József Banyár

Model Options for Mandatory Old-Age Annuities

TÁRSADALOMBIZTOSÍTÁSI
KÖNYVTÁR

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MODEL OPTIONS FOR MANDATORY OLD-AGE ANNUITIES

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FOREWORD – NARROWING THE SUBJECT

At first glance, the past few decades of the life insurance market in Hungary (and probably in the majority of the European, or at least Central-Eastern European countries) give the impression that annuity insurance is a marginal product that is not worthy of special attention, since most people (with the exception of a few specific groups such as disabled miners) clearly do not tend to purchase such products.

A completely different view unfolds, however, if the onlooker regards not the past, but the possible future. It is highly probable that within a decade annuity insurance can move from being a marginal product to a distinctly leading product within the Central-European region. [1] [2]

Considering the importance and complexity of the phenomenon, one can hardly find literature dealing with the subject and this is especially true in the case of Hungarian language publications[3]; we can, however, witness a gradual

[1]The fundamental World Bank publication, which set off the changes to the Central-Eastern European pension systems (World Bank [1994]), only deals with annuities among other topics in its Appendix, although more recent publications forecast the increasing future significance of annuities and annuities markets in view of the rise of defined contribution (DC) systems, in addition to stating that, in contrast with the accumulation phase, problems relating to the annuities phase have so far been neglected. (See e.g.: Wadsworth [2002], Cardinale-Findlater-Orszag [2002], Rocha-Thorburn [2007], Stewart [2007], Antolin-Pugh-Stewart [2008]). Blake, in his much-quoted paper (Blake [1999]) already warned about the increasing importance of annuities and determined a list of the most important related tasks. Davis (Davis [2003]) was also quick to compile a list of tasks relating to the regulation of annuities.

[2]This book was written in 2008-2010, so I reviewed related literature up to and including 2010. After this time I mainly dealt with other projects, so I did not really follow new literature on annuities. In spite of this, I must mention two important books published since then about this topic. The first was written by experts from the World Bank (Rocha–Vittas–Rudolph [2011]) and the second by OECD experts (OECD [2014]).

[3]After the launching of the private pension funds, Réti warned (Réti [1999]) of the incredibly (and needlessly) detailed regulation of the institutional system of private

change within the past few years concerning literature in English. Accordingly, the exploration of the specific features of how annuities operate, the impact of the different factors, the resulting hazards for service providers, consumers and regulators, as well as the exploration of means of protection against these hazards, is important. A separate, important question is what needs to be regulated and what it is worth regulating, and how coherent regulation may be achieved. Due to the complicated inherences there is a major risk of ending up with bad, voluntarist solutions and this should avoided at all costs.

I hope that my book contributes to the abovementioned goals, to the development of sustainable products for the market, to the drafting of regulations that optimally protects customers, and last but not least to promoting a better understanding of the specific ways in which annuities operate.

Although I insert the subject matter into a wider context, in the book I do narrow my subject from several aspects. I deal primarily with life annuities, but this is technically often coupled with annuities certain and the phased or otherwise regular withdrawal of funds, which I therefore also discuss here. Life annuity is a much broader category in a technical sense than its meaning as a commonplace term, for example it also includes the payment of life annuity premiums and certain loan instalments. Here, I only deal with life annuities according to the commonplace meaning (though at the beginning of the paper I insert those into a broader context of life annuities in a technical sense), so I deal with the situation in which a service provider makes regular disbursements to a client, but I do not deal with the inverse situation. Life annuities are suitable for paying orphan's annuity, handicapped annuity, disability annuity, etc., but I narrow down my subject to old-age, i.e. pension annuities. Among these I only detail mandatory life annuities, so my supposition is that at the time of retirement the funds on pension accounts must be mandatorily converted into annuity. This criterion is of great importance, because the mandatory conversion into annuity cuts through a vicious circle, which is the reason why voluntary annuity markets are typically insignificant all around the world (compared to the life insurance market, but also in itself). The core of the vi-

pension funds, although the regulation of the services they provide is extremely flimsy and suggests that it was not thought through properly from an insurance mathematics perspective. This was also admitted by Stahl, who actively took part in the elaboration of the regulations (Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002], Stahl [2005]). Erdős (Barabás-Bodor-Erdős-Fehér-Hamecz-Holtzer [2006]) states bluntly that "In 1997, when the bill on private pension funds was being drafted, the first services to be provided appeared so distant and the bill was written in such a hurry, that the provisions on services were hastily cobbled together."

cious circle is extremely strong auto-selection[4], meaning people who do not expect a long life do not buy annuity, but instead consume the assets and thus retain the possibility of eventually leaving an inheritance. As a result, the annuitant's lifecycle becomes longer than average and forces annuity providers to raise their premiums accordingly, which in turn diverts yet more people from concluding annuity agreements. Consequently, making old-age annuities compulsory eliminates one of the strongest sources of adverse selection, and so this factor must certainly be taken into account when designing an annuity system. In other words, this book would be completely different if I assumed that converting the funds on individual accounts into annuities was voluntary. However, I further narrow my subject by assuming that the mandatory annuity system is coupled with the prohibition of differentiation among insured individuals. This is also a supposition of similar strength to compulsory conversion, but with an opposite impact: while obligation eliminates the possibility of adverse selection, prohibition of differentiation returns it to the equation across a broad spectrum, which must be handled somehow. The options for providers are totally different with possibilities to differentiate (according to gender, qualification, health etc.), or with legislation in place that prohibits it. EU policy recently adopted the prohibition of differentiation, so this must certainly be dealt with, and this is the major reason why the mandatory annuity system disintegrates into models, which I present below.

Above, I presented the ways in which I narrowed down the subject matter. But at the same time, I have also attempted to enforce the opposite trend, i.e. to discuss the subject as generally as possible – within the limitations stated above – I do not intend to narrow the subject to the Hungarian situation, although most of the examples are naturally from Hungary. My work is more-or less generally applicable to pension systems that have mandatory individual accounts and mandatory annuity systems in place. Looking at it from this perspective, the above is primarily characteristic to the Eastern part of Europe and less to the Western part. The major difference between Eastern and Western Europe is that the part of the pension scheme that operates according to a non-pay-as-you-go (PAYG) system, which is (more or less) mandatory (or at least covers almost every employee) and is generally referred to as the funded system, is extremely different. In Western Europe the occupational pension model, in which the pension is provided by institutions that are quite close to (or

[4]English literature uses only the term "adverse selection". Hungarian terminology is more detailed, however, also applying the terms" auto-" and "anti-" selection. The difference lies in the intentions of the insured (the annuitant). Contrary to anti-selection, in the case of auto-selection there are no bad intentions from the annuitant's side. So in the case of annuity the correct Hungarian terminology is generally auto-selection, but in this English version of the book I will use the usual English term: adverse selection.

form part of) the employer from both an organisational and policy point of view is extremely widespread, while in Central and Eastern Europe these are financial providers separate from the employer. In Western Europe, occupational pensions are characteristically of Defined Benefit (DB) type (although Defined Contribution, DC-type pensions are quickly gaining ground), while in Eastern Europe these providers operate almost exclusively according to the DC system. Contrary to the DB system, in the case of DC systems the rules of annuity provision are not clear, so there is room for analysis, such as this book aims to provide.

The aim of this publication is to present the possible types of annuity regulatory systems and models that are adequate from a technical point of view, i.e. are free from contradiction and serve the interests of clients within the context of widely applied conditions (individual account, mandatory conversion to annuity, prohibition of differentiation).

The structure of the study is as follows: the first chapter provides an overview of the most important technical questions relating to annuities, the theoretical basis of annuity calculations and the theoretical relationship between certain types of annuities and other financial products (mainly life insurances). At the end of the first chapter, I will describe the theoretical suppositions on which annuity calculations are founded and the cases in which these are not fulfilled in the practice, meaning what problems may arise with relation to cash flow. The second and the third chapters deal with these problems, which are fundamentally the consequence of prohibiting differentiation, and with possible techniques for the management of these problems. Among the difficulties I highlight the problems of selection and choice, and these are detailed in a separate chapter while attempting to quantify the negative impact of these problems on annuity cash-flow. Some of the techniques applied to treat various problems are synergistic, while others are mutually exclusive, and accordingly it is not possible to construct annuity models voluntarily by defining a few “Lego bricks” from which decision-makers can build arbitrary models. In view of this chapter four provides an overview of the consistent annuity model variants produced by the various possible elements. I find there are a total of six consistent annuity models with inherent alternates for decision makers to choose from, but the various elements of separate models may not be arbitrarily combined. In the final chapter I attempt to provide criteria for choosing between models and provide a brief overview of the regulatory problems. I consider the wider context of annuity models and the regulatory and institutional problems of fund accumulation and I write in general about the pension system framework within which a system of individual, capitalised accounts work properly and which I personally prefer.

1. ANNUITIES IN GENERAL

1.1. Definition and types of annuity

The terms[5] annuity and life annuity may be used in a wider, technical, or in a narrower, regular sense.

In a wider, technical sense an annuity is every form of regular, standardised cash-flow that was preceded by or will be succeeded by some kind of transfer of goods, capital or services as compensation with relation to such and in strict settlement; or some kind of prolonged payment obligation undertaken voluntarily and without compensation. Standardisation means that the parts of the cash-flow (the "payments") follow one another at regular intervals perpetually, for a non-defined or pre-defined period, and/or until the occurrence of a certain pre-determined event or events, and the magnitude of payments is either constant or changes according to a defined pattern. In a narrower sense, I regard annuity only as those regular, standardised cash-flows that are paid by a financial institution (or perhaps directly by the state) to an individual (or to several private individuals within the framework of a contract).

Life annuity is an annuity where the start and/or termination of payments and changes in the magnitude of payments and/or the intervals between annuity payments depend on the death of one or more people (the annuitant or annuitants). The term annuity may also be used in a technical or everyday, i.e. narrower sense. In a technical sense the annuity is the premium paid for life insurance, but in a narrower sense only instances in which the service provided is in itself the annuity may be regarded as life insurance (i.e. the annuity is in itself

[5]In this book I discuss annuities formally, meaning according to their aim and structure, but it would be possible, and of course also very interesting, to discuss them according to their history. Regrettably, there is very little literature available on the subject and I have not found any documents at all in Hungarian or discussing the Hungarian context. The first chapter of Poterba [1997] and of Mackenzie [2006] deals with early annuities (e.g. in antiquity and in modern times). Poterba [1997] primarily focuses on the American history of annuities, which are also included in the publications of the American "annuities association", the Insured Retirement Institute (IRI Annuity Fact Book 2009, IRI [2009]).

a life insurance; this is why it is discussed in life insurance textbooks, e.g. Banyár [2003b].)

Unless specifically indicated, later in the book I use the terms "annuity" and "life annuity" in a narrower, i.e. everyday sense, unless specifically speaking about the mathematics of annuities, since the technical sense is self-explanatory in that context because at that level there is no difference between the narrower and wider meaning of the terms. As the subject of my study is life annuity, the term "annuity" is used as a synonym for "life annuity", unless otherwise indicated. Since I regard life annuity as life insurance, I therefore also use the term "annuity insurance" in the sense of life annuity.

Annuities – both in a narrow and even more so in a wider sense – could have a practically infinite number of forms, so it is extremely difficult to find a grouping in which all possible types are indicated. Consequently, I do not strive for completeness but instead try to provide as complete a description as possible of the various possible types.[6]

The first and most important classification criterion is whether or not the disbursement of the annuity depends on coincidence, which fundamentally means the death of one or more policyholders. If the answer is yes, then we are talking about life annuities; if the answer is no, then we are talking about annuities certain.

Depending on their term, annuities certain may be further divided into annuities of defined and non-defined term. An annuity certain with non-defined term is basically phased withdrawal from the accumulated assets (principle), the duration of which is not pre-defined (although obviously limited by the assets available), but there is no intention to either consume the entire capital or for further accumulation at real value. Annuities certain of defined duration may be an everlasting "perpetuity", or temporary "financial" annuities. In the case of perpetual annuity, the owner of the principal (assets) only consumes and receives as a regular benefit the interest, or return (or only a portion of that), so the principal is preserved forever (or may even increase). An example – among others – is ground rent. In the case of financial annuities the intention is to consume all assets within a pre-defined period of time (or to repay the entire debt, or accumulate the targeted assets).

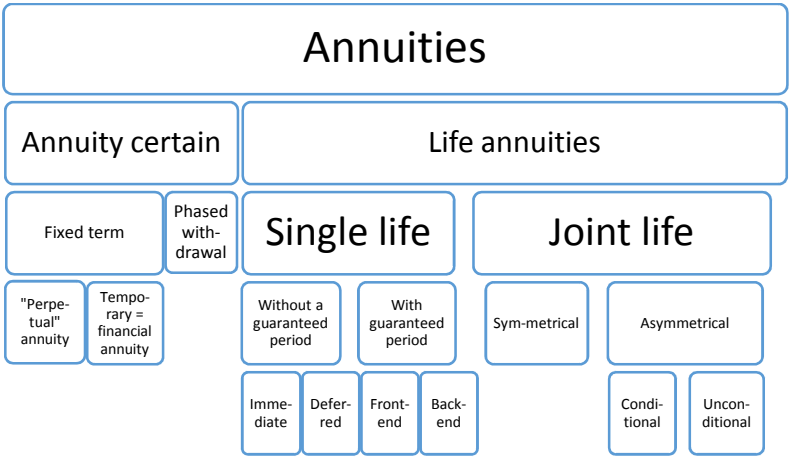
[6]Some form of classification of annuities (different – although not fundamentally – from both each other and from the one applied here) can be found in every life insurance textbook (e.g. Mehr-Gustavson [1987], Black-Skipper [1994], Banyár [2003b]). Of these, the Black-Skipper [1994] classification (and the book itself) is widely referred to and, for instance, is also used by Blake (Blake [1999]). Similarly to the majority of literature available, the recent Encyclopaedia of Actuarial Science (Teugles-Sundt (Ed.) [2004]) instead contains a kind of (not too detailed) annuity listing. For a practical approach to the issue see, e.g. Professional Training Services [1997].

Life annuities may be for one or for more people (meaning there may be one or more insured persons). There may be a great variety of joint life annuities, but in general these may be further classified into groups depending on whether or not we distinguish between insured persons. The first type may be called asymmetric, the second symmetric. For the purposes of this book the most important symmetric, multi-person life annuity is joint life widow(er) annuity with two owners, where the disbursement continues until both insured persons (generally husband and wife) are deceased. According to one variation, the sum assured decreases following the first death (but to a higher amount than half of the sum assured). Multi-person asymmetric annuity insurances may be conditional or unconditional, according to whether or not the start of annuity disbursement is linked to the death of one of the insured persons.

Single life annuities either begin immediately or are deferred, and often have a guaranteed period at the beginning ("front end") or at the end ("back end") of the term, although of course the majority have no guaranteed period at all.

The table below includes the most important types of interest to this study, and perhaps also in general:

Table 1: The most important annuity types



So annuities may be distinguished according to the following criteria (in supplementation to the above, to a certain extent):

- Does it include haphazard elements (annuity certain versus life annuity)

- According to the length of term (defined or undefined term, temporary or life-long in case of life annuities)
- With respect to life annuities:
  - According to the number of insured persons (single life, multi- or joint-life)
  - In case of multi-life annuities according to the relative situation of the insured persons (symmetric and asymmetric annuities)
  - In the case of asymmetric annuities does the commencement of benefit payments depend on an insurance event or not (conditional or unconditional annuities)
  - Does it include guaranteed benefit or not (annuities with or without a guaranteed period)
  - According to the start of the service/benefit (commencing immediately or deferred)

We must note with relation to Table 1 that it is incomplete, and that it does not contain all the possibilities within the types included, meaning the above list of criteria may be extended. Although multi-life annuities characteristically have no guaranteed period (as indicated in the table), theoretically these may be supplemented with a guaranteed period (although this is less logical considering that the usual reason for including a guaranteed period is that it converts a single annuity into a joint annuity, badly, but in a technically simpler manner). Both multi-person annuities and annuities that include a guaranteed period may be deferred, although they generally begin immediately, and having a guaranteed period at the beginning is not a particularly logical solution in the case of deferred annuities. In principle, symmetric life annuities may also be conditional, but it is much more difficult to find practical examples for such cases. The table might be continued, and in a certain sense I do continue it later in the book, although I fundamentally remain within the above framework.

However, some important features that are strongly applied in the book are not included the table:

- **The frequency of the annuity**. In theory any frequency could be determined, though in practice two frequencies are applied most often: monthly and annual. Pension, widow(er), orphan, disabled, etc. benefits are almost exclusively monthly payments.[7]

[7]That is of course linked to the generally monthly schedule of wage and utility bill payments as a fundamental determinant. (Although it must be noted that it is a relatively new phenomenon; until the middle of the last century weekly pay checks were more frequent, then later fortnightly payment was in place for a while, both of which clearly made it easier to budget smaller salaries than a rarer salary payment. Therefore, especially in the case of lower pensions, a more frequent than monthly pension payment rate

- Does the annuity payment happen at the beginning or the end of the annuity period? Depending on these options one can speak about **annuities due and annuities payable in arrears**. This aspect is a purely technical one.
- Though in the case of annuities the default method is that **annuity payments are the same magnitude**, one can imagine a different situation. These days it is only natural that annual indexation of the portion of the yield beyond the computed one, credited based on the capital investment that provides the basis for annuity, is not considered a diversion from annuity payments of identical magnitude. However it is possible that there is some sort of trend in annuities beyond that, e.g. annuity payments increase (incidentally decrease) each year by a predefined amount, or by a pre-defined ratio. It is also possible that this increment is already continuous during the course of the year, or it could occur over a longer period of time than a year. On my part, only these changes are considered imaginable within annuities and I do think that the construction whereby a financial service provider pays the benefit otherwise payable as one sum in three, significantly decreasing instalments, so that the "annuity payment" affords the policy holder a more favourable tax rating cannot be defined as an annuity.
- Another default method is that this single sum (regularly decreasing, increasing) **annuity due is denominated in Euro** (EUR) (in general in some unit of currency), although internationally (not yet in Hungary) there exist investment unit linked "variable annuities" on the model of unit linked (UL) life insurance, where the benefit/sum assured is not defined in currency, but in **units** of variable value (constant, decreasing, or sometimes increasing).
- Although it does not affect the annuity constructions, it is nevertheless important to mention that contrary to the still valid centuries-old custom according to which only age and gender differentiations are applied to premiums (although differentiation according to gender has been banned in Europe since December 2012), health-differentiated, so-called "impaired" annuity premiums have already appeared on certain markets (UK, USA). (Just as the regulator approach according to which certain types of differentiation, e.g. according to gender, must be prohibited, has also appeared).
- Although I will not be dealing with the subject in my book, It is also important to mention that the above definition of annuity is extended in

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may be considered, against which the argument may be the higher related cost compared to the "usual" schedule of utility bills.)

a certain sense by the annuity paid when receiving Long Term Care, because the commencement of "payment" is conditional, and this condition is not the death of the policyholder, but a certain physical or mental condition (policyholders are unable to take adequate care of themselves due to old age). In a certain sense, the (internationally rare[8]) home purchase life annuity, the regulation of which has only recently begun in Hungary, is an extension of the traditional definition of annuity because of the specific nature of the cash-flow, although, again, I will not be dealing with this topic further in my book.

- It is also worth mentioning an early, historic form of annuity, the "Tontina", which in a certain sense provided the basis for contemporary annuities. Its original form was suggested (according to some sources based on southern Italian examples) by an Italian doctor of medicine, Tonti, who served in the French royal court in the mid seventeenth century. According to this, a group of rich men would collectively lend a larger amount, a hefty sum each, to the King for the long term, the interest of which would be annually paid by the treasury. The interest payable to a deceased lender would be distributed among those still alive, so they would gradually receive ever increasing amounts of interest and finally the last survivor would receive the full interest on the total amount lent. When this person also died, the principle would become the property of the King (or in the commercial version would also become the property of the last survivor). So the solution merges annuity and gambling. Tontinas quickly spread throughout the world and gradually the gambling element began to dominate, as a result of which they were eventually banned almost everywhere and today exist almost only in France, where they were originally introduced.
- Finally let me mention an interesting hybrid, a cross-breed of annuity certain and life annuity: annually recalculated annuity certain. This is a kind of annuity certain, the term of which is the expected remaining lifespan of the annuitant. Since these changes annually, it is recalculated annually and the annuity payment is modified (diminished) accordingly.

In the next subsection of this book I will discuss the mathematics of annuities according to a somewhat innovative approach. The basic formulae can naturally be found in literature (partly in my own books). The most frequently referenced actuarial literature publications in English that also deal with the mathematics of annuities are the British Neill [1989] of 1977 (unchanged reprint of

[8]The reverse mortgage construction is much more common internationally.

the 1977 issue) and the American Bowers-Gerber-Hickman-Jones-Nesbitt [1986]. One of the (Swiss) authors of the latter summarised very briefly and consistently at a very high level the mathematics of annuities in a frequently referenced book (Gerber [1995]). From the more recent literature the Canadian Milevsky's book deals explicitly with annuities (Milevsky [2006]).

Neil's book includes the classic explanation of annuity mathematics (including life annuities), the majority of which he presumably also took from tradition. This sort of explanation is also characteristic of the respective Hungarian literature (e.g. Krekó [1994], Michaletzky [1997], Banyár [1994] and [2003]). Neil's book is complemented by Hungarian course textbooks from the early nineties, which are based on translated English actuarial training textbooks (McCutcheon [1991], Neill [1991a] and [1991b].)

Two further elaborations are worthy of note from classical English life insurance literature: the work by Booth-Chadburn-Cooper-Haberman-James [1999], which aimed to be a new summary of English insurance mathematics and the huge, three-volume Actuarial Encyclopaedia (Teugels-Sundt (editor) [2004]).

For Hungarians, the Hungarian literature has the advantage of enabling us to look back on a much longer period than in the case of English literature, since the older literature is more available in Hungarian language. A thick volume of "Political Mathematics" (Weninger [1869]) appeared in Hungarian as early as 1869, attracting great attention and providing an overview of the mathematics of life insurances, including annuities.

The first complete work of Hungarian insurance mathematics using standard notations, and which may be regarded as the fundamental writing on the subject is Political Mathematics Part 2 by Károly Bein, Samu Bogyó and Miksa Havas, with the subtitle: The Theory and Practice of Life, Disability and Pension Insurance, published by the Franklin Association in Budapest in 1907 (Bein-Bogyó-Havas [1907]).

In more recent Hungarian insurance mathematics literature (beyond my own books – Banyár [1994] and [2003]), Béla Krekó (Krekó [1994]) and György Michaletzky (Michaletzky [1997]) wrote in general about the mathematics of life insurances, the latter focusing primarily on annuities.

1.2. The mathematics of annuities

Most books, including the abovementioned ones, usually present the mathematics of annuities by explaining how certain types of annuity can be constructed out of smaller units (certain or conditional lump sum payments with differing conditions) and this is the basis for the various relationships between the premiums of concrete annuity types and other life insurances or financial

products. In this book I will follow a different, precisely opposite logic, that I have developed myself. I do this firstly because it makes the relationship between different types of annuity clearer and secondly because as a result I can present a "global", "birds eye view" of annuities without concentrating on the technical details (which are presented well by the above-mentioned works). Following this logic, I begin with presenting the relationships between the premiums of various annuities.

1.2.1. THE RELATIONSHIPS BETWEEN THE NET PREMIUMS OF VARIOUS ANNUITIES AND OTHER LIFE ASSURANCES

It may be stated that just as the light of the sun can be split up into all the colours of the rainbow, and just as white light contains all colours, so the simplest annuity, perpetuity, contains all possible annuity types and the other financial products related to annuities. Below, we will see how they form a part of it and how the various constructions are related to each other. I restrict my analysis (for the sake of simplicity and clarity) to annuities due, but with minor modifications this calculation can also be used for annuities payable in arrears.[9] The appropriate, standardised notations will be introduced according to the order of explanation (and are listed at the end of the book).

"In the beginning there was the perpetuity", we might say. The simple, standardised form of perpetuity is when somebody receives i interest at the end of each year on his/her EUR 1 capital and spends it. In this way the magnitude of the principal remains (nominally)[10] unchanged for eternity, as does the resulting income, which in view of its regularity we can regard as an annuity. The payments of this annuity are always due at the end of the year, so this is an

[9]The distinction between annuities-due and annuities payable in arrears is quite technical, and from the perspective of the formulae it is not particularly important. The point of the difference is that the particular payments are due at the beginning (annuity-due) or at the end (payable in arrears) of the intervals between two payments. It is obvious that the two annuities differ from each other only in the first (and perhaps the last) payments, the rest of the payments are the same. From the point of view of calculations it is important to know the exact situation. We denote the net (without costs) single premium as „$\ddot{a}$" in case of annuity-due and as „a" in the case of annuity payable in arrears. Our choice also means that in the following we use only the variation $\ddot{a}$, instead of duplicating the (very similar) formulae.

[10]Naturally, our analysis can be relatively easily extended to principal unchanged in real terms. In this case, we must split the nominal interest rate into two parts and the real interest rate will play the same role as the nominal interest rate in this analysis. In practice, land rent as a kind of annuity is the closest to perpetuity and represents a more or less unchanged principal in real terms.

annuity payable in arrears. How we can turn this EUR 1 principal and *i* interest rate into an annuity-due perpetuity? The question can be reformulated: what sum can we deduct from the EUR 1 capital at the beginning of the year, so that with an *i* rate of interest rate the principal will again amount to EUR 1 at the end of the year? If we denote (as is usual) this sum with *d*, then we receive the following equation:

$$(1 - d) \cdot (1 + i) = 1$$

from which:

$$d = 1 - \frac{1}{1 + i}$$

The usual practice is to denote the reciprocal (1+i) with *v*, the so-called discount factor:

$$v = \frac{1}{1 + i}$$

So:

$$d = 1 - v$$

from which firstly:

$$v = 1 - d$$

and secondly:

$$d = 1 - \frac{1}{1 + i} = \frac{i}{1 + i} = i \cdot v$$

So *d* is the discounted value of *i*, and this is indeed logical, because the question could also have been formulated as follows: what rate of interest rate would we get instead of *i*, if we want to receive our payment a year earlier? The answer, naturally, is the discounted value of *i*, i.e. $i \cdot v$.

So we have our first standardised annuity-due, where the annual payment (in advance) on EUR 1 principal at an *i* interest rate is *d*. The notation of the single premium of the perpetuity (for annual EUR 1 payment in advance) is:

$$\ddot{a}_{\overline{\infty|}}$$

So, the equation of a perpetuity with annual payment *d* is

$$1 = d \cdot \ddot{a}_{\overline{\infty}|}$$

This expresses the fact that for EUR 1 the client receives an eternal annual cash-flow of *d* magnitude always in advance at the beginning of the year.

Below, when I write an equation with 1 on one side of it, it expresses what products the client can receive for EUR 1, which is logically equivalent to a single perpetuity part.

Before we continue, it is instructive to express $\ddot{a}_{\overline{\infty}|}$ from the above equation:

$$\ddot{a}_{\overline{\infty}|} = \frac{1}{d} = \frac{1}{i \cdot v} = \frac{1+i}{i} = 1 + \frac{1}{i}$$

where $\frac{1}{i}$ is the well-known formula for a unit of perpetuity payable annually in arrears ($a_{\overline{\infty}|}$). Annuity-due perpetuity to all intents and purposes differs from this only with respect to the EUR 1 paid right at the beginning of the annuity period.

So perpetuity is a cash-flow that lasts for eternity. Naturally, we can "sever" this cash-flow in different places during its lifetime. What happens, for instance, if we only wish to receive EUR *d* in advance annual annuity-due payments on our EUR 1 capital until the time our death? Then obviously immediately following our death (i.e. at the next payment due date) our heir[11] will get back the EUR 1 principal (which he/she may again invest in a perpetuity if they so wish), because the point of perpetuity is that the whole original capital is always "restored" again and again one year after the interest payment. But from another perspective, this means that we have split our uniform perpetuity into two financial products:

1. An annuity-due lasting until our death with annual payment *d*, i.e. a life annuity, and
2. A life assurance with an assured sum of EUR 1 and payment due at the time of death (a whole life policy).

We can purchase these two financial products for exactly EUR 1. The usual notation of the net single premium of an annuity-due life annuity with an annual payment EUR 1 is $\ddot{a}_x$, where *x* is the entry age of the annuitant (until whose death the annuity lasts) at the commencement of the contract, while the notation of the net single premium of a whole life assurance with EUR 1 sum

[11] I am consciously not dealing with the fact that the insurer pays the sum assured to the beneficiaries independently from the probate process, because legally, if there is a formal beneficiary then the sum assured does not become part of the inheritance. It nevertheless constitutes an inheritance to all intents and purposes.

assured is A_x. In both cases the implicit assumption is that the financial institution (in this case a life insurer) invests our capital at an annual interest rate of *i* and does not charge anything for its service.[12] Using these notations we can write down this "segment" of the perpetuity, as:[13]

$$1 = d \cdot \ddot{a}_x + A_x$$

Naturally we can sever the cash-flow of the perpetuity at other places too.
We also distinguish between two other important cases of standardised annuity, and especially with reference to life annuities:

1. The annuity lasts for a predetermined term (for the sake of simplicity for a certain number of whole years), after which the capital is paid out, and
2. The annuity lasts for a predetermined term, but maximum until the time of our death, after which the capital is paid out (or transferred to our heir[s]).

In the first case it must obviously be necessarily true that

$$1 = d \cdot \ddot{a}_{\overline{n|}} + v^n$$

where I denote the (whole) number of years of the term with *n*, the net present value of an annuity certain with an annual premium of EUR 1 with $\ddot{a}_{\overline{n|}}$, and the present value of 1Ft due in *n* years with v^n.

While in the second case it is clearly true that

$$1 = d \cdot \ddot{a}_{x:\overline{n|}} + A_{x:\overline{n|}}$$

where $\ddot{a}_{x:\overline{n|}}$ is the net present value of the EUR 1 annual life annuity-due until death, but maximum for *n* years, while $A_{x:\overline{n|}}$ is the net single premium of a life insurance for death and maturity (an "endowment") with EUR 1 sum assured.

Above, we first severed the perpetuity when someone died. This event can be logically extended to more than one (two in the simplest case) deaths. Then we get the following equation (with two clients insured):

[12] Or at least it makes to do with the "margin" between the interest it actually earned and the interest it pays to the client.

[13] Most of the actuarial books include this equation. See e.g. Bein-Bogyó-Havas p182, Bowers et al. p131, Gerber p36, Krekó p30, Neill p63, but a significant number of the equations shown here are missing.

$$1 = d \cdot \ddot{a}_{xy} + A_{xy}$$

where $\ddot{a}_{xy}$ is the net single premium of a EUR 1 annual annuity-due for two insured persons that lasts until both are alive. (They are x and y years old, respectively, at entry, or below, for the sake of simplicity, annuitants x and y). A_{xy} is the single net premium of a whole life insurance with two insured persons, which pays EUR 1 when either of annuitants x or y dies.
Naturally, joint life annuities also have a temporary variant, in which case the equation

$$1 = d \cdot \ddot{a}_{xy:\overline{n}|} + A_{xy:\overline{n}|}$$

will be true, where $A_{xy:\overline{n}|}$ is the net single premium of a joint life endowment that pays a EUR 1 sum assured when one of the insured persons dies or after a period of *n* years, while the $\ddot{a}_{xy:\overline{n}|}$ single premium joint life annuity lasts until both annuitants are alive and *n* years have not yet passed.
A further extension is the addition of a guaranteed period (*g* years) to the annuity. The following can be stated with regard to the practical reasoning behind these. In the case of the immediately commencing and non-conditional annuities analysed so far it can happen that the annuitant dies not long after concluding the contract and the annuity payments cease immediately after the first is paid out (or in the case of conditional or deferred annuities might not begin at all). This inevitably scares off many potential annuity buyers from purchasing an annuity, who vacillate between spending their accumulated capital and leaving it to their children, and makes this largely dependent on their life expectancy, which they cannot know in advance. By guaranteeing payment of the (usually lifelong) life annuity for a number of years, providers can reduce this psychological barrier to the purchasing of annuities. The guaranteed period can be included at the beginning (front-end) or at the end (back-end) of the term. In case of an annuity with a guaranteed period at the beginning (which we will denote with ${}_{|g}\ddot{a}_x$), the annuitant (or their heir) is guaranteed to receive the payments due for the first *g* years of the annuity even if they die in the meantime. Naturally, this guarantee will have no effect if the annuitant dies after *g* years.

An annuity with a guaranteed period of this kind obviously provides more than an annuity with no guaranteed period, so its premium is also higher. Accordingly, EUR 1 is no longer enough (above the annual *d* payment) to cover the previous whole life insurance, only a modified form of it. This modified form means that the EUR 1 benefit will certainly not be paid during the first *g* years, even if the insured party dies during this period. If the insured party dies

during the first *g* years, the beneficiary will also only receive the EUR 1 at the end of the *g* year period. This is as if the first *g* years represented a unit, and the annual consideration of mortalities only commences once this period has expired. Let us denote this with a *g* in the lower left hand corner (where *g* means the length of the first such period). Then we can write the following equation:

$$1 = d \cdot {}^{|g}\ddot{a}_x + {}_gA_x$$

In the case of an annuity with a guaranteed period at the end (which we will denote with ${}^{g|}\ddot{a}_x$), the beneficiary will receive payments for *g* years following the death of the insured party. This obviously means that the sum assured of the whole life insurance will also be payable for *g* years following the death, i.e. we "prolong" payment of the death benefit for *g* years. This can be express by saying that in case of death beneficiaries are not yet due the EUR 1 unit, but only its value discounted by *g* years, so our equation will change as follows:

$$1 = d \cdot {}^{g|}\ddot{a}_x + v^g \cdot A_x$$

The parts of the above equations can be further segmented. The following equation is obviously true:

$$\ddot{a}_x = \ddot{a}_{x:\overline{n}|} + {}_{n|}\ddot{a}_x$$

i.e. the lifetime annuity can be broken up into an *n*-year ($n < \omega$) temporary annuity-certain and a deferred (life) annuity with deferment period *n*. Accordingly, the equation $1 = \mathrm{d} \cdot \ddot{a}_x + \mathrm{A_x}$ can be also written in the following form:

$$1 = d \cdot \left(\ddot{a}_{x:\overline{n}|} + {}_{n|}\ddot{a}_x\right) + A_x$$

(A deviation from the basic formula is that in this case not every insurance will necessarily result in the payment of benefits, because the deferred annuity ceases without payment if the insured individual dies within the *n* years.)

Analogously to the annuities $\mathrm{A_x}$ can also be split into a "temporary" and a "deferred" sum assured. The "temporary" insurance is the classical term (death) insurance, and we traditionally denote deferred insurance with ${}_{\mathrm{m}|}\mathrm{A_x}$! (Remarks: 1. I must emphasize that I have denoted the period with *m* and not *n*, because the length of deferment does not necessarily have to be the same as in the case of annuities, and 2. ${}_{\mathrm{m}|}\mathrm{A_x}$ should not be confused with the previously introduced ${}_{\mathrm{g}}\mathrm{A_x}$, which means something different. In the first, the vertical denotes a condition, i.e. a contingent benefit [if the insured person dies within

the first *m* years then no payment is due according to this insurance policy], while in the second case there is no condition, we have simply "combined" the first *g* years.)

Accordingly, it is also true that

$$A_x = A^1_{x:\overline{m|}} + {}_{m|}A_x$$

where $A^1_{x:\overline{m|}}$ is the traditional notation of the single premium of a term (death) insurance with EUR 1 sum assured and a term of *m* years. So we can break up our basic equation $1 = d \cdot \ddot{a}_x + A_x$further into the following form:

$$1 = d \cdot \left(\ddot{a}_{x:\overline{n|}} + {}_{n|}\ddot{a}_x\right) + A^1_{x:\overline{m|}} + {}_{m|}A_x$$

(In this formula either the classical term insurance or the deferred whole life insurance will definitely cease without paid benefit, but it is possible that the deferred annuity will also not involve the payment of benefit.)

Although I do not deal with the topic, I must remark that

1. Temporary annuities may also be broken up into the sum of a short temporary and a deferred temporary annuity,
2. Joint life annuities with two or more insured persons can also be broken up similarly to single client annuities.

Annuities with a guaranteed period are complex products so they may naturally also be broken up.

A front-end guaranteed period annuity is clearly the sum of an annuity-certain and a deferred annuity, that is

$${}^{|g}\ddot{a}_x = \ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x$$

So the equation $1 = d \cdot {}^{|g}\ddot{a}_x + {}_gA_x$ changes as follows:

$$1 = d \cdot \left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) + {}_gA_x$$

Back-end guaranteed annuities provide a greater guarantee than front-end guaranteed annuities, because:

1. They include the guarantee provided as front-end guaranteed annuities, since annuity payments are also due during the first *g* years in this case, and
2. In contrast to front-end guaranteed annuities, payments will definitely last longer in this case than without a guaranteed period. In the case of a front-end guaranteed annuity, the guarantee is not effective if the in-

sured party dies after the guarantee period, but in the case of a back-end guaranteed annuity there is no such break.

Back-end guaranteed annuities, as a complex product, may be interpreted in two ways:

1. It is to all intents and purposes an annuity certain with a term of g years (because the insured party is certain to receive payments during the first g years, even if they die immediately after purchasing it) and a "normal" life annuity whose payments are all "shifted" by g years, meaning:

$$ {}^{g|}\ddot{a}_x = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x $$

This interpretation is totally compatible with the "shifted" whole life insurance in the equation $1 = d \cdot {}^{g|}\ddot{a}_x + v^g \cdot A_x$. In this case, the equation receives the following form:

$$ 1 = d \cdot \left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) + v^g \cdot A_x $$

or

$$ 1 = d \cdot \ddot{a}_{\overline{g|}} + v^g \cdot (d \cdot \ddot{a}_x + A_x) $$

2. It is naturally a normal lifelong annuity plus such a whole life insurance, where the sum assured is paid as an immediately commencing annuity certain with a term of g years, so:

$$ {}^{g|}\ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x $$

In this case, the equation $1 = d \cdot {}^{g|}\ddot{a}_x + v^g \cdot A_x$ takes on the following form:

$$ 1 = d \cdot \left(\ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x\right) + v^g \cdot A_x $$

It is taken for granted, that here the sub-total $d \cdot \ddot{a}_{\overline{g|}} \cdot A_x + v^g \cdot A_x$ is equal to A_x.

Naturally these two forms are equivalent, because if we replace $\ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x$ in the equation with $1\text{-}d \cdot \ddot{a}_x$, then we get

$$
\begin{aligned}
{}^{g|}\ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x &= \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot (1 - d \cdot \ddot{a}_x) = \ddot{a}_x + \ddot{a}_{\overline{g|}} - \ddot{a}_{\overline{g|}} \cdot d \cdot \ddot{a}_x \\
&= \ddot{a}_x + \ddot{a}_{\overline{g|}} - \frac{1 - v^g}{1 - v} \cdot (1 - v) \cdot \ddot{a}_x \\
&= \ddot{a}_x + \ddot{a}_{\overline{g|}} - (1 - v^g) \cdot \ddot{a}_x = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x
\end{aligned}
$$

Naturally, it is in theory also possible to guarantee the temporary life annuity, although this solution seems less justified than in case of a lifetime annuity. Whatever the case, the equations are also extensions of the above equations in this case, i.e.

$$
{}^{|g}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_{x:\overline{n|}}
$$

where g≤n, naturally. (We can see that if g=n then ${}^{|g}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{\overline{g|}}$ and ${}_{n|}\ddot{a}_{x:\overline{n|}} = 0$.) This restriction is not necessary in the case of a temporary life annuity:

$$
{}^{g|}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{x:\overline{n|}}
$$

However, in this case it will also be true that:

$$
{}^{g|}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{x:n} + \ddot{a}_{\overline{g|}} \cdot A_{x:\overline{n|}}
$$

Because the annuity-certain is in fact a guaranteed annuity, the guaranteed period can only be properly interpreted in the case of life annuities. It is also possible to apply a guaranteed period to joint life annuities, but again, it is probably less relevant to those in view of the fact that the intention of the insured parties was intrinsically to leave their capital to the other party in the form of an annuity (we could in fact state that a single annuity with a guaranteed period is ultimately a [poor] simulation of a joint life annuity). Nevertheless, if it is also required for both temporary and joint annuities, then a guaranteed period may naturally be added to the annuity according to the above.

There is a clear relationship between the orders of magnitude of the above-mentioned annuities. It is clear that:

$$
\ddot{a}_{\overline{n|}} > \ddot{a}_{x:\overline{n|}} > \ddot{a}_{xy:\overline{n|}} > \ddot{a}_{xyz:\overline{n|}}
$$

and that

$$
\ddot{a}_x > \ddot{a}_{xy}
$$

because (based on the relationships described in the first line above) in the case of $\ddot{a}_{\overline{n}|}$ we are sure to receive payments for *n* years, but in case of $\ddot{a}_{x:\overline{n}|}$ for only a maximum of *n* years, in view of the fact that the insured individual currently aged *x* years could die sooner. If, however, the death of one out of two or more jointly insured individuals can also stop the flow of annuity payments, then the expected period of annuity payment will be even shorter in such cases. The same can be said in the case of lifelong single and joint life annuities.

This also means that the differences between the above-mentioned premiums will be higher than zero. Fortunately, their meanings are also easily interpreted:

$\ddot{a}_x$-$\ddot{a}_{xy}$: *x* receives EUR 1 per annum, but pays this back (negative annuity) while *y* is still alive, meaning *x* only receives payments after *y* has died. If *x* dies before *y*, no payment occurs at all. So, this is a conditional annuity: *x* begins receiving annuity payments once *y* has died. We can denote this with $\ddot{a}_{x|y}$or $\ddot{a}_{x|y} = \ddot{a}_x$-$\ddot{a}_{xy}$.

In this case, our equation will change as follows:

$$1 = d \cdot \left(\ddot{a}_{xy} + \ddot{a}_{x|y}\right) + A_x$$

$\ddot{a}_{x|y}$ may also be regarded as a kind of asymmetric widow annuity. In this case, the financial situation of the two insured parties is different, and client *y* wishes to assure an income for the other insured party, who is their dependent (client *x*) following their own death. The death of *x* does not affect *y* financially.

$\ddot{a}_{\overline{n}|}$-$\ddot{a}_{x:\overline{n}|}$: the annuity is payable for those years of the *n*-year term during which *x* is no longer alive. If *x* survives to age *x*+*n*, then annuity payments do not commence at all. So this is also a conditional annuity, which we shall denote with $\ddot{a}_{\overline{n}|x}$.

The meaning of this is clear: the beneficiary receives annuity payments (e.g. an "orphan" annuity) until a certain age (symbolised by the *n*-year term), but only if the insured party dies before the beneficiary reaches that age, otherwise annuity payments do not begin at all because the conditions are not met.
In this case, our basic equation will alter as follows:

$$1 = d \cdot \left(\ddot{a}_{x:\overline{n}|} + \ddot{a}_{\overline{n}|x}\right) + v^n$$

Of course, this also makes sense if the life annuity is a joint one:

$$\ddot{a}_{\overline{n}|xy} = \ddot{a}_{\overline{n}|} - \ddot{a}_{xy:\overline{n}|}$$

In this case, payment of the "orphan's annuity" also commences if either of the parents (i.e. not just the pre-determined one) dies before the child reaches a certain age.

In these cases it is generally not important to take into account the fact that there is also a chance, although a slim one, of the child dying, but if we do take this into account then we can create a three-person joint life annuity:

$$\ddot{a}_{x:\overline{n}|yz} = \ddot{a}_{x:\overline{n}|} - \ddot{a}_{xyz:\overline{n}|}$$

According to this, there will be no annuity payments while all three of the insured persons are alive, and if all of them survive until the end of the *n*-year term then no annuity payment will occur at all. If *z* or *y* dies, than x will begin receiving annuity payments for a maximum period of *n* years (i.e. the end of the predetermined age: *x*+*n* years), or until the time of his/her death prior to the end of the term. If *x* dies first, annuity payments do not beg in at all. (In this example the parents were y and z, while in the previous one they were x and y!)

In these cases, our basic equation change as follows:

$$1 = d \cdot \left(\ddot{a}_{xy:\overline{n}|} + \ddot{a}_{\overline{n}|xy}\right) + v^n$$

and

$$1 = d \cdot \left(\ddot{a}_{xyz:\overline{n}|} + \ddot{a}_{x:\overline{n}|yz}\right) + A_{x:\overline{n}|}$$

It is also worth examining in general a whole group of joint life annuities and the relationships between their premiums. Remember that $\ddot{a}_{xy}$ is an annuity that immediately ceases if one of the insured persons dies. In practice, however, it is difficult to find a situation (aside from life insurance premiums or loan instalments, which are technically also annuities) in which two people only need an annuity while both of them are alive. The two following two cases are much more probable:

1. An annuity is only needed at all after one of the annuitants dies (and naturally only while the other insured party is still alive), or
2. An annuity is needed while any of the insured persons is alive.

In view of what we already know, however, in both cases we can assemble the required annuity from the ones we have already examined:

1. If we buy two single life annuities for insured persons *x* and *y*, but pay back both premiums to the insurer while both of the annuitants are alive ("negative annuity"), we get precisely the annuity that commences when only one of the insured persons is alive, and lasts until their death. So the net single premium of this is: $\ddot{a}_x + \ddot{a}_y - 2 \cdot \ddot{a}_{xy}$

2. This case differs from the previous one in that only one of the two annuity payments are returned to the provider while both of the insured parties are alive, so the net single premium will be: $\ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}$

Further generalising these two cases we can arrive at a single general formula, from which almost all two-person joint life annuities can be originated (including the two discussed above). If we want client *x* to receive an *A* annuity following the death of the other insured party, and for client y to receive *B* annuity following the death of *x* for the remainder of their lifetime, and C is the joint annual payment received while both are still alive, then we can describe this situation with the help of the following formula:

$$A \cdot \ddot{a}_x + B \cdot \ddot{a}_y - (A + B - C) \cdot \ddot{a}_{xy}$$

The previously discussed two annuities are in fact special cases of this general equation, in which

In Case 1: A=B=1 and C=0

In Case 2: A=B=C=1

The formula can be standardised by choosing one of the (not zero) parameters (e.g. A) to be 1, as I do below.

Giving different values to the parameters A, B and C results in many different joint life annuities. These can be classified into two main groups according to whether or not A=B=1. In the former case the situation of the two insured individuals is symmetrical, because if either dies the other receives exactly the same annuity, while the second case, when A and B are different, is asymmetrical.

In case of symmetrical annuities, the most logical solution would be for the joint annuity (C) to decreased following the death one of the annuitants, although not by half, but by a somewhat lesser degree. (If it were to decrease by half, then we would be talking about two single life annuities rather than a joint life annuity, because if A=B=0,5, and C=1, then $0{,}5 \cdot \ddot{a}_x + 0{,}5 \cdot \ddot{a}_y - (0{,}5 + 0{,}5 - 1) \cdot \ddot{a}_{xy} = 0{,}5 \cdot \ddot{a}_x + 0{,}5 \cdot \ddot{a}_y$). So the most appropriate parameters are the following: 0,5 < A = B < 1 and C=1, in which case the single premium will be:

$$A \cdot \ddot{a}_x + A \cdot \ddot{a}_y - (2 \cdot A - 1) \cdot \ddot{a}_{xy}$$

The aim and the function of these annuities will be absolutely clear if we call them “widow’s” annuity insurance.

1.2.2. A POSSIBLE CALCULATION OF THE NET PREMIUM OF LIFE ANNUITIES (AND OTHER LIFE ASSURANCES)

In case of life annuities, contrary to the annuities-certain, (almost) all payments occur by chance, depending on whether or not the insured party is alive when the payment is due. Accordingly, the calculation of net premiums requires death and survival probabilities. The usual notation of these probabilities is q (in case of death) and p (for survival). More specifically:

q_x is the probability that a person aged x years will die within one year,

p_x is the probability of the opposite: that a person aged x years will still be alive after one year.

It is obviously true, that

$$q_x + p_x = 1$$

We require survival probabilities not just for one year, but also for a longer period, so we introduce a generalization of p_x:

${}_{t|}p_x$: the probability, that a person now aged *x* years will also be alive after *t* years. It will be also true, that ${}_{t|}p_x \cdot {}_{z}p_{x+t} = {}_{t+z|}p_x$.

From these it is easy to calculate the probability that a person who is *x* years old now will die sometime between the age of x+t and x+t+1: ${}_{t|}p_x \cdot q_{x+t}$.

These probabilities can be calculated from a mortality table. Its main column is a series of l_x called the "life table", which shows for the whole integers x=0, 1, …, ω, what proportion of a standardised (usually 100,000= l_0) group of new-born children (aged 0) will still be alive at age 1, 2, … In practice, there is nobody still alive at ages ω+1 or higher (ω is usually 100 years in Hungarian tables).

d_x denotes how many people out of this population will die between their x and x+1 birthdays. Clearly

$$d_x = l_x - l_{x+1}$$

From these, the necessary probabilities can be calculated in the following way:

$$q_x = \frac{d_x}{l_x}$$

$$p_x = \frac{l_{x+1}}{l_x}$$

$$ {}_{t|}p_x = \frac{l_{x+t}}{l_x} $$

$$ {}_{t|}p_x \cdot {}_{z}p_{x+t} = \frac{l_{x+t}}{l_x} \cdot \frac{l_{x+t+z}}{l_{x+t}} = \frac{l_{x+t+z}}{l_x} = {}_{t+z|}p_x $$

$$ {}_{t|}p_x \cdot q_{x+t} = \frac{l_{x+t}}{l_x} \cdot \frac{d_{x+t}}{l_{x+t}} = \frac{d_{x+t}}{l_x} $$

Since people who survive until age *t* will either also survive until age t+1 or die before reaching it, it is obviously true that

$$ {}_{t|}p_x = {}_{t+1|}p_x + {}_{t|}p_x \cdot q_{x+t} $$

because

$$ \frac{l_{x+t}}{l_x} = {}_{t|}p_x = {}_{t+1|}p_x + {}_{t|}p_x \cdot q_{x+t} = \frac{l_{x+t+1}}{l_x} + \frac{d_{x+t}}{l_x} = \frac{l_{x+t}}{l_x} $$

Before discussing the calculation of life annuities let us recall the formula of the annuity-certain. The net single premium of a temporary annuity-due annuity-certain with annual EUR 1 is:

$$ \ddot{a}_{\overline{n|}} = 1 + v + v^2 + \cdots + v^{n-1} $$

This is the sum of a series with *n* members. Each member is the expected discounted value of a payment. The calculation of the expected value is simple here, because the probability of the payment of each member is 1 (the payment is "certain"). The principle of the calculation of the life annuity is the same, but a difference is that only the first member of the series is certain. The probability of the payment of the remaining members is the probability that the insured will be alive at that age:

$$ \ddot{a}_x = 1 + {}_{1|}p_x \cdot v + {}_{2|}p_x \cdot v^2 + \cdots + {}_{\omega-x|}p_x \cdot v^{\omega-x} $$

where ω is the statistically relevant highest possible age.
The formula of the temporary (annuity-due) life annuity is simply:

$$ \ddot{a}_{x:\overline{n|}} = 1 + {}_{1|}p_x \cdot v + {}_{2|}p_x \cdot v^2 + \cdots + {}_{n-1|}p_x \cdot v^{n-1} $$

The deferred annuity's formula is:

$$ {}_{n|}\ddot{a}_x = {}_{n|}p_x \cdot v^n + {}_{n+1|}p_x \cdot v^{n+1} + \cdots + {}_{\omega-x|}p_x \cdot v^{\omega-x} $$

We can construct the formulae for life annuities with guaranteed periods at the beginning and at the end (front-end and back-end) from the formulae for annuities certain and non-guaranteed annuities.

If we introduce the survival probability for two lives: ${}_{t|}p_{xy}$, which means both a person currently aged *x* years and another currently aged *y* years will still be alive in *t* years, we can produce the formula for the single net premium of a two-person joint life annuity.

$$\ddot{a}_{xy} = 1 + {}_{1|}p_{xy} \cdot v + {}_{2|}p_{xy} \cdot v^2 + \cdots + {}_{min(\omega-x;\omega-y)|}p_{xy} \cdot v^{min(\omega-x;\omega-y)}$$

If we suppose that the life span of the two persons is independent from each other, then these probabilities can be derived from the survival probabilities for single lives:

$${}_{t|}p_{xy} = {}_{t|}p_x \cdot {}_{t|}p_y$$

This is a classical actuarial assumption, but it is not necessarily true in practice in view of the fact that joint life annuities are generally bought by spouses whose habits (nutrition, sport, leisure, etc.) are harmonised during their long years together, so because of this, and because of their emotional connection, their deaths are quite often also synchronized ("broken heart syndrome"). Some actuaries have constructed complex mathematical models to handle this phenomenon, the discussion of which goes beyond the scope of this book.

The calculation of joint life annuities with more than two persons and all the other annuities discussed earlier can be made analogously.

The principle for calculating the reserve of the annuities is quite simple in the case of single premium annuities (practically all annuities are single premium): the reserve must contain a sum as if the annuity insurance was concluded now with the present parameters (the age is the present age of the insured, the term is the remaining term). That is, the reserve is equal to the single premium of that annuity. An example for the reserve of a life annuity at the beginning of year *t* of its term:

$$V_t = \ddot{a}_{x+t}$$

and in case of a temporary life annuity:

$$V_t = \ddot{a}_{x+t:\overline{n-t}|}$$

where V_t is the usual notation of the reserve at year *t*.

1.3. Possible problems of annuity cash-flow

The following implicit conditions are (and to a certain extent are necessarily) used when writing down the cash-flow formula of an annuity:

- The actual mortality of the annuitants is identical to the value assumed during calculation, i.e. to the "projected" mortality; this can be split up into four "sub-assumptions":
  - The composition of different sub-groups of annuitants with diverse risk profiles does not vary from the one used in the calculation.
  - There are no unforeseen trends (and especially no greater decrease than calculated) that were not included in the calculation of the mortality table.
  - There is no fluctuation in mortality compared to the one assumed, beyond the annual fluctuation resulting from having a small number of annuitants, (a small number of annuitants, which is not big enough with respect to the following point).
  - The number of annuitants is high enough to enable the whole number of deceased annuitants to exactly generate the necessary fraction of reserves.
- Everybody purchases annuity insurance of identical amount, i.e. the portfolio of annuitants is homogenous.
- Annuity payments are constant in time, they do not change.

Naturally all the above conditions must be allowed for in practice and the resulting problems must be handled either in the formulae, or in calculation or reserve management practices. Hereafter, I attempt to elaborate on problems resulting from allowing for the above conditions, and on the possible methods of treatment. Above I have listed the following problems:

- Differentiating between annuitants (insured individuals),
- Annuitant selection, treatment of selections effects,
- The applied mortality table,
- The treatment of longevity and mortality risk in general,
- Problems caused by having a small number of annuitants,
- Homogeneity of the annuitants' risk profile,
- The issue of indexing the annuity, which is strictly related to the question of what technical interest rate to apply in the annuity formula and to investment return.

Hereafter, I analyse these problems with special emphasis on selection-related problems, in view of their importance, after which I move on to other important questions that arise with relation to annuities, to all intents and purposes a discussion of regulatory problems.

2. THE TREATMENT OF ANNUITY CASH-FLOW RELATED PROBLEMS

2.1. Differentiation (and homogenisation) of the insured

One of the most important methods through which the insurer succeeds in making sure that the calculated premium of the insurance is sufficient to cover benefits is by setting markedly different premiums for different segments of the risk community; in other words the insured are differentiated according to their risk. In the case of life insurances – not including annuity insurance – this is traditionally done by determining two premium tables[14] for each product, one for men and one for women. The different premium is set according to the age of entry, and accordingly the default is differentiation by age and gender.[15] Beyond this, risk assessment (or underwriting) is generally (but not always) carried out, where efforts are made to quantify risks resulting from individual health status and behaviours such as drinking, smoking, consumption of drugs, etc. In addition, occupation (e.g. dangerous vocation – miner, policeman, etc.), sports and hobbies (e.g. extreme sports) are taken into account and it is decided whether it is necessary to increase the premium based on age and gender calculations. In other words, the insured are further differentiated by risk.

The essence of differentiation may also be expressed in a way that it enables the splitting of the risk community, which is inhomogeneous from the point of risk, into homogeneous parts (or at least more homogeneous segments com-

[14]In the EU we have to use past tense in this respect, because of the Gender Directive. The rationale of this directive is questionable and we only hope that the coming Age Directive will not prohibit differentiation according to age, which could lead to the end of life insurance as we know it. Later I will discuss the topic of prohibiting gender-differentiation (which is – mistakenly – considered as discrimination in the Directive).

[15]Naturally the premium depends (typically in a linear manner) on the sum insured, and the term of the insurance (if there is any difference in this respect at all, e.g. in the case of life-long annuities we may assume that they have only one type of term), but this difference is not due to the risk of the insured.

pared to the entire risk community). Consequently, differentiation also means homogenisation at the level of segments of the risk community.

Nevertheless, differentiation cannot always be carried out for various reasons, or it is not always possible to create an adequately high number of risk "sub-communities" with a sufficient number of members. In such cases, homogenisation of the risk community may be an independent solution, which is the opposite of differentiation in a certain sense, (though from another point of view, these assume or complement each other). In the course of homogenisation, risks deemed to be unmanageable, or individuals deemed to be unmanageable risks, are (temporarily or permanently) excluded from the insurance and from the risk community (waiting-time, exclusion, exemption, refusal). Eventually lower and upper limits are defined for the possible sum assured, where attempts are made to remove extreme risks. (I will also detail this type of method in relation to annuities).

Although differentiation and homogenisation are generally applied in parallel, they can replace each other to a certain extent. If there is less differentiation (e.g. due to business or cost considerations, to simplify underwriting an insurance policy), the importance of homogenisation increases. By applying a greater degree of differentiation, risks that would otherwise be excluded, or insured persons who constitute an extreme risk, can also be insured by the insurer.

2.1.1. DIFFERENTIATION IN THE CASE OF VOLUNTARY ANNUITY INSURANCE

The significant difference between the above cases and the case of voluntary annuity insurance is that traditionally no risk assessment is applied (neither exclusion, nor refusal). A different premium is applied based on age and gender (and possibly a lower and/or upper limit is set for the possible insurance amount – see below), but state of health, occupation, drinking, smoking etc. are not assessed. Instead, the basic supposition is that annuity insurance is by definition purchased by those who have the best life expectancy and whose expected remaining life is longer than the average. Therefore, mortality is not calculated from the public mortality table, but rather from the annuitant select mortality table that indicates significantly lower mortality.

The reason for the above practice is that the risk of annuity insurance is precisely contrary to that of all other, traditional life insurances (term, endowment, fixed term, whole life); that is the risk to be managed is not that the insured individual dies much sooner than the average, but on the contrary, that the insured party dies at an older age than average. Therefore, traditional risk assessment methods and their whole approach cannot be applied in this case,

since in the case of annuity insurances the best client is the client who poses the greatest danger in other forms of insurance. (Therefore no exclusion or refusal of the client is applied).

The consequence of the above practice is that buying annuity insurance is mainly worthwhile for those who have reasonable cause to expect a long life. Consequently, on the one hand this practice strictly constrains the potential range of clients, and on the other hand it justifies the insurer's preliminary assumption of the client's long life. Naturally this does not apply in the case of mandatory annuity insurances, since people have no choice with regard to concluding an insurance contract.

Recently, some insurers became disturbed by the consequences of traditional practice since it constrained their client base, so the so-called impaired annuity, with premium differentiated according to state of health, emerged on the British and American markets. The logic is that clients suffering from certain sicknesses and damaged health are offered a preferential tariff, i.e. their state of health is also taken into consideration in the premium (in addition to age and gender). Naturally this already presupposes a certain risk assessment, the logic of which is precisely the opposite of the traditional method: those who "simulate" illness/disability must be weeded out and not those who claim to be healthy. The phenomenon may also be interpreted so that, similarly to other sectors of voluntary insurance, competition in the annuities market has led to differentiation of the insured.[16]

We must realise that while traditional risk-assessment penalizes attitudes and behaviour that is generally regarded as negative (unhealthy lifestyle, addictions), in the case of impaired annuities these proven negative habits may provide the basis for a preferential premium.[17]

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[16]Pre-empting later sections of this book to a certain extent, the question may be raised: why, in the case of voluntary annuity insurances, is differentiation made according to state of health and not according to level of education, which is precisely contrary to what is raised with respect to mandatory annuities in the latter part of this work. The probable reason for differentiation according to educational attainment not occurring in the case of free market annuities is that annuity insurance is voluntarily concluded almost exclusively by people who have completed some form of higher education, while in the mandatory system this is also obligatory for those with a lower level of education. Naturally, the possibility of differentiation according to educational attainment could gradually decrease in future if college certificates become universal.

[17]Though no impaired annuity is necessarily needed for this, as indicated by a piece of news that was published in the Hungarian tabloids and caused quite a stir. According to these reports: "The Dutch Paerel Leven voluntary supplementary pension insurance fund has proposed charging higher premiums to heavy smokers based on the fact that their life expectancy is shorter." See: http://www.stop.hu/articles/article.php?id=284988, 1 March, 2008 Source: MTI

In summary, differentiation in the case of annuities may be envisaged according to those important factors which have a high impact on remaining life expectancy – and therefore on the premium of the annuity. These factors include:

- Place of residence, housing conditions
- Workplace and nature of work (e.g. physical, intellectual, etc.)
- Level of education
- State of health (e.g. disability, sickness)
- Lifestyle, drinking, smoking, etc., sports
- Marital status
- Income (and via this, the magnitude of the expected annuity)
- Adverse selection – provided the insured party has a choice

It is important to note that these factors are not independent from each other; for example, state of health is related to lifestyle, and probably also to housing conditions and level of education, and vice versa, etc. This must be taken into consideration at the final determination of differentiation factors. Below, I examine and rank the possible use of these factors (with the exception of adverse selection, which I detail in the coming chapter).

2.1.2. OPTIONS FOR THE DIFFERENTIATION OF MANDATORY ANNUITIES

According to the above, the pure actuarial approach would require a deeper differentiation in groups of diverse risk, since the calculation becomes increasingly stable by further deepening differentiation. In addition, the stochastic equivalence between the service received and the premium paid becomes valid for increasingly narrow groups, in other words; the deeper the differentiation, the more equitable the annuity. On the other hand, reference made to the stability of the calculation also naturally means that the less differentiated the annuity, the less stable the calculation, and especially in the case of several, competing service providers, and some mechanism is required to manage this instability.

At this time no-one doubts that annuity benefits provided for identical assets must be differentiated according to age. This is particularly fortunate from a regulatory point of view, because the lack of this differentiation would either make a mandatory annuity system unmanageable, or would require drastic measures (basically necessitating a uniform age of retirement) that both the theory and practice of the Hungarian pension system has already left behind. As regards any other differentiation, the various opinions are totally divergent.

Looking at this issue from the point of view of differentiation possibilities as regards annuities, the major obstacles to differentiation (apart from age) are

not independent from each other. There is some overlap between them and therefore these might theoretically be listed separately. Primarily due to the overlapping, I do not highlight the general problem of differentiation by gender as a special obstacle, although the list below covers this issue.

1. Connection to the annuity of Pillar I
2. EU regulations
3. Equity problems – relation to other social welfare systems
4. Incentives for undesired trends
5. Available data, assessability of the given parameter

Let us analyse these in the above order.

Connection to the annuity of Pillar I. Private pension funds in Hungary were established with the aim of enabling the annuity purchased from accumulated assets to complement the annuity received from Pillar I, where the latter remains the larger component within the total pension. Although it was not specified that the annuity of Pillar II must operate according to the same principle as the Pillar I annuity (and in fact several factors such as enabling a choice between different kinds of annuity suggested precisely the opposite), it is (would have been) a logical requirement that the two parts of an individual's pension match each other, i.e. that major parameters like differentiation must be in harmony within the two pillars.

Looking at the annuity of Pillar I from the perspective of differentiation, we may say that it includes a differentiation by age, although not in a perfectly correct way actuarially, but it exists nevertheless. However, as differentiation by gender is becoming increasingly less common[18], we may practically assume that is does not exist. There exists differentiation according to the level of accumulated entitlements (income), i.e. the digressive scale, but this is also going out of fashion. There is no differentiation according to place of residence, level of education, state of health, marital status, lifestyle, drinking, smoking, etc., but there exists differentiation according to job and workplace to the extent that a preferential retirement age is determined for certain occupations (miners, ballet dancers, members of the armed forces).[19] Although this preferential age of retirement is not calculated based on lifetime being different from the average, but to a certain extent on an arbitrary basis and while con-

[18]Becoming less prevalent because the lower retirement age of women may be considered as a form of differentiation by gender, even though this makes the annuity even less correct from an actuarial perspective.

[19]In 2011 the majority of these were cancelled for new entrants. One of the exceptions are ballet dancers.

sidering other factors (e.g. in the case of ballet dancers the opportunity to carry on the given vocation[20]).

So it may be assumed that (using wording that takes into account the characteristics of Pillar II) in Pillar I there is no (current or later) differentiation by gender, or according to the magnitude of accumulated assets, or any other kind of differentiation with the exception of differentiation by workplace and the nature of the work performed, but differentiation does not happen in an actuarially correct manner in this case either.

So, in contrast to Pillar II annuities, Pillar I represents (would have represented) the requirement that it should include no form of differentiation except by age and certain occupation groups. This issue is especially sharply raised when it comes to differentiation by gender, as annuities were differentiated according to this criterion by Hungarian insurers until 21 December 2012. This (i.e. to immediately ensure that this general practice of private insurance companies should not be applied to mandatory annuities) is probably the reason why the Private Pension Funds Act has always stipulated that the mandatory annuity must be calculated using the unisex mortality table.

Though the requirement posed by Pillar I is clear, it is still not necessary to restrict options for differentiation in Pillar II. However, the use of the unisex mortality table is such a strong requirement that no divergence is possible from this perspective.

The EU regulations characteristically do not concern themselves with what options member states allow or prohibit for differentiation in the case of insurance products and annuities, with one important exception: differentiation by gender. The so-called Gender Directive adopted in 2004 ("COUNCIL DIRECTIVE 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services") prohibits the application of different premium calculations according to gender, although not with immediate effect and while taking into account the specific regulatory frameworks of member states. The provision is justified according to items (18) and (19) of the Preamble as follows:

"(18) The use of actuarial factors related to gender is widespread in the provision of insurance and other related financial services. In order to ensure equal treatment between men and women, the use of gender as an actuarial factor should not result in differences in individuals' premiums and benefits. To avoid a sudden readjustment of the market, the implementation of this rule should apply only to new contracts concluded after the date of transposition of this Directive.

[20]Combined with the arbitrary assumption that the individual will not be able to conduct any other occupation to earn a living and consequently must retire.

(19) Certain categories of risk may vary between the genders. In some cases, gender is one but not necessarily the only determining factor in the assessment of risks insured. For contracts ensuring those types of risks, Member States may decide to permit exemptions from the rule of unisex premiums and benefits, as long as they can ensure that underlying actuarial and statistical data on which the calculations are based, are reliable, regularly up-dated and available to the public. Exemptions are allowed only where national legislation has not already applied the unisex rule. Five years after transposition of this Directive, Member States should re-examine the justification for these exemptions, taking into account the most recent actuarial and statistical data and a report by the Commission three years after the date of transposition of this Directive."

In Article 5 of the Directive ("Actuarial factors") it specifically states:

(1) Member States shall ensure that in all new contracts concluded after 21 December 2007 at the latest, the use of gender as a factor in the calculation of premiums and benefits for the purposes of insurance and related financial services shall not result in differences in individuals' premiums and benefits.

(2) Notwithstanding Paragraph 1, Member States (including Hungary) may decide before 21 December 2007 to permit proportionate differences in individuals' premiums and benefits where the use of gender is a determining factor in the assessment of risk based on relevant and accurate actuarial and statistical data. The Member States concerned shall inform the Commission and ensure that accurate data relevant to the use of gender as a determining actuarial factor are compiled, published and regularly updated. These Member States shall review their decision five years after 21 December 2007, taking into account the Commission report referred to in Article 16, and shall forward the results of this review to the Commission".

So in summary: whilst member states must prohibit differentiation by gender in premium calculation, they are free to temporarily decide not to implement it in the case of certain types of insurance. This temporary exemption may theoretically be endlessly extended, but should they fail to do so this exemption will terminate and the unisex tables must be applied.

This was the situation until 1 March 2011, when the Court of Justice of the European Union made this Article invalid with effect from 21 December 2012. This means that in future, differentiation according to gender is not an option in the case of insurance products (Court of Justice of the European Union [2011]). However, in theory this rule is not valid for mandatory annuities because they fall under the sphere of authority of a different Directive to the one governing employer pension funds, but it is valid for mandatory annuities provided by insurance companies. Although it seems that independently from the Gender Directive, and especially because of the reasons explained in the previous chapter, a unisex rate must also be applied in the case of mandatory annuity.

Equity problems – relation to other social welfare systems. Although some of the above-listed aspects of differentiation may be justified and correct, mainly taking into consideration life style, drinking, smoking, sports etc., and accordingly with regard to state of health, they may nevertheless lead to inequalities under the existing social insurance systems. In the case of annuity, people with no unhealthy habits and who lead a healthy lifestyle and participate in sports would be expected to have a longer lifetime, meaning they will have to pay a higher premium for the same annuity. This is fair if this same factor is also taken into consideration (with an opposite effect) in other systems, because a healthy person probably requires less healthcare, so it would be justifiable to decrease their level of healthcare contributions, while people who lead an unhealthy lifestyle should have to pay more[21]. So taking the annuitant's state of health into account is equitable provided it is also taken into consideration by the healthcare system.

Incentives for undesired trends. In strict relation to the above, state of health considerations may appear in the annuity premium of an individual if the annuity provider penalises healthy lifestyles and rewards unhealthy lifestyles and harmful habits such as smoking, drinking alcohol and taking drugs. This in itself may be an incentive for the development of such habits, or at the very least may provide a good excuse for not changing existing bad habits. This undesired incentive may have a milder, but equally dangerous impact from an annuity point of view in that it motivates certain individuals to prove harmful behaviours in order to receive a more favourable annuity. In fact, both effects should be avoided, on the one hand because they are directly harmful, on the other hand because filtering out potential fraud may increase the cost of the annuity determination process.

Available data and the assessability of the given parameter. When insurers traditionally differentiate annuities according to age and gender, it is the result of considerate evaluation and emphasises those factors that have the most important impact on lifespan, such as:

[21] As a curiosity, I mention that many experts question whether prevention diminishes healthcare expenditure. It is possible that the National Health Service spends more on those who have a healthy lifestyle, because they are more likely to show up at screenings and receive treatment at an early stage of illness, etc. By contrast, an unhealthy lifestyle is often related to people who don't participate in screenings, do not request treatment and only go to the doctor when there is already nothing to be done, so there are fewer costs for the healthcare fund. If this is true, then although differentiation according to state of health may be justified within the annuity system, it would also generate incentives for undesired trends.

- It can be easily and reliably assessed at no additional cost
- Reliable data is available on the correlation between the parameter and life expectancy
- It is a clear and stable (calculably variable) characteristic of the insured party

From this perspective, age and gender are incomparably more useful parameters than the following from among the previously listed possible differentiation factors:

- Accommodation, housing conditions
- Workplace, nature of work (e.g. physical, intellectual, etc.)
- State of health(disability, illness)
- Lifestyle, drinking, smoking, etc., sports
- Marital status

Accommodation, workplace, nature of work, customs, habits, marital status and state of health all change and so it is difficult to characterise them by a single parameter in order to compare different individual situations. It is even more difficult to assess them objectively (except for marital status perhaps) and there is a great opportunity for concealment or misrepresentation of the situation, so this data may only be acquired by the insurer in a costly and unreliable manner (although to different extents). Furthermore, we generally do not possess good statistics on the impact of these factors on life expectancy (an in fact they are not independent of each other).

From among the above-listed factors, income (and through this the expected annuity payment) and adverse selection can clearly be easily and reliably assessed at no additional cost to the insurer, but it is also true with relation to these (at least in Hungary), that we do not possess good statistics on the impact of these factors on life expectancy. Although the (highest) level of educational attainment may change during the course of an annuitant's lifetime, we can generally expect it to be a stable and relatively easily identified factor well before retirement, and there are already statistics concerning its correlation to life expectancy. (This is also true with relation to marital status.)

Despite the above-listed obstacles, there are not only actuarial but also "social policy" arguments in favour of differentiation according to certain parameters. In this context, it is important that the regulator takes into account that in the case of a mandatory insurance – for example the mandatory annuity – the prohibition of applying differentiation to premium calculations also means that the regulator creates a forced financial solidarity among certain risk groups within the population; in other words, it forces groups of people to financially

support other groups.[22] This type of redistribution is well known in other fields of social life, but it is important to repeatedly call attention to the fact that the usual ideology of annuities – according to which the state takes from the relatively better off and gives to the less wealthy, or that the common social cost is borne by the rich rather than the poor – does not fully work and to a certain extent the truth is just the opposite. The relatively poor have a shorter life expectancy, so they support the rich who have a higher life expectancy. Only the lack of differentiation between male and female partially contradicts this and regular arguments in favour of redistribution are still valid. This phenomenon is often referred to (in a rather telling way) in literature as "pervert redistribution".

In the context of pervert redistribution we must mention that the prohibition of differentiation between male and female with regard to annuities (generally) works against this (while it maximally effects a "non-pervert" or what we might call a "usual" redistribution) since women, who are generally in a less favourable position on the labour market, find themselves in a more favourable situation during the annuity. But if we also cannot differentiate, for example, according to level of education, then high income earners will be in a better position compared to people with lower incomes, and accordingly in the case of mandatory annuity the extension of the prohibition of differentiation to every other factor beyond age may not necessarily be easily justified. Let me mention here a factor to be detailed later in the book that suggests that the lower the technical interest rate, the more favourable the annuity for the higher income layers of society (because of longer lifetime) than for the lower income layers (with a relatively shorter life expectancy). Similarly, the guaranteed period is more favourable for people with lower incomes. In summary, pervert redistribution may be diminished if annuities are differentiated according to factors (e.g.: educational attainment) that correlate well to income, or if we apply a fixed, but variable technical interest rate, as will be discussed later, or if we apply a buffer-type death benefit, as will also be discussed in a later chapter.

[22]Adopting the view generally accepted in the actuarial profession, I in fact do not consider insurance itself, or the financial transfer necessarily taking place among the insured via insurance, as constituting solidarity. I consider this characteristic of insurance to be natural, and in itself it includes neither force nor solidarity. However, solidarity does appear in insurance, and in the form of forced solidarity, if groups of evidently different risk and with a balanced premium are classified into the same risk community. The magnitude of the forced solidarity is also precisely as much as the amount of money transferred by one risk group to the other due to the prohibition of differentiation.

2.1.3. OPTIONS FOR LEGISLATORS

To avoid fraud, I would suggest we calculate only those of the above-mentioned factors that can be assessed relatively reliably and cheaply, and accordingly we may consider the following differentiation options:

- According to gender
- According to the magnitude of pension assets
- According to state of health
- According to certain occupations
- According to level of education

In addition, adverse selections due to clients having a choice between different types of annuity with different mortality tables (i.e. differentiation according to the choice made by the annuitant) is also a possibility, the theoretical impossibility of which I will prove in the following chapter.

Differentiation **according to gender,** based primarily on political and not technical arguments, may not be considered as a possibility with regard to mandatory annuities for the time being. This raises significant technical issues that may possibly be facilitated by other measures, for example by the mandatory joint-life annuity that will be discussed later. Nevertheless, there may be techniques that can be used to neutralise arguments against differentiation according to gender (if this was not banned), and this is the regrouping of funds between individual accounts, the impact of which on the magnitude of the annuities of women and men is identical to that calculated on the basis of a unisex table, while the actual calculation is made on the basis of tables that are differentiated for men and women. The mechanism of the solution and related arguments has already been detailed in a previous article of mine (Banyár [2002]). Among others I explained that the unisex mortality table, which clearly favours women more than men, compensates for certain inequities against women, but despite this, I do not approve of its implementation. In the very same study I said that even if we stick to the idea that women and men of similar age should receive the same annuity for the same savings, it would be more correct to directly regroup the assets from the pension accounts of men to those of women to the extent that it simulates the application of the unisex table, than to actually apply the unisex table.

If regulations do not homogenise the size of possible annuities, **differentiation according to the magnitude of pension assets** presents itself as a strong possibility. This, to a certain extent, handles the differences between accommodation, physical and intellectual work, level of education and lifestyle. Although we must note that we have no data about the level of positive correlation between life expectancy and the magnitude of accumulated assets assumed by many, including myself, to exist in the case of private pension funds.

This data should be systematically collected (allowing for the fact, although not assuming, that this data does not indicate a correlation between the two factors, as is supposed by some experts, or that they might possibly indicate a negative correlation).

In the long term, the introduction of **differentiation according to state of health** is also worthy of consideration, although for the foreseeable future (until it is also taken into consideration within the healthcare system) it is expedient to avoid enabling characteristics that are widely considered as negative, primarily meaning harmful behaviour such as smoking and alcoholism, to represent an advantage with regard to annuities (even if this may be justified from a risk perspective). However, the level of detail of differentiation according to state of health may be very different; practically speaking we are dealing with a group of factors that may be taken into consideration gradually as we learn more about their impact on life expectancy. Perhaps the first fact that may be taken into consideration is disability, which is well-documented; it is important to note that the structure of disability pension and its adjustment to the system of private pension benefits is an important problem that I do not deal with in this book.

Consideration of the impact on life expectancy of spending many years in a certain occupation, e.g. a miner, may also be raised. However, two important issues need to be mentioned in relation to this factor:

1. The trend of technical development is clear; occupations requiring higher than average physical effort are gradually disappearing, so this differentiation factor is losing importance in the long term.
2. In Hungary the pension system significantly abused this factor for a long time (until 2011) because it defined a number of occupations (armed forces, ballet dancers) with regard to which many years of early retirement were assured in a way that is totally unjustified based on the actual physical stress and can only be interpreted as having remained in the system as a political heritage. For this reason, the gradual elimination of existing methods of differentiation is on the agenda rather than taking into account differentiation according to occupation. In Hungary these kinds of differentiation only officially exist within the pension system in Pillar I and the Act does not provide for whether these must also be taken into account in Pillar II and if so, how. This obviously generates tension between the two systems when the annuity phase commences.

Differentiation according to educational attainment meets most of the necessary requirements: it is easy and cheap to assess, “politically correct” in that it favours the poor as opposed to the well-to-do; statistics are available about

its impact on age, which indicate that according to this factor the life expectancy of the Hungarian population is extremely diffuse. The application of this factor should be among the first to be examined, especially if differentiation by gender cannot be implemented by any method. Naturally, technical questions such as when should a change in the level of educational attainment be taken into consideration as a factor during the course of calculation (i.e. until the annuity commences or perhaps until a certain pre-determined age?), must be answered first.

Similarly to differentiation according to occupation, although perhaps more in the long-term, the same concern may be raised here; namely that this factor would also "run out" in the coming decades. There seems to be a trend nowadays that higher education is becoming increasingly general. This emphasises the requirement of differentiation in that we must leave possibilities for differentiation according to other factors open for the future.

Above I discussed differentiation as differentiation according to premium. Clearly, differentiation according to reserve follows from differentiation by premium, but it may also be logical for differentiation according to reserve to be more advanced (according to several factors) than differentiation by premium. This may be specifically raised when the regulation – due to different, non-technical considerations – basically prohibits differentiation (e.g. by gender, age and other factors). This may lead to the perception that two annuity portfolios of strictly different composition from a risk point of view appear to be identical and the same provision is created for them, which however would not be a good idea from a prudential perspective.

Theoretically, any parameter may be used for differentiation by reserve; no considerations are raised against it such as in the case of premium differentiation according to gender. Accordingly, it is logical that even when using the unisex premium table the gender of the insured should be taken into account during the course of reserving, and a larger reserve should be accumulated for portfolios containing more women even if the same amount of premium was collected with respect to them as for portfolios that include more men.

During the course of reserving, differentiation according to other factors beyond the above would be useful with respect to prudential aspects. The highest education level attained (for example, classifying the population into 2 or 3 groups according to their level of education) is perhaps the most useful of the above factors.

2.1.4. DIFFERENTIATION IN LITERATURE

Blake [1999] mentions adverse selection and mortality risk as major problems of annuity for providers and annuitants. On the basis of asymmetric information, only people who expect a long life are prepared to voluntarily purchase an annuity. According to Blake, the insurer is unable to differentiate among annuitants of diverse risk. Therefore the annuity is calculated on the basis of the mortality of a select group, so for those with worse mortality, annuity would be a bad deal.

Blake's statement contradicts what I stated above, because in my view the insurer is able to differentiate, as indicated by the recent developments on the annuity market which were not yet obvious back in 1999. As I have written, I agree with Blake to the extent that insurers have not differentiated among the annuitants until quite recently, which is probably the consequence of the small voluntary annuity market, which is unattractive to the service providers. Annuitants were offered a high premium as a consequence of the above, which further restricted the market. So this phenomenon existed, but in my view Blake explained it incorrectly and not everybody agrees with it in the literature either because, in a competitive market, a better premium is offered to those of better risk according to the World Bank [1994]. Finkelstein-Poterba [2002] traces the phenomenon back to the decisions taken by insurers; according to them, insurers do not collect data on social-financial status and accordingly they do not differentiate in premium, which causes losses for insured persons of low status who purchase annuities. They thought this statement needed explanation, and according to them the plausible explanation is that verification of such data is expensive. Based on their survey, when information on annuity premiums was collected from many annuity brokers, it turned out that providers do not differentiate much among clients. In addition, they have not specifically prepared to differentiate in the future because to set the premium they have only been interested in age, gender, type of annuity, market type (voluntary, mandatory), assets, frequency of annuity payment, duration of guaranteed period and the benefit payable to possible survivors. There were only a few companies that offer specific premiums for clients in a state of bad health, e.g. smokers.

The World Bank [1994] also believes that adverse selection is the primary problem with regard to private annuities. As with Finkelstein-Poterba, the World Bank also mentions that for insurers, the price of the annuity is actuarially unfair in respect of the good risks (i.e. for people with a shorter than average life expectancy), so these products are only bought by the rich. In other words; annuities are a luxury. The World Bank only mentions but does not explain the phenomenon, although it calls attention to the consequences of redistribution among annuitants. This already appears on the voluntary annuity

market where, in principle, there is no obstacle to providing an annuity with a preferential premium for those of higher mortality. However, this redistribution may pose a special problem if prices cannot be differentiated, especially if annuity purchase is mandatory at these prices. According to the World Bank [1994], such uniform prices mean an ex ante transfer from low risk to high risk and it is particularly unfavourable for low income workers. This means it is not a usual redistribution because the money flows from the lower income (i.e. shorter than average life expectancy) people to the higher income (i.e. longer than average life expectancy) people. Consequently this sort of redistribution is generally called "pervert" in the literature.

Redistribution as a result of prohibiting differentiation was also examined by others. Brown [2003] came to the conclusion that even if a large redistribution item occurs in the mandatory annuity because of uniform premiums it may still be worthwhile for people with higher mortality rates based on utility, if the cost of annuity administration is low. It seems that Brown got these results disregarding the bequest motive because at the end of the study he called attention to the fact that in future researches this impact should be taken into account despite the fact that there is no consensus among economists as regards the significance and modelling of this motive.

In the Hungarian literature, the issue of redistribution was raised in the context of private pension annuity, the conclusion being that efforts should be made to avoid it. In the study by Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] it interestingly does not appear with relation to demands for differentiation, but with relation to the refusal of a Guarantee Fund and the yield-balancing provision as redistributive measures, which is contradictory to the often-quoted basic philosophy of the private pension system that includes self-care, individual savings, etc. (page 486) (I do agree with this, as I will elaborate on later). The study focuses on the non-transparent character of these redistributions as a further negative element. Michaletzky (Michaletzky [1999]) also agrees with the avoidance of redistribution and on this basis he finds the implementation of the unisex table to be problematic (page 103). "The most painful point in setting up the service is the implementation of an adequate mortality table. …. [the unisex table] is clearly an item of the Act, the application of which compromises the principle of the fully funded pension fund and, understandably, gives rise to social criteria."

In the literature, the issue of differentiation appears primarily in the context of mandatory annuities. The World Bank [1994] says that if the annuity is mandatory then the question of what factors premium differentiation should happen according to becomes an important public policy issue concern. Can we differentiate according to income, gender, race or other factors? They mention that, for example, in the United States of America race and gender are prohibited differ-

entiation factors in occupational pensions and in life insurance. (In the United States race is an important factor, in Hungary race would not even be considered as a factor since the population is largely homogeneous.)

Neither English nor Hungarian literature provides orientation regarding the issue of differentiation, nor does it tell us what criteria should be applied. The issue is a problem, and regulations are unclear about the allowed factors (e.g. Michaletzky [1999] and Réti [1999]). They use a special wording stating that "it is not clear who compose a risk community". This wording confuses the problem of differentiation and mortality profit distribution. (E.g. when Michaletzky [1999] lists the options of risk community, he mentions the following: according to type of annuity, investment portfolio and age cohorts. He says it is not clear whether the fund is entitled to define who belongs to a certain risk community or if it is defined by law.) Réti [1999] believes that the risk community should mean every member of the fund collectively, including active and inactive members, which would indicate the distribution of mortality profit rather than differentiation. There is only one negative statement with regard to differentiation that can more of less be regarded as a majority opinion: there should be no differentiation according to gender, although as we can see in the case of Michaletzky above, there are those who find this problematic. Ágoston-Kovács [2007] calls attention to certain pieces of research in Hungarian demographic literature (not necessarily relating to annuities) that could also be used as criteria for differentiation. According to this (Kovács [2007] page 7): "We have known for a long time that widows and divorced people have a higher mortality and a worse state of health than their married peers. With a few notable exceptions, similar patterns are also true for single men and women".

Winkler-Mattar [1999] also believe that the solution to the problem of adverse selection in annuities is the differentiation of annuitants, although they mention as a new perspective that if we differentiate the annuitant portfolio, e.g. offer a more favourable premium to people with higher mortality, it would in principle aggravate the problem of selection in the remaining population. Fundamentally this is the problem of the old providers, but is also a possibility for new market players as mentioned by Ainslie [2000] in respect of impaired annuities provided for people with a damaged state of health, which allow the new entrants to "cherry-pick" the clients of existing providers. According to his data the first such annuity (the so called nursing care annuity) was sold by Eagle Star in 1991.

While in the case of a voluntary annuity this "cherry-picking" is a part opportunity, part danger, depending on whether we are talking about a new or old service provider, in the case of a mandatory annuity it clearly represents a danger for the regulator. The World Bank [1994] has already called attention to this. According to the World Bank, if risk categories are created then annui-

ty providers will attempt to skim the best risks in every category. Accordingly, they take a clear stand that if annuity purchase is mandatory then service providers must be prohibited from selecting among annuitants, or a specific fund must be created to provide cover for "bad risks" (i.e. people who are very healthy). This must be subsidised by either the annuity providers or by the state. For my part I think that this second solution is not the way to go, because it makes the system unnecessarily complicated or obviously transforms taxpayer money to profit in the case of certain providers.

2.2. The issue of the applied mortality table

There is one certainty about mortality tables used in annuity premium calculation and reserving: we must use a projected mortality table that takes into account the foreseeable change in mortality, as tables based on historical data may not be satisfactory. The above differentiation factors should also be considered via the use of mortality tables. However, other issues, including for example whether we should apply a central mortality table or use different ones for each provider, depend on other elements of the annuity system.

2.2.1. THE PREPARATION OF THE MORTALITY TABLE, MORTALITY PROJECTION

One question that needs to be answered is who should prepare the (unisex and differentiated, but definitely projected) mortality tables for annuity premium calculation and for reserve calculation. There are diverse possibilities, the more important ones being:

- A state institute established for expressly this purpose, or an existing institute (such as the Central Statistical Office's Institute of Demographics), which is entrusted with this task,
- The providers involved (life insurers and funds), or an institute established by some of these providers (or possibly several groups of providers),
- Each provider prepares these tables themselves.

In Hungary at present, the Central Statistical Office's Institute of Demographics prepares mortality tables (e.g. broken down according to geographical area, place of residence, level of education), but the composition of the population that participates in a relatively new voluntary private pension system (which is possibly increased according to changing state preferences and then reduced to a minimum, and therefore strongly selected) is clearly a

special case. There is good reason to assume that the life expectancy of private pension fund membership is different to that of the entire population of Hungary, but in the early phase of a scheme this difference is difficult to estimate, and in case of a low number of annuitants this difference can be extremely high. Later, with the growing number of pensioners taking part in this scheme, the difference between participant mortality and the entire public mortality table will probably decrease gradually.

A single provider can only prepare mortality tables for itself with sufficient precision if it has a large enough, stable enough and old enough portfolio of annuitants. But in the initial phase of a scheme no company provide such annuities, so nobody has a sufficiently old portfolio. The portfolio may only really be regarded as stable if the members of the private pension fund system do not have the opportunity to change providers, but this can only be achieved if the whole system was launched with these conditions. If (as is the case in Hungary) people's right to choose between providers was declared when the system was first launched, it would constitute such a big step backwards for schemes that such a step is practically impossible. In a free market many providers have sufficiently large portfolios, but not all of them. Due to all these factors, in a newly launched private pension system we can generally not assume that any of the providers are capable of producing their own mortality tables, so the regulator should reject this possibility at the launch of the system.

In summary we either have an existing or a new institute which is run by either the state or the providers involved. Either way is feasible, but it is expedient for the state to take on the role of initiator via regulation, either by establishing or designating such an institute, or by assigning responsibility to the providers and setting them a deadline. In other words, mortality tables and mortality projections should under all circumstances be prepared centrally (and projections should be regularly checked and recalculated by the state), for which it is expedient to use the mortality data of private pension fund members that has already been collected (which in Hungary, for example, the Central Registry of Pension Funds did indeed collect until 2011).

2.2.2. CENTRAL MORTALITY TABLE OR INDIVIDUAL PROVIDER MORTALITY TABLES?

The next question if whether or not the centrally prepared mortality table must be used obligatorily by every provider (if there are multiple providers at all). The following major possibilities arise:

- Everybody must apply the centrally prepared mortality tables in unchanged form,

- The centrally prepared tables must be applied, but a certain, predetermined level of deviation is permitted,
- Any degree of deviation from the central tables is allowed, but the provider must justify the reasons for deviating from the table,
- The central table is merely provided as an aid for the provider, which is free to deviate from them in any manner without having to provide justification.

The question may also be raised whether the above possibilities apply for both premium calculation and reserving. To cite just one example: should everyone apply the same central table in unchanged form for both premium calculation and for reserving, or should use of the central table only be mandatory for reserving, while diversion from the central table is permitted with respect to premium calculation?

There is no free choice between these possibilities; the choice depends on the other elements of the system. For example, the last option assumes that service providers take all responsibility for calculation and provisioning, if they make a bad decision nobody will come to their aid. This also represents a risk for the clients. This may only be expected from service providers if they can count on a stable range of annuitants, i.e. if clients cannot move freely from one provider to another.[23] If according to the annuity regulations the insured may also move from one provider to another in the annuitant phase it is important to clearly set the magnitude of the reserve that they may take with them, and this is only possible if the mortality tables used to calculate reserving are centrally defined.

When prescribing the mandatory application of the centrally defined table it is important to take into consideration that such a requirement makes the state responsible if the application of this table places the service provider (which is not owned by the state) in a difficult situation. However, the state may not necessarily avoid such responsibility by not making the application of such a

[23]This is the reason why I find the regulation of the Hungarian private pension funds concerning this option, which has been in force throughout the existence of the system, deeply problematic. As Article 6 (1) of Government Decree 170/1997 provides: "…the mortality tables defined in Article 32 (1) of the Pensions Act as well the ones to be used in Article 16 (2) shall be selected or prepared from the mortality tables published by the Central Statistical Office by the fund actuary with respect to the demographic conditions of the fund membership receiving the service". This places the burden of all responsibility concerning the premium calculation and reserving on the fund and its actuary, but without helping them to stabilise the portfolio of insured at the fund. So this regulation was contradictory in itself and it should have been changed.

table mandatory, if the state does not create other conditions adequate for the provision of calculable annuity for the providers.

Making the application of the central table mandatory for reserving has significance for consumer protection too, since it protects clients from irresponsible service providers who intentionally charge a low premium but have low reserves, and who after a certain period of time are unable to provide the undertaken service.

If we look at whether the application of central tables should be made mandatory at reserving or at calculation of the premium, we can state that it is not expedient to require mandatory application of the central table at reserving, but not with regard to premium calculation, because this would provide premium calculation with a kind of illusionary freedom.

If the central table is also required at premium calculation, this explicitly emphasises the responsibility of the state for the adequacy of the calculation, which only works if the state operates a premium offset mechanism in parallel with imposing the mandatory application of the central table. The imposition of the central table at premium calculation (and here we only mean the calculation of net premium), if coupled with the centrally defined technical interest rate, gives an advantage to the client in that they can easily compare the premiums set by different providers. Although this is basically also the case if the central table (and the technical interest rate) is imposed only for reserving, since this in itself strongly determines premium calculation.

Transitory solutions between the application of a mandatory table and a free choice of tables are not clear enough solutions and it is difficult to put forward good arguments in favour of them. These transitory solutions may be applied in cases where the state wants to regulate, but at the same time wants to rid itself of the responsibility in such a way, for example, that it does not provide tables differentiated by an adequate number of parameters for reserving, so adapting to the concrete composition of the given risk community is ultimately left to the provider, together with the risk.

Based on the above, the possibilities are reduced to the following:

1. For reserving, every provider is obliged to apply the central mortality tables (differentiated according to several factors, primarily by gender and educational attainment), but are free to deviate from the central (unisex) tables with respect to premium calculation.
2. The central mortality tables must be used for calculating both reserves and net premium.

Method 1 seemingly provides a certain level of illusionary freedom with regard to premium calculation, and at the same time it allows providers to adapt to the expected risk composition of the risk community. It also signals that

responsibility for the bad composition of the risk community is transferred by the state to the provider, which is not necessarily an equitable method if differentiation according to the above-mentioned important aspects is prohibited.[24]

In the case of method 2, the state must also explicitly recognise what is implicitly recognised by the imposition of the rule: that it takes responsibility for the adequacy of the calculation. In other words – as I have written above – we explicitly recognise the responsibility of the state for the adequacy of the calculation, and parallel with the imposition the state operates a premium-offset mechanism. Prohibiting differentiation gives strong arguments to support the operation of a premium-offset mechanism, since in this case the failure of a provider is not necessarily the provider's fault, but may have been caused by the regulations (because of the prohibition of differentiation).

If the state imposes the application of tables differentiated by an adequately high number of parameters for both premium calculation and reserving, it automatically solves the problem of funds with special composition (which are characteristically closed); these do not require separate regulation. If the tables are not adequately differentiated, it is possible that closed funds must be managed separately, although in this case they must be closed from every aspect, i.e. members must not be allowed to change providers.

The groups of specific management in Pillar I, (policemen, prison guards, soldiers, ballet dancers, etc.) pose a separate problem, but I will return to this issue later.

2.2.3. THE MORTALITY TABLE IN LITERATURE, THE UNISEX TABLE

Literature identifies mortality table problems related to the fact that no reliable mortality data is available in many countries; fortunately Hungary is not one of them. This however is a major obstacle in annuity market development (e.g. Rocha-Thorburn [2007]). Nevertheless not all the necessary mortality-related data is available in Hungary either, as mentioned in papers written by Stahl and Michaletzky when speaking about the development methods of unisex mortality tables. They also provide details of what further factors should be taken into account in an adequate annuity mortality table.

It is with relation to the already mentioned adverse selection that the possibility of using different mortality tables for annuities than the ones applied in

[24]Because in this way the composition of the risk community does indeed become unpredictable, because if, for instance, a provider expects a lot of male clients and so sets a relatively low premium, then this will attract many women from other providers, who were counting on having more female clients and the calculation will no longer be valid, meaning the provider will experience an immediate loss.

the case of other life insurances arises (e.g. Hylands-Gray [1992]). They say that the use of a single basic table is justified for mandatory pension annuities, meaning we should experience no selection effects in this case. Mehr-Gustavson [1987] states (page 533) that in the case of annuities, mortality tables different from the ones applied at other life insurances must be used due to three reasons: "(1) Safety factors built into life insurance mortality tables would have the opposite effect if the table were used for annuity rates. An unsafe rate would result in projecting a lower survival rate than indicated by the basic data. (2) Mortality rates have been decreasing. While this trend is a safety factor for life insurance rates, the effect is a table that is unsafe for annuities. For every year that annuitants live longer than predicted, the insurer suffers a survival loss. (3) Because people in bad physical condition are unlikely to purchase annuities, mortality among annuitants is usually lower than among life insurance buyers. Annuity mortality tables therefore will show fewer deaths and a greater life expectancy at any given age than will mortality tables used for life insurance".

The unisex table of mortality is dealt with by the Hungarian and English language literature in many instances. According to the previously quoted Mehr-Gustavson ([1997] pages 533-534) there is an increasing demand for a unisex table in society, especially in the case of occupational pensions. In the case of individual annuities – as women buy other individual life insurances cheaper – it would be logical for annuities to be more expensive in their case. Despite this, there are increasingly frequent requests for the use of unisex tables. According to the authors it seems that actuarial considerations are ignored in heated social disputes and the actuarial equity is replaced by social equity, so it is possible that unisex tables will also be implemented in the case of individual annuities.

According to more recent literature (Curry-O'Connell [2004]) every annuity is differentiated by gender in England, but this book examines the possibilities and impacts of pricing mandatory annuities using unisex tables.
In their opinion the arguments in favour of the unisex table are as follows:

- Unequal annuities with respect to identical pension asset payments constitutes discrimination,
- The observed difference in life expectancy between men and women is irrelevant, because there is an extremely significant correlation at those ages when the majority of people die,
- Unisex annuities would increase the pension income of women,
- Gender is becoming an increasingly less relevant factor in annuity pricing.

Arguments against are as follows:

- Annuities differentiated according to gender cannot be considered discriminatory in view of the fact that women live longer than men and so the expected income is equal in both cases,
- In any given year insurers are more likely to pay annuity to a woman than to a man who purchased the annuity at the same age, so correlation according to age is irrelevant,
- The unisex annuity would decrease pension income and increase costs.

According to Curry-O'Connell [2004] a further argument in favour of unisex annuities (and this is also the most important argument), is that no differentiation can be made between men and women in labour law. In the USA and in Canada the occupational DB pension is unisex by mandate. Unisex occupational pensions exist in the UK and constitute one third of the market. We should also consider that unisex annuity is provided within the state-run pension system in both the UK and Sweden. They say that if unisex annuities were mandatory, unisex premiums would be better than under the current, voluntary system. They think that despite the fact that a unisex premium would not have a significant impact on pension income they also had not found any argument against mandatory unisex annuities.

Hungarian literature deals extensively with unisex mortality tables and unisex annuities. Réti [1999] believes that the unisex annuity does not mean uniform unisex annuity on the entire market, only within each separate fund, and therefore premiums may be very different at funds with a majority of men and a majority of women. According to Stahl [2000] the two weak points of the private pension system are the normative annuity (which was abolished while this book was being written) and unisex annuity. In respect of the latter, the major problem according to Stahl is that regulations state that premiums must be determined in a unisex manner, while reserving must be differentiated by gender. This leads, according to him, to the problem that not all funds may succeed in balancing income and expenditure. Because in fact, the balanced unisex premium will be the average of the premium of men and women weighted by assets, a value that in fact depends on the fund, i.e. the unisex premium will still be different at each fund even if men and women die according to the national average at every fund. Moreover, this value is uncertain because members can switch from one fund to another. According to Réti, a solution to the problem of unisex annuities is only possible if this problem caused by having different providers is eliminated. He provides two different solutions:

1. Annuity is purchased by every man from insurers, while the fund provides annuities to all women. In this case, the unisex mortality table corresponds exactly to the female mortality table.
2. There is a single risk community with a single state-run provider. The premium may be defined by using a software-based linear programming application. The consequence of the solution is that there is no need for a separate Guarantee Fund.

The study by Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] repeated that the provision of the law, according to which the premium must be determined on a unisex basis, while reserves must be determined in a differentiated manner, represents a problem. According to the study, the problem cannot be solved by the usual principle of equivalence stipulated by the Act. Why it poses a problem is not detailed in this paper, but rather in the next study written by Stahl [2005]. Here, he restates that the problem may be partly managed by a central provider and partly by solving a mathematical programming task, which according to Stahl is a completely different method than the application of the equivalence principle, but the difference between the two methods is not detailed. Therefore we must depend on guesswork concerning the difference: perhaps according to Stahl the equivalence principle is not correct because by calculating using some kind of unisex table it is not clear that the total premium collected would be sufficient for the requirements of a reserve that is calculated using differentiated mortality tables. Therefore, he wants to take into account the composition of the annuitants by gender when calculating the premium, in addition to having them weighted according to their assets. In my view, Stahl's solution is problematic because:

- Unfortunately, there is no guarantee of any kind that the actuary has preliminary information about the gender composition of the annuitants. If there are competing providers, then he will certainly not have the information available; if there is a central provider, then even though he may have a fairly good approximation, he will still not know the exact composition because the decision to retire is made by the individual, and the central provider will certainly not know about it in advance.
- Moreover, people do not retire at the same time, and certainly not in large groups, while, although Stahl formally ignores time, his solution can only be interpreted if he assumes that this is what they do. Since this is not the case, it is an illusion to believe that this programming task can be defined at all. So we are left with the equivalence principle.
- Approaching the problem from another angle: if, using Stahl's assumption, we possess all the data required for the mathematical program-

ming, then there is in fact no need for programming, since this information may be incorporated into the structure of the unisex mortality table, and the equivalence principle would provide a perfectly good result.

- For my part, I in fact suspect that since Stahl was never a practising actuary and was not completely familiar with actuarial methodology, he chose instead to apply a method that he knew well (mathematical programming) for classic actuarial tasks too.

Stahl [2005] stimulated debate among experts and basically the only public debate on private pension annuities in Hungary. Stahl's statements were disputed by Miklós Arató in two articles, and György Németh also commented on them, to which Stahl responded in turn. From a mortality table aspect only Arató [2006a] is important here. He thinks that the prohibition of differentiating between men and women is overemphasised, since there are similar differences on the basis of smoking and non-smoking people, obesity and non-obesity, but nobody wants to differentiate according to these criteria. In fact, his argument is a little surprising, as he does not say what an average actuary would do, according to which the more criteria used for differentiation the more stable the calculation, but instead finds the existing differentiation excessive on the grounds that no other criteria for differentiation are applied. According to Arató (page 272), "It is completely irrelevant whether or not we have prepared a good mathematical programming model for determining unisex annuities if we are in error by 20-30 percent when determining life expectancy". Ágoston-Kovács [2007] later said that the Stahl-Arató-Németh debate centred on problems caused by differences between the unisex table and the life expectancy of the two genders. They also state (page 562) that "…there is still no solution for the definition of private fund annuities and there is not even a generally accepted proposal. The range of 'solidarity' between men and women is not defined; we do not know whether the difference present among genders with respect to life expectancy should be equalised nationally or within each individual fund. If within each fund, it would generate unpredictable transfers among funds, as everyone would try to get into a fund that has more young men. Or possibly efforts would be made to exclude women from the fund, for example by the establishment of closed, occupation-based (e.g. 'miners') funds".

2.3. Longevity and mortality risk in general

When I provided the individual reserving formulae for life annuities above, I implicitly assumed that the actual mortality of the insured is identical to the presupposed mortality (the one in the tables used for calculation). In practice, the actual mortality will be different from the presupposed figures, and therefore mortality loss or gain is generated at the providers. The difference can have two causes:

1. Fluctuation of mortality from year to year (random deviation),
2. Mortality changes according to a different trend compared to the one projected (systematic change).

If there is a systematic reduction in mortality, the trend is referred to as the longevity problem, which can again be divided into two types:

1. A trend within the whole portfolio of insured persons, or
2. At trend at a certain service provider.

The entire portfolio of the insured, independently from the type of differentiation prescribed by regulations and purely from a technical point of view, must be differentiated from a risk aspect, and different mortality tables must be established for the different groups, and naturally, trends of varying magnitude will manifest themselves within these groups. (For example, within a given period the life expectancy of women with university degrees increases at a much higher rate than that of men who left school at 14, etc.) In other words, the projected and actual mortality trends of the entire portfolio's various differentiated groups will differ in different ways.
The mortality trend of the differentiated groups of various providers may differ from the trend observed within corresponding groups of the whole insured portfolio, again for two reasons:

1. The given differentiated group at a certain provider is very small, and therefore the actual trend of this small sub-group differs randomly from the "big" trend,
2. The composition of the insured at a certain provider is not random according to the characteristics ignored at differentiation in that it does not reflect the composition of the entire population, but systematically differs from that (e.g. alcoholics or people who lead healthy lifestyles are especially highly represented at a given provider).

Naturally, the trend or the systematic change first appears as a random diversion (fluctuation), so these two effects can only be separated from each other in the long term. We do know, however, that the smaller the insured portfolio,

the higher the fluctuation will be, and in fact with an adequately small portfolio of insured parties the fluctuation will certainly take place, since the number of insured and the number of deceased is an integer and not every mortality probability fraction can be generated as a quotient of an arbitrarily small integer. So vice versa, the fluctuation of mortality from one year to another decreases in proportion to the increase of the portfolio. And as the portfolio grows there is an increasing probability that the reason for the deviation between the theoretical and actual mortality is a trend rather than a fluctuation. So the bigger the portfolio, the higher the probability that this trend can be recognised and quantified within the shorter term.

I assume that the trend of the entire portfolio and of differentiated groups of the insured will be identified and quantified by a central institute based on collected mortality data. I can see a small possibility that trends manifesting at different providers can be separated from accidental fluctuations, and therefore below I begin with the assumption that there is only one kind of trend; the trend of (differentiated groups of) the entire portfolio of insured, and deviation from this at individual service providers is listed among random fluctuations. However, we must take into account the possibility that the cause of different trends at individual providers may ultimately be the regulations and the restriction of differentiation.

Before discussing the management of mortality risk, the question arises of who ultimately has the right to the eventual mortality profit that results from the difference between the theoretical and actual mortality. I include the word "ultimately" because without it, it would be easy to provide the bogus reply: nobody, we will set it aside to cover later mortality losses. This is a bogus reply because if we do so, mortality losses and profits accumulate on a longer term, but we must eventually still raise the same question with regard to this accumulated result: who should be credited (or bear the burden)? So I reject this answer and I only regard the accumulation of profit or loss as a possible method of problem management.

So who receives the profit that accrues from the death of more insured individuals than was initially calculated? It is easy to answer the question by saying that naturally it belongs to the insured, since the annuity is fully generated from their assets and the provider simply redistributes this and the yield amongst them. This is a completely legitimate answer, but the question was somewhat misleading because I did not ask about the mortality result or the mortality loss, but about mortality profit. If I had asked who bears the mortality loss, the same people who replied "the insured!" to the previous question would most probably have said that it is the responsibility of the provider. It is obvious that these two answers are contradictory, since why should the provider only bear the loss but not receive a share of the profit? Clearly the consistent

solution is that the mortality loss should be borne by whoever is entitled to the mortality profit. We must of course recognise that according to current trends (in the developed world, and since the second half of the 90s in Hungary), there is a greater chance of mortality loss rather than mortality profit (although this ultimately depends on the projection; if we project too rapid an improvement in mortality then the mortality result could show a systematic profit).
The mortality profit/loss may ultimately[25] be credited or debited to three market players:

1. The state (i.e. the taxpayers),
2. Annuity providers,
3. The insured.

The first item may be raised because the state plays a role in generating mortality results partly by prohibiting certain differentiation and partly by preparing and imposing central mortality tables for reserving, and mortality outcome may be the result of both of these. Though we must also consider that if the state is allowed to become involved in the financial matters of an annuity even to a small extent, such state intervention may increase unpredictably, which suggests that it is useful to keep the state away from the annuity issue in financial terms. Furthermore, no matter how long a term we examine, loss and profit will have a balance that is probably not equal to zero. If it is positive, meaning the state profits as a result, we might raise the question of why the state is withdrawing the money of the insured for other purposes. If it is negative, one might raise the question of why annuitants should be supported with taxpayer money. So this solution is best avoided, although whether this can in fact be done depends on the implemented annuity model. If the regulator opts for the central service provider model, an intervention of this nature may occur more easily – and to a certain extent is much more justified – than if the providers are market players who are independent from the state. Nevertheless, even in the central provider model, it is expedient to endeavour to keep the state neutral in financial terms with regard to the annuity system.

If the annuity provider is a profit-oriented enterprise of which the insured is a client, it may be logical to expect that it protects the client from possible decreases in annuity caused by mortality loss; and if it is expected to assume the mortality loss, then it is only natural that it will also be entitled to the profit. However, an expectation with relation to this may be that providers should not accumulate too large a mortality profit in the long term, meaning it should not withdraw too much from the capital of the insured for its own profit (keeping the long-term accumulated mortality loss low is in the interests of the provider and it is the provider's responsibility to monitor it). If the annuity pro-

[25]So if it is carried over in the form of a mortality reserve, then in the long term.

vider is a non-profit enterprise (fund) owned by the insured, there is no-one other than the insured to take the loss, so in this case the possibility of the provider bearing the loss does not arise at all, since the burden of the loss falls back onto the insured via the provider under all circumstances and the profit also belongs to the insured. Although it is possible to imagine a solution whereby the mortality loss is borne by a group of fund members that differs from those who "generated" it (and vice versa in the case of profit), but this is clearly inequitable and should be avoided. So, for instance, passing the burden of mortality loss onto members who are still in the asset accumulation phase of the annuity should be avoided (as should passing them the mortality profit). The asymmetric and therefore inequitable solution of debiting the mortality loss to members who are still in the asset accumulation phase, but allowing annuitants to retain the mortality profit is to be especially avoided. One of the reasons this solution should be avoided is because an inequitable regrouping of this nature makes the membership unstable and provides them with arguments for switching to funds with a more favourable demographic status. Therefore, treatment of this problem might also result in the abolishment of the right to choose between funds, meaning it could give rise to a kind of "vicious circle" (a "regulatory spiral").

Based on the above, the mortality outcome may be borne by the insured or a profit-oriented provider (if the provider is an institution of this nature), or the outcome must be somehow shared between the two of them. To a certain extent, the "purest" solution is if the insured person bears the full mortality outcome, or at least part of it, because in this case the question raised with regard to the state and profit-oriented providers, that too much accumulated mortality loss or profit is problematic, does not arise, since both are owned by the insured. Whomever it belongs to, an effort must be made (i.e. mechanisms must be operated) to assure that the mortality outcome tends towards zero in the long term, otherwise it will result in the systematic regrouping of capital between the insured and the provider, or between different groups of the insured, which should be avoided as an inequitable practice.

Due to the above arguments, below I suggest a number of solutions for the management of problems concerning mortality outcomes that are valid if the mortality outcomes fundamentally belong to the insured.

2.4. Managing mortality risk

Mortality risk may be managed by the following methods:

1. Attempts to take into account in advance potential mortality changes (trends) for each differentiated sub-group (taking into account the fact

that several factors may be considered when calculating the reserve), i.e. to project mortality,

2. The mortality loss of the given year decreases the investment yield of the given year (at a 0% technical interest rate; at a higher technical interest rate, the additional investment yield), while mortality profit increases this yield, meaning the mortality outcome becomes part of the indexation,
3. Reserving to smooth out fluctuations and longer lifetimes compared to the projected life expectancy,
4. Making solvency capital compulsory for providers (that have actual owners),
5. In addition, the risk is adjusted to the financial power of the different providers by splitting the mortality risk into parts and separating normal and peak risks.

Homogenisation – to be discussed later – contributes to the management of mortality risk, so that one differentiation factor (the size of the annuity capital) is cancelled (or at least its potential impact is significantly decreased).

Let us take a more detailed look at these methods.

2.4.1. FORECASTING MORTALITY

I have already dealt with the issue, although certainly not in sufficient detail, and I am not going to engage in details here either. Projecting mortality is a complex issue with a large body of literature, which I shall not detail here. However, as part of the "infrastructure" of private pension annuity, a new or an existing institute must routinely deal with mortality projection. As I mentioned earlier, projection aims to identify long term mortality trends rather than random annual fluctuations.

Feedback is an unavoidable part of the process, i.e. mortality projections must be regularly controlled and updated according to current data. Projections must be prepared for the entire portfolio of insured people as well as for the various risk groups. In harmony with the above, initially it is expedient to differentiate according to the following breakdown (while data must also be gathered according to other aspects to enable the later breaking down of the population into further risk sub-groups):

- By gender and for both genders in the following breakdown,
- According to level of education (at least with three groups: elementary education as a maximum, secondary school graduates and higher education graduates),
- According to the magnitude of pension assets.

We may also raise the question of how often the projection should be corrected. I think that even in the long term correction should not occur more often than on an annual basis, since no significant change is likely to occur within a shorter period and a more frequent projection could be misleading because of possible cyclical annual fluctuations in mortality.

2.4.2. MORTALITY OUTCOME AS PART OF INDEXATION

If the mortality outcome, or a part of it, becomes part of indexation, it means that it is either transferred directly back to the insured or charged to them (i.e. credited or debited) on an annual basis. This solution may be more or less forced (as in the case of an annuity model where providers are institutions without capital, like Hungarian private pension funds), or it may be a voluntary option. I have already elaborated on arguments in favour of the voluntary option. However, the choice also depends on the foreseeable magnitude of fluctuations in mortality, in other words on the expected size of the mortality outcome. This in turn depends on the size of the risk community. If there is a small risk community (e.g. at a small provider, or at any provider at the launch of the system), this should not be used, because the fluctuations are very big. In the case of the biggest possible risk community that includes the whole country, for example in the case of a central annuity provider or in the case of an annuity model with a central annuity pool, its use is almost mandatory, or at least there are few arguments against its use.

The joint treatment of the investment surplus yield and the mortality outcome attempts to smooth out annual mortality fluctuations, which are partly random (or are the result of a small portfolio of insured) and partly the result of deviation from the trend, which, however, can only be separated from random fluctuations over a longer period.

The mortality outcome may be either profit or loss. It seems that there is no specific problem if the mortality profit must be distributed back to the insured. However, the mortality loss is a different issue and the treatment of profit and loss is not necessarily symmetrical. We may raise the question of to what extent the investment surplus yield should bear the mortality loss, or in other terms: what portion of the loss should be debited immediately and directly to

the insured. The issue usually arises if we ask whether the annuity may decrease from one year to the next due to mortality loss if no adequate (surplus) yield is generated to cover the mortality loss in the given year. (Naturally the question is raised not only in this form, but also with regard to whether annuity can decrease as a result of possible negative investment yield – I will deal with this in relation to indexing).

The majority of insured people would probably prefer not to have any nominal decrease in annuity for any reason, for example due to mortality loss. In such cases the corresponding part of the loss is either carried over (hoping that the situation will improve and the current loss can be recovered from investment and/or later mortality profit), or it is absorbed by some sort of reserve. One may ask whether this reserve is created from the capital of the insured persons or whether it should come from the solvency capital of the provider. In the event of carry-over, two questions are raised: how should the loss be funded until such time as it is recovered, and if the loss is not recovered, who should bear the loss ultimately, the insured or the provider? In summary, the problem can to all intents and purposes be treated as if mortality outcome was not a part of indexation at all, with the reserving and the solvency capital discussed below. This also means that making the mortality outcome part of indexation is in itself not sufficient without (one of) the two other methods, so the method must be complemented by one of the two.

Making mortality outcome part of indexation does not mean that it is superfluous to present the investment and mortality outcomes separately, and to indicate what factors add up to produce the indexation. This prevents the eventual mortality profits from hiding bad performance of the provider in the area of investment yield.

In fact, indexation based on the combined investment and mortality outcome diffuses the mortality outcome. It is, however, not self-evident in what groups this should be applied. The major possibilities are:

1. Among the annuitants of all the providers,
2. Among all the annuitants of each provider,
3. Among annuitants in the same risk group at each provider,
4. Among annuitants of identical age at each provider,
5. Solutions 3 and 4 together.

The application or applicability of solution 1 depends on the annuity system. The major possibilities are:

- If there is only one central provider, then this is the obvious solution, although diffusion via the other possibilities may also be considered in this case.

- The whole point of the central pool model, which I will present later, is that the mortality outcome is centrally managed, so it is dispersed within the entire insured portfolio. In this case, the entire combined investment and mortality outcome is not dispersed but rather only and exclusively the mortality outcome. The central pool can be organised so that certain elements of the entire annuity system (e.g. annuity payment, etc.) are centralised, and also in such a way that it operates only as a kind of risk balancing system. Both are complicated systems, but examining them is made easier if several things are centralised, so if it is a solution operated only as a risk balancing system, it is probably not easier but in fact more complicated.
- The pool may also be organised on a voluntary basis, and naturally this also aims to disperse the mortality outcome.

The justification behind a central pool is that is compensates providers with a bad composition of insured persons (and their annuitants) for losses resulting from this bad composition, which they cannot manage through differentiation because of the regulations. One argument against it is its complexity.
Solution 2 is almost self-evident if the range of insured people is fairly restricted, i.e. if it cannot be split further. We might say that in such a situation the provider has no choice but to opt for this solution. However, in the case of a larger portfolio of insured people (beyond a certain threshold) it is more correct to break down the portfolio further, i.e. to opt for solution 3. If the portfolio is even larger (so beyond another threshold) it is worthwhile distinguishing between cohorts (i.e. to apply solutions 3 and 4 together), since a generational impact can also be observed with regard to mortality, and it is justified to say that each cohort should bear its own risk.

A special version of solution 4 may also arise, a dispersion among annuitants who began receiving annuity payments in the same year, but usually only if the annuity model is a version of the central provider model that only permits contracts to be concluded with a single provider (the one that won the tender for that particular year) in one year, and with another provider the next year. Accordingly, in this case the risk community at each individual provider is intrinsically split according to year of retirement as well as corresponding to grouping by provider, i.e. solutions 4 and 2 overlap.

Theoretically, another (totally incorrect and therefore indefensible) solution would be to transfer these losses to later annuitants by increasing the premium.

2.4.3. RESERVING FOR INCREASED LONGEVITY AND TO SMOOTH OUT FLUCTUATIONS

If we allow the entire mortality loss for a given year to cover the mortality loss at the expense of investment yield, irrespective of the fact that this may lead to a decrease in the annuity in the following year, then there is no need for a separate reserve to smooth out fluctuations. But if we want to restrict fluctuation (either by not allowing annuities to decrease from one year to the next, or by not allowing the rate of increase of the annuity to decrease below a certain level, e.g. half the rate of inflation), then a reserve or solvency capital must be set aside. Reserving and/or solvency capital may not only complement but may also replace the need to cover mortality profit at the expense of surplus profit, but this of course comes at a price.
The reserve may have two sources:

1. A portion of the premium paid for the annuity,
2. A part of the regular surplus investment yield.

Source 1 above will probably not be enough, or too much is generated simply because at the beginning of the insurance it is difficult to calculate how much will be required during the term of the annuity. Accordingly, this source may not be used independently from source 2. On the other hand, source 2 must be applied as mortality losses occur, so this solution indirectly leads back to the solution whereby the mortality result is treated together with indexation, and therefore it should be considered whether the application of the direct method would, in fact, be better.
As regards reserving, the question arises whether all kinds of fluctuation should be smoothed or only some of them. The possibilities are as follows:

1. Only those that are the consequence of mortality fluctuation, although this is too abstract a solution for the insured.
2. Those that result from the joint fluctuation of mortality and investment yields. The essence here is that in years of "good yield" the annuity is intentionally increased at a lower rate, by which means we can save for the years of "bad yield", allowing annuities to be raised by a higher extent than the yield. The danger of this solution is that it introduces an arbitrary element into the system, because we can never be certain when we are experiencing a "good year" or a "bad year"; these are relative terms that we can only judge for certain in hindsight, when it is too late. It is easy to imagine a situation in which bad years accumulate, and after the "bad year" in which we use up our reserve there follows an even worse year.
3. Only if the mortality outcome and the yield together would bring the annuity below a certain level (e.g. below the previous year's annuity

level). The advantage of this solution is that the strict and clear rule prohibits arbitrary "smoothing". Naturally, unfortunate circumstances may continuously evolve even using this solution, and the reserve may become exhausted at some point as a result (despite the fact that it is expedient to construct the system such that the reserve is topped-up at the earliest opportunity). In this case, as I have already indicated, there are two options: either the clients take the loss (i.e. the annuity decreases) or the service provider (if it has actual owners) takes the loss against its solvency capital.

There can be two types of reserve:

1. A safety component generated within the individual reserve of each insured individual.
2. A "demography" or "demography and yield balancing" reserve that is handled separately from the individual reserve (and from all other reserves), which, in comparison to the other possibility, we might also describe as a "collective" reserve.

Below I will explain the essence of both solutions, and also why I clearly prefer the first. But first, we must note that although Hungarian regulations included the "yield-balancing" reserve for a long time, criticism of this provision, including my own, appeared very soon after the regulations were adopted. For example, the paper by Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] says (page 486-487): "However, reserves diminish the magnitude of the service and the redistribution that results from reserving is contradictory to the so-often cited basic philosophy of the private pension fund system, i.e. self-sufficiency, individual saving, etc. The yield-balancing provision perfectly illustrates non-transparent and unnecessary regrouping. The wish to have a roughly even yield rate for a prolonged period is a questionable objective, as the real target should be to have the largest possible balance in the individual accounts at any point of time. The yield-balancing provision is certainly contradictory to such a target. … The only consequence of the existence of reserving is redistribution, which is a complicated procedure from an implementation point of view".

The essence of the **solution within individual reserves** is that the client is promised a conditional extra benefit in advance. The condition is that due to unfavourable yield and/or mortality loss there was no need to use the collateral for the extra service to maintain the annuity level prior to the maturity of the service. If the collateral for the extra service partially decreases because of the above events (which we may regard as a kind of "calculation surplus" or "safety supplement" or "safety valve", or "buffer"), then the service itself propor-

tionally decreases before its maturity. If the collateral runs out completely prior to the maturity of the extra service and there was no possibility for topping-up before the extra service was due, then the service is not provided at all. If the collateral for the extra service diminishes in a given year as a result of use, then in the first year in which the total yield and mortality outcome is positive this result must be used to top up the individual reserve to the level necessary to assure the extra service (meaning the indexation of the given year can only be this much lower). The total yield and mortality profit, and even the profit of the consecutive years, must be entirely used for this top-up if necessary, until the extra service is completely covered (until the entire collateral is available for equalisation). If this reserve within the individual reserve runs out completely and is again followed by a bad year, then (depending on regulations) this must either be absorbed by the insured (in other words the annuity of the insured person decreases, which may only happen in the case of a provider that has no solvency capital, such as the Hungarian-type pension fund), or by the for-profit provider, who can cover it against its solvency capital. In summary, contrary to the method of collective reserve, in this method clients does not lose the right to their own money and therefore it is they who receive extra service and not somebody else.

The extra service itself may be payable during the insured individual's lifetime or may be payable after their death, i.e. it may be of two fundamental types:

1. The extra service may take the form of a **mandatory periodical annuity increase**, payable during the lifetime of the annuitant. In this case, the basic annuity must be set in such a manner that it achieves a certain increase (e.g. 5% beyond indexation) at certain regular periods (e.g. every 5 years), but only if mortality (and yield) does not change for the worse. If it does, then this increase serves to balance the worse mortality (or yield). The advantage of this solution is that the entire capital of the insured individual is used for what it was meant to be used for, i.e. for the annuity. However, a disadvantage is that the "buffer" is constantly being used to cover the promised extra services, meaning it continuously diminishes, while it may easily happen that the bad years, for which this buffer was established, occur sometime during a later phase of the term. This is a strong argument for having the payment of extra services begin as late as possible, i.e. after the death of the insured individual.
2. Service due following the death of the insured individual may also be one of two types: annuity or a lump sum service. The first solution is a mandatory back-end guaranteed period, while the other is a type of supplementary whole life insurance. It is true for both solutions that

their advantages and disadvantages exactly mirror those of a service payable during the lifetime of the insured individual. An advantage of both types of service payable after death is that the service is rendered at the very end of the duration, when we can already be certain that the premium was calculated in an adequate manner and the remaining reserve of the extra service may be utilised without consequence. An additional advantage is that if the buffer is emptied, it may in theory be completely topped-up several times during the term of the annuity, providing the yields develop accordingly. However, the disadvantage is that it is not the insured individual who enjoys this extra service (unless it is used to pay for the funeral of the deceased, although this is also a relative exception). However, a further important, but technical advantage of payment following death is that the environment of the insured individual is motivated to report their death, and in this way the insurer avoids paying the annuity after death.

Let us examine these two solutions in detail:

2.1. Mandatory, 1-2 year back-end guaranteed period: In this case, it is compulsory to include a 1 or 2-year back-end guaranteed period in every annuity. If the mortality projection proves correct and there is no negative yield, the service of the guaranteed period is transferred to the client (or to the beneficiary assigned by the client). If not, this extra service may be used to cover the loss that is the result of incorrect projection, in which case the duration of the guaranteed period will decrease, possibly to zero. The advantage of the solution is that the nature of the post-death payment is identical to the one before death, although a disadvantage is that it is hard to find arguments for extending payment of the buffer in such a case, especially if the aim is to use it for the funeral of the deceased. Therefore, the other solution is more logical in this case, which is

2.2. Mandatory supplementary whole life benefit, which is a lump-sum payment. From a technical point of view it is advantageous if this whole life service is not defined as a fixed lump-sum or in relation to the last annuity amount, but instead as a certain proportion of the reserve. This helps the buffer fit into the environment in which it is topped-up from the investment and mortality profit that is divided and used in proportion to this reserve. The rule of thumb for the size of the buffer can be based on what expected lifetime increase it must compensate. For example, using 5% of the reserve as a buffer might compensate a roughly 1-year increase in life expectancy (or more by topping-up multiple times, if the increase is “dis-

tributed well" over time).[26] A non-technical problem with relation to the solution is that it is not certain if every insured person has an heir or is able to assign a beneficiary, although this problem may be bridged if it is used as default for funeral expenses (or for a proportion of them).

In summary, if we use a buffer then most of the arguments support solution 2.2. However, this may also be applied in a somewhat modified form:

1. The buffer is not necessarily used for extra service after the death of a given insured party, but in theory the buffer of the deceased may be distributed among the surviving insured parties. The advantage of the solution is that through this method the entire capital is used for annuity benefit; the disadvantage is that it is a less equitable solution with regard to the insured individual.
2. The buffer may be applicable to a certain extent for smoothing out yield fluctuations, although this requires a somewhat more complicated operational mechanism than the one above. In this case, the buffer is not only topped-up (if some of it was used earlier), but it is topped-up to an even higher level (e.g. up to double the original value). This may occur if the combined mortality and investment outcome results in the annuity

[26]The buffer so defined may be calculated as follows:

If we use a technical interest rate of 0%, then the premium and the reserve of the annuity insurance can be calculated with the help of the remaining lifetime (usually denoted at age x as: e_x). In this case, an insured individual aged x years may receive the following monthly (advance) annuity from start-up assets of „K_0" magnitude (disregarding possible differentiation factors such as the volume of capital for the time being):

$$\frac{K_0}{[12 \cdot e_x + 1] \cdot (1 + \lambda)}$$

where $[12 \cdot e_x + 1]$ represents a whole unit of $12 \cdot e_x + 1$, and λ represents the expense part of the annuity (in proportion to the net premium).

With the incorporation of the buffer this formula changes to the following:

$$\frac{K_0}{[12 \cdot e_x + 1] \cdot (1 + \lambda + p)}$$

where p is the buffer defined in proportion to the reserve, and where partial costs are not charged for the buffer, which wouldn't be justified either.

In the case of a technical interest rate other than 0%, similar, but somewhat more complicated ideas may elaborated, but the following statements remain valid. Although in this case there is a good chance that we will not succeed in topping-up the buffer, or that it can only be topped up on fewer occasions.

increasing above a certain level (e.g. the previous year's rate of inflation) and the buffer has not yet reached its maximum value. In this case, one could say that part of the extra profit is used as a reserve for worse years. The buffer could be used if it were in an over topped-up status (that is beyond the initial, normal value), and the annuity increase is below a certain level (e.g. the previous year's rate of inflation), up to a maximum of the rate of inflation or normal value of the buffer. This is a non-biased method for smoothing out fluctuations, so if it is important to operate such a smoothing mechanism due to various (political) considerations, it should be done using this kind of a more or less objective method. The disadvantage of the solution is that it potentially withdraws much higher reserves from the benefit of a given annuitant than if the buffer were not used for smoothing.

I have not found international examples of the application of buffers, so as far as I know this is my own, original idea.

Interestingly, according to my experience pension fund experts do not intend to apply the above individual provision for smoothing, but are planning to use a **collective** "demographic and/or return equalizing" **reserve** that is kept separate from the reserve of the individual insured persons, despite the fact that it has serious faults compared to the individual reserve. The essence of the idea (similarly to the individual reserve) is that at the very beginning (when the insurance policy is taken out), the reserve that is put aside from a part of the premium is used should the annuity decrease, and it is supplemented at the very first opportunity when this can be done without any decrease in the annuity. The supplementation occurs from the return (and from the eventual mortality profit), up to a certain predefined level (so potentially during several years, when as a result of this supplementation the annuity does not increase), as in the case of the above-mentioned individual reserve. Although the collective reserve belongs entirely to the whole risk community, it is not denominated – this is the difference compared to the one generated in individual reserves. Therefore a serious problem of equity emerges with regard to this kind of reserve, especially if the reserve is accumulated for too long, or is not used for a time, since in this instance it would primarily be used in favour of insured individuals who did not contribute, or did not contribute to an adequate extent, to the generation of this reserve (since some of them have died in the meantime). In other words, in the case of this solution, the client loses the right to their own money, and therefore the problem is better managed by the similar, but more equitably operating individual reserve method (if the idea that the mortality loss should be borne by the insured individual emerges at all, because there is nobody else to bear it).

2.4.4. SOLVENCY CAPITAL

The use of solvency capital differs from the reserve-type solutions above in that the financial consequences of fluctuations must be borne directly by the provider, not by the insured. Naturally, the word "directly" is important here because the provider may include the potential use of solvency capital in this manner (i.e. the costs of calculating the fluctuation) into the insurance premium, so it will be ultimately paid by the insured.

In the case of annuities, this kind of use of solvency capital is really its most important function. However it is useful to first review why the law usually prescribes the use of solvency capital for assurances, to help us understand when the use of solvency capital instead of the above-mentioned reserves can arise at all.

The function of solvency capital in general, is that it enables the insurer to perform its undertaken services for the client with a high level of probability, even if various obstacles arise that might otherwise prevent this. These obstacles may be of the following types:

- **Insurance technical risk** – the payments made by the insurer are higher than calculated either due to transitory risk fluctuation or a long-term risk increase. In the case of annuities this means that as a result of changes in mortality (which are favourable for the insured and unfavourable from the service provider's perspective) the annuity would decrease from one year to the next and the excess yield (if permitted by regulations), or the reserve generated for this purpose, would not be sufficient to balance the fluctuation.
- **Investment risk** – the insurer does not achieve the yield promised to the client. This, in the case of life insurances (including annuity insurances), means the yield adequate to the guaranteed technical interest rate and possibly to the guaranteed increase in benefit (if such exists), so the risk means the risk of a smaller yield than the sum of the technical interest rate and the guaranteed increase in annuity. At a 0% technical interest rate without a guaranteed increase of service this means the risk of a negative yield – if this is not balanced by positive mortality outcome (provided the regulations manage the two together).
- **Risk due to the options and guarantees** undertaken. To a certain extent the above two risks are similar, but there may also be other guarantees. For example surrender (which generally does not exist in life annuities). In the case of life annuities it is generally difficult to identify such a risk, but to a certain degree the possibility that the client will transfer their reserve to another provider (if the regulation opts for an annuity model that includes this possibility) may be regarded as a risk.

Technically this is a risk of surrender and the problem is that a portion of the reserve must be liquidated, which generates a loss.

- **Operational risk,** i.e. if some problem occurs in the operation of the insurer (including a natural disaster, human intervention, damage to the insurer's database as a result of theft, operation becomes difficult or impossible, resources must be channelled to unforeseen places, etc.). This is a risk faced by all financial service providers.

I must also mention that the level of the first insurance technical risk mentioned above, i.e. the size of fluctuation in payments, gradually decreases as the number of insured increases, so the smaller the portfolio of insured the higher the fluctuation. This is why, although regulations usually require solvency capital to increase in proportion to the size of the portfolio, it also sets an absolute minimum to enable the management of fluctuations in small portfolios. I will elaborate on this topic later.

In view of the special nature of Hungarian institutions that provide private pension services and their regulation, we must also mention the **relationship between solvency capital and ownership**. These funds are unique both internationally and with regard to other Hungarian financial providers. What makes them unique is that contrary to other providers (and to a certain extent to common sense), they are alone in not having any capital, including solvency capital! Looking at the above functions of solvency capital, it is obvious that of the abovementioned risks only the operational risks could arise during the asset accumulation phase (this is the phase in which all of the private pension funds and most of the voluntary pension funds in Hungary were when this book was written and until the closure of most private pension funds), meaning that a lack of capital is much less of a problem[27] than it would be for other financial service providers. However, all this changes in the annuity payment phase, as all of the above risks may manifest themselves. On other words, the actual solvency capital regime (regarding the lack of solvency capital as such) of the Hungarian funds is exclusively and solely designed for the asset accumulation phase without any regard for the annuity payment phase (perhaps because at the time of elaboration of the regulations the 15 years after which the annuity payment was due to start seemed incalculably far off in the future).

However unusual the lack of solvency capital at the funds, it must be said that this is entirely in harmony with the ownership conditions of the private and voluntary pension funds, i.e. with the fact that they have no actual owners, only "quasi-owners". The formal owners of the pension funds are the actual

[27]It seems that in Hungary the lack of solvency capital to cover operational risks has been bridged at a system level and with stricter and more detailed regulation and supervision compared to other financial service providers.

members, but equal rights are afforded to those who contributed a hundred million Hungarian Forints to the establishment of the fund as to those who paid the first thousand Forints in premiums yesterday. In such a situation any capital accumulation (which, in practice, is unavoidable to a certain extent, because operation requires equipment, computers, etc. which constitutes capital) would be unfair, since the person from whom it is generated cannot expect any yield or more voting rights[28]. If the pension funds had capital, it would be accumulated from the contribution of the starting generation, and the next generation would possibly not contribute to that, but if the fund is liquidated, then the capital would be distributed among members of the next generation who did not accumulate it. This would be unfair and deeply problematic, compared to which the system chosen by the regulations, i.e. the lack of formal capital, is a much more equitable solution. In summary, the actual ownership structure and the regime of solvency capital are in a good logical relationship with each other.[29] In the capital accumulation phase, this system indeed works (worked!) satisfactorily, with strong supervision. However, this logically coherent system cannot (could not have) manage(d) every problem, e.g.it could have only handled the problem of annuity with certain restrictions.

Solvency capital is needed if we want to avoid the client (member) having to bear certain risks, primarily the risk of eventual bad investment outcome and serial mortality losses which may lead to a decreasing annuity. Based on the above, a financial institution with "quasi-owners" cannot even theoretically have solvency capital. If Hungarian-type pension funds are allowed to provide annuities, we must take into account the fact that all the risks are directly borne by the annuitants (and can only be diminished using the above buffer-technique).

This problem appears particularly sharply when an annuity commences and the annuitant risk community is small, so the mortality fluctuation is necessarily high. Solvency capital has an especially big role to play at that time (see the section on management small portfolios).

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[28]At least formally and legally. In reality, the majority of Hungarian pension funds are de facto (but not de jure!) multinational financial service providers, who do have a strong voice in how the fund operates, but this can be traced back to organisational culture rather than legal reasons.

[29]Out of interest, let me note that from this aspect the Hungarian regulation is inconsistent, since the regulation of institutions very similar to pension funds is not as fair as this, and according to the above is therefore deeply problematic. At insurance associations solvency capital is required, but they have no real (only formal) owners. Nevertheless, this inconsistency would be difficult to correct by cancelling the solvency capital requirement at associations, because insurance risk could not be undertaken at all without solvency capital. It may be remedied only by the elimination of the association form (in the case of insurers). See Banyár [2010].

In reaction to the above arguments, it would be highly problematic if the regulator had ordered that funds may provide annuity but to do so they must accumulate solvency capital from the contributions of members. On the one hand, it would be problematic because of the equity-related reasons explained above and on the other hand because this would not solve the problem. Because risk fluctuations are highest when a provider puts an annuity on the market, and this is when a high level of solvency capital must be available, i.e. there is no time to wait for it to accumulate slowly.

It would also be problematic if funds started to accumulate solvency capital for the annuity payment phase in the capital accumulation phase, since the right to choose between funds does not include allowing people to take the capital accumulated via their contributions to another fund. In addition, there is another significant problem.

An important and deeper connection between solvency capital and ownership is that solvency capital must always be available in its totality, i.e. even immediately after the majority of it was used for the purpose for which it was established.[30] In such a case the owner must immediately top-up the solvency capital to the adequate level and this is indeed what happens[31], because there are owners. At Hungarian-type pension funds without a real owner, it may happen that during a long period the solvency capital is topped-up once to an adequate level, but once it is used the fund may lack sufficient solvency capital for a prolonged period, which in turn endangers the provision of annuities. Without owners, nobody has an interest in putting the missing solvency capital back into the fund, since this is not coupled with any advantage. If the existing members, as owners, were forced to do so, then they would be better off "defecting" from the fund and joining another fund that does not have a lack of solvency capital and where they are not asked to deal with such problems. The only option in such a case is that the supervisory authority prohibits the continued operation of a pension fund that has no solvency capital, so if the regulator required solvency capital for funds with no owners, it would program a high probability of bankruptcy into the system.

Therefore it can be stated that solvency capital is only a characteristic of profit-oriented financial institutions (such as insurance companies), so it is only they who can solve problems concerning solvency capital and not the funds. Institutions without real owners should either not be allowed to provide life annuities or must be transformed into organisations with actual owners.

[30]Gábor Borza drew my attention to this issue (Borza [2010]).

[31]Should they fail to do so, the consequence is a supervisory intervention. Within this framework, the provider is possibly liquidated and clients are transferred to another provider. Solvency capital allows for the seamless management of the portfolio transfer without any damage to clients.

The other possibility is to recognise that risks in respect of annuities are directly borne by the annuitants (in practice this means that in the case of unfavourable mortality outcome the annuity may even decrease nominally from one year to the next – with the use of a buffer this can only happen after several consecutive "bad years").

Based on the above, solvency capital in respect of annuities may and must be required from insurance companies and possibly from other providers that operate in corporate form, and only in the case of annuities provided by such providers may it be required that the annuity does not decrease from one year to the next and/or in the case of bad mortality outcome. The current European, and accordingly the Hungarian, regulations on insurers already prescribes compulsory solvency capital for insurance companies. If the regulation simultaneously permits insurers and pension funds to provide annuity, this requires the operation of two different systems from a solvency capital point of view. (Here, of course, the general question arises as to whether two different systems should be operated in parallel and whether the clients sufficiently understand how the system works).

Accordingly, in the case of annuity the solvency capital on the one hand provides protection to the insured against mortality loss, or in the case of using a buffer against overspill of the accumulated effect of consecutive unfavourable mortality, and on the other hand, with relation to the former, in the event of an extremely unfavourable investment outcome. Beyond this, in the case of a small insured portfolio, where solvency capital must be available if the collected premium is not enough to provide the initial reserve (due to the insured having a different risk composition than originally projected). These can all be regarded as a kind of option, which the insured party purchases from the insurer and in respect of which they must pay an option premium. This option premium may be partly built into the premium of the annuity or into the yield of the annuity reserve. The higher the probability of utilisation, or the uncertainty of utilisation, the higher the option premium. From this perspective it is primarily the technical interest rate that serves as a kind of regulatory parameter. The higher the technical interest rate, i.e. the annual expected yield, the higher the probability that it cannot be achieved in a given year, so the higher the probability that the option will be drawn down and the more expensive the option itself will be, and vice versa. This is an argument for it being compulsory to apply the lowest possible technical interest rate of 0%[32], which at the

[32]Naturally the technical interest rate may in theory be negative too, so 0% is not the smallest possible value and no "natural" low limit can be set for negative interest rate, not even -100%, since for short periods an interest rate below an annual -100% can be imagined, although the annuity is fundamentally set for a longer period than one year. However, a negative interest rate must be regarded as a curiosity and very strong and

same time also means that the option premium is restricted to a minimum, since with such a low interest rate there is a minimum of risk that the yield of the investment will not reach it.[33]

In the case of annuities, the necessary magnitude of the solvency capital depends on several factors, and therefore legislation must also differentiate according to several factors. The major factors influencing the solvency capital requirement:

- Annuities with a guaranteed period require lower solvency capital than annuities with no guaranteed period, since front-end guaranteed periods decrease the risk of the insurer and the use of a back-end guaranteed period is nothing other than the contemporary sale of two insurances (life annuity and whole life) that have opposing risks and partly cancel each other out,
- With regard to annuities with front-end guaranteed periods, a minimum level of solvency capital is sufficient during the guaranteed period (since it involves no insurance risk), if the regulations allow this part of the annuity to be provided by another provider,
- If the annuity can be split over time between two different providers, then a smaller solvency capital is required for the temporary annuity paid at the beginning of the term, while a larger solvency capital is required for the whole life part of the annuity (which may be regarded as the peak risk),
- The solvency capital requirement is increased if the premium of the annuity is officially maximised,
- In the case of unisex annuity requirements, a smaller solvency capital is satisfactory for joint life annuity than for single life annuity, since the uncertainty ceases to exist as a result of unisex annuity requirements.

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specific reasons are required for its application, therefore I maintain that 0% is the smallest possible interest rate, even if this is more of a physical limit than a theoretical one.

[33]This is strongly dependent on the investment strategy, which in turn is related to the interest rate. The lower the interest rate, the more daring the investment strategy may be, i.e. it may be a strategy that brings higher yield in the long term, but increases yield volatility in the short term. So in a certain sense, the 0% technical interest rate not only decreases the probability of a negative yield, but also increases it to a certain extent.

2.4.5. SEPARATION OF NORMAL AND PEAK RISK – COMMON TREATMENT OF PEAK RISKS

In respect of annuities, extended lifetime (longevity risk) may be regarded as the peak risk for the insurer. It is primarily institutions that provide immediate, lifelong annuities that are exposed to this; annuity providers that provide deferred annuities are less exposed (the longer the deferral the lower the exposure)[34], temporary annuity providers are even less at risk and providers of annuities certain are not exposed at all. As the immediate lifelong annuity may be split into the sum of temporary and deferred annuity, this method enables the normal and the peak risk of the annuity to be separated, while the peak risk may also be diminished. So practically: an upper age limit may be determined (e.g. 80 or 85 years) and the immediate life annuity (and its single premium) may be split into these two parts: a temporary annuity until this pre-determined age, and a deferred annuity beginning from this age. The premium of an immediately commencing lifelong annuity can be split up in a similar manner. Since these days it is difficult to imagine an annuity with an unchanged sum assured, i.e. a non-indexed annuity, special care should be taken that the service provided in these two different phases does not "slide apart" during the first phase because of the different indexation applied. There are two ways to protect against this sliding apart:

1. In the first phase, the provider of the first phase manages the reserve of both parts and invests them in the same way until the client reaches the age limit. Accordingly, the indexation of the two parts will be parallel during the first phase and the sum assured runs smoothly into the second phase, and the reserve of the deferred annuity is only transferred to the provider that assumes the peak risk at the beginning of the second phase.
2. The reserve of the deferred annuity is already transferred at the very beginning to the other provider, but investment may be made exclusively into fixed yield (e.g. inflation-linked) bonds at both providers, and so the indexation of the reserves remain parallel to one another.

The deferred annuity provider may be of three types:

1. The same provider that provides the temporary part of the annuity. The provider may do this if it has generated the solvency capital for the entire life annuity, i.e. including for peak risks,

[34]This is only true if we compare the deferred annuity with an immediate annuity in which the entry age of the insured persons is identical. But if we compare, for example, two cases where someone enters into an annuity that commences immediately at the age of 62 or enters into a 12-year deferred annuity at the age of 50, then it is not clear in which annuity the longevity risk is higher.

2. A second provider with bigger financial muscle than the first one, a kind of annuity re-insurer,
3. A central provider established or selected by the state (to all intents and purposes an "official" re-insurer).

In cases 2 and 3 the need for solvency capital is smaller at the temporary annuity provider than if it held the entire risk. What is more, if the temporary annuity is an annuity certain, then this service may be provided with a minimum of solvency capital, and practically speaking the Hungarian-type pension funds are also adequate for this.

The longevity risk in case 1 is undertaken by the original provider (who is capable of doing so because it possesses an adequate level of solvency capital), but in cases 2 and 3 it is undertaken by the re-insurer, which in case 3 ultimately means the state. This encourages the state to issue as accurate as possible mortality projections, because this enables the state to avoid peak risks becoming a great burden.[35] (If the projection is correct, there is no extra burden). With an adequately selected upper age limit it is achievable that the central provider remains a smaller provider (with respect to the reserve), so the major part of the annuity risk remains with the market and is not transferred to the state. However, due to the upper limit of the service, the temporary annuity means a more calculable situation for market players, i.e. it is easier for them to calculate the length of time during which they must bear the extra burdens if they make a miscalculation.

2.4.6. MORTALITY PROJECTION AND MANAGING LONGEVITY RISK IN LITERATURE

According to Mehr-Gustavson [1987: p. 527-528], different mortality tables are already used for annuitants in America, and since 1949 improvements in mortality have also been taken into account. This means that the projection of mortality in the annuity calculation began at that time.

According to an English textbook, which has also been translated into Hungarian (Hylands-Gray [1992]), the calculation of future improvements in mortality must be assessed prudently. The textbook also states that in England a survey of the annuitants' mortality was conducted in 1967-70 and now forms the basis for the applied standard mortality tables. These were not adequate for premium calculation, only for comparison, because the future improvement of

[35] According to some experts it is worrisome that the state influences several important parameters at the same time. Therefore it is possible that it is expedient for mortality projections and the central provider to be clearly separated, if a suitable model is introduced.

mortality was not taken into account. Therefore, an adjustment was applied by setting out from the 1968 table and then deducting $1/20^{th}$ of the year from the age each year (with regard to the relevant generation), and this is how they calculate the premium. In addition, when setting prices they take into account the annuitant mortality experience of the given insurance company. According to Hylands-Gray [1992], the projections retrospectively reflect a mixed image. Based on the figures from 1975-1982, the predicted male mortality was close to the actual level, but in the case of women it was highly underestimated.

Blake [1999] also cited the underestimation of mortality improvement as being one of the major problems for annuity providers, because mortality generally improves over time. This is also backed up by Davis [2003], who cites the 2002 conclusions of the Financial Services Authority (FSA), and who states that the most important risk is that providers underestimate the lifetime of annuitants.

Winkler-Mattar [1999], who aimed to provide advice to the insurers on how to handle risks relating to annuities, see two ways of managing the risk caused by improving mortality:

1. Insurers give clients the option to switch to an annuity, but do not guarantee the price and instead allow clients to move to another provider,
2. The use of generational mortality tables (I assume they mean projected mortality tables according to cohorts!).

Booth-Chadburn-Cooper-Haberman-James [1999] (page 226-7) call attention to mortality improvement in developed countries, meaning annuity calculations must also take into account anticipated future improvements in mortality (but they do not provide details about how this should occur!). This is done by Antolin [2007], according to whom mainly stochastic methods should be used for mortality projection, the basis of which was laid down by Lee and Carter in 1992 (see Lee-Carter [1992]).

Winkler-Mattar [1992] present the method by which projections were conducted before Lee-Carter, and state that since experience indicates that mortality is decreasing, the values of q_x must be decreased when performing projections. However, it is not expedient to do so in a linear manner, because the probability would eventually become negative. This is why the formula $q(x,t) = q_x \cdot e^{-\lambda_x \cdot (t-t_0)}$ is widely used, where λ_x is the age-specific annual improvement trend and t_0 is the year for which q_x was originally valid. This is a simple model, but according to Winkler-Mattar there is no empirical proof of its correctness. Therefore, other methods are also applied, e.g. the "frailty" model, where the population is split into stronger and weaker sub-populations with the presumption that the evolution of medical science has a higher impact on weaker populations. However, it seems that even the total eradication of various illnesses would not

radically prolong lifespan, and in fact nothing of this kind has ever happened as a result of developments in medical science; the increase in lifespan is more the result of other factors.

However, until now none of the methods applied for projection succeeded in properly reflecting the actual trends (as also stated by OECD [2008g]). Barnshaw-Laster-Steinmann [2008, page 11] quotes Chris Shaw's analysis (Fifty Years of United Kingdom National Population Projection – Shaw [2007]), according to which in 1977 a boy born in 2010 was predicted to live for 71 years, but by 2000 this figure had already been modified to 77 years. This shows that it is only possible to indicate mortality with a great deal of uncertainty (i.e. the uncertainty with regard to projections still existed even two decades after Hylands-Gray). Retrospectively all we can state is that according to the 2007 OECD data, life expectancy at birth and at the age of 65 increased by on average 0.6 and 2.6 years, respectively, every 10 years between 1960 and 2000 in the OECD countries (OECD [2008g]).

We may state that in the case of annuities the projection of mortality is obvious in the international literature. Despite that, at least with regard to annuities, it is practically non-existent in the Hungarian literature. For instance, Michaletzky [1999] writes with regard to private pension fund annuities (page 106): "…it would be important to precisely define which mortality tables may/must be used during the evaluation of services; the tables (by gender) that are valid when the annuity service is determined, or the table of the actual year". We can interpret this as meaning that the possible need to use projected mortality tables didn't even cross his mind. Arató [2006a] only states that "…we are 20-30% wrong when determining life expectancy", but he does not mention the importance of projection (although we can imagine that his thoughts also include this fact).

It is no surprise that the phenomenon that makes projection necessary is only mentioned in detail in the international literature, but not in the Hungarian literature. This is the phenomenon of increased life expectancy and the related risk, the so-called longevity risk.

The term "longevity risk" has had a rapid rise to fame. In the book by Mehr-Gustavson [1987], the word longevity doesn't even appear in the index. In the following decade Black-Skipper [1994] was already using the word longevity on pages 427-428, but only in the context that extended lifetimes are making it more important to plan pension schemes for workers.

In 1999, Winkler and Mattar already demonstrate the phenomenon with a number of tables from Swiss Re on the expected increases in life expectancy at birth at and at age 65. The reasons include: better health care, changes in nutrition habits and other changes in lifestyle (e.g. a decrease in smoking). According to the Harvard Medical School, women who also give birth in their 40s live

longer. The self-employed also live longer than others. However, they relativize the longevity phenomenon because they think that in the case of a heterogeneous portfolio the swings in mortality caused by heterogeneity are higher than the changes in longevity, so the longevity risk may not be separated in the case of such a portfolio. However, 8 years later a Swiss Re publication dealing with the problem (Scotti-Effenberger [2007]) already includes a sub-chapter entitled "Demography challenge and longevity risk". According to this, life expectancy at birth in the second half of the 20th century increased by on average 4.5 months annually. This may be primarily, but not only, due to the decrease in infant mortality, and this is gradually decreasing in significance these days. According to the study, the demographic trends will have the following consequences:

- A decrease in state-funded pensions and an increasing shift from DB systems to DC systems,
- Increased mobility will break previous family ties and therefore the young will take increasingly less care of the old,
- In general, we can increasingly count only on ourselves!

According to Antolin [2008a] and the OECD [2008a], pension systems are mainly affected by uncertainties relating to increased life expectancy, and must manage two important risks: the risk of annuitisation timing and the risk of longevity.

This management of longevity risk and how to cover it (hedging) is often discussed in the English language literature. In 1999, Blake (Blake [1999]) spoke only about the "natural" hedging of the annuity longevity risk, which had been well known for a long time in literature, i.e. that the provider should not only sell annuities, but also life insurances with an opposite risk, so the risks would partly balance each other. This is also mentioned by Davis [2003]. We must add that two years later Blake expressed a completely different idea about longevity coverage which has since become very popular in the literature (Blake – Burrows [2001]). Winkler-Mattar [1999] also mention that whole life insurance and annuities compensate each other's risks, but they also note that there is much less possibility here than conventional wisdom believes, because this would only be entirely true if the same person concluded the two insurance contracts. However, according to experience, the clientele of the two insurances is different even at the same provider. Beyond this, Winkler-Mattar [1999] mention that for many years insurers assumed that the increase in longevity is overcompensated by the investment yield (although for instance in Germany the regulations restricted the direct transfer of the yield for this purpose). This may also be interpreted as meaning that my proposal for treating mortality and investment yield together was already a well-known and widespread practice.

According to Winkler-Mattar's calculations [1999] in Switzerland, for instance, in the case of entry at the ages of 60 and 75 an average excess yield of between 0.31% and 0.43% was required to compensate for longevity, meaning that in their opinion the mortality risk is significant compared to the potential investment return used to compensate it.

In a certain sense, Wadsworth [2002] puts forward a radically different proposal (and from another perspective provides a new approach to risk management at the expense of excess yield), according to which the longevity risk, or at least part of it, should be transferred to the client via the providers undertaking fewer longevity guarantees. In his opinion, this risk is already not priced correctly today.

Wadsworth also prepared a report commissioned by the Association of British Insurers (ABI) in September 2003 entitled "The Future of the Pension Annuity Market" (Wadsworth [2005]), in which he determines that longevity risk has a very restricted secondary market and there is little chance for the development[36] of a wholesale market. He also finds the supply of long-term bonds to be generally important with respect to annuities, and can see problems and reasons for action in this regard.

According to Scotti-Effenberger [2007] there is an extremely high and generally unrecognised risk of systematic lifetime increase in relation to annuities in the entire population. According to them, the longevity phenomenon is well understood from a demographic and macroeconomic point of view, but it was not examined in detail from an insurer's risk management perspective. Since this is a systemic risk, it cannot be diversified, that is the traditional techniques of risk management (e.g. reinsurance, portfolio unification) do not work. For this reason, governments are the best equipped to help with risk management and play a key role. Their main tasks include: raising awareness, educating people, issuing the necessary instruments needed to hedge longevity risk, reducing adverse selection and encouraging self-care via tax incentives. One of the reasons governments may assume longevity risk from providers is that they are already exposed to it anyway because of the pay-as-you-go pension systems. Along with governments, this risk may also be traded on financial markets where mortality indices are used as benchmarks. So far very little such securitisation has taken place, but Swiss Re has already issued some of these instruments. The market methods for hedging may include: interest rate swap, participation in business deals that have opposite mortality dynamics, investing in businesses with different mortality dynamics, buying longevity bonds and

[36]It appears that the time for the development of such a market arrived in the United Kingdom in 2010. The IPE reported on 31 January 2010 that 8 banks, insurers and reinsurers had established a Life and Longevity Markets Association (LLMA) to promote coverage for longevity risk and trading. (IPE [2010], The Economist [2010])

the synthetic proxy hedge. The latter two are still in an experimental phase and no such instruments are yet available. It is mainly pharmaceutical companies, old-age homes and biotechnology companies that profit from increased life expectancy. Investing in these may be regarded as a "natural hedge". However, the problem with longevity bonds is that longevity risk is replaced by credit risk, which is not necessarily desirable except if the state issues such bonds. All in all, according to Scotti-Effenberger [2007], the task of insurers with relation to annuities is to work towards better pricing and the minimising of administrative and marketing expenses, as well as cleansing the basic product from unnecessary "frills". They see many more tasks on the part of the state:

- Issuing better mortality tables thus enforcing transparency in longevity,
- Raising awareness of higher life expectancy (according to a UK survey people tend to underestimate their lifetime by about 5 years. According to another survey, if a longer life is expected people are more inclined to take out private pension insurance),
- Making information available about the financial situation of retired people, see the Swedish "Personalregister",
- Improving the financial education of citizens,
- Issuing financial instruments for hedging – unfortunately only a few countries have 30-year government bonds and 50-year bonds are practically non-existent,
- To increase the volume of business and decrease adverse selection by making annuity mandatory,
- To develop and establish a supervisory framework,
- To introduce harmonised and motivating taxation – within an EET[37] system.

According to Barnshaw-Laster-Steinmann [2008], longevity risk, i.e. uncertainty with regard to how long we will live, is made up of two components: an individual and an aggregate component. The individual element may be managed by organising the risk into a pool (which is nothing less than purchasing an annuity), however the aggregate part cannot be diversified and at the moment it is also difficult to hedge. Managing annuity risks (and in DB systems in general) may happen via so-called liability-driven investing (LDI), which means trying to match assets to liabilities so that investments are mainly made in long term bonds. (So LDI is the same strategy that we call ALM in insurance, an interesting question is why this needed to be rediscovered and given a new name? – According to the OECD [2008g], ALM requirements have al-

[37] "Exempt, Exempt, Taxed", meaning the premium and yield are exempt from taxation, but the benefit is taxed.

ready been implemented in many countries, so matching assets and liabilities has become a stronger requirement.) Given a lack of suitable long-term bonds, an LDI strategy and the implementation of inflation-swaps and swaptions allow the matching of term and convexity without investing everything into bonds. In the case of derivatives, the "basis risk", i.e. that they do not precisely match assets, must also be taken into account. From the point of view of an institution, adequate risk management may be "white labelling", when the product of a professional insurer (manufacturer) is sold by another institution (distributor) in its own name. Additionally, this is often also developed jointly. However, for the moment the transfer of longevity risk is not really working, there are unfortunately not as many vendors (longevity bonds) as potential buyers and although a market does exist, it is very restricted and sluggish. One solution could be the secondary market of life insurances, the "life settlements market", which has been used for a long time mainly with relation to risk insurances, but it is not transparent enough and too specialised to be a mass solution. However, the existence of the longevity risk market is indicated by the fact that the Financial Times has published a list of available longevity indices.

Antolin [2008a] (and the very similar OECD report [2008a]) mentions the difficulties of market hedging longevity risk because in his opinion longevity swap options have ceased to exist and consequently instruments to hedge longevity are absent. The experiments of the private market have failed – so far. The planned EIB bond issue also failed in 2004 (see also OECD [2008g], which adds that it would have been too expensive anyway). Antolin [2008a] and OECD [2008a] find that one of the reasons for this is that nobody would really profit from the unforeseen increase in age, so there are not enough issuers (see also OECD [2008g]. Moreover the issuer cannot really exit from such bonds, making them even less attractive for potential issuers. Therefore he suggests, similarly to Blake or Scotti-Effenberger, that the government should issue such instruments. However, an argument against this is that the government already assumes more than enough longevity risk (so in contrast to Scotti-Effenberger [2007]), in his view this factor represent an obstacle). Swaps are not really being considered as a result of the financial crisis.

However, both Antolin [2008a] and the OECD [2008a] note that in addition to longevity bonds, simple long-term government bonds are also missing. So it is the government that should issue such bonds, and as for longevity risk management, it may at least stimulate the market by publishing a longevity index (that the market regards as reliable). (This is repeated by OECD [2008h] and OECD [2008g]). These are all necessary for the market to voluntarily sell annuity products (Antolin calls them "decumulative" products), since the major obstacle is that the stakeholders think they would be unable to cover the risk.

2.5. Managing a small portfolio

The annuity formulae described at the beginning of the book implicitly assume that the portfolio of insured is adequately large, since mortality according to the original presumptions is only possible in this case. Because any kind of mortality or survival probability can only be generated as a quotient of whole integers (the ratio of deceased compared to the number of people at the beginning of the period) in the case of an adequately large number of integers (portfolio).

If the mortality result is included in indexation, it adjusts the reserve via the indexation mechanism. However, in the case of a small portfolio it could occur that no insured people die in a certain year or, by contrast, a large mortality loss occurs that may even surpass the entire surplus yield on the investment; although if at least one insured individual dies then large mortality profits may arise in that particular year (in proportion to the reserve of the whole portfolio). Therefore there may be extreme fluctuations in the annuity of insured individuals belonging to a small portfolio from one year to the next. There are two fundamental methods of protecting against these fluctuations:

1. Using a pool, i.e. via the unification of portfolios. In such a case, the portfolios of different providers are unified and organised into a bigger pool, where these fluctuations will necessarily become smaller,
2. Using solvency capital.

If the regulations allow institutions without solvency capital (like Hungarian-type pension funds) to also provide life annuities, only the first option is available for them (which it is worth making mandatory irrespective of the magnitude of the portfolio). However, at the launch of the annuity system the entire risk community is small, so the problem of a small portfolio will necessarily emerge even if there is a pool, and accordingly pension fund providers may even require government assistance. While the pool is almost unavoidable for Hungarian-type pension fund providers, it may also be useful for providers that have their own capital (insurers), so it is expedient if the regulator allows the formation of pools on a voluntary basis. What is more, it may even become mandatory, so the pool may work as a supplementary risk-equalisation mechanism that adjusts haphazard compositions of the risk community at the different providers. This could not be managed with their natural tools, as differentiation (e.g. according to gender) is not permitted. Naturally it may happen, mainly at the beginning, that even a portfolio which is organised into a pool is not sufficiently large, and therefore the implementation of the second solution (in the case of providers with own capital) may occur in parallel with the first one.

Services provided at the expense of solvency capital (in this instance the smoothing out of fluctuations caused by the mortality result) may be regarded as an option provided to the client, the price (premium) of which is paid by the client either in the cost part of the annuity premium or as extra yield to the insurer. In the case of a large portfolio, and if the technique that mortality results are taken into account during indexation is implemented, and especially if the technical interest rate is 0%, there is a small likelihood of solvency capital being used. But if the portfolio is small, there is a higher chance of it being used due to balance fluctuating mortality results. Differentiating a small portfolio (for instance during reserving) would split the portfolio into even smaller portfolios.

According to the regulations, even if the mortality result is treated as part of the indexation in normal cases and the providers have own capital, in the case of a small portfolio (presumably at launch) it would be expedient to suspend this indexation rule based on the above-mentioned reasons and to instead implement the following approach (regardless of whether the given portfolio is the portfolio of a provider or a pool):

1. Until the entire pool reaches a certain threshold, the entire mortality result belongs to the provider (or in the case of a pool: the providers) and is not included in indexation. In this case, indexation takes place exclusively on the basis of the investment result. Transitory mortality losses are financed from solvency capital by the provider(s) and the mortality profit will be one of the sources for topping up the solvency capital. As long as this takes place, although the reserve calculation may occur by differentiation according to risk groups, the mortality result of the different risk groups are not treated separately.
2. If the average for the entire portfolio reaches the threshold but there are risk sub-groups that don't, then in that given year the risk groups shall be consolidated starting from the smallest group until the number of people in the consolidated group reaches the threshold. The mortality result is accounted for in the consolidated group exactly as below, but not according to the risk groups within that.
3. If every risk sub-group reaches the threshold, the mortality result is included in indexation, the mortality result is handled separately for every risk group and the indexation of that particular group occurs separately from the other groups.

This initial use of solvency capital may be seen as the initial investment of a for-profit capitalised provider, the return of which is the option premium collected for the purpose of providing solvency capital for a future, larger portfolio.

Calculation is needed to determine the abovementioned threshold. The principle of the calculation may be that the portfolio of insured should be large enough to prevent the probable fluctuation of mortality from exceeding a yield level that is sustainable on a longer term.

2.6. Homogenising the annuitants' portfolio

From a risk point of view the homogeneity of the annuitants' portfolio may be examined according to several aspects: e.g. homogeneity by gender (the insured are only women/men), by year of birth (the insured belong to the same cohort), etc. These are very interesting problems, but it is probably impossible to achieve homogeneity by gender (or only by illegal means). In addition, homogeneity according to cohorts can be achieved by differentiation (because no legislator can expect pension savings to be exchanged for annuities in 50 years' time based on a tariff that was set in stone today). Accordingly, from among the possible problems of homogeneity, below I highlight the question of homogeneity according to the size of the annuity and this is what I will be discussing hereafter. This issue is particularly important because of the possible (probable?!) **correlation between the magnitude of the annuity and the expected remaining lifetime**.

Above, by writing down the various formulae for the premiums and the reserves of life annuities uniformly for an annual annuity of 1 EUR, I implicitly assumed that the annuities are of equal magnitude. Naturally the actual size of annuities would probably indicate a relatively large deviation, which would not cause a problem if we assume that there is no correlation between the size of the annuity and remaining life expectancy. If we differentiate the portfolio of insured according to several different parameters (usually: state of health, smoking/drinking etc. and lifestyle), this would probably be the case within the different subgroups and naturally within the entire portfolio of insured. In addition, it is also possible that the level of annuities would become homogeneous to a certain extent within the different sub-groups (even if there is a strong possibility that the average annuity of the various differentiated subgroups might differ significantly from one other). In view of the fact that it would seem that in most countries (including Hungary) this kind of multifaceted differentiation is not possible for non-technical reasons in the case of mandatory annuities, it is still possible that there will be a correlation between the amount of the annuity and remaining life expectancy as a result of differentiating factors that we do not take into account.

For the time being there is no data (at least in Hungary) for the correlation itself or for its rate, but in my personal view there is a strong likelihood of a

large, positive correlation (so the bigger the annuity, the longer the lifespan of the insured). I maintain this opinion despite the fact that many experts have doubts on the basis that concealment of income is prevalent in Hungary, so in reality those entitled to a small annuity will also include many very rich people. I believe that we could of course have clearer data if we did not have to deal with this phenomenon, but a positive correlation can be assumed nevertheless because there are relatively fewer rich people, so having rich among the poor may distort the picture somewhat (mitigate the positive correlation), but would not change it. In addition, this may change as the economy becomes progressively whiter (which may be significantly facilitated by a more equitable tax system introduced by Hungary's competitors and which Hungary is thereby forced to follow).

Given the lack of data, it is of course pure speculation to claim a positive correlation or a lack of one, but the problem is nevertheless worth dealing with and formulating calculations for.

In addition to data collection and the resulting digressive annuity table, there is of course another way of handling the problem, i.e. by eliminating it a priori, or at least significantly diminishing its magnitude. This is the **homogenisation** indicated in the title.

Homogenisation means **restricting the possible difference between the sizes of annuities**, i.e. converting peak annuities to average annuities (in other words: setting a cap for annuities). The justification of such a regulation can of course not be that we use it to create an actuarially more favourable or more easily calculable position, because a situation that is more favourable actuarially can only arise as a consequence of another suitable reason for homogenisation. Such a reason can be found easily if we look at why the law obliges citizens to accumulate savings to cover their financial needs in old age. Why doesn't the state leave people to do this for themselves without making it mandatory[38] and intervene only by providing them with precise information about to what extent they should set aside from their current income? The answer is that people's time preference is generally not adequate, meaning too many people prefer to consume in the present rather than in the future and are there-

[38]It seems to be an enterprise that is not totally impossible. China is proud of having one of the highest savings rates in the world, having achieved such a situation by launching a campaign to make saving a "patriotic obligation" (see: Akerlof-Shiller [2009]). Obviously we cannot predict what sort of campaign would lead to success in a society that is more individualistic than Chinese society, such as Hungarian society, but even if such a campaign were a success it would take decades to actually change the current pension system to reflect this. In addition to which it is unclear whether the campaign played an important role at all, or people simply obeyed the government's orders.

fore not inclined to save enough money on their own, which in turn would mean the appearance of masses of unsupported old people within the state social system, causing its eventual financial collapse. In order to avoid this, the state obliges citizens to accumulate savings in advance.

However, the necessary pre-savings have an upper limit, beyond which the above-mentioned problem does not arise, and so beyond that limit there is no need for any obligation, the state can be satisfied with the common sense of citizens, without which old people who once had a good level of income may have to radically diminish their consumption but will not be destitute and will therefore not have to rely on central redistribution. Forward-thinking individuals should not be obliged by the state to make savings beyond a certain level within such a system (private pension funds), beyond which they may accumulate savings with a better yield (e.g. purchasing and/or renting out real estate, starting up their own business etc.). There are serious arguments in favour of setting an upper limit to private pension fund savings and accordingly on mandatory contributions, thus making the possible annuities more homogeneous.
Homogenisation may have different methods:

1. The system does not allow the generation of such annuities form the word go, and also sets a strong upper limit for total contributions (perhaps even by prohibiting the payment of further monies into the private pension system once a certain level of capital has been accumulated). This is a better solution than the one currently used in Hungary whereby the maximum is linked to the actual income, because it manages the problem of a possible drastic drop in income.
2. Regulation would set the same upper limit for (let's say) monthly annuities, but with regard to annuitisation rather than contributions. The assets beyond the upper limit would be paid to the fund member in a lump sum. Compared to the previous solution, this is better because it manages the problem of uncertainty with respect to the rate of capital exchange into annuities.
3. Another option is that the regulations allow an annuity to exceed the upper limit but exclude the possibility of significant differences in annuities at different providers. This can be achieved by requiring annuitants to split their capital between several providers so that the benefits provided by each provider is smaller than a pre-determined limit (but only as many providers may be chosen as are necessary to achieve this, meaning the annuity cannot be intentionally fragmented). This solution is different from the previous two because it splits the longevity risk between providers while still keeping it within the system.

Homogenisation naturally not only means the exclusion of annuities that are too high, but also those that are too low. Part of it is that the regulations should not permit too low a level of savings to be converted into an annuity. I deal with this aspect separately below. We might refer to this as the question of an "absolute" annuity minimum.

The other aspect (the question of a "relative" annuity minimum) only comes into the picture if – from among various annuity types – the client may also opt for an annuity that has a guaranteed period. If a guaranteed period is possible, the guarantee has its price, which manifests itself in a reduction in the monthly annuity that can be provided in exchange for the accumulated capital. Therefore it is justified that if the regulations permit a guaranteed period then it should set a certain limit (according to capital or annuity) below which the insured individual may only purchase a single annuity without a guaranteed period. This also serves the homogenisation of the risk community, but from a different perspective.

In the literature I have only found reference to homogenisation in Winkler-Mattar [1999], who used it in the same sense as I do. They say that in the case of a heterogeneous portfolio the mortality swings caused by heterogeneity are higher than the ones caused by a long life, meaning longevity risk cannot be evidenced in such a portfolio. They also note that, in contrast to other life insurances, heterogeneity is generally not diminished by reinsurance. However, they suggest the same solution as I do to the problems caused by heterogeneity, i.e. they recommend that the level of annuity payments should be restricted and that in general the insurance company should restrict the level of annuity within its total business, for instance via a suitable commission structure.

The Hungarian literature does not use the expression homogeneity, although Michaletzky [1999] already proposes (with a reference to Stahl, and without indicating the source!) that there might be a correlation between mortality and magnitude of capital; he suggests solving this by preparing a suitable mortality table.

2.7. Indexation

2.7.1. POSSIBLE FORMS OF ANNUITY RESERVE INVESTMENT AND ANNUITY INDEXING

The traditional annuity-formulae assume that the insured person nominally receives the same annuity payments from the start until the end of their life. From another perspective it supposes that the yield on the annuity provision is precisely identical to the expected yield or rather to the technical interest rate, and that it moves neither up nor down with respect to it. In other words, the

annuity does not have to be increased as a consequence of the reimbursement of yield above the technical interest rate (excess yield) and does not have to be decreased with respect to losses resulting from a lower yield than the technical interest rate.

However it is obvious that the technical interest rate will be surpassed by the actual yield from time to time during the course of the term, and therefore the annuity will have to be indexed – and especially if the technical interest rate was set to ensure a great probability of this happening,[39] and if indexation itself is an expectation from a certain perspective.

In reality, the possibilities (and in fact the need) for indexation depend on how the reserve is invested. The possible forms of investment of the annuity reserve are different depending on who bears the investment risk. This naturally may also have an impact on the annuity construction itself, and primarily on the indexation. The possibilities are:

1. Both the provider and the client are exempt from the investment risk; indexation depends on the yield of the assets invested into the reserve. The basis of the investment strategy is that in such cases the provider invests exclusively into (good quality) bonds in an expiry structure matching the expiry structure of the annuity portfolio. So the provider is exempt from market fluctuations, since it invests only and exclusively into bonds that are kept until the date of expiry, which need no revaluation in accordance with changes in interest rates. As long as only fixed yield bonds existed, this strategy also meant that the annuity dues remained unchanged until expiry, and possibly (rarely) even grew at a pre-determined rate. These days bonds with yields linked to inflation are widespread, so it is relatively easy to achieve an annuity that is linked to inflation through the application of this strategy, or perhaps even an annuity that is indexed to inflation plus a fixed percentage (e.g. 2 %), if the provider invests only in bonds with yields that are indexed to inflation.
2. The investment risks are divided between the client and the provider; indexation depends on the excess yield achieved in the previous year. This solution in fact is the return-refund technique which is applied by traditional (i.e. not unit-linked) life insurances. In this case the benefit paid by the insurer increases every year depending on the investment yield achieved in the given year (so the investment risk is partly borne by the client), although the benefit may not decrease nominally (so part of the investment risk is borne by the insurer who guarantees a part of

[39]Except if the technical interest rate was set by the provider in such a way that it represents "the" yield that is guaranteed to the client, while the excess yield above this is the profit of the provider and the yield deficit below this level represents a loss for the provider. This is example 3 below.

the yield). The insurer guarantees not to achieve at least a 0% yield, but to achieve a usually higher yield pre-included in the benefit, the so called "technical interest rate". In these cases, the insurer partly invests in bonds, shares and other types of assets such as real estate. With this solution the rate of indexation cannot be computed in advance and it is not possible to provide a good guarantee for this at a low cost.

3. The annuity provider bears the entire investment risk; the annuity is not indexed and increases by a pre-determined percentage ratio or in absolute terms. This solution is mainly applied in Britain and America, but the bankruptcy of some long-established insurance companies in the 1990s (the most important example being Equitable Life, one of the oldest and most prestigious English insurers) showed what a dangerous strategy it is, although Equitable Life was able to fulfil its obligations despite bankruptcy. The essence of this model is that the annuity payment is not indexed (or more rarely: its level changes at a predictable rate) and the insurers may compete with each other with respect to who will undertake the given series of payments at the lowest price. Thus the competition, in addition to the costs, is in the size of the yield that providers undertake to provide above and beyond the technical interest rate, i.e. the magnitude of the technical interest rate. However, any yield beyond the technical interest rate undertaken with respect to the client belongs to the insurer, which achieves this however it can. In my opinion, this solution cannot be applied in the case of mandatory annuities, not so much because of its danger, but because it is not adequate to allow the application of unforeseen (e.g. inflation-linked) rates of indexation, which is essential to private pension annuities and represents part of old-age security.
4. The client bears the entire investment risk; the magnitude of each annuity payment depends on how the reserves are evaluated when a particular annuity payment is due; indexation in advance is unpredictable. This solution is the transposition of unit-linked insurance facilities onto annuities. The client bears the entire investment risk and the payment depends on the actual daily value of the assets. If the invested assets are secure (e.g. not Greek government bonds), this is similar to solution 1, although the payments fluctuate unnecessarily as a result of daily changes in bond valuations, which are not compensated by the relatively higher yield. If the invested assets are risky, the high yield must be paid for by the high fluctuation of annuity payments, which cannot be allowed in the case of small annuities (such as private pension annuities will be), and accordingly this solution should not be recommended with relation to private pension annuities.

In summary, I only regard two of the above options as being possible in the case of mandatory annuities, and therefore below I assume that we can only chose between these two options.

2.7.2. AN EXPEDIENT EXPECTATION WITH RESPECT TO INDEXATION – THE GUARANTEE PARADOX

It is worthwhile – as a bad example – to cite the Hungarian annuity regulations (Section 4 (7) of Government Decree 170/1997) that were in effect throughout the existence of the Hungarian private pension fund system (although ineffective due to a lack of actual annuity payments). According to the regulations, the sum assured by the annuity must be defined so that the annuity paid by the fund shall be indexed at least to the same extent as the pension paid by the social security system (Pillar 1). Obviously this rule implicitly follows the return-refund indexation technique, but with this technique (and in fact with relation to all of the above indexing techniques) it is not expedient to pose such expectations because:

- The expectation itself has no relation to what the annuity can provide.[40] The annuity may be fundamentally indexed in proportion to the extra investment yield (providing there is no mortality loss).
- Overall it is an incalculable risk for the service provider, because the change in pension provided by the social security system is very much dependent on politics, so the indexation of the social security pension is practically unpredictable according to experience.

As a result of the above, either one cannot find a provider for the mandatory annuity with these conditions, so the state is forced to set up a central provider, or the providers are forced to tie up an irrationally high proportion of the annuity premium (that is the private pension fund capital of the clients) for hedging unpredictable political risks, so the annuity would be smaller than what it would be without this indexation rule. Such a requirement should not be posed to a central provider either, because it cannot guarantee this yield through market investments alone without the involvement of extra state funding, while

[40]This was specifically true as long as the so-called Swiss indexation was in force within the social security system, because that could definitely not be covered by capital market facilities, but even if that were possible, there was still the political risk, i.e. that is the state could deviate from this indexation at any time, as it frequently did. The political risk remains even if the introduced regulation on indexation is now price indexing (as was more or less introduced in Hungary in 2009).

when elaborating the regulations it is expedient to avoid the inclusion of state funding already from the word go.

This example already indicates that contrary to everyday reasoning minimum requirements for guarantees with respect to yields should be set cautiously in the case of annuity regulations. The so-called "yield guarantee" paradox[41] states that the higher the yield we wish to guarantee to the client, the lower the actual benefit will be.

This paradoxical statement may be understood if we think about the way providers can achieve a higher required yield. If the requirement is relatively modest, the provider would probably conduct a more prudent investment policy than it would without this requirement, so it makes investments (primarily into bonds) that are less volatile but generate a lower yield. As a result of this rule, the yield would be lower in the long term than it would be without it. If the requirement exceeds a certain level, the prudent investment policy of the provider will be insufficient in itself. In addition, a portion of the premium is used to meet the requirement and consequently there will be less service.

It is therefore not expedient to pose such expectations with respect to indexation and yield, and instead investments into inflation-indexed bonds and indexation to inflation should be required. Alternatively, in the case of indexation based on return-refund, regulations should be implemented that make the above goal achievable but do not demand that the above paradox is avoided. This is a requirement of a 0% technical interest rate (and expected yield) in the case of an indexation technique based on return-refund, whatever relation the developed indexation has to indexation in Pillar 1. Nevertheless, the 0% technical interest rate means that the annuity increases at the rate of the entire investment excess yield[42] (naturally only if the mortality result is 0 or positive), which usually and on average will exceed the indexation of Pillar 1 at the majority of providers even without such a formal requirement.

[41]This yield guarantee paradox was explained by András Kozek (Allianz Hungária Biztosító Zrt. Vice CEO) on 22 September 2006 in an expert meeting dealing with annuities as follows: "If the legislator forces the insurance companies to assure higher guaranteed yields, the insurers will be able to assure lower total yield for their clients, because the higher the guarantee, the less risky the insurer's investment strategy, and this allows insurers to invest their clients' money in more secure instruments that provide a lower yield".

[42]Or rather the portion of this that is due to the client, i.e. the total net yield on the investment.

2.7.3. INDEXING AND THE POSITION OF INSURED INDIVIDUALS – ADVERSE SELECTION DUE TO THE TECHNICAL INTEREST RATE

When financial experts design the Pillar 1 annuity, they are inclined to think that the choice between a lower starting annuity indexed using a higher index and a higher starting annuity indexed using a lower index is simply an issue of balancing the state budget. The first alternative means a lower burden today because the burden is put off to a later date, while the second alternative means the opposite. Although this is true with respect to the entire portfolio in general, the situation is different when it comes to individual annuitants, because one of the two options will be more favourable for people with different life expectancies. Accordingly, choosing between the two alternatives is not (just) a choice based on financial values, but concerns the question of which segments of the population we should benefit. Option 1 means that annuity payments begin at a higher level but are indexed at a lower level, which is favourable for people with a shorter life expectancy (men, blue-collar workers, people with lower education and/or lower income, etc.). Option 2, i.e. an annuity that begins with lower payments but uses higher indexing, is favourable for people with longer life expectancies (women, white-collar workers, people with higher education and/or higher income), because the two segments of the population can maximize the total annuity received during their lifetime in different ways.

This logic is valid not only in respect of Pillar 1, but also in respect of mandatory annuities, primarily via the choice of technical interest rate. One might say that a higher initial annuity with lower indexation gives something back to those who receive a less favourable annuity because of the unisex table, compared to what they would receive with a differentiated table. In addition, the higher income segment can better bear the devaluation that results from lower indexation.

If clients can choose between different technical interest rates, or rather between providers who offer differing levels of indexation, then people with lower life expectancies would tend to accumulate at providers who offer a higher technical interest rate, while those with longer life expectancies would accumulate at providers who offer a lower technical interest rate,. Overall, the composition of the portfolio of annuitants would be different from the calculated one, so providers who offer a lower technical interest rate would accumulate a risk loss. It is expedient to avoid this and therefore consideration must be given to whether the regulation should allow for competition between technical interest rates (or more precisely: competition between different rates of indexation, although providers can also foresee this effect, so a uniform tech-

nical interest rate can become a reality without regulatory intervention – at the maximum interest rate allowed by the regulations).

However, the above problem may be solved without competition between technical interest rates because different interest rates favour different segments, so the choice itself is not a neutral one. The solution may be that in the case of a private pension fund annuity we calculate using different technical interest rates (so with different indexation rates) at different points of maturity, e.g. 3% until the age of 70, 1.5% until the age of 80, and 0% beyond the age of 80. The interest rate is also fixed in this case, but nevertheless changes without generating adverse selection with regard to differing interest rates.[43] To a certain extent this is a compromise between the above-mentioned two options; it favours people who are better off to a lesser extent than, for instance, a uniform technical interest rate of 0% would do.

2.7.4. THE ROLE AND RATE OF THE TECHNICAL INTEREST RATE

It is worthwhile taking a separate look at the role of technical interest rates, since it is different in the case of diverse investments and in the case of the resulting indexation techniques. I elaborate in detail on the indexation techniques to be considered in the case of mandatory annuities, after which I also speak briefly about two other indexation techniques.

When all the investment risks are undertaken by the insurer and the annuity is not indexed, the technical interest rate is essentially a competitive factor,

[43]The solution is based on István Hetényi's proposal at one of the Pension and Old-Age Roundtable (NYIKA) meetings in 2007, that the indexation of pensions in Pillar I should depend on the age of the insured; the higher the age the higher the indexation. This solution is however rejected by some who argue that Hetényi, being an elderly pensioner, was willing to change the rules driven by selfish motivation and therefore it should not be taken seriously. But in my view, the following arguments can support this proposal: 1. The higher the technical interest rate the lower the indexation, so the difference between the standard of living of pensioners and active people may be very different. This does not cause a problem for those with short life expectancies, but it does for people with long life expectancies. For them, it is worth diminishing or reversing this difference. 2. According to experience pensioners react by decreasing their consumption, which may be fairly easily done in the case of certain goods, since many of the goods are already not on people's shopping lists as they age. However, there are also goods that are consumed more in old age and especially at a high age. These are primarily medical, nursing and pharmaceutical services. Above a certain age the strategy of lowering consumption does not really work especially with respect to increasingly small families, where fewer and fewer relatives can supplement the missing pension or provide care informally.

although this is not visible to clients, who only see that different providers are charging different prices for the same annuity.

When the entire investment risk is borne by the client, as in the case of a unit-linked annuity, the technical interest rate does not have a role to play in principle since there is no guaranteed yield, which is one of the most important characteristics of the technical interest rate. Nevertheless, in a strict "technical" sense a kind of "interest rate" may be defined here, or rather just a rate, which is the rate at which the number of paid annuity units diminishes year by year. In the case of unit-linked annuities, this rate fundamentally plays the same role as the technical interest rate in the case of traditional insurances (disregarding the guaranteed yield), i.e. the investment yield must be at least as much so that the paid annuity does not decrease from year to year, and the more it exceeds this (i.e. "excess yield") the more the annuity may increase.

In the case of **inflation-linked bonds** that pay a yield, the usual interest paid is: inflation + a fixed percentage (e.g. 2%). If it invests in bonds of this nature, the insurer can to all intents and purposes[44] offer two types of indexation to the client:

1. It increases the annuity payment by the rate of inflation every year (i.e. it maintains the value in real terms),
2. Every year, it increases the annuity payment by inflation + a fixed percentage (which is not necessarily as much as the yield of the bond!), i.e. it continuously increases its value in real terms.

In the first case, the maximum technical interest rate is a fixed percentage of the bond yield, since the insurer may at most promise the entire bond yield to the client as a yield. However, if the technical interest rate reaches this maximum then the insurer can only cover its expenses and its profit from the one-off charge paid at the beginning as part of the annuity premium, and must use this to cover everything during the entire term. Therefore it is probably expedient for the insurer not to exhaust the maximum potential of the technical interest rate and to determine it at a lower rate than this fixed interest rate. Naturally (if indexation is fixed) the premium is a competitive factor among insurers, which is affected by costs and the technical interest rate, and this pushes the insurer towards determining as high a technical interest rate as possible.

In the second case, the maximum technical interest rate can be the difference between the bond yield beyond inflation and the fixed interest rate beyond inflation that may be granted to the client. Theoretically it is possible that

[44]Theoretically the insurer could keep the annuity payments unchanged, i.e. not apply indexation, or could promise a rate of indexation below the rate of inflation, but I do not elaborate on these approaches since I do not deem them realistic options at the moment.

the insurer offers the client a higher fixed increase beyond inflation than the bond yield fixed beyond inflation. In this case the technical interest rate will be negative.

With such a policy for the regulation of the technical interest rate, it is sufficient to prescribe that the insurer may not offer more to the client than it can achieve via the bond yield. Although during the course of regulation of the technical interest rate it should also be considered how the very same regulation defines the indexation rule of mandatory annuities. This may be achieved in two ways:

1. Each provider may index the annuity according to one scheme (e.g. precisely to inflation),
2. There is competition between the providers with regard to the level of indexation they apply to annuities. The possible options: to inflation or to inflation + a fixed percentage.

In the event of the 2nd option, the danger of adverse selection must be taken into account since people with shorter life expectancies would opt for lower indexation with a higher starting annuity, while those with longer life expectancies would opt for an initial lower annuity and the corresponding higher indexation. This creates losses for insurers who opt for higher indexation. If insurers recognise this then none of them will opt for this, and so the market will automatically settle in a state that corresponds to version 1, meaning everyone indexes exactly according to the rate of inflation.

In the case of the **return-refund technique**, the regulator cannot regulate the technical interest rate as liberally as in the case of the previous indexation technique. It is expedient to apply two rules in this case:

1. The technical interest rate should be uniform throughout the entire market, regulations should not allow competition in this field,
2. The technical interest rate should be as low as possible (e.g. 0%).

Arguments in favour of a fixed technical interest rate:

- In the case of life insurances, fixing the eventual upper limit of the technical interest rate is a generally applied regulatory practice, because competition pushes insurers towards increasing the technical interest rate and encourages them to endanger their long term interests (i.e. the sustainability of services) while taking into account their short term interests. Therefore the definition of a low eventual technical interest rate may be regarded as prudential expediency.
- In the case of strong market competition it is not significant whether the regulation sets a maximum technical interest rate or a fixed technical interest rate because insurers will “stick” to the maximum possible val-

ue as a result of competition. Their starting point is that the vast majority of clients prefer the certainty of a high value starting annuity to an eventual higher increase in value, because the latter is dependent on excess yield of uncertain magnitude. The situation is naturally different in the case of weak market competition – e.g. a monopoly or colluding oligopoly (cartel).

- If, despite the above, there are different kinds of technical interest rate on the market, it would be suitable for allowing the selection of the insured according to risk criteria, which would make calculation more difficult and more expensive. In the case of a uniform interest rate, this adverse selection effect does not exist.
- A uniform interest rate makes it easier for clients to choose between providers in the annuity phase (if regulations provide for such an option) and helps clients compare the performance of different insurers. Comparability also means that providers cannot hide higher costs by applying a higher technical interest rate.
- Arguments in favour of applying a lowest possible technical interest rate:
- A priority target is maintaining the real value of annuities, which (with this indexation technique) can be best achieved using a low technical interest rate, since there is a good chance that the yield will be higher than inflation every year. Therefore the lowest possible technical interest rate of 0% is preferred.
- A similarly important target of annuities is not only to maintain their value but to possibly increase their value over time. This is also best achieved by applying the lowest possible technical interest rate of 0%.
- The 0% technical interest rate may be regarded as a safe interest rate if recalculation of annuities becomes necessary during the term of the annuity because of improved mortality; this interest rate is the best way to avoid the decrease of the annuity payment and allows for adjustment at the expense of excess yield. (Naturally the possibility of negative yield cannot be excluded).
- However, one must note that this technical interest rate is primarily beneficial for insured individuals who are healthier and who have a longer life expectancy and a higher level of education.

This latter problem is eliminated by applying a fixed interest rate that changes during the term, as proposed by Mr. Heténi and described previously.

Finally it is worthwhile citing – as a bad example – the ineffective Hungarian private pension annuity regulations. According to Section 5 (3) of Government Decree 170/1997: “the technical interest rate applied in the annuity pro-

vider personal pension fund may be at most 1.5% higher than the interest rate resulting from the indexation of the social security pension determined for the actual year". I find this definition fatally flawed for two reasons:

- The technical interest rate cannot be changed annually with respect to annuities that have already started, and therefore it is not justifiable to adjust it to such a volatile standard. In addition, it is also expedient for the technical interest rates of annuities that commence in different years to be similar to each other.
- The technical interest rate set in this manner is almost certainly too high, so it is almost certain that it will be impossible to achieve an investment result of this magnitude, let alone a higher one, to ensure the adequate increase of the annuity. For the sake of comparison: in the case of Hungarian life insurances, the highest possible technical interest rate has been 2.9% since the mid-2000s.

2.7.5. INDEXING AND INFLATION MANAGEMENT IN LITERATURE

In the English language literature, the issue of indexation primarily means inflation management and indexation to inflation, although there are different indexations on the market too. This is fundamentally due to the fact that in Britain and North America the most frequent annuity is still one that has an unchanged sum assured without any indexation, the so-called "level annuity", the major deficiency of which is the lack of stability in value if the annuitant lives for a long time (Blake [1999]). According to Blake, annuitants generally prefer the "level" annuity, because they tend to underestimate their lifetime. In addition its starting level is roughly 30% higher than that of an indexed annuity (Blake [1999]).

According to the World Bank [1994], private insurers are often blamed for not providing protection against inflation. According to them, one of the possible solutions to the problem is to index the benefit to inflation and at the same time to keep reserves in inflation-indexed securities or in foreign assets that are immune to domestic inflation. Chile is mentioned as an example. The report adds that the indexed annuity is naturally lower at the beginning compared to if it were not indexed, and therefore it is not clear whether blue-collar workers would be better off with indexation.

Poterba [1994] finds the reason for the lack of indexation according to inflation on the American market in the fact that the Treasury only recently started to issue sovereign bonds indexed to inflation, so annuity insurers have only been able to begin covering inflation-indexed annuities relatively recently. According to Blake [1999], insurers try to purchase indexed bonds with re-

spect to indexed annuities. He writes as late as in 2006 with reference to the British market that indexed annuities are a relatively new development made possible by the fact that the government began issuing long-term bonds that are indexed to inflation (Blake [2006a]).

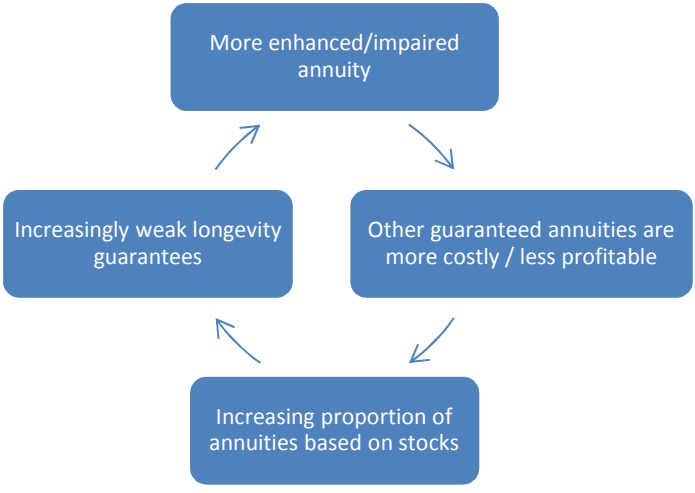
The issue of indexation also emerges with relation to unit-linked annuities (variable, equity-linked, etc. annuities). This option has already been mentioned by the World Bank [1994], although not in relation to inflation management but rather because in their view that this is the solution to the problem of fixing the date of annuitisation, since the price of annuity units fluctuate together with the market. Nevertheless, the disadvantage of this solution is that the risk of fluctuating annuity must be borne throughout the lifetime of the pensioner. According to Poterba [1997], although the market for "variable annuity" has increased most significantly in recent years, these constructions do not necessarily protect pensioners against inflation. He mentions as an example that this could occur if payments are adjusted to the stock market, which then underperforms compared to inflation.

Similarly to the longevity problem, unit-linked annuities can be also considered as a possible solution from the perspective of insurers. This is why Wadsworth-Findlater [2002] offers a new kind of unit-linked annuity instead of the traditional "level" annuities, the cost structure and internal structure of which is visible (the English terminology calls this "unbundled"), permits changes over the course of time and most importantly does not include a longevity guarantee for the client. The concept was elaborated by Wadsworth in a brief article in 2002 (Wadsworth [2002]). According to this, everybody on the annuity market offers almost identical products (this refers primarily to the UK market). This will change in the future and he calls this process the "unbundling dynamic". Its logic is as follows:

- The proportion of "enhanced" and "impaired" annuities will increase, so pricing becomes increasingly sophisticated, which also affects opportunities for pricing the remaining products. He calls this "anti-selection".
- This means that the annuity will be more expensive for the survivors, so the traditional annuity hedged by bonds becomes increasingly less satisfactory. Accordingly, annuities must shift towards the capital market and towards annuities that are linked to investments.
- The immediate consequence of this is less longevity guarantee, since this is already priced inadequately today.

These dynamics are depicted in the following diagram:

Diagram 1: Unbundling dynamics



Watson Wyatt (whose partner Wadsworth deals with annuities) stated in 2008 (Watson Wyatt [2008]) that unit-linked ("variable") annuities are becoming increasingly popular, but that several insurers have suffered major losses due to these as a result of inadequate hedging, despite the fact that the theory behind hedging is relatively well-developed and naturally requires an extensive knowledge of "Greek letters". However, hedging is not enough; there is also a need for adequate product design and periodic monitoring. In respect of product design, the combination of adequate incentives (bonuses) and contra-incentives (penalties, exclusions, limiting the availability of options, etc.) is necessary and the use of options and customer behaviour with respect to lapse must be monitored. With relation to hedging, he mentions the "basis risk", the causes of which are, in his opinion:

- Choosing inadequate hedging because there was nothing better available,
- The fund-manager deviated from the designated path (track), which to a certain extent it is allowed to do,
- Unexpected costs (e.g. taxes).

In relation to unit-linked annuities we must mention that in the Anglo-Saxon insurance market the expression "variable annuity" does not only mean the annuity itself, but also entirely standard unit-linked insurances, whose theoretical target is to convert the accumulated assets into an annuity at maturity. This is what the term means even if experience suggests that the majority of people do not convert their accumulated capital into an annuity at the end of the term, since it is not obligatory. This is why a number of publications with the title "variable annuity" do not in fact deal with actual life annuities. An example is Abbey-Henshall [2007], who even notes that the term is often used with respect to something that is neither "variable" nor an "annuity".[45]

The otherwise vague Hungarian annuity literature deals relatively extensively, but also rather unilaterally, with the issue of indexation. The starting point is generally the provision of the act on private pension funds that links the indexing of private pension annuities to the indexing of social security pensions. Michaletzky [1999] raised, but did not really answer, the question of what happens if the yield is not sufficient for indexation. According to the Act, the pension fund supervisory authority must be informed, but it is unclear what the supervisor is able to do. In contradiction to Michaletzky, Réti [Réti 1999] clearly and correctly notes that the indexation requirement stipulated by law is impractical in insurance technical terms and is contrary to the essence of the private pension system. The only indexation that can have an organic relationship to the private pension system in his view is indexation according to yield, and no other possibility is available. This also means that pension systems based on different principles must have different principles for indexation. In this dispute, Réti mentions Stahl as a counterpart for discussion and thinks that the understanding of indexation sharply contradicts regulation 170/1997. Réti's comments on social security are also interesting, since he is one of the few people who have a profound knowledge of the system in Hungary. According to this, the Hungarian social security pension system has followed systematic indexation from the beginning of the 1970s, although it was only defined in the Act in 1992. Réti also gives the formula of the indexation from which we learn (without him mentioning it) that a 0% technical interest rate is taken as granted. The formula is not provided in traditional actuarial formulae, of which he in fact states that classic actuarial formulae cannot be used for annuities because they assume there is no inflation. These days, annuities cannot be provided based on such an assumption.

At the time, Stahl had not yet respond in writing to Réti's proposals, but in Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] he already notes in

[45]This is why I prefer the term "unit-linked annuity" to "variable annuity".

respect of Swiss indexation[46] regulations (page 485): "Many experts believe that this provision further increases the risks within the system in view of non-foreseeable wage increases. It is also unclear how the insurance companies that provide these services can fend off such risks".

However, in 2005 he clearly turns against the regulation (Stahl [2005]), saying that the basis for annuity indexation must be investment and mortality outcome, which may even permit a decrease of the annuity. This means his proposal includes no guarantee with regard to indexation and consequently there is no need for solvency capital, but he nevertheless proposes that private pension funds can have solvency capital without practically any consequences, including the central annuity provider he proposes here.

Arató (Arató [2006a]), who disputes Stahl, highlights that the Hungarian regulations in force on indexation are far less certain than indicated by Stahl, because in reality the provider does not have to adjust to the Swiss index, which is somewhat more objective, but to the actual pension increases, which are highly exposed to politics and are impossible to anticipate. Therefore, he says (page 274): "I propose that pension funds and insurers provide two types of annuity: one without any kind of mandatory indexation, only recomputed on the basis of the available assets (which means that the annuity may even diminish; this version is basically identical to what János Stahl proposed – B.J.), and another one based on Swiss indexation. This latter would be based upon the actually defined rate of inflation and on the wage index. The pension guarantee fund should operate as a re-insurer and would determine premiums for the Swiss-indexed annuities while taking into account the pension fund's other re-insurances and investment guarantees. Naturally, the starting level of these different annuities would differ significantly; the annuity without mandatory indexation may even be 20-30% higher". Arató proposes two types of annuity so that there is no pressure on the Swiss-indexed annuity to begin with a high sum assured, and so theoretically providers are able to manage the Swiss index from the high deduction at the beginning of the term. Naturally, this annuity would most probably not be attractive so the other option could provide a way out which could practically displace the Swiss-indexed annuity from the market. So in reality Arató proposed how the Swiss index might be formally maintained, possibly as a gesture towards the politicians who devised and promoted it. We can conclude that to all intents and purposes Stahl and Arató agree.

Joining the debate, György Németh also agrees with such an indexation (Németh [2006]), with the significant difference that contrary to Stahl, he

[46]The so called "Swiss indexation", or "Swiss index" in Hungary meant the average of the consumer price index and wage index. Officially, SoS pensions were indexed according to this during the 2000s, and the SoS indexation was the basis for the indexation in the private pension annuity.

thinks in terms of a central pool and not a central provider with respect to mortality, while the pension funds would compete with each other in the field of investing in reserves. So pension funds would apply varying indexation each year, according to the sum of their common mortality and differing investment outcomes. The problem with this proposal is that its implementation would probably be a lot more problematic, since due to the end-of-year mortality regrouping it retrospectively turns out that the investment yield was not generated for as much reserve as was necessary, so the two outcomes cannot be simply added up for each fund, in addition to which mortality regrouping would also require the regrouping of yield, which could easily make this system non-transparent. Arató, commenting on the proposal (Arató [2006]), did not pick up on this (page 569), but in his view: "The author obviously does not know that the mortality of pension fund members may be significantly diverse within the pension funds. Following his proposal dangerous trends might begin at different funds". It is not clear what this comment refers to, since if this suggests that fund members might switch between funds because of mortality differences, then the argument is not justified, as Németh handled that problem with the concept of the mortality pool.

We may conclude that in the mid-2000s Hungarian experts generally rejected the indexation regulations and instead supported the common management of investment outcome and mortality result via indexation.

The existing indexation rules were also rejected by Barabás-Bodor-Erdős-Fehér-Hamecz-Holtzer [2006], with whom Ágoston-Kovács [2007] also agrees. The previously mentioned authors come back to Réti's proposal according to which "The indexation of pensions does not necessarily have to be identical, as this is an element of competition for the insured individuals in the case of non-state systems (this also does not comply with the current rules)".

3. SELECTION PROBLEMS AND THEIR MANAGEMENT

The premiums and reserve formulae of annuities (as for any other life insurances) assume that the included probabilities are valid for the providers' annuities population. This is true in the following cases:

- The underlying annuities population and its risk composition is consistent,
- As observations concern the past and calculations concern the future, the probabilities may be used subject to the condition that there are no trends in the observed mortality.

This latter phenomenon is called longevity risk, the management and mortality tables of which have already been covered. In this chapter, I focus on the first case and how it is achievable that the risk composition of the annuitants' portfolio of different providers is identical to the risk composition of the observed portfolio. In other words; how can selection (adverse for the provider) within the portfolio of different providers be prevented and, should this selection happen, what management methods can be applied?

3.1. Possible selection effects and their management

The problem of adverse (anti- or auto-) selection may be split into two parts:

1. Selection between people who buy or do not buy annuities,
2. Selection among people who purchase annuities.

Experience tells us that a very strong degree of selection has evolved in the voluntary annuity markets. Overall, however, very few people buy annuities although, typically, these people live longer than average compared to the overall population. This increases the annuity premium which squeezes even more people out of the annuity market etc. This phenomenon is a significant obstacle that inhibits the development of the annuities market. Managing this phenomenon has recently begun through differentiation according to state of

health (see impaired annuities). Since this study only focuses on mandatory annuities, a precondition is the conversion of mandatory private pension savings into annuities, and therefore I shall only look at the second type of selection.

Adverse selection among people who purchase an annuity happens mainly for the following reasons:

- Accidentally – this may happen to providers with a very small portfolio, and necessarily cannot reproduce the risk structure of the entire annuitant portfolio,
- For systemic reasons – this may happen with providers that are organised in a specific way (e.g. to suit different employers or professions, see: "Military Pension Fund"), and we may also see this as a problem experienced by a closed risk community. Theoretically, this may also happen in open risk communities, but is based either on deceiving clients or does not last long owing to competition,
- Via conscious selection on the part of clients – people prefer funds that provide choice and are more favourable towards them in an open risk community or when given free choice. This selection may also happen among providers (if the given parameter is applicable to the entire scale of products of a given provider), but also with annuities that have different parameters. Selection among products may, of course, also take place in the first two cases which further aggravates the situation. Selection amongst products may take place along the following parameters:
    - **By choosing amongst annuity types**. Here, of major importance is the choice between two parameters provided by annuity types:
        - A choice between annuities with a guaranteed period and without a guaranteed period, and also the form of the guaranteed period (occurring at the beginning or at the end of the annuity period). The total volume or sum of benefits of the annuity in the case of insured people with varying life expectancies is different, therefore insured people with a shorter life expectancy will tend to choose an annuity with a guaranteed period while people with lower life expectancies will choose an annuity without a guaranteed period,
        - A choice between annuities for a single life or for two (joint) lives (provided differentiation according to sex is prohibited in the premium). In such cases, the woman of a couple will choose a single life annuity, and the man of the couple purchases an annuity for two. It is theoretically possible, though less likely in practice, that the range of products

includes both symmetrical and asymmetrical joint life annuities. Selection occurs if there is a choice between these.

- **By choosing the indexation rate.** The provider can influence this by changing the magnitude of the technical interest rate. A higher indexation rate (i.e. a higher initial premium and a smaller starting annuity) will be selected more often by people who expect a longer lifetime, while the reverse will be true for those with shorter life expectancies. Here, contrary to the previously discussed selection due to the guarantee period, selection is not the result of maximising the total benefit, but rather the benefit received during the annuitant's lifetime.
- **By choosing the starting time of the annuity.** There may be a notable selection impact if clients are given the right to start their annuity later rather than at the date of retirement. Additionally, there are significant changes in possibilities with inheritance before and after the start of the annuity. People with an expectedly shorter lifetime would tend to delay the start of the annuity, because if they died in the meantime, their entire accumulated pension assets would be left to their heirs. This significantly reduces the mortality profit of the provider. Such a selection is weakened by the fact that not every insured person finds inheritance important (although it is a fairly general motivation) and also that the start of the annuity is frequently an economic "requirement" for insured people.

For the provider, the best method for managing adverse selection, i.e. managing things if the risk composition of the portfolio of the insured party moves off in an unfavourable direction from the one designed, is via differentiation. If every risk group pays a premium that corresponds to their own risk, the composition of the insured portfolio may move away from the designed portfolio and the premium paid would automatically cover the received benefits. The problem with selection, from the insurer's point of view, is not that people with diversified risk find themselves in a certain risk community, and this is not the primary reason why they should be differentiated by risk; the issue is that the provider does not anticipate the composition of the risk community and cannot prepare for the effects of selection while regulations prohibit differentiation in general or according to important factors such as gender. So selection problems may need to be managed by other methods, namely by reducing selection factors, or by compensating for their effects.

Selection has three major areas which can be handled using different methods:

1. **Affecting the entire market.** From among the above methods, this means choosing the starting point for the annuity. This may be managed in two ways:
   (1) By disallowing a delayed commencement of the annuity. This may cause system rigidity, which may nonetheless be eased by allowing an interim period (see later) right after this start-off point (although there exists the possibility that the inheritance will be lost upfront).
   (2) By applying a mandatory guaranteed period at the beginning. As a result the life annuity portion will become a deferred annuity and during the deferred time period the insured person can start the annuity whenever they wish (although there can be no further delay of the deferred annuity). (The OECD actively proposes this solution – see Antonin [2008a] and OECD [2008a]).
2. **Amongst providers:** when the risk composition of the annuity portfolio shows diversified movement amongst different providers (if there are several providers on the market). Given a selection with a small portfolio, choice due to there being a closed risk community and due to the indexation rate belongs here. In theory, it may happen that a provider offers different indexation rates concurrently. It may also happen that a provider does not provide every type of annuity from among the ones permitted by regulations, so a choice that needs to be made from among the products offered by one provider may become a selection between providers. This may be dealt with by using the following methods:
   (3) Central provider. This eliminates every selection problem amongst providers as only a single provider remains on the market. Theoretically, this does not deal with the problem of a small portfolio, though with a central provider this can only happen when the system first begins to operate and for a short time thereafter. The problem is not handled if this central provider offers several types of indexation concurrently.
   (4) Making all providers "closed" during the period of asset accumulation and prohibiting all voluntary movement between them (that is, non-voluntary movement; while movement linked to some other factors is permitted[47]). In parallel with this, every provider must calculate an annuity premium relating to their own mortality conditions. The existing Hungarian system stipulates that every provider set an annuity calculated in line with its own

[47]E.g. if there are only sectoral funds, people working in a certain sector of industry must join; and if someone changes jobs and their branch of industry, the person also changes provider, though changing insurers is not the reason for the change but rather the consequence.

mortality conditions while the regulation leaves all funds open in the meantime. In effect this is both ill-considered and inconsistent and good practice cannot be built upon this foundation.

(5) Risk equalisation (premium equalisation), unifying the portfolio and creating a pool among providers. This may be voluntary or mandatory but in certain situations (e.g. in the special case of Hungary with providers that do not have their own capital) the regulator should choose the mandatory pool.

3. **Within the same provider:** selection due to having a choice among different optional annuity products. This may be managed using the following methods:
   (6) Restriction of the product range, eliminating the practical possibilities of choice (e.g. only a single life annuity without a guaranteed period, or a single life annuity with a guaranteed period at the beginning, may be chosen).
   (7) Retaining the product range, though the choice is removed: different segments of the risk community are assigned to defined annuity products ("segmentation of the risk community"). E.g. there are both single life annuities and annuities for two persons, but the first one is purchased only by single people, and the second only by couples.
   (8) Directed selection, where the different annuity products are calculated according to a mortality table that reflects the risks inherent amongst those who opt for a given product. As I show below, this is an illusory option that cannot be implemented in practice.

To conclude, the following solutions exist as regards selection problems:

1. Joint management of the risk of the entire insured portfolio by a:
   a) central provider
   b) pool
2. Restricting the possibilities of choice through:
   a) disallowing delayed commencement
   b) several types of annuities based on a clear segmentation of the risk community
   c) restricting the possibility of changing providers in the asset accumulation phase (making the providers closed)
3. Restricting the product range, implementation of a single type of annuity (e.g. a single life annuity with a guaranteed period at the beginning), or
4. Directed selection (which is an illusory solution)

These solutions, or at least some of them, must be selected and jointly implemented in certain combinations.

A more detailed discussion of the different solutions is presented below.

3.2. Managing different risks together

3.2.1. CENTRAL PROVIDER

There may be different ideas relating to a central provider with functions that do not deal with selection effects but rather with other systemic problems.[48] Naturally, it cannot manage every type of selection issue (e.g. it cannot manage problems relating to choosing amongst different products or choosing the annuity's starting point). Central providers manage selection risk by not allowing the risk community to choose between different providers, so that both good and bad risks present themselves at the same provider.

Theoretically, the central provider may be equally established and owned by the state or have a private owner assigned by the state. However, the monopolistic position in the case of a single, central, privately-owned (and therefore for-profit) organisation would be difficult to justify so in reality only a state-owned provider could be considered here.

At the same time, one can envisage a central system but with a number of providers handling all selection issues. These providers may be for-profit privately owned companies. Providers would have a monopolistic position not in respect of the total portfolio but only from the perspective of having a certain stake within the portfolio of insured people, e.g. people buying an annuity in one particular year, and this would solve the problem of selection.

An opportunity for its realisation is that the government invites tenders for a central provider for a given year. The selected provider would furnish everyone with an annuity (naturally based on mortality projections included in the given groups, and via this would deal with demographic upturns and downturns, e.g. a baby-boom period). This right would be assigned to a provider in respect of every insured person who retires in a given year, until the death of such people. In follow-up years other providers would receive the same assignment. (Possibly a restriction could be made, i.e. one provider could apply for this tender only at certain times thus avoiding the development of a monopoly situation). Using this approach, the composition of a portfolio in the

[48] Only as an example: this may be the "final provider" should nobody be providing annuities on the market, e.g. as a result of rigid regulations, and a provider to whom the savings of "disappeared members" could be transferred, etc.

hands of a provider would reflect the market average in addition to being homogeneous (since everybody retired in the same year).

This is another example where it is not the entire portfolio of annuities that make up a risk community, yet the majority of selection problems are eliminated. This is one advantage of this possibility. The disadvantage is that people retiring in a particular year may find themselves in a worse situation compared to others if they end up with an underperforming provider. An additional disadvantage is that this can pave the way for corruption, for example by collaboration among providers if they conspire among themselves with regard to "who will win" and when. A defence against this would be the strengthening of the Competition Office and an enhancement of competition by allowing providers from outside the country (although from within the EU) to apply for the tender.

Beyond the above facts, in respect to the central provider it is conceivable that centralisation might be implemented, not as regards the entire annuity system, but only in relation to certain elements of it e.g. annuity payments, management of mortality risk (where a central pool would be established), etc. In this case, the function of centralisation will not necessarily be the management of selection effects (with the exception of the central pool) but would have some other, important aim such as cost saving, convenience for clients, etc.

3.2.2. PORTFOLIO UNIFICATION (POOL) AND PREMIUM EQUALISATION AMONGST PROVIDERS

Portfolio unification may be voluntary and therefore (usually) partial; this need not concern every provider although it may happen on a voluntary basis that the market unifies the portfolio. It may also be mandatory and cover the entire market (although theoretically it can also be partial, and only some groups in the market may be forced to unify the portfolio, although this would require very specific reasons).

In the case of voluntary portfolio unification, risk equalisation may include the joint management of investment and mortality risks, yet in this situation it is not worth maintaining separate organisations and instead of unifying the portfolio it becomes more reasonable to merge the organisations.

However, mandatory portfolio unification – basically for these reasons – may only include the management of mortality risk because if investment risk is jointly managed as well, then we are already speaking about a central provider. From another point of view, this means that a mandatory pool may be considered in relation to investment and indexation strategy where risk is inherent in the investment. Namely, if investment in bonds that are indexed to inflation is mandatory (as in Chile), if need arises a central provider is more

reasonable than a mandatory pool, as in this case the "competing" providers can only compete with each other with respect to cost, and this will probably be a priori lower with a central provider due to the scaled return than via different organisations within a system composed of small organisations.

In summary, with respect to mandatory annuities mandatory portfolio unification may only cover mortality risk and it is worth implementing with a classical yield-return investment and indexation strategy – otherwise it basically points to the central provider solution.

Portfolio unification refers to joint management of the mortality risk which can happen in different ways. In its minimal form, it means only the (mortality) profit and loss equalisation generated at the provider when concluding annuity insurance; in its maximal form it means a continuous distribution and equalisation of the profit and loss generated during the entire period of operation.

So the minimal form is a premium equalisation mechanism designed to equalise the initial profit and loss generated at different providers that occurs as a result of prohibiting differentiation, i.e. the compulsory use of the unisex premium table,[49] since it is almost certain that the gender composition of new contracts will differ depending on the provider within a given period, e.g. one year. With respect to equalisation, it is also conceivable that one might take other factors such as level of education into consideration. Equalisation will give a correct result if the otherwise projected unisex mortality table is correct. As there is a need for a uniform scale for equalisation, with this mechanism it is necessary that the market has available a centrally prepared, unisex, projected mortality table. Premium equalisation may technically happen such that the different providers transfer the surplus of the actually collected premium (according to the unisex premium table) and the premium necessary for reserves according to the differentiated mortality table, and the deficit thus generated will be received from this organisation. This also means that, with this mechanism, the differentiated mortality tables necessary for reserving must also be unified for the whole market, so these must be centrally defined and implemented by mandate.

One problem with relation to premium equalisation is that profit and loss generated by different providers within a specific period of time do not neces-

[49]In the EU, differentiation according to gender is prohibited, and this is probably also the case in many countries outside the European Union. In this respect, my message is restricted to those countries where such a prohibition is in force. With a lack of such a prohibition, selection problems can be solved by the oldest and probably the best insurance technique: differentiation of the insured according to the risk involved. An example of this solution is Chile, where these problems arise infrequently, and their method of operation cannot be adapted well to Europe.

sarily offset each other, as it is not certain that insured people with favourable and unfavourable characteristics from the insurer's point of view will take out annuity insurance at the same rate and at all points of time throughout the market. If central mortality projections are correct, the balance of profit and loss will be zero in the long term, but it may not be so in the short term. This would cause a problem in practice if there is a deficit at the central organisation in charge of premium equalisation (i.e. initial mortality losses generated for the entire market, in other words more people with less favourable conditions from the insurer's point of view conclude annuity contracts at a certain point in time). Such a transitory deficit may theoretically be covered by the state by repaying it from transitory surpluses. This may force the state to prepare mortality projections in a suitable manner. However, a problem occurs if a deficit or surplus is systematically generated in the central organisation and a clear problem in connection with the surplus is that insured people may be deprived of it in a questionable manner.

In its minimal form, providers carry the burden of mortality losses that emerge later, or they transfer this (or a part of it) to clients in the form of indexation. It is only logical to distinguish between initial and later mortality losses if the latter can be transferred to the client via indexation (which assumes a return-refund indexation technique). Otherwise one might note that if a pool is operated, it may be done in a maximal rather than minimal form.

In practice, the maximal form includes the above-mentioned minimal form as the mortality loss of the first year may be largely due to risk composition at the time of concluding the contract. In order to be able to continually redistribute mortality losses and gains amongst providers, reliable and uniform evaluations are required. As with the premium blending mechanism, this requires centrally-designed standardised reserving rules and differentiated mortality tables along with a centrally and uniformly defined technical interest rate.

Based on these reserving rules, different providers will define the mortality profit and loss in each financial year and will summarise them in total at the market level. If the balance is positive or at least breaks even, then they will define the magnitude of the uniform mortality profit using a kind of baseline, e.g. according to required reserving, and where there is a surplus this will be transferred to where there is a deficit. If the balance is negative, clearly the surplus is not enough to cover it. Then the deficit would be distributed against the return (or if the given year was also bad with regard to return, then against the reserve, i.e. the benefit size of the annuity) or against the solvency capital function against whoever will cover the final loss (i.e. the insured person or the provider). Clearly, this will relate to the ownership and capital conditions of the provider. If the insured persons themselves are the owners and the provider and thus have no own capital, the loss shall be accounted for by the yield and if

there is an actual owner, meaning there is solvency capital, then the loss may be accounted for via this, while amounts may be regrouped from where the loss is below the market level (including the profit) to where losses exceed it.

It is worth mentioning that if a central organisation is set up on the market for management of the pool, this would most likely imply centralisation of other elements of annuity provision if this also allows for cost saving. Eventually, only the investment will remain with the separate providers, and they will compete over the yield if the indexation rules permit this. This is exactly the same system as used by the Swedish private pension system.

3.3. Narrowing down choice options

3.3.1. PROHIBITING LATER COMMENCEMENT

A possible later start for payment of a private pension annuity will lead to adverse selection if the inheritance regulations are different before and after the start of paying for a private pension. This was the situation under the earlier Hungarian private pension law when before starting the payment of the annuity a beneficiary could be assigned for the entire pension capital, although after starting annuity payment heirs did not receive anything (with the exception of guaranteed period benefit and joint annuities).

This problem can be handled in three ways:

1. An annuity with a mandatory front-end guaranteed period or an equivalent regular withdrawal plus deferred annuity facility where the deferred annuity must be bought at the time of retirement,
2. Changing inheritance regulations so that only the spouse inherits his/her pension account and it is obligatory for married couples to buy a joint, two-person annuity with the other insured person being the spouse. This solution does not deal with selection by single people, although the motive of inheritance probably plays a less important role in their case,
3. Prohibition of the deferred commencement of payments.

The latter has another version, one that grants that if the insured person does not need the annuity at the beginning of his/her retirement period, they may further accumulate money so that later on the annuity will be higher without there being a problem of adverse selection. This may be achieved by suspension of the annuity (including immediately after beginning payments).

A suspended annuity behaves like an annuity even during the period of suspension, so that in the case of death of the insured party there is no more payment (without transferring capital to the heirs). Although the due annuity units

are not paid, these "flow back" to the annuity provision of the insured by increasing the sum and, through this, continually increasing the annuity due to the insured. This is exactly why suspension is also in the interests of the insured person because the longer the period of suspension, the higher the annuity at a later age.

3.3.2. SEGMENTING THE RISK COMMUNITY

Adverse selection owing to choice among products can also be managed without radically diminishing the product range if it is compulsory to segment the risk community according to products. Adequate justification is necessarily needed for such segmentation and I have identified only one such segmentation possibility: if married couples are obliged to buy a joint annuity and singles are obliged to buy a one-owner annuity. As a two-owner annuity for couples is a "natural unisex situation", this solution would generally diminish the risk-related uncertainty of mandatory unisex annuities, though this makes it questionable whether single people should be required to purchase a unisex annuity.[50] (The issue in Europe has been settled, at least for the time being, although major question marks remain.)

For this solution, joint annuities must naturally be standardised. One of the possibilities is a symmetrical joint annuity, where one of the insured receives 70% of the starting annuity if the other member of the couple dies. The other insured person is strictly and always a married spouse. Different joint annuities may naturally be conceived of as well, but only one kind should be available on the market, otherwise adverse selection due to choice among products will appear.

Generally, members of a couple do not retire at the same time, though when one of them retires, then a standard joint annuity must be determined for him/her based on the accumulated annuity capital, and when the other member of the couple retires the same kind of annuity must be defined for him/her as well. The advantage of this way of operating is that it would solve the survival (widow/widower) annuity problem as well because after the death of one member of the couple, the other would get x% of the commonly-held annuity.

[50]In the end, the most important argument behind a unisex annuity is the characteristics of the division of labour within the family (i.e. women remain at home with their children and therefore receive fewer pension entitlements) and women suffer disadvantages as a result. These disadvantages are compensated for by the pension-capital transfer from men to women via a unisex annuity. This kind of compensation isn't justified in the case of single women, however.

Naturally, further detailed rules are needed for the operability of a system that would handle the following cases:

- What happens if the retired spouse dies before the other retires?
- What happens if the joint annuity payment has started and the couple later divorce?
- What happens if a single person, whose one-owner annuity payment has already begun, should marry?

There are ways to engage these issues and possible regulatory options, so in the order above:

- The widow simply receives x% of the annuity of the deceased insured spouse, but the payment may be held back (i.e. it is suspended according to another rule) until the survivor retires. In such a case, the survivor receives a single person's annuity via his/her own capital unless he/she marries again. If the widow dies before retiring, the suspended annuity will cease to exist without it being paid.
- The annuity is divided into two annuities for single persons (further elaboration is needed to define the proportions in line with one of the possibilities so that persons receive an equal annuity). If one of the partners has not yet retired, the annuity payment is suspended in relation to him/her.
- The annuity is converted into a joint annuity.

The distinguishing and standardisation of annuities is possible if reserving, the mortality tables used for reserving and the technical interest rate are all standardised, so this solution assumes this to be the case.

The above regulation does not fit logically with a regulation where the beneficiary can be freely assigned in the accumulation phase (see e.g. the Hungarian private pension regulations). Logically, it fits a regulation where the heir is the spouse in the accumulation phase and the pension assets of the deceased will be credited to the survivor's pension account as an inheritance.

An amendment of the regulations on the accumulation phase would also be reasonable in order to harmonise it with the above-suggested rules in the annuitant phase so that in the case of divorce, the portion of the balance of the denominated accounts generated by the couple during the period of the marriage will be halved.

3.3.3. REDUCING CHOICE AMONGST PROVIDERS

The issue of the development of closed funds (and more generally private pension providers) must be looked at differently when a private pension system is being designed and within the conditions of an existing private pension system.

The regulator may à priori decide to establish closed funds e.g. on a technical or industrial sector basis. Here, stable risk communities will be created where the annuity calculation may be based on one's own mortality experiences and there is no adverse selection (or at most if members are allowed to choose between different types of annuity).

Yet if a fund system has already been launched on a market where practically all funds are open and everybody may enter and leave funds as they please, although the regulator might still leave providers to develop the mortality table (as the Hungarian regulator did[51]), the possibility for fund-choice must be diminished á posteriori so that the system approaches the closed funds model.

Overall, the goal is that payment of the annuity should begin in a stable risk community with a computable risk. This does not require the prohibition of every kind of choice with respect to the insured and providers. The risk community would be stable even if this prohibition would arise sometime before someone's retirement. Let us say that insured people may move freely among funds until the age of 50, but thereafter they may not switch funds and will receive their annuity from this fund (or from the insurer that has contractual relations with it) until the end of their life (and thus they also cannot change providers during their annuitant period). Thus enables the provider to calculate the annuity for a risk community with a stable composition; so it might, for example, happen that an insurer is connected to different funds with diverse risk compositions and provides an annuity for all of them, but charges different premiums in the case of each fund.

However, we must consider whether it is worth taking a step backwards from a model based on free movement between providers in order to solve selection problems, or whether it is perhaps more expedient to choose one of the other methods discussed here to manage the problem.

[51]In Hungary, this situation was probably not generated intentionally. It seems that in Hungary the funds are regulated exclusively based on the characteristics of the accumulation phase, and rules for the annuity phase are superficial, or its characteristics were not taken into account. During the asset accumulation phase no selection problem is caused by clients having a free choice between funds.

3.4. Narrowing down the product range

It is obvious that the possibility of choosing between different products may lead to adverse selection. In the following, I examine the mechanism of this adverse selection and what losses it can incur to the provider. The examination will be done via a choice permitted by existing Hungarian regulations amongst immediately commencing annuities for single persons; thus I assume that the insured may choose among three annuities:

1. simple (i.e. without a guaranteed period)
2. front-end guaranteed period
3. back-end guaranteed period

3.4.1. PRELIMINARY CONSIDERATIONS

The need to discuss this problem arose as a result of the **Hungarian private pension regulations**, so I will first provide a short overview of this.
According to the provisions in force pursuant to the Private Pensions Act, the annuity may be a:

- classic life annuity
- annuity with front-end guaranteed period
- annuity with back-end guaranteed period
- joint annuity with two or more lives

Regarding parameters, the situation is:

- The law does not stipulate the **length of the guaranteed period** so, in theory, it could be of any length and the client may choose from any available value.
- A **joint annuity with two or more lives** is ill-defined. The law does not stipulate whether the retirement age, and thus entitlement to a pension, needs be reached in order receive a payout. Theoretically, a pension may be paid out without an individual being entitled to a pension, e.g. if the annuitant is younger than the retirement age. Also, the law does not state what happens to the annuity following the death of one of the annuitants (e.g. is a joint annuity possible where payouts stop after the death of one of the annuitants?). The Act does not separately refer to possibilities for the guaranteed period. Also, one cannot know whether or not there is a restriction on the number of annuitants.
- Although the Private Pensions Act does not specify the frequency of the annuity (weekly, monthly, quarterly, etc.), it may be assumed that it should adjust to the frequency of the annuity from the first Pillar, i.e. it

must occur on a monthly basis. The question still remains: should it be 12, 13 or perhaps more "months"?[52] Given a lack of stipulations, we must assume that any solution is possible.

Below, I shall show that a choice amongst products in the above manner without pre-determined restrictions leads to severe adverse selection, something that one would wish to avoid, so choice options need to be reasonably restricted or narrowed down.

I first look at **the logic of adverse selection as a consequence of choice between products**. When examining the criteria people look at when choosing an annuity, two extremely objective "targets" are imaginable, namely that in comparison to the payable premium, the client maximises

1. the payout received during his/her lifetime,
2. all benefits (irrespective of whether they are received during his/her lifetime).

The reason for the latter may be the rational consideration that the client wants to leave a portion of their capital as an inheritance, i.e. they would like to look after someone such as the surviving partner or there might be the irrational but existing fear that "when I die, the insurer will pocket my money". Yet a combination of these extremes is also possible, i.e. if someone maximises the benefit received during their lifetime but also wishes to look after survivors to a certain extent. However, the guaranteed period is not the best form of caring for survivors (as at the time of drawing up the contract, one cannot know from when they may need to be looked after, so a "solution" within these extremes is likely to be very restricted, and accordingly it is reasonable to focus on the most objective targets).

The more people adhere to this objective target of maximising all benefits, the larger the selection problem will be. There is no selection problem whatsoever if everyone seeks to maximise the benefit received during their lifetime as, in this case, everyone will obtain a simple annuity and nobody would see it as worthwhile to pay for a guaranteed period to leave an annuity for someone after their death. The transitory option – i.e. somewhere between the two – clearly mitigates the problems arising because of this with the provider (which are bigger if everyone maximises things according to extreme assumptions made). In the computation I shall thus use this supposition, which also means that my results will give the worst possible results from the insurer's point of view.

[52]In Hungary, during the 2000s pensioners received 13 months of annuity for some years. In 2006, during the election campaign, one party even promised pensioners a 14th month pension. The 13-month annuity was abolished in 2009.

Different types of annuity are worth looking at by themselves from the perspective of "potential" adverse selection. If we take a simple annuity as the basic case, we can say that the difference in mortality profit and loss between the best and the worst (from an annuity point of view) groups of insured persons (with the shortest and longest life expectancies) is lower in all the other annuity types than in this basic case. Namely:

- A life annuity with a front-end guaranteed period is the sum of an annuity certain and a deferred annuity. Within the premium, the mortality selection affects only the deferred annuity part, and its weight is smaller with respect to the identical monthly annuity than in the case of immediate annuities.
- An annuity with a back-end guaranteed period is the sum of an immediate lifelong annuity and a whole life insurance. The annuity and the whole life insurance behave contradictorily from a risk point of view so the effect of selection is mitigated by the fact that they are present in one contract.

However, this also means that the risk composition of the insured portfolio is most important in the case of a simple annuity, and the longer the guaranteed period, the more its significance decreases (although it does not cease to exist), i.e. the smaller the magnitude of potential loss due to an unfavourable portfolio composition. In other words, it would be most beneficial to the provider if everyone were obliged to buy an annuity with a relatively long guaranteed period.

The question from the point of view of adverse selection due to choice is which group of insured persons should buy which type of annuity if the objective target of clients is to maximise all the benefits compared to the premium paid. In general, we may state the following:

- The more certain somebody is of not having a long life (e.g. because he is a man and also has an illness), the more worthwhile it is for them to purchase a guaranteed period annuity, and the less worthwhile it is to purchase a single life non-guaranteed period annuity, as the guaranteed period would certainly increase the received benefit, meaning it would be an effective guarantee in their case.
- Those who expect to lead a long life (e.g. healthy women) will not see paying the guaranteed period surcharge as being worthwhile; they are better off choosing a single life non-guaranteed period annuity as they will receive the front-end guaranteed period benefit without requiring the guaranteed period, and in addition they will receive a bigger benefit than the expected value for the entire risk community.

These are only my preliminary ideas in the context of this topic, which will be further elaborated on after the analysis below, during which I rely on the following purely technical **assumptions** to support my calculations, which do not influence the essence of my findings:

- the annuity is annual and annuity-due,
- g, i.e. the guaranteed period, is a whole number,[53]
- g is the same in relation to both front-end and back-end annuities.

The following assumptions do affect the substance of the investigation and are very "strong" assumptions that one would do well to partially resolve or mitigate in a later investigation:

- First of all, I assume that the client is precisely aware of his/her remaining lifetime (denoted with "h"). This assumption is necessary to a certain extent as the essence and the cause of adverse selection is that the client possesses certain information about his/her expected lifetime that the provider does not have (and even if the provider would possess that information, it could not take it into account). The assumption is an exaggeration, but it also shows us what the least favourable situation is from the provider's perspective. So, with this assumption, the number received for the impact of adverse selection may be regarded as the maximum loss to the provider.
- Similarly, the worst case scenario for the provider is if every single client maximises all of the benefits ("B") compared to the premium paid ("P"), so they maximise the $\frac{B}{P}$ quotient. The opposite assumption whereby a client would prefer the maximum benefit to premium quotient during their lifetime need not be investigated, as the result has little impact. It is easy to admit that, in this instance, everyone will purchase a simple annuity as it is not worth paying for the guaranteed period, which is worth nothing to the client. Every other assumption between these extremes may only be more complex and more subjective and also increases the complexity of the analysis, so I will not deal with them here (although it is an issue that is worthy of later investigation).

[53]The value of g is obviously either at least 0 (if it is irrelevant whether there is a guaranteed period or not) or at least 1 (if we compare guaranteed period annuities, we must assume that there is a guaranteed period), or 2 (if we are speaking about front-end guaranteed period annuities that guarantee something, because a "simple" primary annuity is nothing other than a g=1 year front-end guaranteed period annuity.

In what follows, I make no assumption for the technical interest rate other than it being at least 0%.

It is worth taking into account the possible magnitude of all the received benefits (i.e. *B*). This obviously depends on the type of annuity, the length of the guaranteed period (*g*) and the remaining lifetime of the client (*h*). As above, I have assumed annual annuities, so *B* "jumps" at the integer values of *h*, while the value of *h* is obviously continuous. Therefore the integral part of *h*, [h], plays an important role in the formulae. As the annuity payments take place at different times, the time-value of *B* must be taken into account for computation, so the different payments must be reduced to a kind of common denominator. In principle, different interest rates should be considered for discounting, but I shall assume it is only worth discounting with the use of a technical interest rate. Should we do something else, the result would be very subjective and we might come to almost any conclusion from the calculations. So the possible values of *B* as a function of the relevant parameters are:

- For a ("simple") primary annuity without a guaranteed period:

$$B = 1 + v^1 + v^2 + \cdots + v^{[h]} = \frac{1 - v^{[h]+1}}{1 - v} = \ddot{a}_{\overline{[h]+1|}}$$

- For an annuity of with a *g*-year front-end guaranteed period:

if $h < g$:

$$B = 1 + v^1 + v^2 + \cdots + v^{g-1} = \frac{1 - v^g}{1 - v} = \ddot{a}_{\overline{g|}}$$

if $h \geq g$:

$$B = 1 + v^1 + v^2 + \cdots + v^{[h]} = \frac{1-v^{[h]+1}}{1-v} = \ddot{a}_{\overline{\overline{[h]+1|}}}$$

- For an annuity with a *g*-year back-end guaranteed period:

$$B = 1 + v + v^2 + \cdots + v^{[h]} + v^{[h]+1} + v^{[h]+g} = \frac{1 - v^{[h]+1+g}}{1 - v} = \ddot{a}_{\overline{[h]+1+g|}}$$
$$= \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{\overline{[h]+1|}}$$

Below, I answer the following questions using the above assumptions:

1. Which type of annuity does the insured individual choose (from among these three)?
2. If the insured individual chooses a guaranteed period annuity, which of the possible guaranteed periods will they choose?
3. What is the rate of adverse selection at the insurer as a result of these choices?

Answers are given partly by an abstract analysis of annuity formulae (details of which are not presented in this book) and partly by calculations with which I am able to control answers given to the third question only by calculation, although the theoretical answers given to the first two questions are also controlled by these calculations. In addition to the above, and in a separate section, I investigate the question of whether adverse selection due to choice may be eliminated by increasing the premium (or via the use of a selection table that primarily considers the mortality of those choosing a given annuity, which is the same thing).

In both cases the investigation has two steps – first for cases with a net premium, after which I generalise the results for gross premiums. At the beginning of the investigation I note general considerations concerning the cost portion of the gross premium and also regarding loading, i.e. the relative value in respect of the net premium, meaning I shall make an effort to determine the broad values between which they should lie.

3.4.2. WHICH ANNUITY DO THE INSURED CHOOSE? EXAMINING SELECTION EFFECTS

3.4.2.1. General considerations – loading thresholds in gross premium cases

In advance, and very generally, the following can be said about loading:

- In the case of different annuities, the relative value of the $\frac{B}{P}$ ratio does not change if the loadings are the same, that is if $\lambda^s = \lambda^e = \lambda^h$,[54] then the rule of selection among different annuities will be identical in the case of both gross and net premiums.
- Since the client receives the most benefits from a back-end guaranteed period, then from a front-end guaranteed period and finally from a simple annuity (provided that g is identical for both guaranteed period annuities and is not 0 or 1), the following inequality must be true with respect to the gross premium:

$$\left(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x\right) \cdot (1 + \lambda^h) > \left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x\right) \cdot (1 + \lambda^e) > \ddot{a}_x \cdot (1 + \lambda^s)$$

Of this latter inequality, certain thresholds are added on top of the possible magnitude of loadings of different annuities; though the problem of thresholds may be raised more generally, and issues relating to different types of thresh-

[54] Loadings for different annuity types: λ^s for a simple annuity, λ^e for an annuity with a front-end guaranteed period and λ^h for an annuity with a back-end guaranteed period.

old arise in the case of gross premium (contrasting with the problem of the net premium). This problem does not manifest itself in connection with the net premium and the only question to be answered is which of the two or three annuities are worth getting if one knows one's remaining lifetime and it is taken for granted that a person is interested in maximising all the benefits received from the provider. If we assume that everyone knows how long they will live, the question of choice only makes sense if we presume that clients are being obliged to purchase an annuity from the money they have accumulated for that purpose. Should this not be the case, a reasonable solution for many would be to not buy an annuity at all but to budget for their remaining lifetime. However, if we disregard the assumption that there exist "all knowing" annuitants, then in the case of net premium the idea that one of the annuities is not worth getting (in general, or in comparison with other options) does not arise, not only because buying an annuity is mandatory, but also because the annuity calculation is "fair", i.e. it is based on an equation between the premium and the expected value of the benefit.

This is not the case with the gross premium, where three further questions may be raised with respect to the absolute and relative magnitudes of the cost (loading) portion of annuities, this being in addition to the above-discussed problems related to the net premium:

I. Is it worthwhile for anyone to buy one of the annuities i.e. does it have an excessively large loading (apart from the fact of whether or not it is mandatory to buy some form of annuity)?
II. Is it worth purchasing the given annuity type in comparison to the other annuity types, i.e. does it have excessive loading compared to others?
III. From what loading value is it worth purchasing one annuity compared to another for a client with a given length of lifetime remaining?

In the first two cases, the investigation concerns the "final" limits, i.e. is there anybody at all existing among eventual clients with a different remaining lifetime (who were the same age at point of entry) for whom it is worth buying one given type of annuity? In the third case, the question is more specific, therefore the threshold of type III must be within the thresholds of type II, and type II within that of type I. Below, I investigate the size of threshold types I and II, and will later use this for delineating the thresholds for type III.

These three issues have been raised because of the need to include loading in the investigation – and its magnitude obviously has rational limits; the question is where these limits are.

If we first examine type I thresholds and the definition of the threshold up to which premium limit it is worth purchasing an annuity, we can see that the

criteria for defining such a threshold in this case is not completely obvious. Nevertheless, it may be logical to say that if the maximum possible benefit from an annuity is smaller than its premium, it is not worth obtaining. Although if it is mandatory to buy an annuity, the issue is not whether or not it is worth buying a specific annuity, but it is still a good idea to be aware of the theoretical upper price limit.

The possible maximum benefit of the annuity is not self-evident as its value depends on how long the annuitant with the longest life is going to live. Naturally, nobody can say, so we can only define a theoretical value. Thus, in order to calculate the maximum possible benefit, the (statistically relevant) maximum possible age must first be decided and also by what discount coefficient we shall reduce the annuity units to a common denominator. In my view, the logical answer to both questions is "the coefficient that was taken into account in the calculation". So the maximum age for the mortality table should also be the highest age here, though we do know that there is a (small) chance of someone having a higher actual age than this as we do not really know what the "biologically possible" maximum age is. For discounting, it is reasonable to take into account the technical interest rate used in the calculation as a client would also be able to achieve that if they were managing their own assets and didn't purchase an annuity.

From this, the following inequalities may be established, and from these we may determine the threshold of type I:

- The possible maximum benefit of a back-end guaranteed period annuity:

$$1 + v^1 + v^2 + \cdots + v^{\omega-x} + v^{\omega-x+1} + \cdots + v^{\omega-x+g} = \frac{1 - v^{\omega-x+1+g}}{1 - v} = \ddot{a}_{\overline{\omega-x+1+g|}}$$

  so for the gross premium it must be true that:

$$\ddot{a}_{\overline{\omega-x+1+g|}} > \left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) \cdot (1 + \lambda^h)$$

  and via this the rule applicable for the loading maximum is:

$$\frac{\ddot{a}_{\overline{\omega-x+1+g|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} > 1 + \lambda^h$$

- The possible maximum benefit for a front-end guaranteed period annuity (assuming ω-x>g, without which we would not, in effect, be referring to a life annuity but an annuity certain) is $\ddot{a}_{\overline{\omega\text{-}x+1|}}$; it must hold true for the gross premium that:

$$\ddot{a}_{\overline{\omega-x+1|}} > \left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) \cdot (1 + \lambda^e)$$

From this, the rule applicable for maximising the loading is:

$$\frac{\ddot{a}_{\overline{\omega-x+1|}}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} > 1 + \lambda^e$$

- The possible maximum benefit from the simple annuity is the same as that of the front-end guaranteed period, so for the gross premium it must be true that:

$$\ddot{a}_{\overline{\omega-x+1|}} > \ddot{a}_x \cdot (1 + \lambda^s)$$

And via this we have a rule applicable for the maximum loading:

$$\frac{\ddot{a}_{\overline{\omega-x+1|}}}{\ddot{a}_x} > 1 + \lambda^s$$

In answering question II, the observation made at the beginning of this section gives us some support. According to this, there is a strict hierarchy of benefit among three different single annuities (if we assume that *g* is the same for two guaranteed period annuities and is at least 2); the most is provided by a back-end guaranteed period annuity, the next highest by a front-end guaranteed period annuity, and the least of all is provided by a simple annuity. So the following inequality must exist with respect to gross premiums:

$$\left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) \cdot \left(1 + \lambda^h\right) > \left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) \cdot (1 + \lambda^e) > \ddot{a}_x \cdot (1 + \lambda^s)$$

In other words: if these inequalities do not exist it is not worth the client buying certain types of annuity so the loading of a simple annuity compared to the loading of the front-end guaranteed period annuity cannot exceed a certain level, otherwise only the front-end guaranteed period annuity is worth purchasing; and this situation is the same with respect to simple and back-end guaranteed period annuities.

Further progress as regards the question can be made by exploring the benefit hierarchy amongst annuities:

- From two annuities, the one with a higher benefit will be bought by the client if its gross premium is not obviously higher than the gross premium of an annuity with a lower level benefit, i.e. if the loading is too small compared to the loading of the other annuity.
- However, nobody will purchase the higher-benefit annuity if the gross premium is much higher than the provided excess benefit, i.e. if the loading is too high.

The above inequalities assist in determining a loading that is "too small". Let us look at these relative lower limits; if any of the previous three inequalities are not satisfied, we may be certain of the result.

- if $\left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x\right) \cdot (1+\lambda^e) \leq \ddot{a}_x \cdot (1+\lambda^s)$, and in fact even if $\left(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x\right) \cdot \left(1+\lambda^h\right) \leq \ddot{a}_x \cdot (1+\lambda^s)$, then nobody will purchase a simple annuity
- if $\left(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x\right) \cdot \left(1+\lambda^h\right) \leq \left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x\right) \cdot (1+\lambda^e)$, then everyone will purchase a back-end guaranteed period annuity instead of a front-end guaranteed period annuity.

Therefore I assume that:

$$\left(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x\right) \cdot \left(1+\lambda^h\right) > \left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x\right) \cdot (1+\lambda^e) > \ddot{a}_x \cdot (1+\lambda^s)$$

will be true, which may also be stated in these alternate forms:

$$\frac{1+\lambda^e}{1+\lambda^s} > \frac{\ddot{a}_x}{\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x}$$

and

$$\frac{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}{\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x} > \frac{1+\lambda^e}{1+\lambda^h}$$

or differently,

$$\frac{1+\lambda^h}{1+\lambda^e} > \frac{\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_x}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}$$

It is naturally true that

$$\frac{1+\lambda^h}{1+\lambda^s} > \frac{\ddot{a}_x}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}$$

but this is already a consequence of the previous two.

Further considerations are required for an exploration of "too large" loadings. It is obvious that the premium for "dominant" annuities (i.e. the other two regarding simple annuity, and back-end guaranteed period annuity in the case of the front-end guaranteed period annuity) can be increased to such an extent that buying them (in comparison with other annuities as alternatives) becomes á priori irrational for a client. Calculation of the threshold can begin from the $\frac{B}{P}$ quotient, so let us examine the question according to annuity pairs. In the examination I have utilised the observation that it will be in no one's interests to

purchase an annuity of higher benefit (what we referred to above as "dominant"), if it is not worth purchasing by those who can benefit the most from this higher level of service compared to annuities that provide less benefit. So:

- With respect to simple and back-end guaranteed period annuities, the question of when it will never be worthwhile getting a back-end guaranteed period annuity may be seen in the light of when it will always be true that:

$$\frac{B}{\ddot{a}_x \cdot (1+\lambda^s)} > \frac{B}{\left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) \cdot (1+\lambda^h)}$$

Clearly it is true that the most difficulty occurs if the benefit is a minimum, as in this case the back-end guaranteed period annuity would normally be better for the client. An extreme case of this is when a client dies immediately, so they only receive EUR 1 annuity in the case of a simple annuity and $\ddot{a}_{\overline{g+1|}}$ annuity in the case of the back-end guarantee. This is when the difference in the benefits of the two annuities will be maximum. So the issue is when the following inequality becomes true?

$$\frac{1}{\ddot{a}_x \cdot (1+\lambda^s)} > \frac{\ddot{a}_{\overline{g+1|}}}{\left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) \cdot (1+\lambda^h)}$$

Clearly if:

$$\frac{1+\lambda^h}{1+\lambda^s} > \frac{\ddot{a}_x \cdot \ddot{a}_{\overline{g+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

Thus it can be stated that the inequality

$$\frac{\ddot{a}_x \cdot \ddot{a}_{\overline{g+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} \geq \frac{1+\lambda^h}{1+\lambda^s}$$

must be true.

- Regarding simple and front-end guaranteed period annuities, the annuitant will not take out a front-end guaranteed period annuity for the same reasons (as the difference in benefits between the two annuities is biggest in connection with annuitants who have the shortest life expectancy), if:

$$\frac{1}{\ddot{a}_x \cdot (1+\lambda^s)} > \frac{\ddot{a}_{\overline{g|}}}{\left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) \cdot (1+\lambda^e)}$$

In other words:

$$\frac{1+\lambda^{e}}{1+\lambda^{s}} > \frac{\ddot{a}_{x} \cdot \ddot{a}_{\overline{g}|}}{\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_{x}}$$

So the inequality

$$\frac{\ddot{a}_{x} \cdot \ddot{a}_{\overline{g}|}}{\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_{x}} \geq \frac{1+\lambda^{e}}{1+\lambda^{s}}$$

must always come true.

- The difference between the benefits of the front and back-end guaranteed period annuities gradually increases in the first *g* year, then it remains constant after the expiry of the front-end guaranteed period; i.e. at present value it is biggest in real value in year *g*. So the question is when does the inequality below:

$$\frac{\ddot{a}_{\overline{g}|}}{\left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_{x}\right) \cdot (1+\lambda^{e})} > \frac{\ddot{a}_{\overline{g+g}|}}{\left(\ddot{a}_{\overline{g}|} + v^{g} \cdot \ddot{a}_{x}\right) \cdot (1+\lambda^{h})} = \frac{\ddot{a}_{\overline{g}|} + v^{g} \cdot \ddot{a}_{\overline{g}|}}{\left(\ddot{a}_{\overline{g}|} + v^{g} \cdot \ddot{a}_{x}\right) \cdot (1+\lambda^{h})}$$

become true?
Or rather:

$$\frac{1+\lambda^{h}}{1+\lambda^{e}} > \frac{\left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_{x}\right) \cdot (1+v^{g})}{\ddot{a}_{\overline{g}|} + v^{g} \cdot \ddot{a}_{x}}$$

So the inequality

$$\frac{\left(\ddot{a}_{\overline{g}|} + {}_{g|}\ddot{a}_{x}\right) \cdot (1+v^{g})}{\ddot{a}_{\overline{g}|} + v^{g} \cdot \ddot{a}_{x}} \geq \frac{1+\lambda^{h}}{1+\lambda^{e}}$$

must always come true.
In summary, the threshold values are:
The "absolute" maximum of loading, i.e. the threshold values of type I, are:

- in the case of back-end guaranteed period annuities:

$$\frac{\ddot{a}_{\overline{\omega-x+1+g}|}}{\ddot{a}_{\overline{g}|} + v^{g} \cdot \ddot{a}_{x}} > 1+\lambda^{h}$$

- for a front-end guaranteed period annuity:

$$\frac{\ddot{a}_{\overline{\omega-x+1|}}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} > 1 + \lambda^e$$

- for a simple annuity:

$$\frac{\ddot{a}_{\overline{\omega-x+1|}}}{\ddot{a}_x} > 1 + \lambda^s$$

It is of course logical to assume that the costs are positive, so $1 + \lambda^h > 1$, $1 + \lambda^e > 1$ and $1 + \lambda^s > 1$ become true, but this has no significance going forward.

The relative values of the loading must fall between the following extremes (type II threshold values):

$$\frac{\ddot{a}_x \cdot \ddot{a}_{\overline{g+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} \geq \frac{1 + \lambda^h}{1 + \lambda^s} > \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

$$\frac{\ddot{a}_x \cdot \ddot{a}_{\overline{g|}}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} \geq \frac{1 + \lambda^e}{1 + \lambda^s} > \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

$$\frac{\left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) \cdot (1 + v^g)}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} \geq \frac{1 + \lambda^h}{1 + \lambda^e} > \frac{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

As it certainly becomes true that

$$\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x > \ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x > \ddot{a}_x$$

(at least if the guaranteed period is effective, i.e. $g \geq 2$), thus it will be true for every type II lower threshold value that it is smaller than 1, i.e.:

$$1 > \frac{1 + \lambda^h}{1 + \lambda^s}$$

$$1 > \frac{1 + \lambda^e}{1 + \lambda^s}$$

$$1 > \frac{1 + \lambda^h}{1 + \lambda^e}$$

The question is what will the criteria be for choosing between annuities within the above thresholds? And can we give a more specific answer to the loading values while also taking into account the remaining lifetime of the client?

3.4.2.2. The choice between simple and back-end guaranteed period annuities

First, I shall investigate the case of the **net premium.** The question is; when does the inequality

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_x} \geq \frac{\ddot{a}_{\overline{[h]+1+g|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

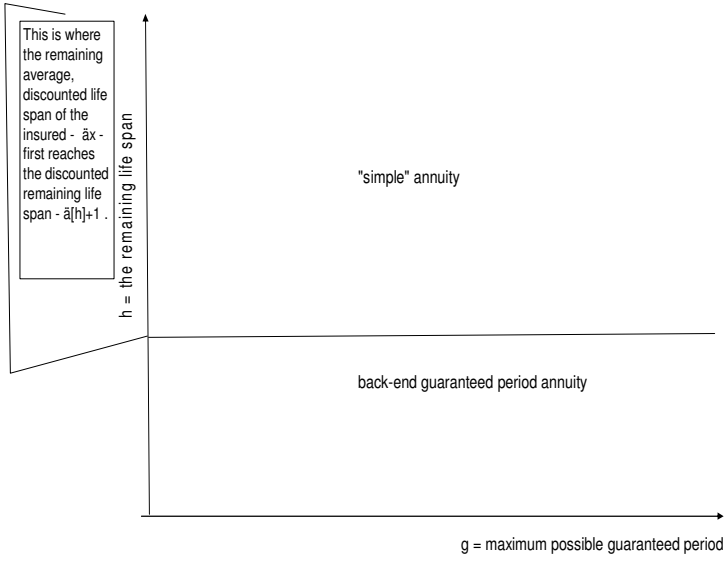
become true – and when is its inverse true?
The result is that if

$$\ddot{a}_{\overline{[h]+1|}} \geq \ddot{a}_x$$

is true then the client will choose a simple annuity rather than a back-end annuity.
Since we can (approximately) call $\ddot{a}_{\overline{[h]+1|}}$ the "discounted remaining lifetime"[55] and $\ddot{a}_x$ the "discounted average remaining lifetime", we can say in summary (and somewhat liberally) that the threshold for choosing between a back-end guaranteed period and a "simple" annuity will always be the discounted general remaining lifetime, irrespective of the magnitude of the guaranteed period. Independently of the duration of the guaranteed period, the client will always choose a "simple" annuity in the case of a discounted remaining lifetime above this, and below it the client always chooses a back-end guaranteed period annuity, provided the calculation is of the net premium and both variations were prepared using the same mortality table. Here, the "simple" annuity will certainly generate a loss, and the back-end guaranteed period annuity will certainly be profitable for the insurer.
This choice is presented in the diagram below:

[55]Since with a 0% technical interest rate, $\ddot{a}_{\overline{[h]+1|}}$ can be simplified to $[h] + 1$ and $\ddot{a}_x$ to $e_x + 0.5$, the dividing line will be the inequality $[h] + 1 \geq e_x + 0.5$. This may be otherwise illustrated as $[h] \geq e_x$-0.5. This is a similar, although not identical rule to the one which states that the remaining lifetime must be higher than the expected remaining lifetime.

Diagram 2: The annuitant's choice between "simple" and back-end guaranteed period annuities



If we extend our analysis to the case of the **gross premium,** we need to ask when the

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_x \cdot (1+\lambda^s)} \geq \frac{\ddot{a}_{\overline{[h]+1+g|}}}{\left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) \cdot (1+\lambda^h)}$$

inequality is true, and when its inverse is true.
Obviously, this is if

$$\ddot{a}_{\overline{[h]+1|}} \geq \frac{1+\lambda^s}{1+\lambda^h} \cdot \ddot{a}_x$$

is true.
So with the gross premium the net premium rule is modified, and the above line is shifted by the ratio of the loading, i.e. above the

$$\ddot{a}_{\overline{[h]+1|}} = \frac{1+\lambda^s}{1+\lambda^h} \cdot \ddot{a}_x$$

line, the client chooses a simple annuity, while below it they will choose a back-end guaranteed period annuity.

The limits discussed above must also be considered with the loadings.

3.4.2.3. A choice between simple and front-end guaranteed period annuities

In the case of the **net premium,** the relationship between the benefit and the premium differs depending on how *h* and *g* relate to each other in the case of front-end guaranteed period annuities.
If $h \geq g$, then

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_x} \geq \frac{\ddot{a}_{\overline{[h]+1|}}}{{}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}}$$

since

$$\ddot{a}_x \leq {}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}$$

so in this case the client will certainly choose a simple annuity, for if they are certain of surviving the guaranteed period, why should they pay its price? In this case, the guaranteed period will have a zero value for them.
If $h < g$, then

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_x} \geq \frac{\ddot{a}_{\overline{g|}}}{{}_{g|}\ddot{a}_x + F_g}$$

will be true if

$$\ddot{a}_{\overline{[h]+1|}} \geq \frac{\ddot{a}_x}{{}_{g|}\ddot{a}_x + \ddot{a}_g} \cdot \ddot{a}_{\overline{g|}}$$

is met, meaning the client will purchase a simple annuity if the discounted remaining lifespan of the client is higher than a portion of the (discounted value of the) guaranteed period (the longer the guaranteed period, the smaller the portion).
Therefore the borderline of choice is the curve:

$$\ddot{a}_{\overline{[h]+1|}} = \frac{\ddot{a}_x}{{}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}} \cdot \ddot{a}_{\overline{g|}}$$

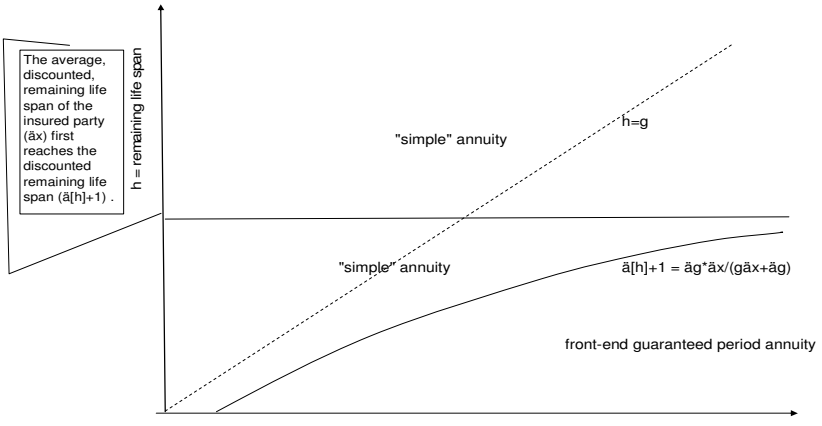
which splits the cases. This curve is always below the line

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$$

But if we increase *g*, the curve converges on this line.

However, if $h = 0$ we get $g = 1$ from the above equation, so the curve starts from this point on the *g* axis.

Diagram 3: The annuitant's choice between a simple annuity and a front-end guaranteed period annuity



As regards **gross premiums** – similarly to the choice between the simple annuity and annuities with a back-end guaranteed period – the result also depends on the ratio

$$\frac{1+\lambda^{e}}{1+\lambda^{s}}$$

which may have three different values:

1. too small, in which case the client will certainly not purchase a simple annuity,
2. too large, in which case the client will not take out a front-end guaranteed period annuity, but will instead purchase a simple annuity, and
3. somewhere in between the two thresholds.

In summary, in the case of the gross premium, the dividing line between the two types of annuity will be similar to that of the net premium with the condition that the curve is corrected by the ratio of loadings and given an extreme magnitude of loading (setting aside the formal demonstration), the only rational decision is a choice between one of the two annuity types.

3.4.2.4. The choice between back-end and front-end guaranteed period annuities

With guaranteed period annuities, when **net premiums** are investigated we can assume that *g* is at least 1. In this case, the relationship between the benefit and the premium of annuities with a front-end guaranteed period may be different depending on the relationship between *h* and *g*. In the case of annuities with a back-end guaranteed period, this ratio is always the same. This is why we must investigate two scenarios:
Let us see which annuity is chosen by the client.
If $h \geq g$, then

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} < \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

will be true if:

$$\ddot{a}_{\overline{[h]+1|}} < \frac{\ddot{a}_{\overline{g|}} \cdot \left({}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}\right)}{\ddot{a}_{\overline{g|}} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:\overline{g|}}}$$

So the dividing line of choice will be the curve:

$$\ddot{a}_{\overline{[h]+1|}} = \frac{\ddot{a}_{\overline{g|}} \cdot \left({}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}\right)}{\ddot{a}_{\overline{g|}} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:\overline{g|}}}$$

which will always be above the $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$ line.
If $h < g$, then

$$\frac{\ddot{a}_{\overline{[h]+1+g|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} = \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} > \frac{\ddot{a}_{\overline{g|}}}{{}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}}$$

becomes true if:

$$\ddot{a}_{\overline{[h]+1|}} > \ddot{a}_{\overline{g|}} \cdot \frac{v^g \cdot \ddot{a}_x - {}_{g|}\ddot{a}_x}{v^g \cdot \ddot{a}_{\overline{g|}} + v^g \cdot {}_{g|}\ddot{a}_x}$$

This time, the client will choose a back-end guaranteed period annuity instead of a front-end guaranteed period. Otherwise, the client chooses a front-end guaranteed period annuity.

The threshold for the choice is the curve

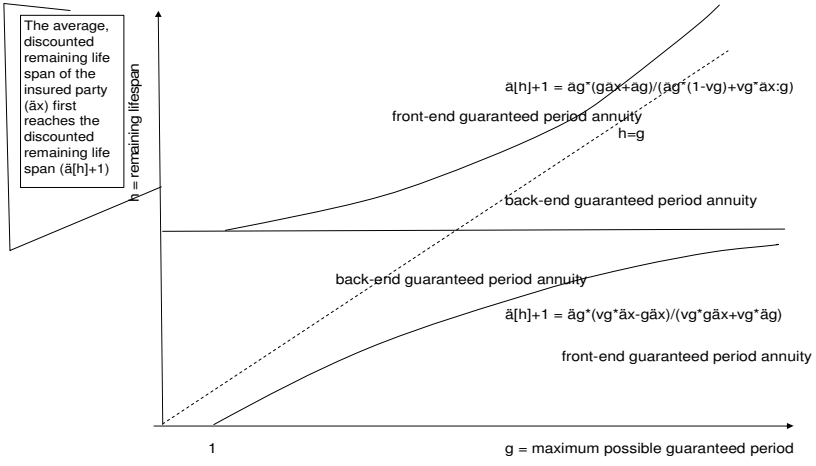
$$\ddot{a}_{\overline{[h]+1|}} = \frac{\ddot{a}_x - \frac{{}_{g|}\ddot{a}_x}{v^g}}{{}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}}$$

which will always be below the $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$ line.
In summary, the client chooses a back-end guaranteed period annuity if *h* falls within the following interval:

$$\frac{\ddot{a}_x - \frac{{}_{g|}\ddot{a}_x}{v^g}}{{}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}} < \ddot{a}_{\overline{[h]+1|}} < \frac{\ddot{a}_{\overline{g|}} \cdot \left({}_{g|}\ddot{a}_x + \ddot{a}_{\overline{g|}}\right)}{\ddot{a}_{\overline{g|}} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:\overline{g|}}}$$

and will select a front-end guaranteed period annuity outside this interval. The upper limit of the interval is always above the $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$ line, and the bottom limit will be below this line.

Diagram 4: The annuitant's choice between back-end and front-end guaranteed period annuities



With the assumption of **gross premium**, the illustration above will be different in the following ways:

The type I thresholds are:

$$\frac{\ddot{a}_{\overline{\omega-x+1+g|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} > 1 + \lambda^h > 1$$

$$\frac{\ddot{a}_{\overline{\omega-x+1|}}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} > 1 + \lambda^e > 1$$

The type II thresholds are:

$$\frac{\left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) \cdot (1 + v^g)}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x} \geq \frac{1 + \lambda^h}{1 + \lambda^e} > \frac{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

In conclusion, with the above loading thresholds, the client will choose a back-end guaranteed period annuity if *h* falls within the following interval:

$$\frac{\ddot{a}_{\overline{g|}}}{\frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} - v^g} \geq \ddot{a}_{\overline{[h]+1|}} \geq \frac{\ddot{a}_{\overline{g|}}}{v^g} \cdot \left(\frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} - 1\right)$$

Outside the above interval, or if the loadings fall outside the range defined by the thresholds, the annuitant will go for a front-end guaranteed period annuity.

3.4.2.5. Summarising: The choice between annuities (in the case of a net premium)

Summarising the above, the following statement can be made. We can deduce the **following three rules** from the three paired examinations above**.**

1. Clients choose between simple and back-end guaranteed period annuities such that above the line $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$ they choose a simple annuity, while below this a line they choose a back-end guaranteed period annuity.
2. Clients choose between simple and front-end guaranteed period annuities such that above the

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

curve they choose the simple type, while below it they will go for a front-end guaranteed period annuity. The

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

curve is always below the $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$ line, and the

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

curve starts from point g=1.

3. Clients choose between back-end and front-end guaranteed period annuities such that between the curve

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x - \frac{{}_{g|}\ddot{a}_x}{v^g}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

and the curve

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_{\overline{g|}} \cdot \left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right)}{\ddot{a}_{\overline{g|}} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:n}}$$

they will select a back-end guaranteed period annuity, while outside these thresholds they will choose a front-end guaranteed period annuity. The upper limit of the band is always above the line $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$, while the bottom limit will be below this line.

If we apply these three rules together, we come to the conclusion that:

- According to rules 1 and 2, the annuitant will take out a simple annuity above the line $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$. Rules 1 and 2 also mean that above the line, the simple annuity dominates both the front-end and back-end guaranteed period annuities, so rule 3 has no significance.
- Below the line $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$, however, according to rule the back-end guaranteed period annuity will dominate over the simple annuity, and below the line the annuitant will certainly choose a guaranteed period annuity according to rules 2 and 3. According to rule 3, the

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_{\overline{g|}} \cdot \left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right)}{\ddot{a}_{\overline{g|}} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:n}}$$

curve is always above the $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$curve, so this curve does not have an effective role to play in any choice made below the line.

- According to rule 3, in the case of the area below the line $\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_x$ the client will choose a back-end guaranteed period annuity above the

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x - {}_{g|}\ddot{a}_x \frac{}{v^g}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

curve and will choose a front-end guaranteed period annuity below it. According to rule 2 the

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

curve is the threshold. Above it, the client will choose a simple annuity, or in accordance to rule 1 a back-end guaranteed period annuity that dominates it, while below the line the client chooses a front-end guaranteed period annuity. Since

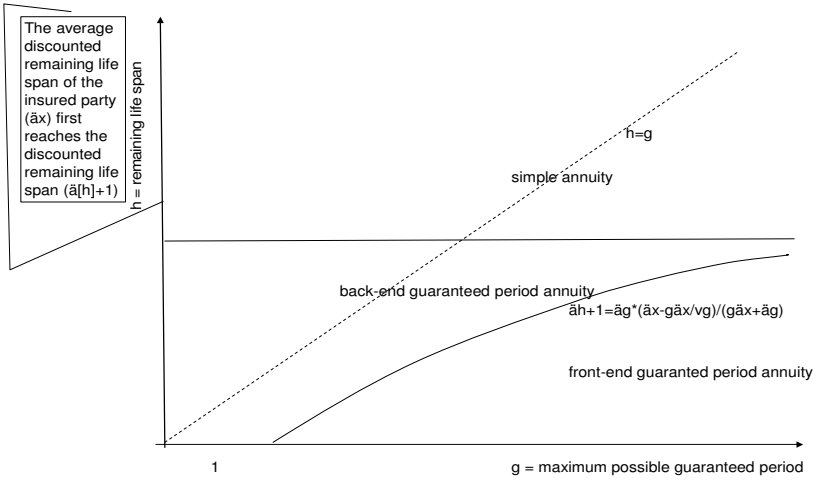
$$\ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x} > \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x - {}_{g|}\ddot{a}_x \frac{}{v^g}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

the borderline between the front-end and back-end guaranteed period annuities will be the curve

$$\ddot{a}_{\overline{[h]+1|}} = \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x - {}_{g|}\ddot{a}_x \frac{}{v^g}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

We can also summarise the results in a diagram.
In case 1, two of the above-mentioned four curves will be effective, and these two will divide the g-h plane in the manner below (I have only indicated the h=g radius for the sake of orientation):

Diagram 5: The annuitant's choice between 3 annuities



Ignoring the formulae, the above results may be explained as follows (with a net premium[56]). I have assumed a 0% technical interest rate in the explanation because disregarding the interest rate greatly improves transparency:

- If the client knows they will get back at least the price of the simple annuity as a benefit (i.e. if their remaining lifespan exceeds the expected value), then it is not worth paying for the guaranteed period as the $\frac{B}{P}$ quotient is already greater than one. In the case of a front-end guaranteed period annuity, a guaranteed period that is shorter than the expected lifetime will not offer the client any extra benefit for the extra premium, while in the case of a longer guaranteed period than the client's life expectancy, the premium (which is definitely bigger than the guaranteed benefit) will certainly be larger than the attainable benefit. It is also not worthwhile buying a back-end guaranteed period annuity, as the client can purchase every extra 1-year back-end guaranteed period (for EUR 1 extra benefit) for a EUR 1 premium, meaning the ratio of extra benefit to extra premium is 1, which is less favourable than what

[56]Only an "elegant" explanation is possible here. As we can see above, this explanation – with a very broad interval – is also true with regard to the gross premium.

has already been achieved with the simple annuity, so it is not worth buying an extra service of this kind.

- In general we may say that if the client lives longer than the possible length of the guaranteed period, it is not worth them purchasing a front-end guaranteed period annuity as they do not receive any extra benefit for the extra premium paid with respect to the guaranteed period.
- In the case of the simple annuity, if a client knows that they will not get back the premium as a benefit (because their life expectancy is shorter than average), then it is more worthwhile buying a back-end guaranteed period annuity than a simple annuity. In this case, the premium for 1 year of extra benefit is EUR 1, so the ratio of the extra benefit and the extra premium is 1, which is higher than the less favourable benefit-to-premium ratio in the case of the simple annuity, so purchasing a one-year back-end guaranteed period increases this ratio. This is also true for every extra year, so it is worth choosing an annuity with the highest possible back-end guaranteed period. Within this, if the longest possible guaranteed period is shorter than the remaining life expectancy, it is not worth getting a front-end guaranteed period annuity, so in this instance the optimum choice will be an annuity with a back-end guaranteed period (of maximum length).
- Based on the above, it is only worth choosing a front-end guaranteed period annuity if the guaranteed period is greater than the remaining lifespan, but the remaining lifespan is itself shorter than average. In this case, however (within this) it is worth buying an annuity with the longest possible guaranteed period because the magnitude of the benefit (the numerator) is the same as that of the guaranteed period, while the denominator will be the guaranteed period plus "something else", i.e. the quotient will be smaller than one. This "something else" is nothing other than the premium of the annuity deferred by the guaranteed period. In such a case, if we increase the guaranteed period by one, then the numerator is increased by one and the denominator is increased by less than one, as a 1-year increase in the front-end guaranteed period increases the premium by less than one (because all that has happened is that an uncertainly due unit of benefit with a premium smaller than 1 now has a certain unit of premium, i.e. equal to 1). The ratio of extra premium to extra benefit is higher than one, i.e. adding the extra guaranteed period increases the benefit-to-premium ratio.
- We cannot, however, be certain that in this case it is worthwhile for the client to purchase a front-end guaranteed period annuity. What is certain is that a back-end guaranteed period annuity is better in this case than a simple annuity, but in the majority of cases it is also better than

the front-end guaranteed period annuity. Supposing that the maximum possible guaranteed period is identical in both front-end and back-end guaranteed period versions; we can say that if the client is likely to live almost until the end of the guaranteed period, it is more beneficial for the client to obtain a back-end guaranteed period annuity than a front-end guaranteed period annuity. This can be seen from the perspective that a front-end guaranteed period only slightly increases (relatively) the premium of the annuity compared to that of the simple annuity so it can be taken as being (approximately) the same. As the benefit-to-premium quotient is smaller than for front-end guaranteed period annuities, and in the case of a back-end guaranteed period both the numerator and the denominator increase roughly by the guaranteed period, the extra benefit premium quotient will be 1, which increases the benefit-premium quotient compared to the front-end guaranteed period variation. However, if we lessen the client's remaining life at a given guaranteed period, after a while we may say that it is not rewarding for the client to pay the extra premium of the back-end guaranteed period for the extra guarantee provided by the insurer compared to the front-end guaranteed period annuity as, while the ratio of the premium of the two types of guaranteed period annuities is unchanged, the ratio of the guarantee provided by a back-end guaranteed period annuity is continuously decreasing, so after a while it will not be rewarding for the client. If we examine a case where the guaranteed periods of a front-end guaranteed period and back-end guaranteed period annuity are different, then we might say that if the front-end guaranteed period is longer than the life expectancy plus the back-end guaranteed period, then it is definitely not worth choosing the more expensive back-end guaranteed period. It may be more rewarding to choose the front-end guaranteed period annuity even above this limit for a while compared to the back-end guaranteed period one, but the back-end guaranteed period annuity will eventually "win".

3.4.3. WHAT GUARANTEE PERIOD WILL THE ANNUITANT CHOOSE?

Let us first take a look at **back-end guaranteed period annuities.** In the case of the **net premium**, according to the above the back-end guaranteed period annuity will only be chosen by the insured person if the $\ddot{a}_{\overline{|h|+1|}} < \ddot{a}_x$ inequality is met. Here, the client must maximise the

$$\frac{\ddot{a}_{\overline{[h]+1|}} + v^{[h]+1} \cdot \ddot{a}_{\overline{g|}}}{\ddot{a}_x + A_x \cdot \ddot{a}_{\overline{g|}}}$$

quotient. As it can be easily demonstrated that this function increases in proportion to g, i.e. an increase in g also causes an increase in the benefit-to-premium quotient, the client will always choose the biggest one from among possible alternatives in the case of a back-end guaranteed period.

In a relatively more complicated way, we can demonstrate that this will also be true with relation to gross premiums.

Examining **front-end guaranteed period annuities** in connection with the **net premium** according to the above, the client will only opt for the front-end guaranteed period annuity if the h<g inequality is met. Here, the client maximises the

$$\frac{\ddot{a}_{\overline{g|}}}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

ratio. Its reciprocal

$$\frac{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}{\ddot{a}_{\overline{g|}}} = 1 + \frac{{}_{g|}\ddot{a}_x}{\ddot{a}_{\overline{g|}}}$$

is a clearly decreasing function of g since if g increases the denominator increases and the numerator decreases, so the original coefficient increases according to the increase of g, i.e. the client aims to purchase the longest possible guaranteed period annuity from among the front-end guaranteed period annuities available.

Assuming a gross premium it is obvious that the quotient

$$\frac{\ddot{a}_{\overline{g|}}}{\left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) \cdot (1 + \lambda^e)}$$

also increases together with the increase of g, i.e. given a gross premium it will be true that a client will choose the one with the highest guaranteed period from amongst the guaranteed period annuities available.

In summary: from the point of view of choice amongst possible guaranteed periods, the size of the loading has no significance, or it will only have significance if the loading is identical with respect to every guaranteed period. If it is different, it is naturally possible that the smaller guaranteed period annuity will be the more favourable choice if its loading is adequately smaller than that of the longer guaranteed period annuity.

3.4.4. THE MAXIMUM POSSIBLE RATE OF LOSS DUE TO ADVERSE SELECTION

To assist my calculations, I have created a unisex extinction order based on one of the male-female mortality tables, where I have weighted the mortality probabilities with the ratio of the population of given ages. (The choice of extinction order [mortality table] occurred within relatively wide boundaries because I was not interested in the impact of the extinction order itself, so I worked with different kinds of optional extinction orders.) On this basis I have calculated the net premium of all possible annuity types with all kinds of guaranteed period – from 1 to 40 years – and have looked at how differences between the gained benefits and the premium compare to the premium with respect to individuals with different life expectancies. I then assumed that the client will choose the option with the highest value, and accordingly so I could estimate the maximum loss to the provider that might result from this choice.

In the model I have developed, the mortality tables (between 1990 and 2000) that provide the basis for the unisex table can be changed so that in each case I compute the unisex table by weighting the gender distribution of the population of the given year. The age of entry can be chosen between the ages of 50 and 70, the technical interest rate can be changed (although I have chiefly utilised the results received at 0%), and one can also amend the number of years of guaranteed period that can be taken into account as a maximum (between 1 and 40 years[57]). The results published below were calculated using the 1990 select mortality table and with an entry age of 62 (otherwise, the mortality table only changes the results to a minimal extent in my experience).

For the maximum degree of adverse selection as a function of the maximum possible guaranteed period and the technical interest rate, we get the following results:

[57]Or rather, in the case of front-end guaranteed periods, up to a maximum age of 101, meaning a guaranteed period of 39 years can be taken into account with respect to an entry age of 62, as can also be seen in the results.

Table 2: The maximum possible rate of adverse selection

Technical interest rate	The maximum possible guaranteed period (years)									
	1	2	3	4	5	10	15	20	30	40
0.0%	0.00%	0.00%	0.00%	0.00%	-0.46%	-3.58%	-8.01%	-12.59%	-14.78%	-12.56%
0.5%	-0.26%	-0.49%	-0.72%	-0.93%	-1.56%	-4.72%	-9.09%	-12.61%	-13.93%	-12.38%
1.0%	-0.55%	-1.05%	-1.52%	-1.96%	-2.78%	-6.61%	-10.62%	-13.92%	-15.01%	-13.99%
1.5%	-0.93%	-1.77%	-2.55%	-3.27%	-4.32%	8.93%	-13.31%	-16.05%	-17.27%	-17.00%
2.0%	-1.37%	-2.61%	-3.74%	-4.77%	-6.08%	-11.19%	-15.82%	-18.84%	-20.56%	-20.76%
2.5%	-1.86%	-3.53%	-5.04%	-6.40%	-7.99%	-14.08%	-18.79%	-22.06%	-24.16%	-25.06%
3.0%	-2.45%	-4.63%	-6.58%	-8.33%	-10.23%	-17.41%	-22.62%	-25.88%	-28.54%	-29.88%
3.5%	-3.06%	-5.76%	-8.15%	-10.28%	-12.50%	-20.41%	-26.17%	-29.76%	32.95%	-34.64%
4.0%	-3.57%	-6.70%	-9.47%	-11.93%	-14.13%	-23.27%	-29.48%	-33.07%	-36.72%	-38.70%
4.5%	-4.01%	-7.51%	-10.60%	-13.34%	-15.78%	-25.73%	-32.04%	-36.15%	-40.13%	-42.26%
5.0%	-4.44%	-8.31%	-11.71%	-14.72%	-17.40%	-28.11%	-34.80%	-38.89%	-43.20%	-45.49%

Explaining the diagram: if the insurer calculates the premium of different single annuities and does not take into consideration the effect of adverse selection caused by choice, clients will choose the best modality for themselves and the premium collected by the insurer will then be that much lower than the benefit paid. So, for example, with a 10-year maximum possible guaranteed period and a 0.0% technical interest rate, the insurer collects 3.58% less premium as a result of adverse selection than it pays out in benefit. (N.B. such a calculation does not include a possible loss for the insurer if the actual mortality of the insured does not correspond to the one calculated).
The results can be evaluated as follows:

- With a 0% interest rate and low guaranteed period (4-5 years), the choice made will have no adverse selection impact in practice,
- With a 0% interest rate and a longer guaranteed period, the adverse selection impact will be greater for a certain period of time (in the table it is -14.78% for a 30-year period, meaning the total premium collected by providers will be this much lower compared to the benefits paid out), after which it decreases somewhat,
- A similar trend prevails with other technical interest rates with the addition that the higher the technical interest rate, the higher the guaranteed period at which adverse selection has a maximum effect.
- The higher the technical interest rate, the stronger the effect of adverse selection. Overall, the interest rate has a very strong impact on adverse selection.

3.5. Directed selection

If insured people are able to choose between different types of annuities at a given provider, those with the shortest life expectancy will choose a front-end guaranteed period annuity, and those with a longer, but shorter-than-average life expectancy will opt for a back-end guaranteed period annuity, while those with a longer than average life expectancy will choose a simple annuity. If we calculate all three types of annuity using an identical mortality table, the result of this selection will be that the insurer makes a profit on the guaranteed period annuities and a loss on the simple annuities, and with a bit of luck the profits and losses of the various annuity types will offset each other.

One way of solving the problem might also be that the annuities are calculated with different mortality tables so that they are adjusted to the mortality of those who, based on rational considerations, voluntarily opt for it. If this is possible, insurers could choose to take advantage of selection instead of struggling against it, and in this way they would get a risk community selected on the basis of risk aspects where the clients themselves, using rational considerations, voluntarily select the risk community they wish to join. Insurers might even help clients and inform them about which annuity is most beneficial, and if someone is uncertain about how long they might live, insurers could suggest they have a medical check-up, which will help them assess their life expectancy. After this, the insurer would simply allow them to make a choice. In this way, risk assessment can be eliminated with the help of precise calculations and "trained advisors", and what is most advantageous for whom is clearly visible. This also helps to avoid problems resulting from concealing data from assessment.

The principal question is whether this scenario is at all possible, and would such an effort to find a way of operating perhaps lead to contradictions? Because if it turns out that this is not the way to go, precisely the opposite must be done. It is possible for the insurer to lessen the risk to itself by applying the above-described adverse (auto-) selection of annuitants so that the annuities actually provided are drastically diminished compared to the possible annuities. If, for example, a provider only sells guaranteed period annuities, it means that its risk will be radically reduced since:

1. A guaranteed period annuity can á priori be provided with a smaller risk than one without a guaranteed period, and what's more
2. this attracts prospective annuitants with shorter life expectancies (primarily men) and makes it less attractive to people with longer life expectancies (mainly women), and in this way the insurer is indirectly selecting its own risk community.

Because of the consequences of item 2, whether this possibility should be permitted needs to be seriously considered. It is better if, in this case, the law excludes this possibility and ensures that an annuity provider is obliged to provide all of the annuities permissible by law.

I show below that this solution is not possible, so the targeted selection will not work, and so I conduct an examination, as I did above, into the option of simple, front-end and back-end guaranteed period annuities.

The issue of whether adverse selection owing to choice may be eliminated by using adequate mortality tables might be restricted to a question of whether adverse selection may be eliminated by increasing or reducing the premium, as a client takes into account the different mortality tables only via the premium.

The answer here is not clear, for if we change the mortality table, the premium for the annuity changes along with the composition of those who will opt for such an annuity over others. The issue then is whether this change leads to every type of annuity remaining on the market for an adequate range of clients or whether, in reality, certain types of annuity will lose ground.

In the case of 3 single-life annuities, this question can further be simplified upon examination; can adverse selection loss suffered due to the choice of a simple annuity be eliminated by increasing the simple annuity's premium or by lowering the premium for guaranteed period annuities? From this point of view, it is of secondary importance whether adverse selection presents itself if the insured can only choose between two types of guaranteed period annuities, and if the answer is "yes", whether adverse selection might be eliminated by increasing or reducing the premium. As this question is not a practical one, I shall not provide an answer here.

So, the question is: **can adverse selection be eliminated by increasing the premium of the simple annuity, or not**?

First, I will look at the case of a **simple annuity and an annuity with a back-end guaranteed period**; and in the case of a **net premium**, those people will choose a simple guaranteed period for whom it is true that

$$\ddot{a}_{\overline{[h]+1|}} \geq \ddot{a}_x$$

i.e. people who will certainly receive as much in benefit as the amount of premium they pay, since in this case, and only in this case will

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_x} \geq \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

be true. If we increase the price of the simple annuity from $\ddot{a}_x$ to $\ddot{a}_x^{'}$, and in the meantime leave the price of the back-end guaranteed period annuity unchanged (i.e. in the case of the simple annuity we assume a greater remaining

lifetime), then it is obvious that those who have so far chosen a back-end guaranteed period annuity (i.e. in whose case

$$\ddot{a}_{\overline{[h]+1|}} < \ddot{a}_x$$

was true) would continue buying this because they have no reason whatsoever to change.
In the case of those for whom

$$\ddot{a}_x \leq \ddot{a}_{\overline{[h]+1|}} < \ddot{a}'_x$$

is met,

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}'_x} < \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

will always be true, since

$$\frac{\ddot{a}_{\overline{[h]+1|}}}{\ddot{a}'_x} \leq 1 < \frac{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{\overline{[h]+1|}}}{\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x}$$

So some clients who chose a simple annuity with a lower premium would now shift to a back-end guaranteed period annuity. In addition, those who change annuities will all be clients in whose case the received benefit will be smaller than the premium paid for the simple annuity; simply annuities will continue to be only purchased by people who receive a bigger payout than the premium of the simple annuity.

Based on the above, we can ascertain that if we increase the price of a simple annuity, we exclude clients from buying a simple annuity who would have chosen it at a lower price, but for whom the new, higher price exceeds the anticipated benefit. Only those clients for whom the anticipated benefit will again be higher than the price will choose the simple annuity with the new price, while others will convert to buying a back-end guaranteed period annuity. This means that the simple annuity continues to make a loss, so raising the price of the simple annuity will not result in simple annuities becoming profitable for the provider, and such attempts would eventually lead to simple annuities being edged out of the market.

Assuming a gross premium does not significantly change the above; the logic applied with regard to the net premium can also be applied here, so adverse selection cannot be eliminated by increasing the premium.

Examining **simple and front-end guaranteed period** annuities **in the net premium case** (although assuming gross premiums will again not change the logic in this case), based on the above the threshold between the simple and the front-end guaranteed period annuities is the

$$\ddot{a}_{\overline{[h]+1|}} < \ddot{a}_{\overline{g|}} \cdot \frac{\ddot{a}_x}{\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x}$$

inequality (where $h < g$ will obviously also be true and the relationship

$$\ddot{a}_{\overline{[h]+1|}} < \ddot{a}_x$$

will also be valid). If the inequality is true, the client will choose a front-end guaranteed period; if it isn't they will choose a simple annuity.

It is obvious that the best simple annuity clients would choose the front-end guaranteed period annuity, so the simple annuity will generate a loss, although not to the extent we might see in the case of simple and back-end guaranteed period annuities, because in this case the simple annuity will also be purchased by people for whom the received benefit will be smaller than the premium paid. The question is whether the profitability of the simple annuity may be restored by increasing the premium of the simple annuity, i.e. if we increase the premium from $\ddot{a}_x$ to $\ddot{a}_x'$.

In the context of an increasing premium, we must immediately ask whether there is some cap on it. It is obvious that the premium of the simple annuity cannot be increased above the premium of the front-end annuity as there would be no argument in favour of buying a simple annuity instead of a front-end guaranteed period annuity in such a case. In fact, we can be a little more precise: the premium of the simple annuity must be lower than the premium of the guaranteed period annuity as this is the sole reason the client chooses an annuity that offers a lower benefit from among two annuities with the same premium.

If the premium of the simple annuity were raised to the level of one with a front-end guaranteed period, and if we were to suppose (in contrast to the above) that only those individuals purchase a front-end guaranteed period annuity for whom the guaranteed period actually guarantees something on the basis of their remaining lifetime, then we may state that in the case of a provider that offers two (i.e. simple, and front-end guaranteed period) annuities, all adverse selection could be eliminated. Under such conditions, the provider actually only sells an annuity with a front-end guaranteed period while in the case of an annuity the problem of adverse selection owing to choice cannot emerge, or at least not at the level of the provider. So the insurer collects premium from the entire risk community and it pays out the same sum in benefits. However, the clients – also based on the above – are divided among the two formally different types of annuity whereby individuals whose life expectancy remains below the guaranteed period choose the guaranteed period annuity, while the rest opt for the simple annuity. In these circumstances, people who choose a guaranteed period annuity obviously pay more in premium than they receive as benefit, and therefore those who opt for a simple annuity obviously

pay less if the entire risk community is in balance with respect to premium and benefit. So the loss generated by the simple annuity cannot be eliminated by raising the premium.

The question arises whether the reverse could provide a solution, i.e. **can we stop adverse selection by lowering the premium of guaranteed period annuities?** In other words, the question is whether the range of simple annuity buyers will grow in response to an inverse strategy i.e. in response to decreasing the premium of either of the (front-end or back-end) guaranteed period annuities. The answer is obviously no, because for those individuals for whom it was worth buying a guaranteed period annuity to begin with it will have become even more worthwhile. In addition, a segment of simple annuity buyers would shift to becoming guaranteed period annuity buyers, i.e. those with the shortest remaining life expectancy from among the simple annuity buyers. But relatively speaking, these were the most favourable clients from a simple annuity perspective, so the simple annuity might generate an even higher loss, meaning the loss cannot be eliminated in this manner.

In summary: in a given situation where clients know their remaining lifespan and all of them wish to maximise the total benefit received compared to the premium paid, it would be impossible to see the single annuity as profitable in itself as neither increasing the premium of a single annuity nor decreasing the premium of some of the guaranteed period annuities would prevent the single annuity maximum from losing ground on the market. In other words, this means that **it is impossible to prepare selection tables that reflect the mortality of individuals who opt for different annuities, and therefore directed selection does not work as a solution.**

3.6. Selection in literature

Practically all the literature refers to it as being self-evident that the mortality of annuitants is lower than that of the entire population (see e.g. a more than 100-year-old book by Bein-Bogyó-Havas [1907], page 162). A large portion of the literature makes reference to "adverse selection" via an analysis of this phenomenon (see e.g. Banyár [2003]). According to Hylands-Gray [1992], potential annuitants conduct a kind of adverse (auto-) selection, and an individual with a bad state of health would rarely (voluntarily) purchase annuity insurance. In their view, not only is the mortality of people who do purchase annuities better than that of the average population, but in addition, signs of a temporary initial selection can be observed (such phenomena can also be identified with other life insurances, if for no other reason than because of the risk

assessment performed by the insurer, which is of course not a characteristic of annuities).

Interestingly, the thick volume of Booth-Chadburn-Cooper-Haberman-James [1999], which is meant as a summary of modern actuarial science, does not include either the expressions "adverse" or "selection", and moral hazard is not specifically elaborated either, although they do say that in the case of annuities (page 226-227) risk assessment is not necessary since the individuals who apply to purchase annuities are "self-selected", and therefore their life expectancy is longer than the average for the population.

In the literature, infrequent reference is made to the selection impact of choices made between different annuity types, but such references do exist. For example, Watson Wyatt Partners [2002] point out that in the case of non-indexed (unchanged) annuities, this is definitely advantageous for those with shorter life expectancies.

Cardinale-Findlater-Orszag [2002] say that the literature relating to annuities focuses on individuals' costs and adverse selection. The most complete and most often quoted summary of the topic comes from Amy Finkelstein and James Poterba in two studies (Finkelstein-Poterba [2002] and [2004]). The Finkelstein-Poterba [2002] study examines not only the selection that occurs via a voluntary annuity choice as typically discussed in literature, but also selection that is the result of choices between different products, which in their opinion is becoming increasingly prevalent because of the expansion of DC systems. They did not perform their investigations in the US annuity market (which they regard to being too small), but rather in the bigger UK market. In their view, the UK market is good because both voluntary and mandatory annuity markets exist, so lifetime extension among annuitants may be examined in both of these markets.

Finkelstein-Poterba [2002] find the roots of adverse selection in the fact that there is information asymmetry between clients and insurer. The client chooses whether or not to purchase an annuity, and if so then which one, on the basis of better information than the insurer has available. The study distinguishes between two types of selection: one at the time of entry and selection within the market. Individuals with a long life expectancy have better incentives regarding annuity purchase (entry selection) and also when choosing the type of annuity. They favour so-called "backloaded" annuities that have an unchanged real value (i.e. indexed to inflation), as opposed to nominally fixed annuities. In contrast, individuals with a short life expectancy will buy guaranteed annuities (either with a guaranteed period or other payback guarantee), which allows them to leave a legacy. Naturally, adverse selection only occurs in cases where information is clearly asymmetrical; otherwise the insurer adjusts the price to the individual risk. In their view, the insurer may take into account the mortali-

ty of those who take out this type of insurance, although here I eventually prove that this is not possible! Active and passive selection are to be distinguished. In the first case, someone decides about purchasing an annuity on the basis of their life expectancy, while in the second, although they cannot know their expected lifespan, they make a purchasing decision with regard to a factor that can correlate to this, namely their wealth. Both cases have similar effects.

Finkelstein and Poterba (unlike myself) examine adverse selection that occurs in an existing annuity market (which I could not do, since no such market exists here in Hungary). They presume that insurers know the rate of selection, and this is reflected in the annuity premium. This is of course a rational assumption, for it exists within a competitive annuity market with a long history, and with old and experienced annuity providers since in such markets providers may well have empirically arrived at their adequate pricing policy, and strong, safety-based overpricing is excluded by competition. The method of their examination is that they look at diversions from pricing using a selected "standard" mortality table with various annuity provisions. This diversion is called the "money's worth" of the annuity, and it is proportional to the estimated discounted value to be paid as a "standard" according to the annuity's premium. This value is "one" if the premium of the annuity is actuarially fair. The costs, adverse selection, etc. bring it to below one. If we use a national mortality table, it is even further below one, although even if we calculate using the annuitants' mortality table, we will still be below one because of costs.

The authors found proof in the annuity premiums for both of the examined selections. Concerning selection within the market, they saw that the money's worth of the annuity almost always decreases with the advancement of age as well as with increases in the premium's magnitude. Another experience was that the more the guaranteed period increases, the greater the money's worth (so, as expected, guaranteed period annuities are principally purchased by individuals who might expect a shorter lifetime).

They present two kinds of proof for entry selection. The first is a comparison of national mortality tables to the annuity mortality tables issued by the "Institute of Actuaries", through which it is clear that survival probabilities are highest in the voluntary annuity market, lower in the mandatory annuity market and lowest in the population as a whole. The second piece of proof is precisely the fact that selection on the mandatory market is half that of the voluntary market with respect to any annuity product. The consequence of this (for them) is that making annuity mandatory would be useful, amongst other things, in order to reduce the level of selection.

Finkelstein and Poterba repeated their study two years later (Finkelstein-Poterba [2004]), when they had access to the database of a United Kingdom insurer and were able to test their assumptions with relation to this. They saw

that adverse selection according to the amount of the sum assured was very small, although other studies generally focused on the relationship between this factor and adverse selection. Yet strong adverse selection was found to relate to other parameters. Strong proof was found with regard to the fact that people who expect a longer lifetime buy more back-loaded (e.g. indexed, possibly indexed to inflation plus) annuities both on the voluntary and mandatory markets, while guaranteed period annuities tend to be bought more by individuals who expect to have a shorter lifetime, although the difference in lifetime related to buyers' guaranteed period and back-loaded annuities in the mandatory market was not statistically significant. On the mandatory market, those who purchase a bigger annuity may expect a longer lifetime, but this is not the case on the voluntary market. The explanation here may be that, á priori, the very rich are present in the voluntary market, so there is no great mortality variation in this case. In the voluntary market, mortality differences between guaranteed period and non-guaranteed period annuitants are only a little narrower than between men and women, yet differences associated with the initial sum assured are minor.

In relation to any evaluation here, the authors note that their results may also be interpreted in the light that those who buy an annuity alter their behaviour so that they can live longer, meaning the results may also be explained by moral risk! Another possible explanation is that we are not talking about asymmetrical information or a competitive market, but instead about symmetrical information and a non-competitive market, which may lead to similar results. However, this probability is not seen as being possible. The third possible explanation is that individuals have diverse preferences; they do not belong to the same risk type and none of them are monitored fully by the insurers (I would say it depends on the target function, as I note later). Such items of preference may include: interest rate, risk avoidance or the desire to leave an inheritance, and these may correlate with the risk type.

The literature widely acknowledges and notes a solution for significantly decreasing adverse (auto-) selection, or possibly even eradicating it, by making annuity purchase mandatory. Naturally, this may only be the case in mandatory systems or in those DC systems where there is a tax incentive. I have earlier quoted Finkelstein-Poterba [2002] from this point of view, yet this idea had already appeared at the World Bank [1994] and Hylands-Gray [1992]. They also remark that individuals with a bad state of health probably opt for the longest guaranteed payment period; so adverse selection for the insurer may also occur here, although it is doubtful that this would be of significant magnitude. For this reason, there was a debate on making annuity mandatory in the United Kingdom (Wadswort [2002]), and the idea is included in the OECD recommendations (Antolin [2008a] and OECD [2008a]), although they do

state that making it mandatory is regarded as a "rough" solution and recommend that the mandatory transition to annuities be the default rather than a regulatory obligation, which according to experience will achieve almost identical results (which is called "soft compulsion" or a "nudge" – see Thaler-Sunstein [2008]). In their view, this will be especially effective if financial education is enforced parallel to it in the interests of promoting annuities.

The idea of directed selection also appears in the literature, as I have described (and disproven) above. According to the World Bank [1994], redistribution as the result of a uniform premium can be mitigated by offering insured individuals with different risks different types of contract and allowing them to choose between these. Clients with a lower life expectancy will choose a contract that includes a death grant. The argument is that if every type of contract is priced so that they are self-supporting, this diminishes the problem of adverse selection and undesirable redistribution.

The issue of selection has not been systematically processed in the Hungarian literature, with the exception of my own work. Instead, only scattered ideas can be found. For example, Kolos Ágoston and Erzsébet Kovács highlight the adverse selection impact of compulsory unisex tables in what they also see as becoming manifest in options among annuities (Ágoston-Kovács [2007]). According to them, "when applying the uniform (unisex) mortality table, the mortality of men and women is averaged in proportion to the number. This does not point to a perfect solution due to individual decisions, for example men buying two life annuities and women buying one life annuity reverses the pre-set proportions". They are also concerned with the fact that due to mortality differences between genders "unpredictable migration may begin between funds, since everyone tries to get into a fund where there are more young men". Or they may try to keep women away from such funds by setting up closed "professional" funds like the "miners' fund", for example.

There is little discussion in the literature about the selection impact of choosing an otherwise mandatory pension annuity at its start, although references are made to this as a well-known phenomenon. For example, Winkler and Mattar [1999] propose to insurers that reducing risk postponement at the start should be allowed only to a certain extent. In respect of this subject, it is worth mentioning that András Simonovits devoted a whole series of studies to this issue in recent years (the first two being Simonovits [2001] and Simonovits [2002]). He is critical of the free option at the start of an annuity and of the so-called "actuarially fair" setting of an annuity, yet his criticism is mainly targeted at the efforts of Pillar I (most prominently towards the NDC solutions). Although his statements could be generally applied to the DC type of funded Pillar annuities, I cannot do so because I have serious problems with Simonovits's approach, and I have written a separate study on this (Banyár [2011b]).

4. MODEL OPTIONS FOR MANDATORY OLD-AGE PENSION BENEFITS

Above I have reviewed the major annuity-related problems and the possibilities for managing these problems. During the course of discussing them it was clear that not every item can be voluntarily combined with other items and certain items presuppose other ones, while others exclude each other and there are also neutral items. Therefore, when developing the annuity system the legislator may choose among the different annuity systems or models. However, before starting to analyse the possibilities we must ask whether it is a must to choose a model and whether it is sufficient to let providers and clients offer or choose the annuities they wish. In my view, this is not possible in the case of mandatory annuities, but the reasons why are worthy of explanation.

4.1. The no model model – Entirely liberalised annuity service provision as a possibility?

We must begin by clarifying that the notion of "fully liberalised" annuity service is not quite precise, since in practice the annuity market, which operates with only minor restrictions, is also generally regarded as "fully" liberalised, and accordingly "full" liberalisation is also scalable. At one end of the scale is the annuity market that operates to all intents and purposes without regulation, while at the other end there is the "liberalised" annuity market, which operates with several, but very general rules.

The annuity market, which operates with only minor restrictions, can certainly not be logically linked to the system of "mandatory" annuity, the basis of which is that the state requires citizens to continuously save a targeted and defined portion of their income during their active carrier for consumption in their old age. This requirement is based on the presumption that the vast majority of citizens are short-sighted and undisciplined, and prefer current consumption to old age security, and this is the reason why a state that does not pose such a requirement would eventually be confronted with a large, low-income old-age population that ends up relying on social benefit. If this is what we presume of the citizens, if this is the reason for obliging them to accumu-

late, then this presumption is not in accordance with the regulation that permits them to do whatever they want with their money at the time of retirement, because it can be presumed that even at that time the majority would spend it in a short-sighted manner, meaning the original problem is reproduced. In other words, fully liberalised annuity provision is in fact the logical approach in lieu of a mandatory labour pension and mandatory pension savings. Such a pension system is also possible, meaning when there is no central pension system at all, or if there is just a basic pension that provides only a minimal service for everyone, and there are international examples of this and it also has many advocates, but the detailed analysis of this system is outside the scope of this book, as here we are dealing with possible models for the application of mandatorily accumulated capital.

So for mandatory savings, in the case of "fully liberalised" annuity service there is still a need for at least one rule: mandatory pension savings (or a mandatory fixed minimum portion of that) must be converted to an annuity.

Naturally this rule in itself is not sufficient, since the meaning of the word "annuity" – as I have indicated – is relatively broad, it includes, for example, both life annuities and annuities certain. Since pensioners are more and more dependent on pension income as their age progresses, only the life-long life annuity is suitable as a mandatory annuity (i.e. temporary life annuity is inadequate!). Consequently, this rule is worthy of further development: mandatory pension savings (or a mandatory fixed minimum portion of them) must be converted to life-long (but not necessarily immediately commencing) life annuity.

Yes, but if there exists such an absolutely minimum regulation, then it is almost certain that the legislator will find himself in a kind of "legislative spiral", so it becomes unavoidable to further elaborate on what is meant by "life-long life annuity". This is because providers (who continue to serve the short-term interests of short-sighted consumers) will try to pay out the savings, or the highest possible portion of it, in a lump sum and within the shortest possible time to circumvent the aim of the regulations by sticking to them formally. This is possible e.g. by paying out the life-long annuity in decreasing amounts, for example at each payment they pay out 50% of the reserve to the client until the end of his/her life (though this very quickly decreases the "life annuity" to a nominal amount). So it is useful to add that life annuities must not be of decreasing amount, etc.

By making it mandatory, the state implicitly also takes responsibility, meaning it cannot allow irresponsible providers who steal the clients' money or are later unable to pay the promised benefits to appear on the market,. This forces the state to restrict the range of possible providers, to supervise their activities and, by setting requirements on available capital, to make them able to provide the undertaken service.

The state can probably not avoid taking a stand on questions such as according to what criteria (e.g. age, gender, occupation, place of residence, size of assets, etc.) providers may differentiate between clients. In the European Union for example it is currently forbidden to differentiate between genders, and no matter to what extent I regard this rule as disputable, one must take it into account.

It is also probable that the state must make some kind of statement concerning the indexing of annuities: is it compulsory or not, and if so then what is it based on (rate of inflation, wage index, etc.)? If the state prescribes something with relation to this then it launches another spiral of legislation: can the longevity risk be devolved to the client or not?; how can it be avoided (through more regulations) that the provider finds itself in a difficult financial situation as a result of an obligation for which the responsibility lies not with the provider, but with the state, etc.?

The justification for a liberalised annuity service may be twofold; a "practical" and an "ideological" reason. According to the "practical" justification, letting the market develop the annuity service system saves the state from having to perform the legislative work. The "ideological" justification states that competing providers provide the best possible result for the client; competition results in the best possible service for the client at the lowest possible price.

Serious counter-arguments can be raised against both justifications. All of the abovementioned counter-arguments contradict the "practical" justification, because if the state has an obligation with regard to the pension system, then the state inevitably gets into a spiral of legislation. The argument against the "ideological" justification is the example of the Hungarian life insurance market (and probably many others), where expensive products that do not meet the clients' needs have been offered for a long time by competing providers. The reason for that is obvious and it would clearly work similarly in the "liberalised annuity market": in the case of liberalised annuity, service providers would do their best to include the highest possible costs in the premium of the product. This can be best achieved if they offer products that are incomparable with those of their competitors and overvalue the significance of the difference through advertising. So unregulated competition that extends too many parameters in reality means less competition, because it exploits the low level of information available to clients. In the case of all goods, intensive competition means competition between standardised products at a central location, which is best symbolized by how the stock exchange operates. There would be no stock exchange without standardization, and accordingly the ideological argument is built on a market vision that is contrary to the real operation of the market.

Altogether I think that the fully liberalised annuity service model is not compatible with mandatory labour pensions and the system of mandatory pension savings; such a model can only operate in countries where the state does not impose any kind of obligation with relation to pension savings. Where such an obligation exists and this model is nevertheless applied, the expectations towards the state and the consequent regulatory spiral easily leads to a state of affairs in which we may begin with a fully liberalised annuity provision system, but will quickly end up with a haphazardly regulated annuity provision system. Therefore it is expedient to avoid this trap and to review the expectations that can be made of a mandatory annuity-provision system, and to include this within a well-considered regulatory system.

4.2. The possible elements of annuity systems and their context

In the above two chapters I have analysed how the selection of the different key elements (scope of choice of the insured individuals, applied indexation technique, etc.) determine the other elements of the annuity system, the resulting problems and the tools for managing them. In summary, due to the internal context of the solutions an annuity system cannot be built up such that the above elements are arbitrarily attached to each other; they can only be combined with each other in a determined way. The above elements define several interrelated annuity systems, but the decision-makers usually only choose from these annuity systems; they are not free to mix the various elements.
Below, I summarise the above-detailed relationship between the elements, and on these grounds I attempt to identify the possible annuity models.

4.2.1. SYSTEM ELEMENTS AND OPTIONS RESULTING FROM THE HANDLING OF SELECTION PROBLEMS

First, let us look at **selection resulting from choice of provider and its possible management.** There is no selection problem in a fully liberalised annuity service system, as the clients' unlimited choice is also coupled with unlimited possibility for differentiation on the part of the providers (including the refusal of the potential client). However, if the regulation restricts provider options, meaning (apart from a few other parameters) differentiation is forbidden, then the corresponding rule on the part of the clients is that regulations do not afford them the opportunity to choose the provider. If the client does so regardless, the loss must be spread among the providers.
So the possibilities are as follows:

Table 3: Possible selection problems arising from the free choice of the provider and differentiation possibilities

Selection problems due to choosing the provider		Differentiation amongst insured individuals	
		Unlimited opportunity	Limited
Insured individuals	May not choose between providers	No selection problem	No selection problem
	May choose between providers	No selection problem	Selection problems!

Hereinafter I do not analyse the possibility that unlimited differentiation among the insured individuals is possible,[58] as according to the above this possibility does not exist in the majority of countries. Although if differentiation among insured individuals is limited, the question arises whether clients may choose the provider, or in other words: are there competing providers on the market?

There is no selection problem if there are no competing providers on the market. This may happen in two cases:

1. There a single, central provider,
2. There are several providers, but they are distinct and it is clearly defined which clients belong to which provider.

However, if there are competing providers on the market then the selection problem caused by the prohibition of differentiation may be equitably managed in two different ways:

1. Through consolidation, via an obligatory and centrally organised pool,
2. By dividing the product(s) into two parts and transferring the risky part to another provider.

Equitable means that theoretically regulations may transfer the management of problems generated by the regulation itself to the provider, so the legislator may say that the negative consequences of regulation should either be covered by solvency capital, or if the provider is able, it may pass them on to the clients. Naturally the legislator may only allow this latter solution to a certain extent, so if at the very beginning the problem is not managed the legislator may find itself in a spiral of regulation similar to that described above, and the result may be haphazard regulation without clear incentives, or it may lead to restricting providers, which in turn motivates them to leave the market. So I hereafter do not raise this possibility in either of the solutions.

[58] Although a regulation based upon such a principle will certainly work with much fewer restrictions, so such regulations could be easily created based on this present study.

There may be two types of pool:

a) Minimal (premium equalizing mechanism). This may be applied if indexing is based on the refund of excess return, through which mortality loss is (partly or wholly) transferred to the clients,
b) Maximal, in which case annual mortality results are distributed throughout the entire term.

If the annuity is split in time into a temporary and a deferred annuity part, then the selection risk is not identical in the two parts. If the duration of the temporary annuity (i.e. the deferral period of the deferred annuity) is small, the vast majority of the selection risk appears at the deferred annuity. The proportion between the two may be changed by increasing the duration of the temporary annuity. In such a case the selection risk of the deferred part decreases. However this can be eliminated totally from the temporary part if instead of temporary life annuity, the temporary part is converted into an annuity certain (or into a phased withdrawal, which is identical from this aspect).[59] Naturally, the price of this it is that annuity payments would be somewhat smaller, and to an increasing extent the longer the temporary part.

So via the phased withdrawal + deferred annuity facility, whereby the deferred annuity is transferred to another provider, we can eliminate selection risk for the term of the annuity certain. The deferred annuity can basically be transferred to two types of organisation:

1. To a central provider,
2. To a well-capitalised market player.

So we acquire the following possibilities:

1. Central provider,
2. Closed providers,
3. Competing providers
    a. Organised into a mandatory pool, the entire annuity is paid by the various providers
        i. Minimal pool: premium equalising mechanism (plus transferring the mortality profit to the clients via indexing),
        ii. Maximal pool.
    b. Competing providers may only pay temporary annuity and the deferred part of the life annuity is transferred to a well-capitalised market player,
    c. The same as above, only the deferred part is transferred to a central provider.

[59]This is to all intents and purposes a life annuity with a guaranteed period at the beginning (front-end).

From among the above, the possibility for a closed provider exists only prior to the establishment of a private pension system, and since the legislators involved generally did not come to such a decision I will not elaborate on it further. However, in this case the statements concerning the central providers are in essence valid, since the closed provider performs its activity as a central provider from the aspect of its own well-defined range of clientele.
So, from the aspect of annuity models the following possibilities exist so far:

Table 4: Possibilities from the aspect of annuity models 1

Competing or non-competing providers	Central provider	Competing providers			
Does it provide the full annuity or only the first part of it (phased withdrawal)?	Full annuity	Full annuity		Only the first part	
Managing provider selection problems	Non-existent	Via a premium equalisation mechanism	In a mandatory maximum pool	The deferred annuity is transferred to a central provider	The deferred annuity is transferred to a well-capitalised market player

According to the above, **selection as a consequence of choosing the annuity type cannot be managed**, in other words if there is such a possibility it must be eliminated (if such an option already exists, and if it doesn't then it should not be offered at all), and instead a precise definition of the type of annuities on offer is required. More than one type of annuity is only possible if no choice is possible, meaning who can purchase the various annuities is pre-defined. In practice this is a single life annuity for single individuals, and the joint lives annuity for married couples, both of which are of course well-defined. But the choice made by the legislator between these options has an equal impact on every annuity model, or in other words it can be freely chosen (except that such a choice must not be allowed!), and so I have not indicated it in the table, since this does not separate the possible models.

Similarly, **selection possibilities as a result of choosing the rate of indexing can also not be managed**, and consequently must also not be offered. This means predefining the indexing rate for the entire market with respect to indexing strategies for investment in index-linked bonds, and pre-determining the technical interest rate for the whole market with respect to return-refund indexing strategies. (Just a reminder: in this case liberalisation would be formal, because due to competition the providers would be forced to apply the lowest premium and accordingly the lowest possible rate of indexing [the highest technical

interest rate]. In the case of a central provider, the choice between various kinds of indexing would result in certain loss for the provider.)

In view of the above, a **free the choice with regard to starting the annuity** may only be allowed if this is an annuity with a guarantee period at the beginning, or more generally if the annuity is made up of two parts: a life risk-free part up to a determined age (annuity certain or phased withdrawal), and a deferred life annuity part. Accordingly, the key is whether the annuity is a simple life annuity (one part, commencing immediately), or a phased withdrawal + deferred life annuity, i.e. one of the above-mentioned ways in which we manage selection that results from choosing the provider. In this latter case the commencement of the phased withdrawal can be delayed arbitrarily, its payment can be suspended, and naturally this part can also be left as a legacy.

At the same time, it is true in both cases that:

1. Concluding a contract for simple (immediately starting) life annuity and deferred life annuity may not happen at a time selected by the client; that must be tied to a general rule (expediently: the date of retirement),
2. The suspension of the already started annuity may be allowed; that does not cause adverse selection.

Accordingly, below I assume that the first criterion is met and there is a possibility for suspension, and I do not indicate this separately. Although the possible models are divided according to whether the possible annuity is simple or deferred (in this latter case it operates as a phased withdrawal until the maturity of the deferral and is regulated separately!). Naturally, where such a division of the annuity already was introduced above, this problem is also handled by that, however the splitting of the annuity is also a possible issue with respect the other models, due to this problem.

So, from the aspect of annuity models the following possibilities exist so far:

Table 5: Possibilities from the aspect of annuity models 2

Competing or non-competing providers	Central provider		Competing providers					
Does it provide the full annuity or only the first part of it (phased withdrawal)?	Full annuity		Full annuity				Only the first part	
Managing provider selection problems	Non-existent		Via a premium equalisation mechanism		In a mandatory pool		Transfers the deferred annuity to a central provider	Transfers the deferred annuity to a well-capitalised market player
Is the annuity composed of one or two parts?	one	two	one	two	one	two	two	two

4.2.2. WHO BEARS THE MORTALITY LOSS (INCLUDING THE LOSS DUE TO INCREASED LONGEVITY)?

Either the client or the provider may bear the mortality loss (i.e. what remains despite the pool), or they may share it among themselves. However, the provider may only bear the loss if its ownership does not coincide with its client nature, since if it does, the loss is still born by the clients – it will be redistributed among different groups of clients in an unfair manner. So the bearer of the mortality loss depends on whether or not the owners of the provider are separate from the clients. The former case describes for-profit providers who bear the loss directly against the mandatory solvency capital. In the second case there is no rationale to for-profit orientation or required solvency capital (which, for instance, is not even a requirement according to Hungarian fund regulations, consistently with itself).

So the possibilities are:

1. The provider is a for profit organisation and
   a. fully bears the (final) mortality loss,
   b. the mortality loss is shared between the provider and the client.

2. The provider is a non-profit organisation (i.e. owned by the clients[60], meaning it is owned by its "members", has no capital, or is a central provider). In this case the (final) mortality loss is born entirely by the client.

In a consistent regulation the mortality profit naturally also belongs to the person who bears the mortality loss, e.g. in case 1/a it belongs to the provider and in case 2 it belongs to the client. According to the above, case 1/b is rational if the traditional, return-refund technique is applied and if the index is the combined investment-mortality return index. In this case the provider guarantees that the annuity remains unchanged at nominal level, and it is in this (and only this) that it shares the mortality loss with the client. This also applies to the case of the minimum pool, which accordingly should only be used in the case of for-profit providers (the maximum pool is much more expedient in the case of non-profit providers).
In the second case it is useful to apply the buffer technique in order to moderate annuity fluctuations. This might also be considered in case 1/b.

Having a central provider is not expedient if it is a for profit organisation, and so in that case the logical solution is for the clients to bear the mortality loss (although it is at its lowest in this case, because this is where the diversifiable part of the mortality risk is most balanced and only the systematic part, the "trend", remains).

Theoretically both competing for-profit and non-profit providers might transfer the deferred annuity to either a central or a well-capitalised provider if the annuity is split into two parts, but non-profit providers tend to transfer it to the central provider, while for profit providers usually prefer to transfer it to well-capitalised market players, so – for the sake of simplicity – this is what I presume hereinafter.

So, from the aspect of annuity models the following possibilities exist so far:

[60] At least if the regulation is consequent.

Table 6: Possibilities from the aspect of annuity models 3

Competing or non-competing providers	Central provider		Competing providers							
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The full annuity		The full annuity						Only the first part of the annuity	
Managing provider selection problems	Non-existent		Via a premium equalisation mechanism		In a mandatory pool				Transfers the deferred annuity to a central provider	Transfers the deferred annuity to a well-capitalised market player
Is the annuity composed of one or two parts?	one	two	one	two	one		two		two	two
Is the provider a for-profit or non-profit organisation?	non	non	for	for	non	for	non	for	non	for

A logical question with respect to a central (and so obviously non-profit) provider is whether it should be identical to the Social Security (hereinafter: SoS) organisation, which pays the benefit from Pillar I of the pension system. As the SoS is not engaged in investment, since a pay-as-you go system is in place, there is no great advantage if the central provider and the SoS are one and the same and accordingly I do not deal with this possibility, with one exception. Namely, it is conceivable that the insured individual is not entitled to receive SoS and private pension parallel to each other, but one comes after the other in such a way that the SoS pension is payable at an older age than the insured party's current age. This is practically speaking one version of a two-part life annuity, where the first part is provided entirely by the private pension system and the second part is provided entirely by the SoS system. So this can be regarded as the situation in which someone wants to retire at an earlier age than the retirement age in force today, in which case they should accumulate savings and use them to bridge the gap until their SoS pension is due, when they officially retire. So the table may be modified as follows:

Table 7: Possibilities from the aspect of annuity models 4

Competing or non-competing providers	Central provider		Competing providers							
Does it provide the full annuity or only the first part of it (phased withdrawal)? withdrawal)	The full annuity		The full annuity						Only the first part of the annuity	
Managing provider selection problems	Non-existent		Via a premium equalisation mechanism		In a mandatory pool				Transfers the deferred annuity to a central provider	Transfers the deferred annuity to a well-capitalised market player
Is the annuity composed of one or two parts?	one	two	one	two	one		two		two	two
Is the provider a for-profit or non-profit organisation?	non		for		non	for	non	for	non	SoS	for

We can conclude from case 1/b that the question of who owns the mortality risk is also related to the indexation technique.

4.2.3. POSSIBLE INDEXING TECHNIQUES

Two of the possible indexing and investment techniques (in close correlation) may be considered in the case of mandatory annuities:

1. Indexing to inflation
2. Indexing based on extra return.

In the case of indexing to inflation the investment may exclusively be made into inflation-indexed bonds, whose maturity structure is identical to the anticipated maturity structure of the annuity portfolio. In these cases the regulations prescribe indexing to the exact level of inflation (or perhaps inflation + a predetermined %), and does not permit providers to change this rate of indexing. In these cases it is not necessary to fix the technical interest rate, as that fundamentally depends on the (fixed) return of bonds above inflation and on the cost level of the provider. In the case of indexing to inflation the provider cannot abuse the fact that the client is linked to it, and therefore there is no reason for the regulator to allow the client to change provider in the annuity phase. Indexing to inflation does not go well with delegating mortality loss to clients, as this can basically be achieved through changing the indexation, so the basic presumption here is that the mortality profit belongs to the provider. If still there is a need to delegate mortality loss to the clients, it is appropriate to apply the buffer technique.

In the case of indexing based on extra return, the rate of the index depends on the extra investment return achieved by the annuity provider. It is not expedient to set a required minimal return, because that would impose on the provider a guarantee which is very expensive and altogether probably not worth paying for the client. Furthermore, only for-profit providers with actual capital can have this set as a requirement, so I assume that with this indexing method there is no set minimal rate for the indexation.

In this case, the technical interest rate must be fixed. If the regulator finds it important that there should only be a minimal chance for indexing below the level of inflation, then this fixed rate is small, for example 0%. In this case, the starting annuity payment provided for identical capital will be smaller than in the case of a higher technical interest rate, though the annuity increases more rapidly, which is more advantageous for annuitants with a longer life expectancy than for someone with a shorter expected lifetime.

As the client in this case is exposed to the annual investment performance of the provider, it is important that providers also compete for already acquired clients, so in this case it is reasonable to provide clients with the possibility of changing providers during the period of maturity. We might think that by setting a higher technical interest rate one may at least partly protect oneself against a lower return, but from the point of view of the provider it is no different from the requirement of a high indexation level; both the required guarantees and the same arguments against them are valid as in the case of a mandatory high indexation rate. The possibility of changing the provider during the term is coupled with having to prescribe the technical interest rate[61] and the mortality tables used for reserving (i.e. this is the second argument in favour of setting the former).

The technique is in harmony with making the mortality profit part of indexing. In the case of a non-profit provider the entire mortality loss must be delegated to the insured individuals, so the annuity could even decrease nominally, while it is reasonable to set restrictions on this, e.g. to prescribe that the annuity may not decrease nominally, in which case the mortality loss is shared.

[61] Certain experts think (e.g. Ferenc Csordás) that changing provider during the term does not require the standardisation of the product, since in a technical sense that is a product-revision. Though this may be technically true, I still do not deem it an adequate solution, because it is too complicated and not transparent for the clients, as well as making changing provider practically impossible. The lack of standardisation also poses a danger to client, namely that the magnitude of the provision is uncertain, e.g. if one of the providers deducts a large cost from the premium and does not put it into the reserve, and does not transfer it either, then the new provider regards the transferred reserve as premium and deducts the cost from that again. So by changing provider the client's moneys and annuity might quickly run out.

Of the two indexing techniques, one (inflation-linked) places the emphasis on the security of pensions, the price of which is that the pensioner does not receive the benefits of an economic upturn, while the other (return refund) concentrates on sharing the results of economic growth with pensioners at the expense of some security. A consequence of this is that when the economy is weak pensioners are better off because the interest on bonds is higher, while in a period of economic boom pensioners are worse off, since the state does not have to pay as much for loans. So indexing to inflation protects pensioners against economic fluctuations, but at a cost of not sharing the benefits of an economic upturn.

In the case of competing services the mandatory pool is not compatible with indexing to inflation, since it leaves no room for competition, so in this case I assume indexing based on extra return.

So, with respect to annuity models the following possibilities exist so far:

Table 8: Possibilities from the aspect of annuity models 5

Competing or non-competing providers	Central provider	Competing providers											
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The entire annuity	The entire annuity						Only the first part					
Managing provider selection problems	Does not exist	Via a premium equalising mechanism		In a mandatory pool				Transfers the deferred annuity to a central provider				Transfers the deferred annuity to well-capitalised market player	
Is the annuity composed of one or two parts?	One	Two	One	Two	One		Two		Two			Two	
Is the provider a for-profit or non-profit organisation?	non		for		non	for	non	for	non		SoS		For
Indexing	To inflation (inf.)	Extra return (er.)		Extra return					inf.	er.	inf.	er.	inf.	er.

4.3. Possible annuity models

Based on the final table above we may distinguish between 14 annuity models, although it is expedient to merge some models (and to handle the internal branches as internal alternatives). I think that wherever the annuity may consist of one or two parts, and where both indexing methods may be considered,

these should be looked at as internal alternatives, so I would add these two rows to the end of the above table:

Table 9: Possibilities from the aspect of annuity models 6

Competing or non-competing providers	Central provider	Competing providers						
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The full annuity	The full annuity				Only the first part of it		
Managing provider selection problems (pool or transfer of deferred annuity)	Does not exist	Minimal pool	Maximum pool			Transfers to a central provider		Transfers to a well-capitalised market player
Is the provider a for-profit or non-profit organisation? (Who ultimately bears the mortality loss?)	non	for	non	for		non	SoS	for
Is the annuity composed of one or two parts?	Both are possible	Both are possible	Both are possible			two		two
Indexing	Inflation	Extra return	Extra return			Both are possible		

This reduces the choice to seven models, each of which has one (and according to the above only one) internal alternative either from the point of view of splitting the annuity or from the point of view of indexing. For the sake of further simplification I also unite the two maximum pool models, i.e. cases when the service provider is non-profit and for profit. These models may be given the following names (further details to follow):

1. The central provider model
2. The insurer annuity model
3. The central pool model (with pension funds or fund managers)
4. The pension fund annuity model
5. The alternative SoS model
6. The OECD model

Missing from among the possibilities, at least in its pure form, is the Chilean model promoted by the World Bank, which combines indexing to inflation with competition between for-profit providers. The reason this is possible in Chile is because differentiation according to gender is not prohibited there, so the selection problem that requires handling does not arise to begin with. The selection problem can be managed either by the (minimum or maximum) pool, and consequently through indexing based on extra return, or by splitting the annuity. Naturally, if indexing to inflation is applied then the OECD model may also be regarded as a special version of the Chilean model.

The name "OECD model" is slightly misleading because the OECD mostly recommends spitting the annuity such that there is only phased withdrawal until a high age (they mention 80-85 years), followed by the deferred annuity. So, all models that apply this kind of splitting, could be called an "OECD model". I did not do so because, at least in my view, my other models include some that are more "striking" than splitting the annuity, so I stuck to this characteristic when naming them. This is partly true for models 3 and 4, as both (or rather the non-profit version of model 3) could be called "pension fund" annuity models, although in the case of model 3 I found the central pool to be a much more important characteristic than the fund's participation in the annuity service.

Below I investigate these models individually in greater detail (not in the order listed above!). I will also examine certain general problems that are true with respect to several models. These will be examined at the model description where they first occur.

4.3.1. THE CENTRAL PROVIDER MODEL

Based on the above, I will first of all summarise the most important elements of the model and their internal relationships in the following table:

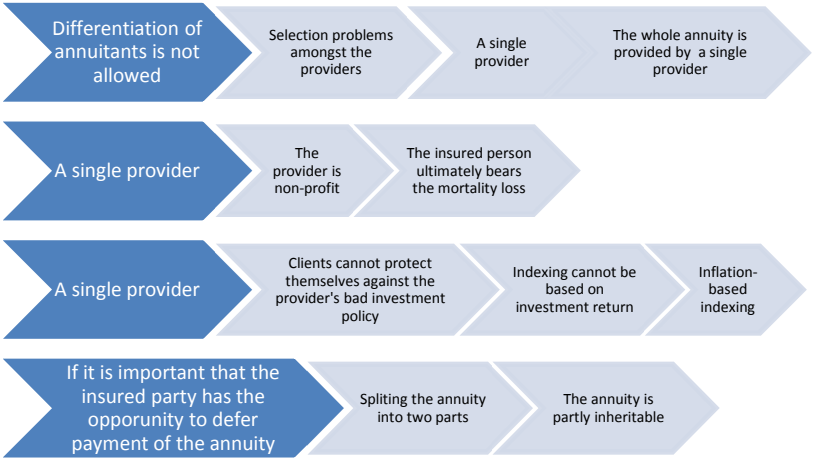
Table 10: The most important elements of the central provider model

Competing or non-competing providers?	There is a single central provider, so selection problems resulting from choosing the provider do not occur.
Does it provide the full annuity or only the first part of it (phased withdrawal)?	Naturally, the sole provider is "forced" to provide the full annuity.
Is the annuity composed of one or two parts?	The single provider does of course not mean that the annuity cannot be split into two parts, not because of selection considerations, but to allow insured individuals opportunities for legacy and deferral. This, however, is an alternate option here; both are possible but one of the two must be chosen, so one or the other option applies to all of the insured parties. This means that when they retire they both receive a uniformly or immediately commencing annuity (which may, however, be suspended), or a deferred annuity plus the opportunity for phased withdrawal, which can be handled flexibly within a predetermined, broad framework.
Is the provider a for-profit or non-profit organisation? (Who ultimately bears the mortality loss?)	It is expedient for the central provider to not be privately owned, so the mortality loss (or the entire mortality gain) must ultimately be borne by the clients themselves. Since indexing based on extra return should be excluded in the case of a central provider, mortality returns cannot be spread among annuitants via regular indexing The possibilities:

	• Loss and profit is accumulated against the capital of the central provider for a period, and therefore the service is adjusted from time to time (less frequently than annually) • The buffer technique is applied, thanks to which the above solution need only be applied rarely, or not at all • An option premium is paid to the state, which assumes this risk
Possible indexing	From among the two possible indexing techniques, the one where the index is not pre-set, but is a post factum value depending on the investment performance should be excluded, because with a central provider in a monopoly position the client loses every possibility to stimulate the provider to achieve the best possible performance. Therefore, only indexing based on investment in bonds with yield indexed to inflation should be applied in this case.

The above options can be presented in the following diagram:

Diagram 6: The internal relationship between the elements of the central provider model



If there is a single central provider, it cannot be a for-profit, privately owned provider. It would be difficult to justify the transferring of a monopoly to a market player, especially when, as is the case throughout practically the whole of Central and Eastern Europe, no market players even exist yet. In this situation the monopoly would be given to a market player, tasked with developing the service, which has not yet proven that it deserves this privileged position. Another argument against the state commissioning a market player to develop

a monopoly (via preliminary tendering, for instance), is that in such a situation the state would intrinsically find itself being blackmailed in view of the fact that it cannot act particularly forcefully against the provider if, for instance, it does not provide a service of acceptable quality or provides the service at high cost, because the provider could threaten to withdraw itself from the market, which would put the state in a difficult position as being responsible for the service but having no means to provide it. So we can predict that the state would have difficulties regulating a market player with a monopoly, and accordingly this situation should be avoided. In view of these arguments I hereafter assume with regard to this model that the central provider is not a profit-orientated organisation.

There are still alternatives with regard to ownership. The owner may be:

1. The clients, as in the case of Hungarian pension funds, or
2. The state.

If the state is the owner, it is logical to ask whether the central provider should be established by extending the activities of an existing state-owned organisation, or should instead be a new, independent provider. In the first case the logical candidate is the actual SoS, the pay-as-you-go system. Both solutions are reasonable, but the SoS can only be a solution if the central provider is not required to perform investment activities (see "alternative SoS model"). If it must conduct investment activities too, there is no advantage in building it into an existing organisation, it is better to have a separate organisation.

Although if a separate organisation is established, one should consider allowing the stakeholders to monitor it, so there should be a kind of Hungarian-type pension fund that provides stakeholders with at least a theoretical opportunity to have a direct say in the management of their affairs. An argument against this solution is that according to experience the activity of members of Hungarian pension funds (voluntary and private funds alike) is minimal, so the opportunity to have a say may be illusory, in addition to which is creates the theoretical possibility for an active minority to acquire control over the organisation.

Managing mortality loss is basically similar in both ownership structures, since in each case its final bearer is the insured individual. If the state has ownership the ultimate bearer of the loss could theoretically be the state, i.e. the taxpayers, but this is not an equitable solution regardless of whether the state gains or loses overall as a result. In every case it is in principle also possible to distribute the loss among different generations of annuitants, but this is also not an equitable solution and therefore not a solution that should be proposed.

A state-owned provider (including centralised providers that are owned by the members) means the nationalisation of the private pension model. This would seem to be a step backwards in a situation in which the accumulation phase is based on the competition of non-state-owned providers. Therefore it is expedient to examine when and why the idea of a central provider may be raised. In two major cases:

1. If it is more *expedient* for the state to organise the service rather than have it operate in the form of competing market providers,
2. If, due to a lack of competing market players, the state is *forced* to organise the service itself.

Breaking down the above cases even further:
It is more expedient for the state to organise the annuity service if

a) it was in fact already not expedient to entrust the service to non-state-owned, competing providers during the accumulation phase,
b) the annuity payment phase differs from the accumulation phase in this respect.

At the time, when the private pension asset accumulating institutions were established practically everywhere throughout Central and Eastern Europe, the main goal was to somewhat mitigate the unavoidably threatening demographics-related financial pressure on the pay-as-you-go system by partly capitalising the pension system. The capitalisation brought forward the deficit that would have appeared later in the pay-as-you-go pension system, as well as spreading it over a longer period of time, thus giving the state a better chance to better manage it. The state exploits this chance properly if the capital within the capitalised system does not fundamentally mean its own debt bonds (or at least not bonds newly issued because of the establishment of the new system). If this is the case it means the state did not in fact exploit this opportunity, or with a little more good will one might say that it hasn't fully exploited the opportunity "for the moment". This also means that nothing happened with respect to the main goal: an implicit sovereign debt was simply replaced by an explicit one, which in many respects is a worse situation than the starting point.[62] If this happens, as was also the case in Hungary, then to all intents and

[62]Just as a matter of interest: even if the state made it mandatory for private pension capital accumulation institutions to invest in their own government bonds, it would be a total own goal. There are of course international examples that aren't own goals: some African countries have made it compulsory for pension institutions to invest the capital of pensioners (e.g. civil servants) into government bonds that have a relatively low return compared to inflation. From the state's perspective this is equal to robbing pen-

purposes all criticism of the private pension system with regard to the fact that it was established unnecessarily, is justified. But at the same time, in this case the problem is not with the competition itself (although competition is naturally restricted), but with Pillar II, the mandatory funded system itself: if it was created in this way then there was no point establishing it at all.

The system would also have been no better had a single, state-owned provider been established. However, if we say that in the long term private pension institutions do not invest in their own government bonds (i.e. they can invest in foreign ones), then it is justified to suggest that the clients, who bear the investment risk, should have an opportunity to choose in exchange for assuming this risk. Choice can of course not only be offered by competing providers, but also by a single provider, but there is no cost pressure on it in this case. Consequently, it is expedient to entrust competition to competing providers rather than to differing portfolios within a single provider. Additionally, it is not a good idea for the state to be the (majority) owner of competing providers, so I think that at the time it was the right decision (in Hungary, for instance) to entrust the service to non-state-owned, competing providers during the accumulation phase.

This is of course a very general statement, because it does not take into account

a. what precisely the competing providers are competing in, and
b. whether it is in fact worth requiring competition with regard to every single component of this activity?

In the accumulation phase, there are two areas of competition (assuming that the provider does not provide a guarantee on the return, as also occurred in Hungary in the case of private pension funds):

1. Net returns achieved by the provider
2. Costs charged for by the provider

Private pension funds sometimes also name a vague third factor “the quality of the service”, but this cannot really be operationalised because the service is clear, so the basic functions (records, notification of members, etc.) can in essence only be performed in one of two ways: well or badly, but the fact that the provider should perform these services well is a fundamental requirement.

The second field of competition is the costs charged by the provider, which basically means administration, and organising and operating the membership records system. This is characterised by increasing returns to scale because the fixed costs are high. This means the larger the provider, the lower the costs

sioners, so it has a specific goal, but in my study I have excluded such motivations from my sphere of examination.

charged. This may result in a provider having a clear competitive advantage with respect to the investment, but not necessarily within the field of administration. Consequently, we cannot know if it was justified to set up a system (as occurred in Hungary in 1997) in which every element is open to competition; it is possible that a model with a centralised membership records and administration and competing investment providers would have been more expedient.

Within the annuity payment phase, the possible fields of competition (depending on how the law defines annuity services) are:

1. Net return achieved by the provider
2. Costs levied by the provider
3. The "tailor-made" nature of the annuity provided
4. Longevity risk management
5. The long-term solvency of the provider

So competition can potentially extend to many more fields in the annuity payment phase than in the accumulation phase. However, there are a great number of arguments in favour of the state restricting competition within these fields via regulation. In the annuity payment phase (in contrast to the accumulation phase) it is much more important to have a calculable return adjusted to inflation than for the provider to achieve a high return. In a mandatory system there cannot exist an endless variety of annuities, so regulations should probably severely restrict the range of possible varieties, and accordingly competition between "tailor-made" annuities (which, as I have already indicated above, are aimed at avoiding competition rather than enabling it).

Regulations often transfer longevity risk to the provider, which it then compensates in its costs, so competition in this field is transferred to cost competition. The long-term solvency of the provider is much more important in the annuity payment phase than in the accumulation phase because in this latter case there is no commitment, while in the payment phase there is a commitment, which the provider may eventually be incapable of fulfilling. So we are left with cost competition and competition with regard to long-term solvency. The returns to scale discussed in the context of the accumulation phase are also applicable with relation to costs, or rather the administration charge, meaning its central management should also be considered in this case.

Long-term solvency is influenced in a positive way if:

1. the organisation is backed by a solvent owner or sponsor such as the state,
2. if the risks are in a single pool, so the random risk fluctuations cancel each other out.

So, in the case of annuities a central provider is a logical alternative that does not contradict the goals of the private pension system. However, one argument against it is a politically-related factor and this is if the population has exaggerated expectations with respect to the national budget as an instrument of welfare, which in turn has a detrimental effect on economic growth. From this perspective it is expedient to mitigate the national budget from a proportion of welfare expenditure (i.e. the pension system). This mitigation exists in the model of competing non-state-owned providers, but not in the central provider model; whatever the regulations, the central provider becomes, both practically speaking and in wider sense, a part of the national budget. In addition, compared to the system of competing providers, it also makes it much more possible for the private pension system to become a stage for irresponsible political promises, as we have seen on many occasions with respect to the SoS pension system.

Naturally the central provider model does not only emerge if the legislator finds it more expedient, but also if there aren't a sufficient number of enterprises that are prepared to rationally organise the private pension annuity, so in the case of a mandatory private pension system the state may be forced to set up a central provider. Such a constraint is better avoided; if the state establishes a central provider it should do so because it is more expedient and not because it has no other option. So the state should avoid becoming its own reason for there being no voluntary annuity providers. The state may cause this if it poses inconsistent and therefore infeasible requirements on providers.

An example from the, otherwise inconsistent, Hungarian annuity legislation in force until 2009: it is (should have been) compulsory for the provider to increase the level of annuity payments annually at least in correspondence with Swiss indexation. As there is no capital market facility whose return is easily correlated with this index (and within this, with the wage index), providers could only have met this requirement if they establish an extremely high safety reserve, i.e. by setting a very low starting annuity. As the legislator requires not only an adequately indexed annuity but also one of adequate level, in this situation the legislator's solution could have been to choose cost-maximising, a constraint which, together with the inadequate indexation rule, makes it almost impossible for prudent providers to enter or remain on the market.

In the central provider model the premium reserve is invested exclusively into bonds, whose return is dependent on inflation. Naturally this investment and indexing technique may not only be used in this model, but wherever we use it the problem of nationalisation emerges, although somewhat differently than in the case of institutional centralisation via the establishment of a central provider. Specifically in the case of Hungary (and many other countries), investment in inflation-linked bonds means mandatory investment into govern-

ment bonds for the foreseeable future, i.e. the cloaked nationalisation of annuity service. Therefore, the question is raised whether, instead of cloaked nationalisation, it would not in fact be more expedient to openly nationalise annuity service, i.e.to set up a central annuity provider?

The consistent models I found ultimately suggest that the answer is yes, because I only regard indexation to inflation feasible (beyond indexing to the excess return refund, as an alternative) in models where the annuity is split and the deferred annuity, meaning the peak risk, is either transferred to a central provider or to a strong market player. In fact in this latter case it is somewhat doubtful whether it is logical and consistent to "also" apply inflation indexing. Nevertheless it is possible that in the long term investing in inflation-linked bonds might not mean investing in government bonds, or at least not in the given country's own government bonds, as

- In theory all EU Member States will eventually join the Eurozone and accordingly they will have access to the foreign (Euro-based) inflation-linked sovereign securities and company bond market,
- It is possible that, following the example set by the state, the market will also begin to issue inflation-linked bonds of adequate quantity and quality, as happened in the case of Chile.

Naturally, Eurozone inflation-indexed bonds would not necessarily solve the problem, since the rate of the inflation may differ within the various countries of the Eurozone. Within a common market there is of course a tendency to equalise inflation rates if the level of development of the different countries doesn't differ to a significant extent. But until a country catches up to the Eurozone average (like most Central and Eastern European countries that have implemented private pension reforms) prices there will remain characteristically lower than in the more developed countries, which in turn means that prices catch up too, so for a prolonged period in these countries the rate of inflation will be also be higher than in the more developed countries of the Eurozone. Indexing to inflation within the pension system should be linked to domestic inflation and as a result using the inflation-linked bonds of other countries could cause a (not necessarily irresolvable) problem.

It is also important to note that in the case of investing in inflation-indexed bonds requirements not only include the fact that the yield of these bonds should be linked to inflation, but also that

a. they must be denominated in the same currency as the liability, i.e. the currency in which the annuity is paid,
b. the maturity structure of the bonds must correspond to the maturity structure of the annuities (asset-liability matching), meaning they cannot be of any kind of maturity, only short term bonds. (However this

strict ALM requirement can be mitigated if the state engages in the long-term issuing of such bonds in adequate quantities).

This latter requirement is needed so that the market fluctuations in bond values can be completely disregarded during the evaluation of annuity provisions and, to all intents and purposes, only bonds that are kept until maturity are included in the investment portfolio.

The notion of the central provider emerged only recently in English language literature, often only as an idea to be rejected. For the World Bank [1994] it was still clear that a decentralised savings system also requires private annuity providers. Examining the institutional structure of the annuity market, Blake [1999] already mentions the state monopoly as a possibility. In his view, this could solve a number of problems he brought up, its advantages being: returns to scale, being more capable of bearing improvements in mortality, indexing and that the interest cycle could be better balanced. He mentions as the major disadvantage that there are few international examples of efficiently managed state-owned institutions. This may over-compensate the advantage of the increased return to scale.

For the authors of the OECD, who collected ideas from many countries that were preparing to introduce an annuity system, the idea of a central provider also emerged. In 2007, Fiona Stewart only raises the fact that the state could itself sell annuities as a possible option. A year later Pablo Antolin is also looking for a suitable provider, and since he finds that insurance companies, who would be best prepared to provide such a service, are not really interested in the possibility, he raises the idea of a single national provider, although in his view this could result in private providers being pushed out of the market (Antolin [2008a] and OECD [2008a]). In his next study (Antolin [2008b] and OECD [2008d]), he adds that its disadvantages include making the taxpayers the final guarantor and that in such a scenario the state reassumes the investment and longevity risk that it had previously left out of its portfolio. In addition there is also a political risk, which is more difficult for a state provider to withstand than for private providers.

In the Hungarian market the idea of a single central provider was raised by János Stahl in 2000 (Stahl [2000]), and at the time only as a possibility to solve problems caused by the unisex table, although in 2005 it was raised as a complete concept (Stahl [2005]). In 2000, he still found it necessary that the idea of a central provider and the NDC-based reform of Pillar I are kept separate from each other, stating that (page 221): “The fact that the previously envisioned system is modified so that there is only a single provider, which is most certainly some kind of state institution, is naturally not identical with the idea that everyone’s pension contribution is kept on denominated accounts,

and the interest is based on the yield of some government bond." In other words: this is a warning that having a single provider does not mean that the reserve is mandatorily invested into government bonds and certainly not that this is done explicitly.

In 2005, he already considers who that central provider might be and thinks (Stahl [2005]) that such a provider could be established jointly by the pension funds or that it might be created based on a Guarantee Fund which has lost its function. In the debate that began with Stahl's article [2005], György Németh (Németh [2006]) finds Stahl's proposal for a central annuity provider to be justified, but in his view it needs refinement. During the course of this "refinement" Németh proposes a completely different concept, which I discuss later under the name "central pool model".

The notion of the central provider was considered as a "theoretically possible, but practically out of question" possibility right up until the summer of 2009. Then, at a session of the Ministry of Finance Working Group set up to regulate annuities, the idea was unexpectedly embraced by Péter Holtzer (who represented for-profit providers in the committee and at the time was President of the Roundtable for Pension and Old-age Affairs, a government-established body). It was thanks to his influence that the bill, which included (though not as he had originally recommended) this proposal for the annuity model, along with another one, was put before Parliament in 2009. He had originally suggested full renationalisation as the right solution (in fact to all intents and purposes the version about which Stahl warned did not properly fulfil the role of central provider!), where the single annuity premium is paid into the budget and, much like in the case of Pillar I, the annuity is then paid directly from there. According to his vision there is no investment, no printing of government securities, no separate apparatus, and in essence not even a central provider, only a central service. The solution, from a certain perspective, would no doubt have been cost-effective, but would have re-included in the system the political risk that had been previously eliminated by the establishment of the private pension system (Pillar II), and would have circumvented the possibility of investments being made in non-government (foreign) securities at some time in the future, i.e. the chance that the Hungarian state would not even be involved in the private pension system via government debt. (Of course, a certain level of state involvement is necessary to the central provider model.)

4.3.2. THE OECD MODEL

Based on the above, the model's most important elements and their internal relationships are as follows:

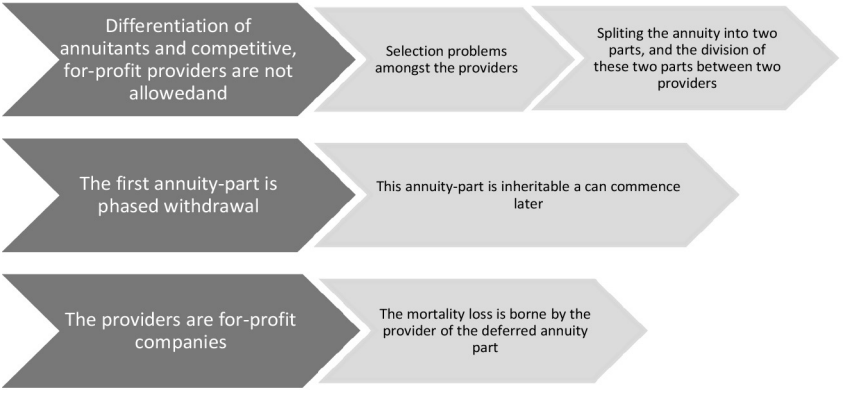
Table 11: The most important elements of the OECD model

Competing or non-competing providers	Clients may choose, with certain restrictions, from among several competing providers.
Does it provide the full annuity or only the first part of it (phased withdrawal)?	Competing providers only provide the first part of the annuity within a phased withdrawal structure that clients are relatively free to organise (i.e. not, or not necessarily within a life annuity framework). This partly solves selection problems between competing providers that result from the prohibition of differentiation.
Is the annuity composed of one or two parts?	As a consequence of the previous point, the annuity is composed of two parts, where the first part is a phased withdrawal that allows insured individuals to defer commencement of the annuity with the majority potentially left as a legacy (providing they do not live it up in the meantime), i.e. "the client's money is fully at his/her disposal". However, the insured individual must immediately purchase the deferred part of the annuity on retirement.
Managing provider selection problems	The remaining selection problems, which are intrinsically fewer than if the provider sold the client an immediately commencing life annuity in itself, and which are borne by the deferred part of the annuity, are transferred by the provider (via transference of the deferred annuity) to a well-capitalised market player and as a result it is fully exempt from them.
Is the provider a for-profit or non-profit organisation? (Who ultimately bears the mortality loss?)	Based on the above, (by passing the deferred annuity to a central provider instead of a well-capitalised market player) the provider could also be a non-profit entity, but this is such a significant characteristic that it paves the way for another annuity model. So in this model the provider is a for-profit company (e.g. from the possibilities available according to the previous Hungarian environment[63] a pension fund transformed into a joint-stock company). Accordingly, the mortality loss, which appears exclusively at the deferred part of the annuity, is fundamentally borne by a well-capitalised provider at the expense of its solvency capital, which is charged as a kind of option fee in the first part of the premium.
Indexing	Theoretically it can be solved via both indexing techniques and there are arguments for and against both solutions. An argument for indexing to inflation is that the two annuity parts can be relatively seamlessly adjusted to each other, and that with respect to the deferred annuity part it is difficult to change providers mid-term, but then again, without this indexing technique "already acquired" clients would be vulnerable to their providers, who from their perspective are in a monopoly position. An argument against inflation-linked indexing is that it negates the need to set up competing providers, meaning it is only raised if competing providers can already be taken as given.

[63] In 2009, Hungary's Parliament adopted an act that included this possibility, but this act eventually never took effect.

The abovementioned relationships are represented in the diagram below:

Diagram 7: The internal relationship between the elements of the OECD model



The essence of the model is that clients retain a high level of control over their accumulated money, including the possibility of leaving the as yet unused portion of the annuity as an inheritance (in the case of annuity certain or phased withdrawal), while this autonomy does not endanger their livelihoods at a potentially very old age, because they are obliged to purchase a kind of "emergency cover" from a portion of the money in the form of a deferred annuity. The division of the money into temporary and deferred annuities depends on the possible age at which the deferred annuity eventually commences. Below I present a few calculations for the approximate proportions.

In the OECD model, but not only in that, an important element of the model is that each immediately commencing lifelong annuity may be divided into two parts, a so-called "temporary" annuity that lasts for a determined period of time, and a deferred annuity, where the period of deferral corresponds to the term of the temporary annuity. Together, the two annuities form precisely an immediately commencing, lifelong annuity.

According to the above, the formal splitting of the annuity is expedient if:

1. the two annuity parts are managed differently, i.e. they are performed by different organisations or different providers, as in the case of the OECD model, or
2. if the intention is to keep a portion of the annuity at the disposal of the clients, so a guaranteed period annuity is the general rule rather than an

immediately commencing simple life annuity. In such cases the annuity part can be managed separately and relatively "liberally" during the guaranteed period.

For the provider, the biggest risk that needs to be managed with respect to the annuity is the longevity risk. If the original annuity is split, the original longevity risk is also split, but it is not spread equally between the two parts of the annuity. It is easy to recognise that the longevity risk is mainly related to the deferred part of the annuity, as its extent cannot be precisely defined, because annuitants could live for a very long time compared to the provider's original plan. In contrast, during the temporary part of the annuity the extent of the provider's possible losses are limited in this respect: the worst that can happen is that annuitants do not die during the term of the annuity, in which case the life annuity behaves like an annuity certain. To demonstrate the possible extent of loss, I indicate the relationship between the net premiums of temporary annuities certain and temporary annuities using the 2000 and 2006 Hungarian male and female population mortality tables for the calculation:

Table 12: The single net premium of a temporary life annuity as a percentage of the guaranteed annuity for entry age 65 years

Life table	2000				2006			
Technical interest rate	0,0%		2,9%		0,0%		2,9%	
Period/maturity	15	20	15	20	15	20	15	20
Male	71%	61%	73%	65%	74%	64%	76%	68%
Female	83%	74%	85%	78%	86%	78%	87%	81%

It is obvious that the higher the mortality the higher the potential mortality loss (i.e. the difference between the premiums of the two types of annuity), namely it is higher for men than for women and higher when calculating with the 2000 mortality table than with the 2006 mortality table (which shows an improving trend compared to 2000). As the annuity is expectedly calculated using a significantly more favourable mortality table than the population mortality table, the possible loss will probably be even smaller in the temporary part.

In addition, the longevity risk may be totally eliminated in the temporary annuity part if we calculate with annuity certain from the start and not with a temporary life annuity. If the two annuity parts are viewed together, this is nothing more than the supposition of life annuity insurance with a guaranteed period at the beginning. The possible annuity payment is of course smaller than in the case of a pure life annuity, but the better the mortality the smaller the decrease, as is also indicated by the table.

The relative weight of the two annuity parts is shown in the table below, in which I present the proportion of the premium of the deferred annuity parts, which primarily bear the longevity risk, within the premium of the original annuity.

Table 13: Deferred life annuity premiums as a percentage of the simple life annuity for entry age 65 years

Life table	2000				2006			
Technical interest rate	0,0%		2,9%		0,0%		2,9%	
Deferral	15	20	15	20	15	20	15	20
men	18%	7%	13%	4%	20%	8%	15%	5%
women	25%	11%	19%	7%	27%	12%	20%	8%

It can be stated that the better the mortality conditions the higher the relative premium of the deferred annuity. So in the case of women it is better everywhere than in the case of men, and in the case of a generally better mortality table (as in the improvement in 2006 compared to 2000) it is higher than in the case of a generally worse mortality table. However, the higher the technical interest rate the smaller this proportion (since as time passes the higher technical interest rate makes the further elements progressively less valuable during the cash-flow phase), and it is naturally also lower the longer the deferment (so lower at 20 years than at 15 years). However, even in the extreme case presumed within the table (i.e. assuming the 2006 female mortality table, a 0% technical interest rate and 15 years of deferral[64]), the deferred annuity still only corresponds to 27% of the premium. True, this is an underestimated value to the extent that the actual annuity will probably have to eventually be calculated using a (from a mortality perspective) better projected mortality table than the 2006 mortality table.

An obvious tool for consuming private pension savings is to prolong the temporary annuity. But this has different types, which raises the question of precisely what kind of annuity it should be, and also whether this consumption should necessarily be made in the form of an annuity (and, of course, whether it is necessary to consume the savings at all). The following options result:

[64] So the insured individual would begin receiving the annuity from one of the providers at the age of 65 and it would last for 15 years, until the age of 80, providing the annuitant survives until that age, after which another provider begins paying the deferred annuity, which now lasts until the end of their life.

- Temporary life annuity
- Temporary annuity certain
- Phased withdrawal
- Withdrawal in a lump sum

The different possibilities have the following advantages and disadvantages:

Table 14: Possible variations in temporary annuity

Possibility	Advantage	Disadvantage
Temporary life annuity	• since the reserve of the previously deceased is distributed among the still living annuitants, this assures the highest level of annuity payments among all the possibilities • it schedules the consumption of monies by the insured individuals and thus diminishes the risk that they will consume ahead of time (presuming undisciplined pensioners with restricted foresight, or who are under increasing financial pressure)	Inflexible, meaning: • it is non-inheritable (since the money of the deceased is divided among the remaining insured individuals) • the possible dates of the retirement should be restricted in view of the danger of adverse selection • may only be provided by an insurer-type institution
Temporary Annuity certain	• flexible, meaning: o inheritable without problem o there is no danger of adverse selection, so the date of retirement may be chosen totally flexibly • it schedules the consumption of monies by the insured individuals and thus diminishes the risk that they will consume their savings ahead of time • it may be provided by any kind of financial provider (e.g. today's Hungarian private pension funds)	• since the money of the deceased is not distributed among the remaining annuitants, it provides a somewhat lower level of annuity compared to temporary life annuity
Phased withdrawal	• even more flexible than a temporary annuity certain, permitting more possibilities for change during consumption • it may be provided by any kind of financial provider (e.g. today's Hungarian private pension funds)	There is no default schedule during the consumption phase, so there is a danger that the insured person will experience certain periods of financial shortages (assumes a relatively disciplined pensioner)

Withdrawal in a lump sum	• the most flexible solution since anything can be done with the savings e.g. annuities (even including lifelong annuities and annuities that are differentiated according to gender) can be bought on the free market or one can start up an enterprise, etc. (i.e. it assumes a disciplined pensioner with adequate foresight, or who is not living under increased financial pressure) • it may be provided by any kind of financial provider (e.g. today's Hungarian private pension funds)	• if the pensioner is not disciplined enough, they may easily run out of money before the commencement of SoS (Pillar I) annuity payments • there may be pressure on the pensioner from their environment to not use the money received as a lump sum to cover their own needs but, for example, their children's needs

The regulator may decide which of the above possibilities to impose from top to bottom. The rule is that possibilities before the imposed ones must also be allowed, while those after it must not. So if the regulation prescribes temporary life annuity, all other possibilities must be excluded. If, however, the regulations allow for the withdrawal of the money in a lump sum, there is no reason to prohibit the other possibilities.

In the OECD model and in the other models that split the annuity into two parts, allowing phased withdrawal, with certain restrictions, is the default, which accordingly permits both temporary annuities certain and life annuities, if the insured party so choses, but excludes withdrawal in a lump sum. The latter, for instance, if the legislator wishes to homogenize the annuity, may be permitted above a certain amount. A possible and expedient rule for phased withdrawal could be that each year following retirement the annuitant can withdraw a proportion of the value of the reserves at the beginning of the year corresponding to how many years are left before the commencement of the SoS pension. The advantage of this solution is flexibility, which does not endanger the potential livelihood of the pensioner (that could arise due to lack of discipline, financial requirements or pressure from their environment) while leaving open the possibility for inheritance, which according to Hungarian experience is a widely known and required feature of the private pension system, its main attraction, one might say. A further advantage is that providers may retain existing pension funds (providing such institutions exist of course – this is the pension fund annuity model!), so there will be default providers on a systemic level, contrary to the case in which life annuity is prescribed by law. Because if life annuities are made compulsory the pension funds must either be transformed into insurance company-like providers (with actual owners and solvency capital), or pensioners must be forced to change providers when they retire (i.e. to leave their previous pension funds), which means there will be no

default provider and, as a potential danger, there will be no provider at all if nobody volunteers to enter the market.

Contrary to the alternative SoS model, where the duration of the temporary annuity is tied to the ratio of Private Pension Pillar to SoS Pillar, this is totally optional in the case of the OECD model and in the case of every model where capital is transferred within the private pension system. However a few considerations should be made when choosing the option, the most important being what magnitude of loss the annuity system is able to absorb as the price of free choice and the opportunity to inherit. A temporary life annuity is "cheaper" than an annuity certain (or phased withdrawal, which has a similar character) in the sense that the reserve of the deceased annuitants goes towards the annuity of the remaining insured parties and is not inherited, so it "stays in the system". The longer the temporary annuity certain, the more money leaves the system, so the assessed temporary annuity certain may be smaller in accordance. Table 12 provides an idea about the magnitude of this loss. It is worth noting with reference to the table that it probably overestimates this loss, because the annuitant mortality table would probably be more favourable than the 2006 population mortality table, meaning the difference between temporary life annuity and annuity certain will in fact be smaller than in the above table. Based on this I think we may consider a 15-year temporary annuity without any problem, but a longer duration might also be considered.

A further question is whether the term of the temporary annuity should be optional, or whether the provider should set the term. Furthermore, if this is to be regulated, then should we define the term of the temporary annuity or the age of commencement of the deferred annuity (e.g. how old the annuitant will be when the deferral comes to an end)? The clear answer to this latter question is that determining the age of the commencement of the deferred annuity is more in harmony with the client's right to choose, since, if the regulation allows the most flexible version, phased withdrawal, which can be suspended and restarted, the term of the temporary annuity cannot be clearly defined.

So the question regarding the term of the temporary annuity can be reworded to state: should the client be able to choose the starting age of the deferred annuity, or is it expedient for it to be fixed by legislation for the entire market. In favour of this predetermination of the starting age (e.g. uniformly at the age of 80 or 85) is the fact that this would to a great extent eliminate the risk of adverse selection,[65] which would result in an increase in the premium of deferred annuities.

[65] I note that the trend of adverse selection is not quite clear. Arguments support that primarily those might choose a high age for the commencement of the deferred annuity who expect a long life, since the protection of the deferred annuity will be effective from a higher age in their case. However, people with a shorter life expectancy may

I have already stated that (to prevent adverse selection) the annuity must be defined at the time of retirement. Apart from this, in this model as well as in any other model in which the annuity is split, regulation should set two objectives:

1. It must assure that the two annuity parts are connected so that during the entire period of retirement the pension paid from the private pension pillar should as far as possible be balanced in real terms, or at least should not decrease.
2. It must assure that in the temporary annuity phase, when the annuitant has a wide scale of choice, this does not result in the consumption of the major part of their savings during the first part of the phase.

The first objective may be achieved partly via indexing regulations and partly by rules concerning the calculation of annuity payments. In the case of the latter, the goal of regulation should be to assure that when the annuity commences the benefits from the two annuity parts are still in harmony, while the former must assure that this harmony continues and the two parts of the annuity do not "slide apart".

The obvious rule when calculating the level of annuity is to calculate the annuity payment as an immediately commencing life annuity at the time of retirement with respect to the full capital. After this, the so determined monthly annuity payment is used to calculate the capital portion to be used for the deferred annuity. The question of course arises: what kind of annuity should this be? There are two possibilities: a simple life annuity and an annuity with a guaranteed period at the beginning (front-end), where the term of the guaranteed period is identical to the period of deferral of the deferred annuity.

The other question, if the client may otherwise choose what annuity to purchase during the period of deferment, i.e. during the term of the temporary part of the annuity, is whether this decision should effect whether the annuity payment is calculated on the basis of a simple annuity or a guaranteed period annuity, or whether it should instead be calculated on the basis of one or the other independently from this decision. In other words: should the immediately commencing annuity calculated at the time of retirement always have no guaranteed period, or should this depend on whether the insured person chooses a temporary annuity, a guaranteed period annuity (so smaller payment at the beginning) or an immediately commencing annuity?

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think about minimising the amount spent on deferred annuity because they would end up not using it, so they might want to instead maximise the capital they leave as an inheritance.

Since, as in the case of almost every choice, the problem of adverse selection emerges, it is expedient for the insured person to again not have the opportunity to choose in this case. Though if the regulator does not give annuitants the right to choose, then it must decide itself which immediately commencing annuity should be used as the basis for the annuity payment. In the case of simple annuity, a higher annuity payment will be determined for a higher age, and if the client chooses phased withdrawal, the monthly annuity payment will be smaller than this during this withdrawal phase, so this solution places greater emphasis on security at a very old age. In the case of guaranteed period annuity, however, if the client chooses temporary life annuity during the deferral period, the level of annuity payments could decrease when the deferred annuity commences.

If the client chooses a temporary annuity that was presumed at the calculation of the deferred annuity payment, the magnitude of the two annuity parts are in harmony with each other at the beginning. So the rules of indexing must prevent the shifting apart of the two annuity parts that are in synch with each other at the start. This occurs automatically with inflation-linked indexing, at least in the event that the insured individual chooses a temporary life annuity or an annuity certain in the first annuity phase, since by choosing they underline the fact that the schedule determined by the insurer for the withdrawal of their money is important to them. Nevertheless, indexing to inflation is contrary to the logic of phased withdrawal as in this case the insured individual supports the option of free choice, but this is significantly impaired by the restriction of the investment, as is characteristic of inflation-linked indexing. So in this case the regulator must decide what it views as more important: the seamless joining of the two annuity parts or the free choice of the insured individual.

At the same time, the suitable joining of annuities can also be assured via return refund indexing if the reserve of the deferred annuity is not immediately transferred to the well-capitalised provider, but the provider of the phased withdrawal invests it in the same manner and with the same return as in the case of the first annuity part. In this case an agreement is only concluded with the well-capitalised provider with regard to the tariff and the fact that the reserve of the deferred annuity will only be transferred to it if the insured person dies (and accordingly it will not have to provide a service), or if the insured person survives until the end of the deferral period.

The second objective, that the insured person does not consume the entire reserve during the initial period, can be assured by a few simple rules. In the case of temporary life annuity and annuity certain it is sufficient to assure that these annuities cannot be annuities that diminish with time, and cannot come to an end before the deferred annuity commences. It is not important to prescribe

that they may not come to an end sometime later; it will probably not be important to a reasonably-minded annuitant if they receive two different types of private pension annuity at some point in the middle of their retirement. It is also not important to define the starting date; it can occur at any time following retirement. In fact, if the insured person has sufficient accumulated capital it may even begin prior to retirement.

The case of phased withdrawal is a little more complicated. The same applies to the start as to temporary life annuities and annuity certain. Since the point here is that the annuitant may voluntarily suspend and restart it, and the withdrawal of money does not necessarily have to take place on a monthly basis or in equal amounts (although these will probably be the default), more general rules are required than in the cases above. Disregarding the fact that the capital may fluctuate significantly during the term depending on the performance of the investment, so if (apart from money withdrawal) we assume an unchanged (or uniformly increasing) level of capital, the regulator has the following possibilities at its disposal, depending on what it would like to achieve:

1. If the objective is that the money that can be withdrawn should not diminish from one month to the next, a possible rule is that the annuitant cannot withdraw more money from the capital during a given month than the balance at the beginning of the month divided by the number of months remaining until the commencement of the deferred annuity. If someone regularly withdraws the same amount and never interrupts the withdrawal, this rule to all intents and purposes provides them with an annuity certain. If they interrupt the withdrawal at some point, the money thus saved may be withdrawn uniformly over the remaining months. The disadvantage of this solution is that it makes it impossible for someone to withdraw a large amount all at once, for example after having not withdrawn anything for a long time.
2. If the objective is to enable the withdrawal of the (indexed) level of the starting annuity payment each month, then a possible rule is that the annuitant may at any time withdraw a sum corresponding to the difference between the existing capital and the following product: the starting level of the deferred annuity indexed according to the investment return achieved until that time, multiplied by the remaining number of months. This rule also permits one-off, larger withdrawals.

So, if the second rule is accepted the insured party may be allowed to withdraw larger amounts as a lump sum from his/her account from time to time, by suspending the withdrawal of money and thus accumulating adequate “savings”. The question is whether the withdrawal of larger amounts of money is already

possible at the time of retirement. As this question is a general one and applicable to every pension model, I will provide the answer in the general section. Another question is whether temporary annuities that have already commenced can be changed within the three possible types. There is no theoretical obstacle (at the most administrative and cost-related obstacles) to shifting (back and forth) between annuity certain and phased withdrawal. However, ongoing temporary life annuities cannot be converted into the two other annuities because of the danger of adverse selection. This danger is much smaller in the case of a reverse shift, so theoretically there is not obstacle to shifting from an annuity certain or from a phased withdrawal to a temporary life annuity (although few people will be interested in this possibility).

This model emerged in Hungary much earlier than it was published by the OECD as a somewhat spontaneous "popular requirement" and the possibility arose independently in various forms both in Hungary and abroad. I formulated it in 2007, when I observed that a broad expectation with regard to private pension funds is that they should be inheritable, and the possible smallest part of it should be shared in risk community with others. At the time, many people said that in their view phased withdrawal is a self-explanatory solution, which they regard as a direct consequence of the logic behind private pension funds, and they could not imagine any other type of annuity. The "experts" "translated" this into a feasible system by adding deferred annuity to the phased withdrawal as an "emergency cover".

This formally appeared in Hungarian literature via Kolos Ágoston (Ágoston [2008]), and in international literature it is mostly advocated by the OECD. With regard to the OECD this model is also an official recommendation for countries that are still only designing the annuity system of their private pension pillar. This was officially put forward in Paris on 12 November 2008 at the OECD Seminar on "The Payout Phase of Pensions, Annuities and Financial Markets", and can be found in the related publications (Antolin [2008a] and OECD [2008a]). According to these, two important risks must be managed: the risk related to the timing of the conversion into annuity and the longevity risk. The most important payment options with relation to the accumulated capital are lump sum, scheduled withdrawal and life annuities, and their combinations, between which a balance must be found between flexibility and protection. Therefore, the OECD believes the best solution is to combine phased withdrawal with a deferred annuity (purchased at retirement). This latter would commence at about the age of 85 and would require about 15% of the accumulated money.

Although Antolin does not cite antecedents, the idea has a short history in technical literature, although the Canadian Moshe A. Milevsky did not propose his idea of Advanced-Life Delayed Annuities (ALDA) as a model of payment

of formal DC systems. This may be regarded as a new idea because in 1999 even Blake said (Blake [1999]) that the deferred annuities market is so small, especially in case of a very long deferral, that in practice the market doesn't even exist. If these are nevertheless available for purchase, then the conditions are generally extremely bad. Furthermore, the market is also shrinking as a result of mortality uncertainties.

In his article "Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities (ALDA)" (Milevsky, Moshe [2005]) Milevsky mentions J. B. Stephenson's 1978 article as an antecedent ("The High-Protection Annuity. Journal of Risk and Insurance 45(4): p. 593-610"). In his article, Milevsky proposes a deferred annuity with an ultra-long deferment period, which in addition would also be a regular premium and would be indexed to inflation. In his view, its targeted community could be members of the North American population who do not have a traditional DB pension. He shows that with a monthly payment of USD 1 during the pension savings period the client would receive a monthly benefit of USD 20-40, provided that the waiting period (deductible) is long enough. In the long term this could diminish the psychological obstacles to purchasing a voluntary annuity in an environment in which promoting DC plans at the expense of DB plans and the very low level of voluntary annuity purchases call for a new solution. Annuity payments would commence at a very high age of 80, 85, or 90.[66] He mentions that one of the novelties of the idea is also the regular nature of the premium, as this product already existed as a product with a one-off premium, but was a commercial failure.

Milevsky's idea evoked a range of literature. According to Gong-Webb [2007] the product has 3 major advantages: 1. households maintain their liquidity until its commencement, 2. it is preferred by households, and 3. with its help the consumption of the capital can be managed using simple rules of thumb. However, in their view it is still questionable whether this product does in fact diminish people's aversion to converting to annuities. Therefore they propose that ALDA should be made the default in 401(k) plans, which is practically the same as the OECD model. However, they note that this solution is disadvantageous for people with relatively short life expectancies.

[66] My own idea: unforeseen changes in mortality could be managed by adjusting, i.e. increasing, the starting age.

4.3.3. THE ALTERNATIVE SOS MODEL

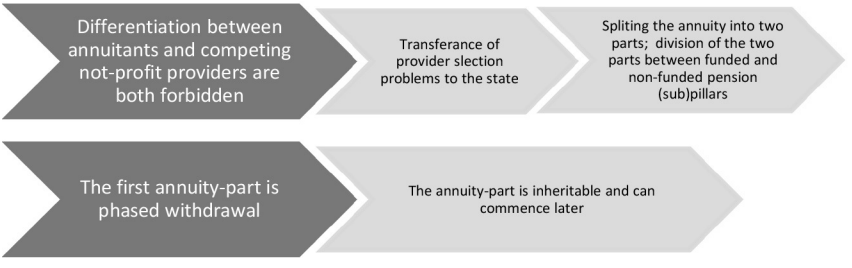
Based on the above, the model's most important elements and their internal relationships are as follows:

Table 15: The most important elements of the alternative SoS model

Competing or non-competing providers	The service is provided by several competing providers, between whom the clients may choose (with certain restrictions).
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The competing providers only provide the first part of the annuity, mainly as a phased withdrawal that is relatively freely determined by the clients (so not, or not necessarily, as a temporary life annuity). However, this first part is the first part of the whole SoS system annuity, so no annuity is received parallel to it from the SoS system. However, the deferred annuity is fully provided by the SoS, so no more annuity is received from the private pension system during its term. This structure among the competing providers completely manages selection problems that arise from the prohibition of differentiation (or rather some level of selection problems may remain if the clients can choose temporary annuity instead of scheduled withdrawal).
Is the annuity composed of one or two parts?	As a consequence of the above, the annuity (which in this case is the total mandatory old age pension) is composed of two parts, where the first part is phased withdrawal from the private pension capital that permits annuitants to defer the commencement of payments, and potentially enables the majority of the capital to be left as a legacy (providing they do not consume it in the meantime). (This is often referred to as "the client has full control of his/her assets".)
Managing provider selection problems	In this structure there are no remaining problems with relation to choosing between providers.
Is the provider a for-profit or non-profit organisation? (Who ultimately bears the mortality loss?)	The issue of mortality loss is not raised in the case of this model with respect to the fact that deferred annuity is a pay-as-you-go product, with relation to which this question is not usually examined. Competing providers may be both for-profit and non-profit providers, but in a situation where non-for profit providers already exist, there is no reason for them to be replaced by profit-orientated providers according to this model. It must, however, be noted that not examining the mortality loss is not a self-explanatory feature of pay-as-you-go systems. Pay-as-you go systems with (primarily NDC-type) individual accounts are capable of raising the question of managing longevity risk, and the fact that this must always be charged to the given cohort.
Indexing	Theoretically it can be solved using either indexing technique. There are arguments for and against both solutions. It is in fact also possible that the insured individuals are allowed to choose the investment, and in turn the indexing technique. However, in the case of this model it is almost certain that the two annuities "shift apart" to some extent, since indexing almost certainly occurs according to different principles with relation to the two annuity parts.

The abovementioned relationships are represented in the diagram below:

Diagram 8: The internal relationship between the elements of the alternative SoS model



The essence of the model is that the state pension (SoS annuity) and the private pension (mandatory annuity) do not exist in parallel, but the two annuities follow one another, meaning a pensioner receives annuity from one only source at a time and not from two sources simultaneously. Naturally either the SoS or the mandatory annuity could be the first of these annuities, but I exclude one of these possibilities to all intents and purposes. Although it is theoretically possible for the SoS annuity to commence immediately after retirement, last for a certain period or until a certain age, and then (if the pensioner is still alive at the time of maturity) be replaced by the private pension annuity, but this would be an inexpedient solution. Not so the other possibility, i.e. its reverse, when pensioners first consume their private pension savings after which, from either a pre-determined age or at the end of the term, the SoS annuity commences. Naturally, this pre-determined age is higher than the actual age of retirement and the usual official age of retirement.

The above two options are not symmetrical because the point of splitting the annuity is for the longevity risk to be either partially or fully assumed by the provider that provides the deferred annuity. From this perspective, we can to all intents and purposes exclude the first possibility, i.e. that this is the provider that provides the private pension annuity (insurance companies, funds and competing providers in general), because the SoS, backed by an explicit or implicit state guarantee, is capable of bearing the longevity risk to a much better extent, and more importantly at a much lower cost, than competing market providers. For them (and ultimately for their clients), assuming the longevity risk is much more expensive, in addition to which it is also uncertain whether all of them will be capable of bearing this risk, i.e. whether one or more of

them might go bankrupt, leaving behind old people without care, who will eventually end up requiring the assistance of the state after all.
So in the alternative SoS model I assume that the pension assured by the SoS system (e.g. by the mandatory pay-as-you-go and funded systems together) is composed of two parts: following retirement, pensioners first consume (or not as the case may be, it's their decision!) their private pension savings, after which the SoS pension commences from a later age and lasts until the end of their life.
The question naturally arises: how long should the private pension last and when should the SoS pension begin? Should the transition between the two be defined according to age or according to the duration of the private pension? The two theoretical possibilities are as follows:

- when the annuitant reached a pre-determined age, e.g. at the age of 75, or
- at the end of the pre-defined, fixed duration of the private pension annuity.

After a short analysis it can be easily recognised that it is only expedient to select the first option, in view of the fact that transition at a pre-determined age has the following advantages:

- it saves the SoS annuity (and in fact the whole system, if the temporary annuity is an annuity certain) from adverse selection on the part of the pensioner (i.e. with respect to the date of retirement).
- private pension annuity fits in well with various types of SoS system, in contrast to a pre-defined transition based on the term, which only fits in well with SoS systems that maintain individual accounts (NDC, or point system), because otherwise equity problems (and via this incentives problems) may arise. The essence of the equity problem is that people who select a later date for retirement should benefit from a suitable increase in their SoS pension, but this can only be achieved through the accurate calculation of entitlements, i.e. via an individual account. (Naturally, despite this it is worth making the SoS system an individual account system "in all events", i.e. even in the case of a pre-defined transition, because of the advantages of such systems.)

In the case of both transition times it is true that the system promotes a later retirement (since until retirement money is accumulated on the private pension account, and can be inherited). If the ratio of the funded pension is sufficiently high within the system, the transition defined according to age will in all events take place at an age at which the majority of the pensioners actually retire, so whether or not the system promotes the deferral of the SoS pension is unimportant, meaning that in this case there is again no advantage to a transi-

tion time determined according to term. However, a major disadvantage of determining the time of transition according to when the term of the private pension ends is that this cannot be fitted well to the SoS pension. By fitting I mean that the transition from one annuity type to the other should not result in pensioners experiencing a change in their level of annuity payments, or at least not a major jump. If we suppose that each annuity part is actuarially correct, then in the case of a transition defined according to age the SoS annuity can be foreseen relatively well by the pensioner. If they decide not to begin consuming their private pension savings, then their (potential) private pension annuity will increase continuously compared to the SoS annuity, but this is not a problem because they are not required to consume it. If, however, the turning point is defined according to the term of the private pension annuity, then we must assume that the pensioner will consume their private pension savings (beginning at some point, which could also be the official retirement age, according to the discretion of the pensioner), meaning a private pension annuity level can be calculated for them. If the term of the private annuity is pre-predetermined, then the annuity level received during this period will not necessarily be in harmony with the actuarially correctly defined SoS annuity level, because this will be different at different ages.

Accordingly, in summary I think that the transition between private pension annuity and SoS annuity must definitely be determined according to age. The next question, of course, is what should this age be? The principle is obvious:

1. The term of the temporary annuity must be added to the official (minimal) retirement age,
2. The term of the temporary annuity must be determined so that average assets accumulated on the individual private pension accounts roughly assure the same monthly annuity (life or certain, depending on regulations), as provided by the deferred SoS annuity based on the accumulated entitlement.

Together with the determination of the transition, the next logical question is how to switch to such a system. This problem can be broken down into three categories:

1. Only the SoS pension exists, but the intention is to split it into a temporary private pension and a deferred SoS pension,
2. Both private pensions and deferred SoS pensions already exist, and the intention is to increase the ratio of private pensions (i.e. to raise the age limit at which the SoS pension commences),
3. Both private pensions and deferred SoS pensions already exist, but parallel to each other.

We can generally state that transition to this model is greatly facilitated if the logic of SoS pensions and private pensions correspond, so if for instance they both operate in a DC system, or NDC system in the case of the SoS pension. For my part I examine only this case, while noting that if the SoS pension does not operate according to an NDC system and we desire such a transition, than we should begin by transforming the SoS pension to an NDC system. Then, disregarding the other problems that result from the transition, such as the fact that those involved need time to get used to the fact that the system they have been familiar with until then has suddenly changed, technically speaking the transition can take place overnight with respect to people who have not yet retired:

1. The method of transition is that the existing, uniform SoS pension is split into two parts at the same time as the entitlements accumulated until that time on the individual (NDC) accounts are also split into two and according to the same ratio (i.e. as if the originally uniform annuity had been split into two parts from the very beginning). The state recognises as explicit sovereign debt the entitlements kept on the NDC accounts until that time and which from then on will go towards the private pension, and the state formally issues sovereign debt bonds with relation to them (within an adequate maturity structure), which then serve as the basis for the private pension annuity. Naturally, in this case the private pension annuity can only be a (temporary) life annuity, because the original SoS annuity, or rather the NDC balances that serve as a basis for the new annuity, are only sufficient to this extent.
2. The logic behind raising the age limit is the same, at least if temporary annuity was provided until then.
3. The transition from existing parallel annuities should also not cause too significant a problem with respect of those who have not yet retired, providing that the transition to the parallel solution was performed properly at the time. This cannot be said, for example, in the case of the Hungarian reforms of 1997, since the people who switched to the private pension system lost all of their previous entitlements accumulated under the SoS system. The consequence is that the relative proportion of capital available to the two types of annuity is different at the time of retirement for different generations: the older the generation, the lower the ratio of private pension capital (assuming of course that everything else, e.g. returns, is equal). In such cases the legislator should not determine a uniform transition, but one in which the transition age increases year by year (for a time).

If the target of the temporary annuity is an annuity certain rather than a life annuity, then the problem of lack of coverage may be solved so that in comparison to the above, the transition:

- is postponed by a few years after the splitting of the contribution, and of course the contribution payable to the private pension fund must also be determined at a level that also cover the costs of legacy, or
- inheritability must be made available gradually: first only for a short time following retirement, then later gradually extended to the entire term of the temporary annuity.

The two phases of the pension annuity, i.e. the parts of the annuity that are derived from the funded system and the pay-as-you-go system, should be harmonised, meaning they should be determined in such a way that the monthly annuity from both pillars is the same. The two parts of the annuity may differ from each other for one of two reasons, so these factors should be taken into consideration when harmonising them:

1. The funded and pay-as-you-go annuities defined at the time of retirement are already different,
2. The two parts of the annuity are indexed differently during the funded annuity period, so they "shift apart".

Harmonisation only appears as a problem if the funded system actually provides an annuity-type service, though even in the case of phased withdrawal it is likely with regard to the majority of people affected.

At the time of the retirement, the harmonisation of the two annuity parts can be achieved by regulating the age of retirement, as I have already discussed. It is a great help If the pay-as-you-go system is an NDC one, since at the time of retirement (i.e. when payment of the funded annuities commences) the annuity of both pillars can be calculated according to the same principle on the basis of the accumulated (actual or notional) capital. As the accounts of funded and pay-as-you-go systems carry interest according to a different logic until the time of retirement, it is not certain that the monthly pension calculated for the two pillars will be identical; the starting pension from the funded system may be smaller or larger than the later one from the pay-as-you-go system. If the pension from the funded system is lower than the one from the pay-as-you-go system, it is not a problem in itself I think, because pensioners will experience an increase in their pension, which will generally be perceived as a positive change. However, if the pension from the funded system, i.e. the starting pension, is higher we should avoid allowing a pension decrease when the switch to a pay-as-you-go pension occurs, which can be realised simply if the system

allows pensioners to (at their discretion) withdraw the collateral of the excess annuity as a lump sum at the time of retirement, or later.

The "shifting apart" of the two annuity parts can be prevented if the rules on indexing are identical for both pillars. The following indexation possibilities may be considered:

- Indexation within the pay-as-you-go system is to a certain extent a matter of decision. The most customary indexing is based on the price index or wage index, or on a combination of the two. The Hungarian pension system seems to be increasingly moving towards price indexing, although this is strongly dependent on politics and therefore uncertain. The NDC system attracts wage indexing to some extent, because it allows a fairly simple connection between increased income and increased obligations, since one of the most important factors of increasing income within the system is the wage index. Indexing the NDC also depends on whether the target of the NDC system is self-supporting, i.e. on whether or not ongoing pension payments and ongoing income remain in harmony in the long term.
- Indexation within the funded system must depend on investment performance. The rate can be linked to some of the macro indexes (wage index, price index, growth of GDP, stock exchange index, etc.) if the investment itself is fixed and occurs using facilities that result in a level of return that depends on the given macro index. From this perspective, it seems that in practice there are two kinds of macro indices:
  1. Shares in a composition corresponding to a given stock exchange index. However, the stock exchange index is too variable to be a suitable basis for the indexation of pensions, not to mention the fact that with such indexation the pensions of the funded and pay-as-you-go systems can certainly not be harmonised,
  2. Inflation-linked bonds. There are currently no bonds whose returns are linked to other indices on the international market (apart from disaster bonds, which are obviously irrelevant here).

So the shifting apart can be best avoided if indexing to inflation is applied in the pay-as-you-go system (which is the usual international practice), while in the funded system regulations prescribe indexing to inflation parallel to investing in inflation-linked bonds.

This model was raised or "invented" by several experts more or less independently from each other, or rather, according to various news sources it is being continuously raised and reinvented, generally not in written form. So far I have found two sources of such proposals in written form (although these

papers are not necessarily the source of the idea and they are generally not referred to in this context).

The model is first mentioned in World Bank [1994], just in the form of a short idea (not detailed), who raised it explicitly with respect to the annuity problems of the DC system. According to the paper, the remaining lifetime is difficult to forecast, even on the level of cohorts, and this is why the providers cover their annuity exposure with reserves or life insurances. In their view, a possible solution is that private pensions should only be used during the first period of the pension, e.g. during the first twenty years, while the national pension system would take over at a higher age. In national pay-as-you-go systems the cost of a higher life-span is transferred implicitly to the next generation, which the state can realise via its power to levy taxes.

The second instance occurred explicitly with reference to the Hungarian circumstances in Point 20 of the paper written by Barabás-Bodor-Erdő-Fehér-Hamecz-Holtzer [2006]. According to this, "Annuity payments that last from the time of retirement until death may also raise certain questions. This was not the case until we had only one national pay-as-you-go social security system. Nowadays though, we already have a private pension fund too (and in fact other recognised pension products may also become available in future). There are very few countries in the world where one can find annuity markets that operate competitively and therefore effectively. Hungary is not one of them, and we are unlikely to be so lucky in the future. We will have an oligopolistic, overpriced, non-transparent annuity market (why should this be the only exception?), where we will have to convert our pension fund savings into annuities. But then why do we have to force people to buy annuities at all? If we think it is unnecessary and allow people to access their capital when they retire in the form of a bank annuity (phased withdrawal), then there is no reason at all for the social security system and the second pillar to begin annuity payments at the same time. So we should consider whether we should perhaps rethink the principle of annuities paid out in parallel and instead switch to a system of annuities that are paid one after the other such that the longevity risk is covered by the social security system, whose risk community is better qualified for this in view of its size, while with respect to fund payments young pensioners should be given the opportunity, within certain restrictions and incentives, to schedule their annuity payouts; the general principle of payouts from retirement until the time of death remains, but gains a different interpretation with respect to the various subsystems".

4.3.4. THE INSURANCE (RETURN-REFUND) ANNUITY MODEL

In the literature, I (Banyár [2007]) drafted this model, and it was approved as a regulation concept by the then Ministry of Finance. The Hungarian annuity regulations adopted in December 2009 partly include this model (and partly the central provider model). The literature presented in the chapter on indexation, specifically Réti [1999], can be cited as antecedents to the model. In Réti's view, indexation can be nothing other than return-based (Réti does not yet mention the problems of managing mortality gain, or that this cannot take place on the basis of a pension fund). This is also Stahl's standpoint (Stahl [2005]), according to whom indexation should occur on the basis of the combined investment and mortality performance. Although as Stahl also allows the nominal decrease of the annuity, his proposal can perhaps be interpreted as belonging to the pension fund annuity model.

Changing providers during the term, which is an important element of the model, is also my own concept. The literature only alludes to it in passing. Réti [1999] and Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] also raise the idea that the provider may be changed during the annuity payment phase, but only as something that would seem to be included in current law; they do not discuss the possible problems and consequences. The latter paper says only that: "Some of the provisions of the Act suggest that according to the intention of the legislator movement among the providers is also allowed during the course of annuity payment. This may have an effect of adverse selection." The idea is also raised by Stewart [2007], who mentions it among other possibilities without elaboration as a possible way of encouraging competition.

Based on the above, the model's most important elements and their internal relationships are as follows:

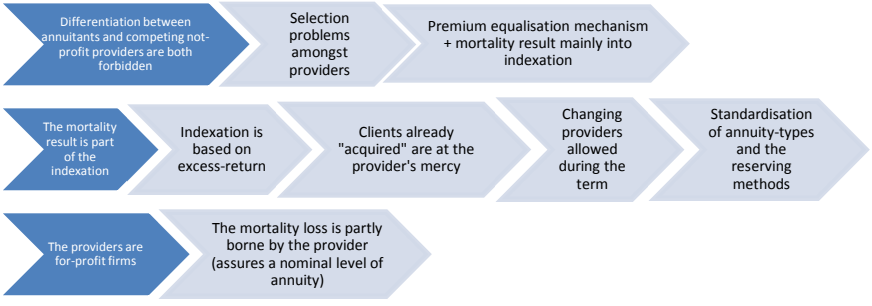
Table 16: The most important elements of the insurance annuity model

Competing or non-competing providers	Clients may choose – with certain restrictions – from among several competing providers.
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The providers provide the entire annuity, so selection problems between providers may emerge, which need to be managed.
Managing provider selection problems	Provider selection problems are managed at the beginning of the term via a central minimal pool (a premium equalisation mechanism), while selection problems that emerge during the course of the term, e.g. selection effects hidden within the mortality loss, are partly transferred to the clients via the indexation mechanism. From another perspective, this also means that the mortality profit also belongs to the client, but clients only bear the loss up to an extent that it does not endanger the nominal value of the annuity.

Is the provider a for-profit or non-profit organisation? (Who ultimately bears the mortality loss?)	The providers are privately owned, for-profit organisations (insurance companies, or institutions that operate and are regulated similarly to insurers). They could also bear the entire mortality risk, but its "option fee" would be too high for the clients, so they only bear a part of this risk.
Is the annuity composed of one or two parts?	Both versions are possible but the regulator must choose, not the individual clients.
Indexing	Excess return-refund indexing, partly because otherwise there wouldn't be much point in having competing providers, and partly to enable the management of the mortality result and provider selection problems via indexing. The technical interest rate must be fixed in the case of this indexing technique so that adverse selection occurring due to different technical interest rates can be avoided. Since with this indexing technique the clients would be exposed to the provider who acquired them, because the provider does not promise (cannot promise!) a rate of indexing in advance, competition for clients must also be maintained during the annuity payment phase, meaning that changing provider must also be allowed in this phase. However, this is only possible if reserving is standardised, meaning it occurs according to a predefined technical interest rate and mortality table.

The abovementioned relationships are represented in the diagram below:

Diagram 9: The internal relationship between the elements of the insurance annuity model



We also receive the above model if the starting point is that the market makes it difficult to issue bonds that are linked to an economic index (e.g. price index), or if such bonds do exists, they cannot be counted on to have an appropriately long term or be sufficiently available in future. Accordingly, a fixed benefit annuity (indexed to an economic index such as inflation) is not an option, only annuities with varying payments.

Technically, the annuity operates somewhat differently in small and large risk communities. The problem of possibly having a small risk community does not arise with respect to the central provider and central pool models (or it does, but only for a short time when the whole system is initially launched), and its significance is small in models that combine a deferred annuity with phased withdrawal. However, in this model the risk community might be shared between a great many providers throughout, so the risk community could be small (mainly after the launch of the whole annuity system or the appearance of individual providers), and so the mortality result could be extreme, meaning the initial premium equalisation mechanism cannot handle all the problems that the other solutions can. This means that separate rules are required in the case of small risk communities.

In other words, the "normal" functioning of the system assumes a large risk community in which there are no extreme fluctuations in the mortality result. In the event of small risk community the highly fluctuating mortality result cannot be made part of the indexation, but must belong to the provider, so in this case the mortality loss is borne by the provider against its solvency capital. This is another reason why it is important that the provider is a for-profit organisation with solvency capital.

Below I elaborate on the operation in a "normal" case (i.e. supposing an adequately big risk community) and in the case of small risk community.

With regard to the **"normal" case** it is worth first reviewing the most important technical assumptions with respect to the annuity, which are as follows:

- The premiums are unisex, and are only dependent on starting age and the sum assured, but reserving is differentiated according to gender and education. (As an alternative, differentiation of the premium according to education should also be considered; this would be of significant help in preventing the system from being accused of pervert redistribution, i.e. that the poor are subsidising the rich via the annuity. Naturally, the reserve would still be differentiated according to education.)
- The indexing of the annuity depends on the combined investment and mortality performance, where only a maximized part can be deducted from the gross investment yield to cover the asset management fee. The mortality performance is calculated separately for each risk group according to gender and education, and is spread within the group.
- Both the unisex table used for premium calculation and the tables used for reserving, which are differentiated according to gender and education, are generated centrally. Of these, the use of central tables is mandatory for reserving. Each mortality table refers to projected mortality.

- The central mortality tables are regularly refreshed based on experience and changed expectations. The most recent tables must always be used for reserving. These tables may also contain future generation effects, meaning separate projections are prepared for different age groups, and these are adjusted separately.
- The technical interest rate is determined centrally and it is recommended to use the 0% rate (or a decreasing rate that eventually drops to 0% during the term of the annuity, see above!).

The annuity is recalculated every year, if the insured individual is still alive at the beginning of the year. The insured individuals are managed separately according to risk group, i.e. according to gender and education, meaning that separate mortality tables are applied for them when reserving and the annuities of the different risk groups are re-calculated accordingly. The re-calculation factors are:

- The starting point is the individual premium reserve ("annuity account") of the insured party (annuitant) at beginning of the previous year.
- This decreases during the course of the year by the annuity payments, and it increases by the share of reserve assigned to the annuitant from the premium reserve of those who died during the year[67]. This is how the calculated premium reserve is computed by the end of the year or by the beginning of the next year. This will be different from the actually necessary premium, because in the meantime the mortality projection will have changed (if modified), and the annuity payment will probably also be different from its possible value because of the investment and mortality result.
- If the institute entrusted with preparing them issued a new mortality projection at the beginning of the year as required, the reserve required for the surviving insured persons is defined on the bases of the actual annuity payment, the age of the insured party and the new mortality table. If there is no new mortality projection, then it is determined according to the previous point.
- If the required reserve calculated in this manner is higher than the reserve available (which has decreased compared to the beginning of the previous year because of the paid annuities and increased with respect to investment profit, in addition to which the reserve of the deceased has now also become available to the surviving annuitants), then the

[67]Without a 0% technical interest rate, it would increase by the supposed yield of the technical interest rate.

missing reserve is supplemented by the insurer from its solvency capital, and the level of the annuity payment remains as it was in the previous year.

- If the reserve requirement thus calculated is smaller than the reserve available (as will presumably be the case in the majority of situations, partly on account of the projection and the 0% technical interest rate), then the annuity payment, and naturally also the reserve, must be increased in the given year by a similar ratio.

As the result of the above method, in comparison to the previous year the annuity for the given year increases precisely by the investment yield, if:

- Precisely as many people died in the previous year as forecast by statistics,
- The average annuity of the deceased is identical to the average annuity of the surviving annuitants,
- The mortality projection did not change compared to the one used in the previous year.

The annuity increases at a higher rate than the investment yield if the number of deceased annuitants exceeds the expected number, or if the average of their annuities is higher than the average for the total annuitant population, or if the mortality projection is modified such that the remaining life expectancy decreases, and vice versa if a change occurs in the opposite direction. Naturally, these factors may not necessarily move in the same direction within a given year, and our final annuity increase will be the resultant of all these. If the factors move in an identical direction, e.g. if the investment yield was low, the actual mortality has decreased, especially among those with higher annuity payments, and the projected mortality also decreased, then it may happen that the annuity decreases compared to the previous year (if there is no solvency capital).

In case of a **small risk community**, the system operates a little differently. Every differentiated sub-group of the risk community (according to gender and education) must achieve a certain minimal number of members at a provider, or if some of the providers decide to unify their annuitant portfolios, i.e. to organise a pool, then this is applicable to the entire pool.

If all the insured individuals of a provider or a pool together do not achieve the minimum number of members in a given year (as characteristically occurs at the launching of an annuity system or if a new provider enters the market at a later date), then in that given year the total mortality result belongs to the provider, i.e. the loss will not be settled against the reserve of the clients, and the profit is not distributed between them. If this results in a reserve deficit, it

must be covered from the provider's solvency capital. The reserve of clients belonging to different risk groups must also be calculated using the proper differentiated mortality table in such cases.

If the total number of insured individuals of a provider (or pool) exceeds the minimum, but it has a risk group whose numbers are lower than the necessary minimum, then the smallest differentiated sub-group must be temporarily combined with the second smallest (and third smallest and so on) subgroup until their total reaches the minimum number. If at this point there is still one or more subgroup(s) in which the number of members reaches the required minimum, then that (those) must be managed according to the normal conditions, while the combined, small sub-groups must be managed in a consolidated manner, which means that the mortality result of the consolidated group must be handled together, while the separate mortality tables continue to be used for reserving.

The above examination must be conducted every year, and on these grounds the consolidated group may cease to exist, or there may be a need for consolidation in the following year.

In view of the competition that exists during the annuity payment phase, i.e. in view of the opportunity afforded to the client to change providers, it is important that the various annuity-related activities of the different providers be harmonised in a timely manner, i.e. that what providers do and when during the course of the year should be pre-determined. It also follows from this that we must think in terms of calendar year rather than insurance year, because the former is uniform for everyone.

A possible schedule for annuity-related activities is as follows. The significant changes generally take place in the first quarter of the calendar year:

- In the first two months of the year the insured persons must report their intention to move to another provider, and this transfer will coincide with the increase of the annuity payment.
- At the beginning of the third month, once the transitions are known, the size of the risk groups is checked and, if necessary, a consolidated risk group is determined.
- For this procedure, we consider a deceased annuitant to be one who died during the previous calendar year, the total sum of their actual individual account/reserve is distributed, from which we deduct the sum of annuity that was actually paid out and add the combined mortality and investment return distributed during the previous year.
- The actual number of the deceased (and of course their concrete identities), is determined in March, together with the investment return of the previous calendar year.

- By the end of February the most up-to-date mortality projection must be prepared (if we agree on an annual adjustment. However, the adjustment of the mortality projection occur less frequently, but this does not change the logic, it must still be completed by this time).
- At the end of the first quarter (in March) we define the combined investment and mortality profit, and the size of the buffer, taking into account those who change providers (both to and from the provider).
- From the beginning of the second quarter (beginning in April), annuities may actually be increased (if they increase all).

Another manageable factor is that new entrants and those who died during the year are not included in the above computation, since from this perspective they will only be taken into account at the beginning of the following year. Naturally, the provider immediately begins paying annuity to new entrants, and the annuity payments of the deceased are immediately terminated.

An important element of this model is the **premium equalization/settlement mechanism**. This requires a fix point to which the market can correlate, so a centrally prepared, unisex projected mortality table must exist on the market. This is not a problem, of course, since it is already available in this model (because of allowing annuitants to switch providers mid-term).

When a client purchases an annuity from a provider, they pay a net premium calculated on the basis of the centrally determined unisex mortality table plus a share of the costs. To meet its mandatory reserving requirement the provider immediately calculates the individual reserve by using the differentiated mortality table corresponding to the gender and education of the client. There is a difference between the actually collected net premium based on the unisex mortality table and the similarly calculated necessary reserve, which may be even larger than the unisex premium (e.g. in the case of a woman with a university degree) or lower (e.g. in the case of a man with only elementary education). The difference must be transferred to a central body, a clearing house that operates the settlement mechanism, and from which providers receive transfers. There may be a choice of operating method, i.e. whether payment should occur immediately or regularly after accumulating for a certain period, and there are arguments for and against both.

As I have already mentioned, in the case of this solution the problem may arise that the losses and profits generated during a given period at different providers may not necessarily offset each other, since on the one hand the unisex table may not be accurate, and on the other we cannot assume that clients who represent good or bad risks for providers are concluding annuity contracts in the same proportion all over the annuity market at every moment.

If the central mortality projections were correct, the balance of profit and loss will be zero in the long term, but that almost certainly will not be the case in the short term.

What I would propose as the best solution to this problem would for the final balance generated by the premium settlement mechanism and the preparation of central mortality projections to be related, and for one of the guarantees that projections are suitable be that this balance must tend towards zero in the long term. Temporary deficits are covered by the state, but are repaid from the transitory surplus, but these surpluses must indeed be transitory, meaning money must not be systematically extracted from the annuitants.

However, since attaining a zero balance of the premium settlement mechanism does not require the life expectancy projection to be correct, this rule is not a proper test of mortality projections, only of whether the unisex table is correct or not. In fact, a neutral balance could also be achieved within the system by manipulating the life expectancy projection, meaning further regulations are required to test the adequacy of projections.

4.3.5. THE CENTRAL POOL MODEL

The model was first proposed by György Németh (Németh [2006]), where the central institute of the pool is the so-called "government actuarial", which would redistribute the mortality result among the providers (in his proposal these are clearly non-profit pension funds). Németh finds that there is no justification for differences in the annuity premium offered by pension funds if there is no possibility for differentiation, and indeed these can lead to operational distortions, which can be prevented by having a single provider (or in reality by using a central pool). In his view this does not mean a single organisation, but a single set of rules – contrary to Stahl's proposal promoting the idea of a central provider.

Based on the above, the model's most important elements and their internal relationships are as follows:

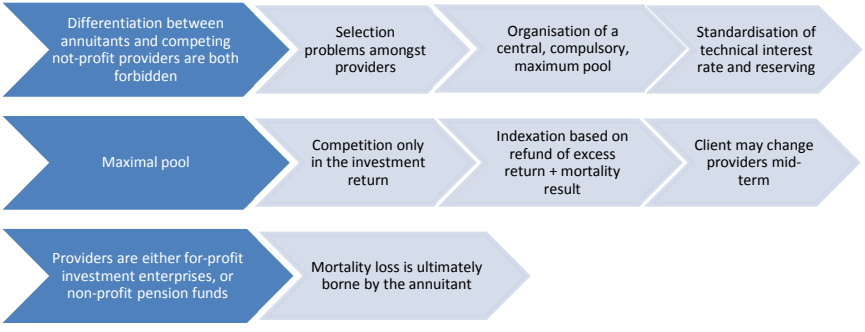
Table 17: The most important elements of the central pool model

Competing or non-competing providers	The service is provided by several competing providers, among whom the clients may choose (with certain restrictions).
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The different providers provide the full annuity, so the problem of selecting between providers emerges and must be managed. However, stating that providers "provide the annuity" is somewhat inaccurate in the case of this model, in the light of the following.

Managing provider selection problems. (The final bearer of the mortality loss)	Provider selection problems are managed with the help of a maximum pool, so the mortality result is distributed amongst the providers every year. This Inevitably means that both the technical interest rate and reserving are fixed, i.e. determined centrally. The mortality result so remaining within the entire market basically belongs to the annuitants.
Is the provider a for-profit or non-profit organisation?	The providers may theoretically be non-profit companies (as in the case of Hungarian-type pension funds, for example), as well as for profit companies (fundamentally investment ventures, meaning they cannot ultimately bear the mortality loss because they are not insurance company-type institutions).
Is the annuity composed of one or two parts?	Both solutions may be possible depending on the decision of the regulator, but only one of them may exist on the market at a given time.
Indexing	Definitely excess return refund, as this is the only solution where it makes sense to have investment competition between providers. Consequently the clients may change provider even once annuity payments have begun, which is not particularly difficult to organise in view of the centralised nature of the service from a certain aspect. The indexing of individual clients depends on a combination of the yield achieved by their provider and the actual mortality result, and this sum could even be negative, although we can assume this happens only rarely.

The abovementioned relationships may be presented in the diagram below:

Diagram 10: The internal relationship between the elements of the Central Pool Model



Obviously, each provider must precisely determine the amount of money (the individual premium reserve of their insured individuals) they must invest so it is reasonable to restrict the option of changing providers to a certain period of

the year, preferably the very period in which the pool determines the mortality result of the previous year, and in turn the index. In addition, clients can also view the performance of their investors at this time. In this model the full annuity service may be centralised, and in fact it is expedient to do so, including for client records. In fact, individual providers are not even required to know who their clients are. If they don't know, the intermediaries may be eliminated from the system, which will save on costs, although the role of intermediaries is not quite clear, since it is often intermediaries who make clients aware of the fact that they can leave under-performing providers.

An internal alternative of the model is that the provider may be a non-profit organisation (e.g. Hungarian-type pension fund), or a for-profit organisation, but it is worthwhile reviewing the arguments both for and against these solutions.

In the Hungarian system the argument in favour of pension funds would have been that these were existing institutions, which, in lack of solvency capital, were however in themselves ill-equipped for providing a suitable life annuity. This solution would have allowed the annuity service to be organised on the basis of the existing pension funds.

However, in the case of the central pool it is also worth centralising administration and membership records, partly because this helps the calculation of the mortality result, and partly because if a central institution is established anyway, then it is expedient to have it perform related activities. If we compare this with the fact that in the "heyday" of the private pension system the majority of pension funds in Hungary did not conduct investment activities, but entrusted investment companies to perform this task, then – had this model been introduced – the pension fund would have "emptied". That is why, in Hungary, with relation to this model it would also have been worthwhile to consider "disconnecting" the pension funds from the annuity system, and to entrust investment companies directly with performing investment activities.

So the idea that competing providers should be for-profit institutions is strongly promoted in this model, which primarily means insurance companies and investment enterprises. However, in the case of insurers the central pool represents a very strong reduction in risk, so this is not a tailor-made solution for those institutions. The minimum pool, which is generally a part of the insurer annuity model, is much better suited to insurance companies. This leaves investment enterprises as possible providers. But of course these cannot bear the mortality loss, not even ultimately (not even if so-called "longevity bonds" exist on the market!), so in contrast to the other models, in this model the mortality loss is ultimately borne by the annuitants even in the case of for-profit companies. However, in view of it having the largest possible risk community, haphazard fluctuations are almost entirely filtered out of the mortality risk, and accordingly in this case this only means the longevity risk.

4.3.6. THE PENSION FUND ANNUITY MODEL

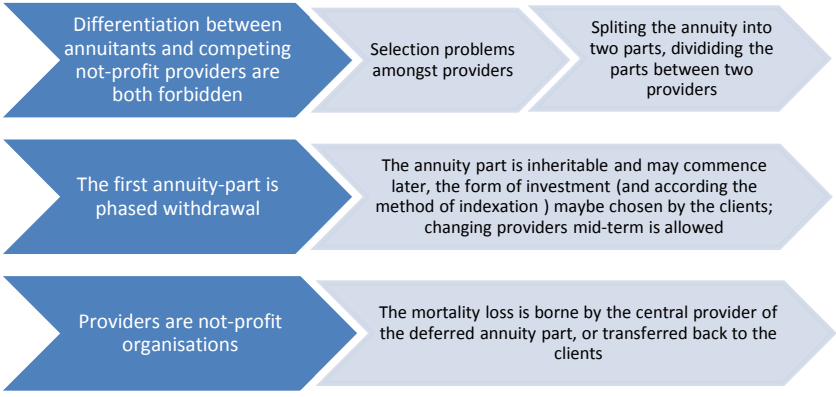
Based on the above, the model's most important elements and their internal relationships are as follows:

Table 18: The most important elements of the pension fund annuity model

Competing or non-competing providers	The service is provided by several competing providers, among whom the clients may choose (with certain restrictions).
Does it provide the full annuity or only the first part of it (phased withdrawal)?	The competing providers only provide the first part of the annuity, within the framework of a phased withdrawal (i.e. not as an annuity). This partly manages the selection problems that result from prohibiting differentiation.
Is the annuity composed of one or of two parts?	Consequently, the annuity is composed of two parts, where the first part is a phased withdrawal that allows insured individuals to postpone the start of the annuity and to leave the majority as a potential inheritance (i.e. "clients have full control of their money"). However the insured individual must immediately buy the deferred part of the annuity on retirement.
Managing provider selection problems	The provider delegates the remaining selection problems, which are a priori smaller than if the provider gave clients an immediately commencing life annuity and which are borne by the deferred annuity part, to a central provider by transferring the deferred annuity, meaning the provider is fully exempt from such risks.
Is the provider a for-profit or non-profit organisation? (Who ultimately bears the mortality loss?)	The provider is a non-profit company, (under Hungarian circumstances) a pension fund as the name of the model indicates. However, since it cannot under any circumstances provide a life annuity, it can also certainly not bear the mortality loss; this can only appear at the central provider. The central provider manages mortality risk as described with regard to the central provider model.
Indexing	Theoretically, indexing may be solved by either indexing technique, and arguments may be raised for and against both solutions. Arguments in favour of indexing to inflation include the fact that it enables the relatively seamless linking of the two annuity parts, and that it is difficult to organise a change of providers during the term in the case of deferred annuity part, although without this "already acquired clients" are exposed to the provider being in a monopolistic position (from the client's perspective) in the case of the other indexing technique. An argument against is that in this case it is not particularly expedient to have the service provided by competing providers, or rather this only arises as an option if the existence of competing providers can already be taken as granted. In the case of indexing based on excess return refund it is justified to afford clients the possibility to both change providers and to choose according to investment portfolio.

The abovementioned relationships are presented in the diagram below:

Diagram 11: The internal relationship between the elements of the Fund Annuity Model



I have already written down everything that is important with relation to the details of this model[68] in the case of previous models above, so I do not intend to repeat those. It is important to note that since the pension fund cannot bear mortality risk, (because it lacks solvency capital), the client can again not choose a (temporary) life annuity during the deferred period according to this model; clients may only spend their savings (or leave them as a legacy) within the framework of a phased withdrawal. Another reason that providers cannot

[68] Despite the fact that for a long time the Hungarian pension profession took as granted the fact that private pension annuities would be provided by pension funds, the pension fund annuity model is not elaborated on in the literature. This is related to the fact that the rules on private pension annuities in general were not set down in Hungary until 2009 (in an Act that did not come into force). For a long time there was an illusory way of thinking about the annuities provided by the pension fund, it was assumed that it was in no way different from the annuity provided by insurers – at most the legislators set certain restrictive conditions as for example in the case of Swiss indexation, but these were not deemed problematic. The exception was János Stahl, who found both unisex annuity and Swiss indexation problematic; therefore in [2005] Stahl proposed indexation based on the combined investment and mortality result, allowing the nominal diminishing of the annuity, which is the essence of the fund annuity model. However, he also proposed that there should also be a central provider, so he did not systematically elaborate the pension fund annuity model.

be responsible for providing full life annuities in the case of pension fund annuity Is that, with no solvency capital, there is no penalty for under-pricing, meaning that in the short term it is worth attracting clients by making irresponsible promises; the worst that can happen is that providers will be incapable of fulfilling their promises in the long term. The problem of possible unpunished, under-priced life insurances can only occur in this model, so this possibility must also be excluded by the regulator!

4.4. Model elements relatively freely applicable to several models

Above I have elaborated on those elements of the annuity system that are not optionally variable, and whose selection assumes the selection of additional elements, and in turn excludes the selection of others. Nevertheless there also exist relatively neutral solutions that, under certain conditions, may be included in several annuity models, or even all of them.

Homogenisation of annuities according to annuity size (partial homogenisation, naturally), which reduces the danger of providers suffering a large mortality loss, because although they may have projected mortality properly, larger annuitants systematically live longer than smaller annuitants, may be raised in respect of every annuity model. The emergence of the problem may also be traced to the prohibition of differentiation, since the size of the annuity presumable correlates well with factors that probably cannot be taken into account, such as level of education or state of health. Homogenisation increases the precision of calculations in the case of every annuity model, but this is particularly true for models that apply immediately commencing life annuities, and so in these cases the implementation of homogenization, i.e. the maximising of possible annuities, is highly recommended.

The issue of homogenisation is raised not only from this technical perspective, but also as a question relating to annuitants having full control over their pension capital. The question is, is it worth requiring insured individuals to convert their savings to an annuity above a certain pension level. I think the answer is no, because the aim of making it compulsory is to make sure everybody attains a certain level of pension, i.e. to safeguard people from behaving in a short-sighted manner. Realising a higher level can be left to the devices of the annuitant, while short-sightedness does not cause social problems beyond a certain level of pension. Luckily, raising the issue of homogenisation from two different perspectives converges to the same solution.

Naturally, we must also decide what method to use to set the maximum annuity. There are two options:

1. Independently for the annuity of the funded system. In the alternative SoS model this of course corresponds to the entire pension annuity.
2. By consolidating the annuities of the funded and pay-as-you-go systems, meaning a maximum is set for funded annuity payouts, and the excess partly supplements payments made via the funded system. Ad absurdum this may be 0, if the pension from the pay-as-you-go system also reaches this minimum.

Option 1 is easier to manage within the funded system, as this requires no data from the pay-as-you-go system and probably also leads to more homogeneous annuities than option 2. However, despite this it is possible that the problem of inhomogeneity is better managed by option 2 because there is a general tendency for people with larger savings to also have a longer life and also receive higher pensions from the pay-as-you-go system, meaning the annuity they receive from the funded system will be below average. Naturally, when selecting option 2 one must take into account the fact that although this may result in the annuity from the funded system being zero, but only above a certain minimum level (see the section on "minimal annuity").

Whatever the decision of the legislator; the capital remaining above the minimum level required to assure the maximum annuity may be withdrawn by the insured party in a lump sum, and may be used at their discretion. (One possibility it to use it to purchase a non-mandatory annuity on the free market, but they could also leave it as an inheritance or invest it, etc.) In the annuity models that begin with a phased withdrawal, this issue is raised such that the first amount withdrawn may be a larger sum, which may be defined according to these principles.

A rule of this kind can be included in every annuity model without difficulty. Naturally, a logical question is why clients should be obliged to accumulate money within a mandatory pension system at all, if they are then not obliged to use this money as an annuity within the framework of that system. There are experts who say that if there is an upper limit with respect to current income for payment into the funded system (as was the case in Hungary), then this in turn also defines the maximum annuity, and so this issue doesn't need to be dealt with separately. Although this may be partly true, in the case of predetermined maximum income there is a significant spread of possible annuities, so they do not give a precise maximum because:

1. The level of income may change or fluctuate: during the lifetime of an individual (and especially in the case of women who leave work to raise children) incomes may sometime exceed the maximum and then there may be a period with no income at all (and consequently no pension contribution).

2. The magnitude of the annuity payment also depends on the given age of retirement. Payouts will be considerably higher for people retire later, compared to people who take out early retirement.
3. The remaining life expectancy also changes during the course of time, and accordingly the amount of capital needed to achieve the same minimum pension level also changes unpredictably.

Furthermore, a minimum pension of this kind must also be indexed, and this does not necessarily concur with the return of already accumulated pension assets. Due to all these factors, it is to all intents and purposes impossible to precisely calculate in advance whether the later annuity will or will not exceed the maximum, and accordingly a certain excess will almost certainly occur in the case of some insured individuals, even in the case of a suitably determined annuity maximum. At the same time, the two rational considerations that:

1. it is expedient to make sure that future pension are as close as possible to this maximum, and that
2. we should avoid needlessly forcing insured individuals to accumulate more capital than is necessary for purchasing the maximum annuity

could lay the foundations for the future reform of mandatory funded pension systems. In a system that takes the above criteria into consideration, pension contributions to the mandatory funded system do not have an income-linked maximum, but the accumulated pension capital does (calculated with a margin in view of the above uncertainty factors), and this is "reverse calculated" from the maximum annuity level. If private pension savings reach this level, payment to this system terminates. Through this we can avoid situations in which an insured individual with a promising carrier ends up with a low pension only because, although they may have often earned a high income during the course of their life they did not pay a high enough pension contribution during such periods, while at other times they had a low income or no income at all.

An argument against this solution is that pension contributions must primarily be paid at a young age, when the money is theoretically also needed for other things, but there are also arguments against this (e.g. the consumption level of young people "balances out" at a lower level, or their lower net income inspires them to greater efforts, which is easier to do at an early age than at an older age, etc.).

Logically, we could of course state that since the question of a mandatory maximum annuity has been raised, then the question of a minimum annuity could also be raised in a similar fashion, but this is a technical issue for the regulator and shall not be discussed here.

Joint life annuity could also be a possible element of any annuity model. The fact that annuitants cannot choose between annuity types and that this is instead chosen for them by the model in the interests of eliminating the risk of adverse selection as a result of being able to choose between annuity types, was the default in the case of every model. If there is only one type of annuity in a model then this condition is met. However, it is also met if there are several types of annuity, but they are clearly assigned to the individual client groups. In practice this means that single people are given single life annuities and couples[69] are given joint life annuities. Accordingly, joint life annuities may theoretically also be included in any annuity model.

The abovementioned models practically speaking included two types of annuity:

1. An immediately commencing, single life, simple annuity (life-long, with no guaranteed period), or
2. A deferred, single life annuity with the possibility of phased withdrawal during the deferral period (from the individual reserve that is kept separate from the premium of the deferred annuity).

I think that joint life annuities must also correspond to the type of single life annuity used in the model, meaning is should be an immediately commencing, simple, joint life annuity, or a deferred joint life annuity.

It is worth precisely planning the exact structure of the joint life annuity. I have proposed one possibility above, but that can be further fine-tuned by taking into account various specific life conditions (a big age difference between the spouses, a big difference in the commencement of the pension, one member of the couple is still working, while the other is already retired, etc.), and the regulations concerning the annuity should be refined accordingly.

Similarly, the impact on the annuity type must also be taken into consideration in the case of divorce and marriage and the regulations must be refined accordingly, with specific regard to the possible unusual situations (widows remarrying, divorce before retirement or after one member of the couple retires, etc.).

The joint life annuity in itself solves the problem of long-term widow's pensions within the funded system. It is important to note that the term "widow's pension" is often used to refer to two different benefits, which commence immediately after the death of one member of the couple:

1. A temporary annuity, usually in the event of death while still working, and a
2. Lifelong annuity, in the event of death following retirement.

[69] Or people living together in any documented manner that is difficult to change and precisely defined by law.

A joint life annuity solves the problem of type 2 widow's annuity. In theory, the Hungarian private pension system handles the type 1 instance via the inheritance of pension accounts, but it does not necessarily manage the problem in the best possible way, because there may be many different amounts on the pension account of the deceased spouse, in comparison to the lost income of the deceased spouse, depending on which phase of the lifecycle they happen to have died. In addition, heirs may often be chosen freely (according to Hungarian private pension regulations, for instance), meaning this amount might not necessarily go to the widow. Therefore, the implementation of the following two rules should be considered instead:

1. The temporary widow's pension due in the event of death while still of working age is in reality is a life insurance; it would be expedient to explicitly include this in the system and separate its premium from the pension account,
2. The pension account is inheritable, but in the case of a married couple the heir is always the spouse, and the accumulated pension assets are transferred to the spouse's pension account and will later increase their old age pension.

In most annuity models it is expedient to implement an **individual reserve (buffer)** to smooth mortality fluctuations, and possibly to bear the final mortality risk (up to the extent of the buffer, naturally). This solution should be considered with respect to all models in which the ultimate bearer of the mortality loss is the insured individual – so primarily in the central provider model, in the pension fund annuity model and in the central pool model. It should also be considered in cases where the for-profit provider provides an immediately commencing annuity (so in the insurance annuity model), i.e. when

a. the fluctuation of the mortality loss has an impact on the bigger portion of the term, meaning that overall the accumulated loss may be bigger than in models that use a deferred life annuity, and/or
b. as a general rule the mortality loss is born by the provider, but as a result it charges too high an "option fee" and accordingly decreases the annuity payout by too much.

NOTATIONS

i :	technical interest rate
v :	discount factor
d:	1-v
x:	the entry age of the insured party/annuitant (also *y* in the case of joint life annuities)
t:	number of (integer) years since the commencement of the insurance/annuity
m:	length of the deferment (integer years)
n:	term of insurance (e.g. a temporary annuity) in whole years
l_x:	number or people still alive at age *x* from a population with a starting age (l_0) of 0 = life table
d_x:	number of deaths at age x within the same population. $d_x = l_x - l_{x+1}$
ω:	the maximum possible (highest statistically relevant) age used in the life table ($l_\omega > 0$, $l_{\omega+t} = 0$, for all t>0 integers)
q_x:	probability of death $q_x = \frac{d_x}{l_x} = \frac{l_x - l_{x+1}}{l_x}$
p_x:	probability of survival $p_x = \frac{l_{x+1}}{l_x}$
${}_{m\|}p_x$:	probability of survival by *m* years ${}_{m\|}p_x = \frac{l_{x+m}}{l_x}$. Naturally ${}_{1\|}p_x = p_x$, and ${}_{0\|}p_x = 1$
e_x:	expected remaining lifetime at age x. $e_x = \frac{l_{x+1}+l_{x+2}+\cdots+l_\omega}{l_x} + \frac{1}{2}$, or $e_x = {}_{1\|}p_x + {}_{2}p_x + \cdots + {}_{\omega\text{-}x\|}p_x + \frac{1}{2}$
$a_{\overline{\infty\|}}$:	single net premium of an immediate, perpetual annuity payable in arrears with annual EUR 1 payment
$\ddot{a}_{\overline{\infty\|}}$:	single net premium of an immediate, perpetual annuity-due with annual EUR 1 payment. $a_{\overline{\infty\|}} = \ddot{a}_{\overline{\infty\|}} - 1$
$\ddot{a}_{\overline{n\|}}$:	single net premium of an immediate, *n*-year term annuity-due, annuity-certain with annual EUR 1 payment

$\ddot{a}_x$: single net premium of an immediate life annuity-due with annual EUR 1 payment for an annuitant aged *x* years

$\ddot{a}_{x:\overline{n}|}$: single net premium of an immediate, *n*-year term temporary life annuity-due with annual EUR 1 payment for an annuitant aged *x* years

${}_{m|}\ddot{a}_x$: single net premium of deferred (*m* years deferment) life annuity-due with annual EUR 1 payment for an annuitant aged *x* years

$\ddot{a}_{xy}$: single net premium of an immediate joint life annuity-due with annual EUR 1 payment for annuitants aged *x* and *y* years. The annuity payments last until the first death

$\ddot{a}_{xy:\overline{n}|}$: single net premium of an immediate, *n*-year term temporary joint life annuity-due with annual EUR 1 payment for annuitants aged *x* and *y* years. The annuity payments last until the first death, but for a maximum of *n* years

${}_{|g}\ddot{a}_x$: single net premium of an immediate life annuity-due with *g* years guaranteed period at the beginning, with annual EUR 1 payment for an annuitant aged *x* years

${}_{g|}\ddot{a}_x$: single net premium of an immediate life annuity-due with *g* years guarantee period at the end, with annual EUR 1 payment for an annuitant aged *x* years

$\ddot{a}_{x|y}$: conditional life annuity. The payments commence at the death of *y* and last until the death of *x*. (If *x* dies earlier than *y*, the $\ddot{a}_{x|y} = \ddot{a}_x - \ddot{a}_{xy}$ annuity ceases without any payments)

$\ddot{a}_{\overline{n}|x}$: conditional, *n*-year term temporary life annuity. The payments start at the death of *x* and last until the end of the *n*-year term (its remainder). $\ddot{a}_{\overline{n}|x} = \ddot{a}_{\overline{n}|} - \ddot{a}_{x:\overline{n}|}$

$\ddot{a}_{\overline{n}|xy}$: conditional, *n*-year term temporary joint life annuity. The payments start at the death of *x* or *y*, and last until the end of the *n*-year term (its remainder). $\ddot{a}_{\overline{n}|xy} = \ddot{a}_{\overline{n}|} - \ddot{a}_{xy:\overline{n}|}$

$\ddot{a}_{x:\overline{n}|yz}$: conditional, *n*-year term temporary joint life annuity for three persons. The payments start at the death of *y* or *z*, and last until the end of the *n*-year term (its remainder), but maximum until the earlier death of *x*. $\ddot{a}_{x:\overline{n}|yz} = \ddot{a}_{x:\overline{n}|} - \ddot{a}_{xyz:\overline{n}|}$

V_t: individual reserve of an annuity or other life insurance at the *t* anniversary of the term before paying the outstanding payment

A_x: single net premium of a whole life insurance with EUR 1 sum assured.

A_{xy}: single net premium of a whole life insurance with two annuitants and EUR 1 sum assured

${}_{g}A_{x}$: single net premium of a special whole life insurance with EUR 1 sum assured. The first insurance period is not a year, but *g* years, which means the death benefit can be paid *g* years after the commencement of the insurance at the earliest

${}_{m|}A_{x}$: single net premium of a "deferred" whole life insurance with EUR 1 sum assured. Its payment is conditional: the sum assured is paid only if the death of the insured party happens after *m* years (deferment) following commencement

$A^{1}_{x:\overline{n|}}$: single net premium of a term (death) insurance with EUR 1 sum assured, and *n*-year term.

$A_{x:\overline{n|}}{}^{1}$: single net premium of a pure endowment insurance with EUR 1 sum assured and *n*-year term. $A_{x:\overline{n|}}{}^{1} = {}_{n|}p_{x} \cdot v^{n}$

$A_{x:\overline{n|}}$: single net premium of an endowment insurance with EUR 1 sum assured and *n*-year term $A_{x:\overline{n|}} = A_{x:\overline{n|}}{}^{1} + A^{1}_{x:\overline{n|}}$

$A_{xy:\overline{n|}}$: single net premium of an endowment insurance with two insured parties, EUR 1 sum assured and an *n*-year term. Payment on the first death.

P_{x}: annual net premium of a whole life insurance with EUR 1 sum assured. $P_{x} = \frac{A_{x}}{\ddot{a}_{x}}$

$P_{x:\overline{n|}}$: annual net premium of an endowment insurance with EUR 1 sum assured. $P_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}}$

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