapley value.*

Proof. The Shapley value meets the axioms PO, ETP, CSE and WFP: It is left for the reader.

Suppose that ψ is a value on $\mathcal{G}_{UR}^N$ such that it meets the axioms PO, ETP, CSE and WFP. The proof goes by induction on $N(v) := |\{\alpha_T \colon \alpha_T \neq 0\}|$, where $v := \sum_{T \in \mathcal{N}} \alpha_T u_T$ and $\mathcal{N} := \{T \subseteq N \colon T \neq \emptyset\}$.

If $N(v) = 0$, meaning $v = 0$, then by PO and ETP $\psi_i(v) = 0$, $i \in N$, that is, ψ is well-defined.

Suppose that ψ is well-defined for each game $v \in \mathcal{G}_{UR}^N$ such that $N(v) = k$. Take a game $v \in \mathcal{G}_{UR}^N$ such that $N(v) = k + 1$. Take a maximal element T^* from $\{T \in \mathcal{N} \colon \alpha_T \neq 0\}$ ($v := \sum_{T \in \mathcal{N}} \alpha_T u_T$). Then by Lemma 5 $w := \sum_{T \in \mathcal{N} \setminus \{T^*\}} \alpha_T u_T$ is an upstream responsibility game, and $N(w) = k$, hence by the induction hypothesis ψ is well-defined at w.

If $i \notin T^*$, then by the axiom CSE we have that $\psi(v)_i = \psi(w)_i$, that is, $\psi(v)_i$ is well-defined.

For all $i, j \in T^*$ by the axiom WFP we have that $\psi_i(v + \alpha u_T) - \psi_i(v) = \psi_j(v + \alpha u_T) - \psi_j(v)$, meaning we have a system of independent linear equalities with degree of freedom 1. Then by that $\psi(v)_i$ is well-defined for each $i \notin T^*$ we have that $\sum_{i \in T^*} \psi(v)_i$ is well-defined, therefore, we we have a system of independent linear equalities with degree of freedom 0, meaning $\psi(v)$ is well-defined.

Since the Shapley value meets the axioms PO, ETP, CSE and WFP we have that ψ is the Shapley value. $\square$

Next, by means of examples we demonstrate that in Theorem 12 the axioms are independent.

Example 13. PO is missing: Take the value which is the double of the Shapley value. Then it is easy to see that this value meets the axioms ETP, CSE and WFP.

ETP is missing: Let $N = \{1, 2\}$ and ψ be the Shapley value plus $(1, -1)$. Then it is easy to see that this value meets the axioms PO, CSE and WFP.

CSE is missing: Let $\psi(v) = \frac{v(N)}{|N|}$, $v \in \mathcal{G}_{UR}^N$ (egalitarian rule). Then it is easy to see that this value meets the axioms PO, ETP and WFP.

WFP is missing: Let $i^* \in N$ be a player, and let ψ be such that for each $v \in \mathcal{G}_{UR}^N$

$$\psi(v) := \begin{cases} Sh(v), & \text{if } \exists i, j \in N,\ i \neq j,\ i \sim^{v - \alpha_N u_N} j \text{ or} \\ & \alpha_N = 0, \\ & \\ Sh(v - \alpha_N u_N) + \alpha_N \chi_{\{i^*\}} & \text{otherwise,} \end{cases}$$

where Sh is the Shapley value and $\chi_{\{i^*\}}$ is the characteristic vector of $\{i^*\}$.

If v is such that $\nexists i, j \in N$, $i \neq j$, $i \sim^{v - \alpha_N u_N} j$ and $\alpha_N \neq 0$, then ETP does not matter at v, and CSE does not matter between v and $v + \beta_N u_N$ for any $\beta \in \mathbb{R} \setminus \{0\}$. Moreover, by Lemma 5 the property of $\nexists i, j \in N$, $i \neq j$, $i \sim^{v - \alpha_N u_N} j$ is equivalent with $\nexists i, j \in N$, $i \neq j$, $i \sim^{v - \alpha_N u_N + \beta_T u_T} j$, $T \in \mathcal{N} \setminus \{N\}$, $\beta \in \mathbb{R}$ and $v + \beta_T u_T \in \mathcal{G}_{UR}^N$.

Therefore, this value meets the axioms PO, ETP and CSE.

Next we show that we can change ETP to NN in Theorem 12.

Theorem 14. *A value on the class of upstream responsibility games (the players set is N) meets the axioms PO, NN, CSE and WFP if and only if it is the Shapley value.*

Proof. The Shapley value meets the axiom NN: It is left for the reader.

In the proof of Theorem 12 we used the axiom only at the step showing that $\psi(0) = 0$. Notice that by PO and NN we also have that $\psi(0) = 0$. Therefore we can apply the proof of Theorem 12 with this minor modification.

□

Next, by means of examples we demonstrate that the axioms are independent in Theorem 14.

Example 15. PO is missing: See Example 13.

NN is missing: See the ETP is missing example in Example 13.

CSE is missing: See Example 13.

WFP is missing: Let $i^* \in N$ be a player, and let ψ be such that for each $v \in \mathcal{G}_{UR}^N$

$$\psi(v) := \begin{cases} Sh(v), & \text{if } \alpha_{\{i^*\}} = 0, \\ Sh\left(v - \sum\limits_{i^* \in T} \alpha_T u_T\right) + (v(N) - v(N \setminus \{i^*\})\chi_{\{i^*\}} & \text{otherwise.} \end{cases}$$

It is clear that ψ meets PO and NN. Moreover, let $i \in N \setminus \{i^*\}$ and $S \subseteq N \setminus \{i\}$. Then if $i^* \notin S$, then $\psi(v)_i = Sh\left(v - \sum_{i^* \in T} \alpha_T u_T\right)_i = Sh\left(v + \beta_S u_S - \sum_{i^* \in T} \alpha_T u_T\right)_i = \psi(v + \beta_S u_S)_i$, $\beta_S \in \mathbb{R}$ such that $v + \beta_S u_S \in \mathcal{G}^N_{UR}$. If $i^* \in S$, then even $v - \sum_{i^* \in T} \alpha_T u_T = v + \beta_S u_S - \sum_{i^* \in T} \alpha_T u_T$, that is, $\psi(v)_i = \psi(v + \beta_S u_S)_i$, $\beta_S \in \mathbb{R}$ such that $v + \beta_S u_S \in \mathcal{G}^N_{UR}$.

Let $S \subseteq N \setminus \{i^*\}$. Then $\psi(v)_{\{i^*\}} = (v(N) - v(N \setminus \{i^*\}) = (v(N) + \beta_S - v(N \setminus \{i^*\} - \beta_S) = \psi(v + \beta_S u_S)_{\{i^*\}}$, $\beta_S \in \mathbb{R}$ such that $v + \beta_S u_S \in \mathcal{G}^N_{UR}$. Therefore, this value meets the axioms PO, NN and CSE.

4.3 An axiomatization based on the dual representation

Next we introduce a further axiom we use in this subsection.

Definition 16. *A value ψ on the class of games $A \subseteq \mathcal{G}^N$ meets*

- *DCSE (Dual Coalitional Strategic Equivalence), if for all $v \in A$, $\alpha > 0$ and $T \in \mathcal{N}$ such that $v + \alpha \bar{u}_T \in A$ it holds that $\psi_i(v + \alpha \bar{u}_T) = \psi_i(v)$, $i \in N \setminus T$.*

The DCSE axiom is a dual version of CSE by Chun (1989), meaning, while CSE applies when the difference between two games is measured by an unanimity game, the DCSE applies when the difference between two games is measured by a dual of an unanimity game.

Proposition 8 implies the following result:

Theorem 17. *A value on the class of upstream responsibility games $\mathcal{G}^N$ meets the axioms PO, ETP and DCSE if and only if it is the Shapley value.*

Proof. The goes as in Young (1985) or Chun (1989). □

It is straightforward to check the independence of the axioms in Theorem 17, the egalitarian rule meets PO and ETP, the double Shapley meets ETP and DCSE, and the Shapley plus a non-zero but zero-sum vector meets PO and DCSE.

For that in this case we cannot change ETP for NN see the following example:

Example 18. Let $i^* \in N$ be a player, and let ψ be such that for each $v \in \mathcal{G}_{UR}^N$

$$\psi(v) := \begin{cases} Sh(v), & \text{if } \alpha_N = 0, \\ Sh(v - \alpha_N \bar{u}_N) + \alpha_N \chi_{\{i^*\}} & \text{otherwise.} \end{cases}$$

It is clear that ψ meets PO and NN. Let $i \in N$ and $S \subseteq N \setminus \{i\}$. If $i \neq i^*$, then $\psi(v)_i = Sh(v - \alpha_N \bar{u}_N)_i = Sh(v + \beta_S \bar{u}_S - \alpha_N \bar{u}_N)_i = \psi(v + \beta_S u_S)_i$, $\beta_S \in \mathbb{R}$. If $i = i^*$, then $\psi(v)_{i^*} = \psi(v - \alpha_N u_N)_{i^*} + \alpha_N \chi_{i^*} = \psi(v + \beta_S \bar{u}_S - \alpha_N u_N)_{i^*} + \alpha_N \chi_{\{i^*\}} = \psi(v + \beta_S \bar{u}_S)_{i^*}$, $\beta_S \in \mathbb{R}$, that is, ψ meets the axioms PO, NN and DCSE.

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