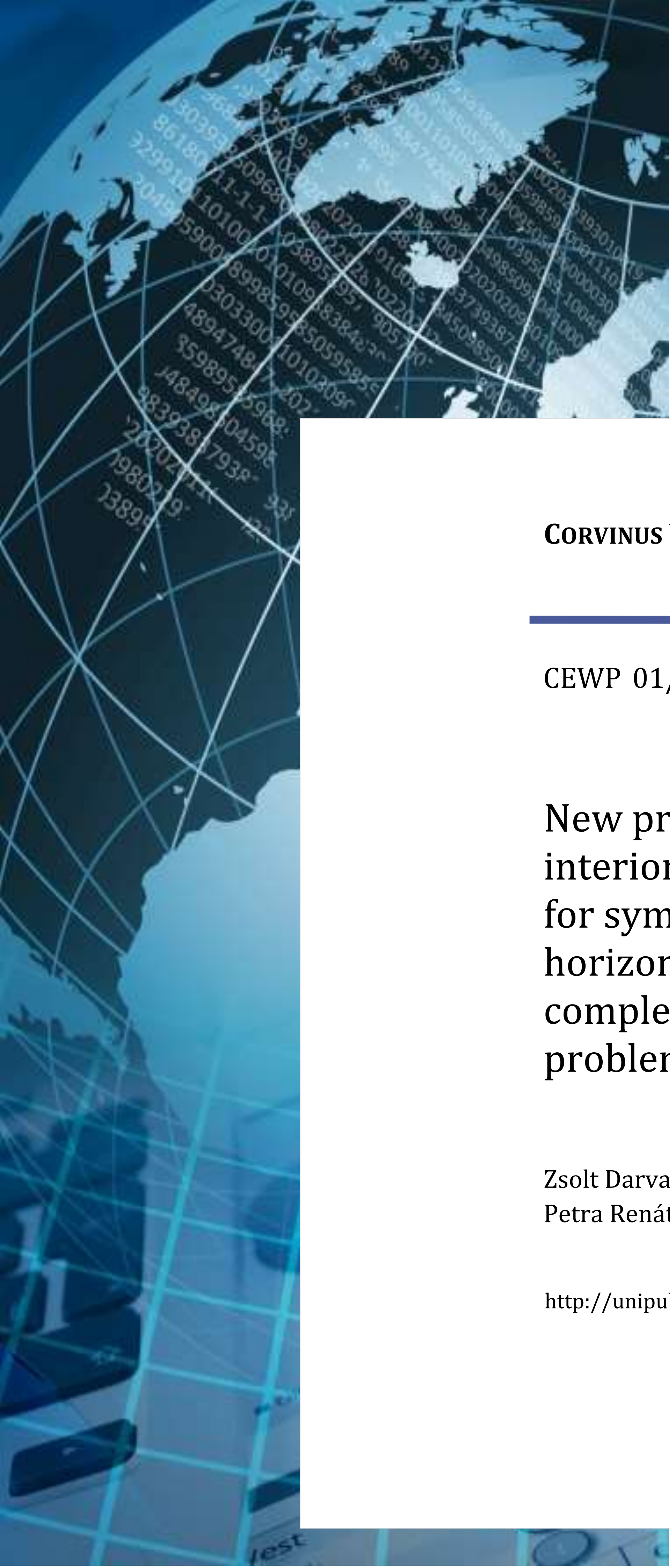

CORVINUS UNIVERSITY OF BUDAPEST

CEWP 01/2021

New predictor-corrector interior-point algorithm for symmetric cone horizontal linear complementarity problems

Zsolt Darvay,
Petra Renáta Rigó

http://unipub.lib.uni-corvinus.hu/6323

New predictor-corrector interior-point algorithm for symmetric cone horizontal linear complementarity problems

Zsolt Darvay and Petra Renáta Rigó

01.03.2021

Abstract In this paper we propose a new predictor-corrector interior-point algorithm for solving $P_*(\kappa)$ horizontal linear complementarity problems defined on a Cartesian product of symmetric cones, which is not based on a usual barrier function. We generalize the predictor-corrector algorithm introduced in [13] to $P_*(\kappa)$-linear horizontal complementarity problems on a Cartesian product of symmetric cones. We apply the algebraic equivalent transformation technique proposed by Darvay [9] and we use the function $\varphi(t) = t - \sqrt{t}$ in order to determine the new search directions. In each iteration the proposed algorithm performs one predictor and one corrector step. We prove that the predictor-corrector interior-point algorithm has the same complexity bound as the best known interior-point algorithms for solving these types of problems. Furthermore, we provide a condition related to the proximity and update parameters for which the introduced predictor-corrector algorithm is well defined.

Keywords Horizontal linear complementarity problem · Cartesian product of symmetric cones · Predictor-corrector interior-point algorithm · Euclidean Jordan algebra · Algebraic equivalent transformation technique
JEL code: C61

1 Introduction

Interior-point algorithms (IPAs) are an efficient tool for solving optimization problems, since Karmarkar [20] published his IPA. The most important results on IPAs for solving linear programming (LP) problems are summarized in the monographs written by Roos, Terlaky and Vial [33], Wright [39] and Ye [40]. IPAs for solving linear complementarity problems (LCPs) have been also introduced. In general, LCPs belong to the class of NP-complete problems [8]. However, Kojima et al. [22] proved that if the problem's matrix has the $P_*(\kappa)$-property, then the IPAs for LCPs have polynomial iteration complexity in the size of the problem, the bit size of the data, the final accuracy of the solution and in the special parameter, called the handicap of the matrix. IPAs have been extended to more general problems such as symmetric cone optimization (SCO) problems. SCO covers LP, semidefinite optimization (SDO) and second-order cone optimization (SOCO) problems. Faraut and Korányi [16]

Zsolt Darvay
Babeş-Bolyai University, Faculty of Mathematics and Computer Science, Romania
E-mail: darvay@cs.ubbcluj.ro

Petra Renáta Rigó
Corvinus Center for Operations Research, Corvinus Institute for Advanced Studies, Corvinus University of Budapest, Hungary; on leave from Department of Differential Equations, Faculty of Natural Sciences, Budapest University of Technology and Economics
E-mail: petra.rigo@uni-corvinus.hu

summerized the most important results related to the theory of Jordan algebras and symmeric cones. Güler [18] noticed that the family of the self-scaled cones, introduced by Nesterov and Todd [26,27], is identical to the set of self-dual and homogeneous, i.e. symmetric cones.

The symmetric cone horizontal linear complementarity problem (SCHLCP) has been recently introduced by Asadi et al. [4]. The SCHLCP possessing the $P_*(\kappa)$-property includes symmetric cone optimization (SCO), second-order cone optimization (SOCO), semidefinite optimization (SDO), convex quadratic optimization (CQO), LP and LCPs as special cases. Several IPAs have been proposed for solving Cartesian SCHLCPs. Mohammadi et al. [25] presented an infeasible IPA taking full Nesterov-Todd steps for solving this kind of problems. Later on, Asadi et al. [3] proposed an IPA for solving Cartesian SCHLCP based on the search directions introduced in [9]. In 2020, Asadi et al. [2] introduced a new IPA for solving Cartesian SCHLCP based on a positive-asymptotic barrier function. Moreover, Asadi et al. [5] presented a predictor-corrector (PC) IPA for $P_*(\kappa)$-SCHLCP. Asadi et al. [6] also defined a feasible IPA for the $P_*(\kappa)$-SCHLCP using a wide neighbourhood of the central path.

There are several approaches for determining the search directions, that lead to different IPAs. Peng et al. [32] used self-regular barriers in order to introduce large-update IPAs for LP. Lesaja and Roos [23] provided a unified analysis of kernel-based IPAs for $P_*(\kappa)$-LCP. Vieira [37] used different IPAs for SCO problems that determine the search directions using kernel functions. In 1997, Tunçel and Todd [36] presented for the first time a reparametrization of the central path system. Later on, Karimi et al. [19] analysed entropy-based search directions for LP in a wide neighbourhood of the central path. In 2003, Darvay presented a new technique for finding search directions for LP problems [9], namely the algebraic equivalent transformation (AET) of the system defining the central path. The most widely used function in the AET technique is the identity map. Darvay [9] applied the function $\varphi(t) = \sqrt{t}$ in the AET method in order to define IPA for LP. Kheirfam and Haghighi [21] defined IPA for $P_*(\kappa)$-LCPs which is based on a search direction generated by considering the function $\varphi(t) = \frac{\sqrt{t}}{2(1+\sqrt{t})}$ in the AET. In 2016, Darvay et al. [14] used the function $\varphi(t) = t - \sqrt{t}$ for the first time in the analysis of an IPA. Later on, this IPA was generalized to SCO [15]. An infeasible version of the proposed IPA was also introduced in [31]. Darvay et al. [12] considered the function $\varphi(t) = t - \sqrt{t}$ in the AET method in order to define primal-dual IPA for solving $P_*(\kappa)$-LCPs. In [29] the author presented different IPAs for LP, SCO and sufficient LCPs that use the AET technique.

In this paper we generalize the PC IPA proposed in [13] to Cartesian $P_*(\kappa)$ SCHLCPs. We also provide a general framework for determining search directions in case of PC IPAs for Cartesian $P_*(\kappa)$ SCHLCPs, which is an extension of the approach given in [30]. We present the analysis of the introduced PC IPA and we give some new technical lemmas that are necessary in the analysis. Furthermore, we prove that the PC IPA retains polynomial iteration complexity in the special parameter κ, the size of the problem, the bitsize of the data and the deviation from the complementarity gap. We also give a condition related to the proximity and update parameters for which the introduced predictor-corrector algorithm is well defined. Note that in [33] the authors gave condition for the update parameters for which the proposed PC IPA for LP is well defined. Moreover, Darvay [10,11] considered PC IPAs using the function $\varphi(t) = \sqrt{t}$ in the AET technique and gave conditions for the proximity and update parameters for which the PC IPAs are well defined. It should be mentioned that in this paper we provide the first result related to IPAs using the function $\varphi(t) = t - \sqrt{t}$ in the AET technique which is well defined for a set of parameters instead of a given value for the proximity and update parameters.

The paper is organized as follows. In the next section, we present the Cartesian $P_*(\kappa)$-SCHLCP. In Section 3 we present the generalization of the AET technique for determining search directions to Cartesian $P_*(\kappa)$-SCHLCP. Subsection 3.1 is devoted to give a general framework for defining search directions in case of PC IPAs for Cartesian $P_*(\kappa)$-SCHLCP. In Section 4 the new PC IPA for Cartesian $P_*(\kappa)$-SCHLCP is introduced, which is based on a new search direction by using the function $\varphi(t) = t - \sqrt{t}$ in the AET technique. Section 5 contains the analysis of the proposed PC IPA. In Section 6 some concluding remarks are presented.

2 Cartesian $P_*(\kappa)$-symmetric cone horizontal linear complementarity problem

For a more detailed description of the theory of Euclidean Jordan and symmetric cones, see Appendix. Let $\mathcal{V} := \mathcal{V}_1 \times \mathcal{V}_2 \times \cdots \times \mathcal{V}_m$ be the Cartesian product space with its cone of squares $\mathcal{K} := \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_m$, where each space $\mathcal{V}_i$ is a Euclidean Jordan algebra and each $\mathcal{K}_i$ is the corresponding cone of squares of $\mathcal{V}_i$. For any $x = \left(x^{(1)}, x^{(2)}, \cdots, x^{(m)}\right)^T \in \mathcal{V}$ and $s = \left(s^{(1)}, s^{(2)}, \cdots, s^{(m)}\right)^T \in \mathcal{V}$ let

$$x \circ s = \left(x^{(1)} \circ s^{(1)}, x^{(2)} \circ s^{(2)}, \cdots, x^{(m)} \circ s^{(m)}\right)^T, \quad \langle x, s\rangle = \sum_{i=1}^{m} \langle x^{(i)}, s^{(i)}\rangle.$$

Beside this, for any $z = \left(z^{(1)}, z^{(2)}, \cdots, z^{(m)}\right)^T \in \mathcal{V}$, where $z^{(i)} \in \mathcal{V}_i$, the trace, the determinant and the minimal and maximal eigenvalues of the element z are defined as follows:

$$\begin{aligned} tr(z) &= \sum_{i=1}^{m} tr(z^{(i)}), \quad \det(z) = \prod_{i=1}^{m} \det(z^{(i)}), \\ \lambda_{\min}(z) &= \min_{1\le i\le m} \{\lambda_{\min}(z^{(i)})\}, \quad \lambda_{\max}(z) = \max_{1\le i\le m} \{\lambda_{\max}(z^{(i)})\}, \end{aligned}$$

The Frobenius norm of x is defined as $\|x\|_F = \left(\sum_{i=1}^m \left\|x^{(i)}\right\|_F^2\right)^{1/2}$. Furthermore, consider the Lyapunov transformation and the quadratic representation of x:

$$\begin{aligned} L(x) &= \operatorname{diag}\Big(L(x^{(1)}), L(x^{(2)}), \cdots, L(x^{(m)})\Big), \\ P(x) &= \operatorname{diag}\Big(P(x^{(1)}), P(x^{(2)}), \cdots, P(x^{(m)})\Big). \end{aligned}$$

In the Cartesian SCHLCP we should find a vector pair $(x, s) \in \mathcal{V} \times \mathcal{V}$ such that

$$Qx + Rs = q, \quad \langle x, s\rangle = 0, \quad x \succeq_K 0, \quad s \succeq_K 0, \qquad (SCHLCP)$$

where $Q, R : \mathcal{V} \to \mathcal{V}$ are linear operators, $q \in V$ and $\mathcal{K}$ is the symmetric cone of squares of the Cartesian product space $\mathcal{V}$. Consider the constant $\kappa \ge 0$. The pair (Q, R) has the $P_*(\kappa)$ property if for all $(x, s) \in \mathcal{J} \times \mathcal{J}$

$$Qx + Rs = 0 \ \text{ implies } \ (1 + 4\kappa) \sum_{i\in I_+} \langle x^{(i)}, s^{(i)}\rangle + \sum_{i\in I_-} \langle x^{(i)}, s^{(i)}\rangle \ge 0,$$

where $I_+ = \{i \ : \ \langle x^{(i)}, s^{(i)}\rangle > 0\}$ and $I_- = \{i \ : \ \langle x^{(i)}, s^{(i)}\rangle < 0\}$.

We suppose that the *interior-point condition* (IPC) holds, which means that there exists (x^0, s^0) so that

$$\begin{aligned} Qx^0 + Rs^0 &= q, \qquad x^0 \succ_K 0, \\ \langle x^0, s^0\rangle &= 0, \qquad s^0 \succ_K 0. \end{aligned} \qquad (IPC)$$

The central path is given as

$$\begin{aligned} Qx + Rs &= q, \qquad x \succeq_K 0, \\ x \circ s &= \mu e, \qquad s \succeq_K 0. \end{aligned} \qquad (1)$$

where $\mu > 0$. The sublass of Monteiro-Zhang family of search directions is characterized by

$$C(x, s) = \Big\{u | u \text{ is invertible and } L(P(u)x)L(P(u)^{-1}s) = L(P(u)^{-1}s)L(P(u)x)\Big\}.$$

Lemma 1 *(Lemma 28 in [34]) Let $u \in \operatorname{int}\mathcal{K}$. Then,*

$$x \circ s = \mu e \quad \Leftrightarrow \quad P(u)x \circ P(u)^{-1}s = \mu e.$$

Considering $u \in C(x,s)$, $\widetilde{Q} = QP(u)^{-1}$, $\widetilde{R} = RP(u)$ and using Lemma 1, we can rewrite system (1) in the following way:

$$\begin{aligned} \widetilde{Q}P(u)x + \widetilde{R}P(u)^{-1}s = q, \qquad P(u)x \succeq_K 0,\\ P(u)x \circ P(u)^{-1}s = \mu e, \qquad P(u)^{-1}s \succeq_K 0. \end{aligned} \tag{2}$$

If the IPC holds, then system (2) has unique solution for each $\mu > 0$, see [4] and [24].

3 Search directions in case of interior-point algorithms for Cartesian $P_*(\kappa)$-symmetric cone horizontal linear complementarity problems

We present the generalization of the AET technique to $P_*(\kappa)$-SCHLCP (cf. [2,30]). Let us consider the vector-valued function φ, which is induced by the real-valued univariate function $\varphi : (\xi^2, +\infty) \to \mathbb{R}$, where $0 \le \xi < 1$ and $\varphi'(t) > 0$ for all $t > \xi^2$. We assume that we have $\lambda_{\min}\left(\frac{x \circ s}{\mu}\right) > \xi^2$. In this way, the third equation of system (2) can be written in the following way:

$$\begin{aligned} \widetilde{Q}P(u)x + \widetilde{R}P(u)^{-1}s = q, \qquad P(u)x \succeq_K 0,\\ \varphi\left(\frac{P(u)x \circ P(u)^{-1}s}{\mu}\right) = \varphi(e), \qquad P(u)^{-1}s \succeq_K 0. \end{aligned} \tag{3}$$

We define the search directions by using the technique considered in [30,38]. For the strictly feasible $x \in \operatorname{int} K$ and $s \in \operatorname{int} K$ we want to find the search directions $(\Delta x, \Delta s)$ so that

$$\begin{aligned} \widetilde{Q}P(u)\Delta x + \widetilde{R}P(u)^{-1}\Delta s = 0, \qquad P(u)x \succeq_K 0,\\ P(u)x \circ P(u)^{-1}\Delta s + P(u)^{-1}s \circ P(u)\Delta x = a_\varphi, \qquad P(u)^{-1}s \succeq_K 0, \end{aligned} \tag{4}$$

where

$$a_\varphi = \mu\left(\varphi'\left(\frac{P(u)x \circ P(u)^{-1}s}{\mu}\right)\right) \circ \left(\varphi(e) - \varphi\left(\frac{P(u)x \circ P(u)^{-1}s}{\mu}\right)\right).$$

In [29] an overview of different functions φ used in the literature in the AET technique is presented.

Throughout the paper we use the NT-scaling scheme. Let $u = w^{-\frac{1}{2}}$, where w is called the NT-scaling point of x and s and is defined as follows:

$$w = P(x)^{\frac{1}{2}}\left(P(x)^{\frac{1}{2}}s\right)^{-\frac{1}{2}} = P(s)^{-\frac{1}{2}}\left(P(s)^{\frac{1}{2}}x\right)^{\frac{1}{2}}. \tag{5}$$

Let us introduce the following notations:

$$v := \frac{P(w)^{-\frac{1}{2}}x}{\sqrt{\mu}} = \frac{P(w)^{\frac{1}{2}}s}{\sqrt{\mu}}, \quad d_x := \frac{P(w)^{-\frac{1}{2}}\Delta x}{\sqrt{\mu}}, \quad d_s := \frac{P(w)^{\frac{1}{2}}\Delta s}{\sqrt{\mu}}. \tag{6}$$

From (6) we obtain the scaled system:

$$\begin{aligned} \sqrt{\mu}QP(w)^{\frac{1}{2}}d_x + \sqrt{\mu}RP(w)^{-\frac{1}{2}}d_s = 0,\\ d_x + d_s = p_v, \end{aligned} \tag{7}$$

where

$$p_v = v^{-1} \circ (\varphi'(v \circ v))^{-1} \circ (\varphi(e) - \varphi(v \circ v)).$$

In order to be able to define IPAs, we should define a proximity measure to the central path. For this, let

$$\delta(v) = \delta(x, s, \mu) := \frac{\|p_v\|_F}{2}. \tag{8}$$

Furthermore, we define the τ-neighbourhood of a fixed point on the central path:

$$\mathcal{N}(\tau, \mu) := \{(x, s) \in \mathcal{V} \times \mathcal{V} : Qx + Rs = q,\ x \succeq_K 0,\ \ s \succeq_K 0 :\ \delta(x, s, \mu) \leq \tau\},$$

where τ is a threshold parameter and $\mu > 0$ is fixed.

As it was mentioned in the Introduction, the search directions can be determined by using another approach, based on kernel functions, see [7,32]. Achache and Pan et al. [1,28] showed that one can associate corresponding kernel functions to several functions φ used in the AET technique. In [29], the author also presented the relationship between the approach based on kernel functions and on the AET technique. However, it is interesting that if we apply the function $\varphi(t) = t - \sqrt{t}$ in the AET, we cannot obtain a corresponding kernel function in the usual sense, because it is not defined on a neighbourhood of the origin, see [15]. Darvay and Rigó [15] introduced the notion of the positive-asymptotic kernel function. In this sense, the kernel function associated to the function $\varphi(t) = t - \sqrt{t}$ used in this paper is positive-asymptotic kernel function:

$$\psi : \left(\frac{1}{2}, \infty\right) \to [0, \infty), \quad \psi(t) = \frac{t^2}{2} - \frac{t}{2} - \frac{1}{4}\log(2t - 1). \tag{9}$$

For details, see [2,15,29]. In the following subsection we generalize a method for determining the scaled predictor and corrector systems in case of the PC IPAs which is given in [30].

3.1 Determining search directions in case of predictor-corrector interior-point algorithms

We generalize the method introduced by Darvay et al. [13] and presented in [30] in order to determine the search directions in case of PC IPAs. The PC IPAs perform a predictor and several corrector steps in a main iteration. The predictor step is a greedy one which aims to find the optimal solution of the problem as soon as possible. Hence, a certain amount of retirement from the central path is obtained after a predictor step. In the corrector step the goal is to return in the τ-neighbourhood of the central path.

Firstly, we deal with the corrector step, which is a full-Nesterov step. Hence, the scaled corrector system coincides with (7). We should decompose a_φ in system (4) in order to obtain the scaled predictor system:

$$a_\varphi = f(x, s, \mu) + g(x, s),$$

where f and g are vector-valued functions and $f(x, s, 0) = 0$. We set $\mu = 0$ in this decomposition. In this way,

$$\begin{aligned} \widetilde{Q}P(u)\Delta x + \widetilde{R}P(u)^{-1}\Delta s &= 0, \qquad P(u)x \succeq_K 0, \\ P(u)x \circ P(u)^{-1}\Delta s + P(u)^{-1}s \circ P(u)\Delta x &= g(x, s) \qquad P(u)^{-1}s \succeq_K 0, \end{aligned} \tag{10}$$

The scaled predictor system is the following:

$$\begin{aligned} \sqrt{\mu}QP(w)^{\frac{1}{2}}d_x + \sqrt{\mu}RP(w)^{-\frac{1}{2}}d_s &= 0, \\ d_x + d_s &= (\mu v)^{-1} \circ g(x, s). \end{aligned} \tag{11}$$

Using this system the predictor search directions can be easily obtained. In the following subsection we introduce the new PC IPAs for solving Cartesian $P_*(\kappa)$-SCHLCPs.

4 New predictor-corrector interior-point algorithm

We deal with the case, when $\varphi(t) = t - \sqrt{t}$. In this case

$$p_v = 2(v - v^2) \circ (2v - e)^{-1}. \tag{12}$$

In the corrector step we take a full-Nesterov step. Hence, the scaled corrector system coincides with system (7) with p_v given in (12):

$$\begin{aligned}\sqrt{\mu} Q P(w)^{\frac{1}{2}} d_x^c + \sqrt{\mu} R P(w)^{-\frac{1}{2}} d_s^c &= 0,\\ d_x^c + d_s^c &= 2(v - v^2) \circ (2v - e)^{-1},\end{aligned} \tag{13}$$

We can calculate the corrector search directions $\Delta^c x$ and $\Delta^c s$ from

$$\Delta_x^c = \sqrt{\mu} P(w)^{\frac{1}{2}} d_x^c, \quad \Delta_s^c = \sqrt{\mu} P(w)^{-\frac{1}{2}} d_s^c. \tag{14}$$

Let $x^c = x + \Delta^c x$ and $s^c = s + \Delta^c s$ be the point after a corrector step. In the predictor step we define the following notations:

$$v^c = \frac{P(w^c)^{-\frac{1}{2}} x^c}{\sqrt{\mu}} = \frac{P(w^c)^{\frac{1}{2}} s^c}{\sqrt{\mu}},$$

where w^c is the NT-scaling point of x^c and s^c. The scaled predictor system in this case is

$$\begin{aligned}\sqrt{\mu} Q P(w^c)^{\frac{1}{2}} d_x^p + \sqrt{\mu} R P(w^c)^{-\frac{1}{2}} d_s^p &= 0,\\ d_x^p + d_s^p &= -v^c.\end{aligned} \tag{15}$$

We obtain the predictor search directions $\Delta^p x$ and $\Delta^p s$ from

$$\Delta_x^p = \sqrt{\mu} P(w^c)^{\frac{1}{2}} d_x^p, \quad \Delta_s^p = \sqrt{\mu} P(w^c)^{-\frac{1}{2}} d_s^p. \tag{16}$$

The point after a predictor step is $x^p = x^c + \theta \Delta^p x$, and $s^p = s^c + \theta \Delta^p \mathbf{s}$, $\mu^p = (1 - \theta)\mu$, where $\theta \in (0, 1)$ is the update parameter. The proximity measure in this case is

$$\delta(v) := \delta(x, s, \mu) = \frac{\|p_v\|_F}{2} = \left\| (v - v^2) \circ (2v - e)^{-1} \right\|_F. \tag{17}$$

Our PC IPA algorithm starts with $(x^0, s^0) \in \mathcal{N}(\tau, \mu)$. The algorithm performs corrector and predictor steps. The PC IPA is given in Algorithm 4.1.

Algorithm 4.1 : **PC IPA for Cartesian SCHLCP using** $\varphi(t) = t - \sqrt{t}$ **in the AET**

Let $\epsilon > 0$ *be the accuracy parameter,* $0 < \theta < 1$ *the update parameter and* τ *the proximity parameter. Assume that for* (x^0, s^0) *the (IPC) holds such that* $\delta(x^0, s^0, \mu^0) \le \tau$ *and* $\lambda_{\min}\left(\frac{x^0 \circ s^0}{\mu^0}\right) > \frac{1}{4}$.

begin
  $k := 0$;
  while $\langle x^k, s^k \rangle > \epsilon$ **do begin**

    (corrector step)
    compute w *using (5);*
    compute $(\Delta^c x^k, \Delta^c s^k)$ *from system (13) using (6);*
    let $(x^c)^k := x^k + \Delta^c x^k$ *and* $(s^c)^k := s^k + \Delta^c s^k$;

    (predictor step)
    compute w^c *as the NT-scaling point of* x^c *and* s^c*;*
    compute $(\Delta^p x^k, \Delta^p s^k)$ *from system (15) using (16);*
    let $(x^p)^k := (x^c)^k + \theta \Delta^p x^k$ *and* $(s^p)^k := (s^c)^k + \theta \Delta^p s^k$;

    (update of the parameters and the iterates)
    $(\mu^p)^k = (1-\theta)\mu^k$;
    $x^{k+1} := (x^p)^k, \quad s^{k+1} := (s^p)^k, \quad \mu^{k+1} := (\mu^p)^k$;
    k:=k+1;

  end
end.

5 Analysis of the predictor-corrector interior-point algorithm

5.1 The corrector step

The corrector step is a full-NT step, hence the following lemmas can be easily obtained by using the results published in [2]. Consider

$$q_v = d_x^c - d_s^c, \tag{18}$$

hence

$$d_x^c = \frac{p_v + q_v}{2}, \quad d_s^c = \frac{p_v - q_v}{2}, \quad d_x^c \circ d_s^c = \frac{p_v^2 - q_v^2}{4}. \tag{19}$$

Lemma 2 gives an upper bound for $\|q_v\|_F$ in terms of $\|p_v\|_F$.

Lemma 2 *(Lemma 5.1 in [2]) We have* $\|q_v\|_F \le \sqrt{1+4\kappa}\ \|p_v\|_F$.

Let x, $s \in \operatorname{int} \mathcal{K}$, $\mu > 0$ and w be the scaling point of x and s. We have

$$\begin{aligned} x^c &:= x + \Delta x = \sqrt{\mu} P(w)^{1/2}(v + d_x^c), \\ s^c &:= s + \Delta s = \sqrt{\mu} P(w)^{-1/2}(v + d_s^c). \end{aligned} \tag{20}$$

It should be mentioned that x^c and s^c belong to int $\mathcal{K}$ if and only if $v + d_x^c$ and $v + d_s^c$ belong to int $\mathcal{K}$, because $P(w)^{1/2}$ and its inverse, $P(w)^{-1/2}$, are automorphisms of int $\mathcal{K}$, cf. Proposition 1 part (ii) from Appendix. The next lemma proves the strict feasibility of the full-NT step.

Lemma 3 *(Lemma 5.3 in [2]) Let* $\delta := \delta(x, s; \mu) < \frac{1}{\sqrt{1+4\kappa}}$. *Then,* $\lambda_{\min}(v) > \frac{1}{2}$ *and the full-NT step is strictly feasible, that is,* $x^c \succ_K 0$ *and* $s^c \succ_K 0$.

Lemma 4 gives an upper bound for the proximity measure after a full-NT step.

Lemma 4 *(Lemma 5.6 in [2]) Let* $\delta = \delta(x, s; \mu) < \frac{1}{2\sqrt{1+4\kappa}}$ *and* $\lambda_{min}(v) > \frac{1}{2}$. *Then,* $\lambda_{min}(v^c) > \frac{1}{2}$ *and*

$$\delta(x^c, s^c; \mu) \le \frac{\sqrt{1-(1+4\kappa)\delta^2}}{2(1-(1+4\kappa)\delta^2) + \sqrt{1-(1+4\kappa)\delta^2} - 1}(3+4\kappa)\delta^2.$$

Furthermore,

$$\delta(x^c, s^c; \mu) < \frac{3-\sqrt{3}}{2}(3+4\kappa)\delta^2.$$

Lemma 5 provides an upper bound for the duality gap $\langle x, s\rangle$ after a full-NT step.

Lemma 5 *(cf. Lemma 5.8 in [2]) Let* x^c *and* s^c *be obtained after a full-NT step. Then,*

$$\langle x^c, s^c\rangle \le \mu(r + 2\delta^2).$$

Furthermore, if $\delta < \frac{1}{2(1+4\kappa)}$, *then*

$$\langle x^c, s^c\rangle < \frac{3}{2}\mu r.$$

In the following subsection, we present some technical lemmas.

5.2 Technical lemmas

First, consider the following two results that will be used in the next part of the analysis.

Lemma 6 *Let* $\bar{f} : (\bar{d}, +\infty) \longrightarrow (0, +\infty)$ *be a function, where* $\bar{d} > 0$ *and* $\bar{f}(t) > \xi$, *for* $t > \bar{d}$ *where* $\xi > 0$. *Assume that* $v \in \mathcal{V}$ *and* $\lambda_{min}(v) > \bar{d}$. *Then,*

$$\left\|\bar{f}(v) \circ (e - v)\right\|_F \ge \xi \left\|e - v\right\|_F.$$

Proof *Using Theorem 2 given in Appendix, we assume that* $v = \sum_{i=1}^r \lambda_i(v)c_i$. *Moreover,* $\bar{f}(v) = \sum_{i=1}^r \bar{f}(\lambda_i(v))c_i$. *Then,*

$$\left\|\bar{f}(v) \circ (e - v)\right\|_F = \sqrt{\sum_{i=1}^r \left(\bar{f}(\lambda_i(v))\right)^2 \lambda_i^2(e - v)} \ge \xi \left\|e - v\right\|_F.$$

Lemma 7 (Lemma 7.4 of [15]) *Let* $\widetilde{f} : [d, +\infty) \longrightarrow (0, +\infty)$ *be a decreasing function, where* $d > 0$. *Furthermore, suppose that* $v \in \mathcal{V}$, $\lambda_{\min}(v) > d$ *and* $\eta > 0$. *Then,*

$$\left\|\widetilde{f}(v) \circ \left(\eta^2 e - v^2\right)\right\|_F \le \widetilde{f}\left(\lambda_{\min}(v)\right)\left\|\eta^2 e - v^2\right\|_F \le \widetilde{f}(d) \cdot \|\eta^2 e - v \circ v\|_F.$$

In the following part we consider some inequalities that will be used later in the analysis [2]. Using the first equation of (7), we get

$$Q\left(P(w)^{1/2}d_x\right) + R\left(P(w)^{-1/2}d_s\right) = 0.$$

The definition of the $P_*(\kappa)$ property results in

$$\left\langle P(w)^{1/2}d_x,\, P(w)^{-1/2}d_s\right\rangle \ge -4\kappa \sum_{i\in I_+}\left\langle P\left(w^{(i)}\right)^{1/2}d_x^{(i)},\, P\left(w^{(i)}\right)^{-1/2}d_s^{(i)}\right\rangle, \qquad (21)$$

where $I_+ = \left\{ i : \left\langle d_x^{(i)},\, d_s^{(i)} \right\rangle > 0 \right\}$. Using that each $P\left(w^{(i)}\right)^{1/2}$, $1 \le i \le m$, is self-adjoint, we get

$$\left\langle P\left(w^{(i)}\right)^{1/2} d_x^{(i)},\, P\left(w^{(i)}\right)^{-1/2} d_s^{(i)} \right\rangle = \left\langle d_x^{(i)},\, d_s^{(i)} \right\rangle .$$

If we substitute the last equation into (21) and using that $P(w^{\frac{1}{2}})$ is self-adjont, we obtain

$$\langle d_x,\, d_s \rangle \ge -4\kappa \sum_{i \in I_+} \left\langle d_x^{(i)},\, d_s^{(i)} \right\rangle . \tag{22}$$

Furthermore, one has

$$\sum_{i \in I_+} \left\langle d_x^{(i)},\, d_s^{(i)} \right\rangle \le \frac{1}{4} \sum_{i \in I_+} \left\| d_x^{(i)} + d_s^{(i)} \right\|_F^2 \le \frac{1}{4} \left\| d_x + d_s \right\|_F^2 = \frac{\|p_v\|_F^2}{4}, \tag{23}$$

and therefore $\langle d_x,\, d_s \rangle \ge -\kappa \|p_v\|_F^2$. Thus,

$$\begin{aligned} \|q_v\|_F^2 &= \|d_x - d_s\|_F^2 = \|d_x + d_s\|_F^2 - 4 \langle d_x,\, d_s \rangle \\ &\le \|p_v\|_F^2 + 4\kappa \|p_v\|_F^2 = (1 + 4\kappa) \|p_v\|_F^2 . \end{aligned} \tag{24}$$

We prove a lemma which is a generalization of Lemma 5.3 given in [13] to Cartesian SCHLCP. We assume that (Q, R) has the $P_*(\kappa)$-property.

Lemma 8 *The following inequality holds:*

$$\|d_x^p \circ d_s^p\|_F \le \frac{r(1 + 2\kappa)(1 + 2\delta^c)^2}{2},$$

where $\delta^c = \delta(x^c, s^c, \mu) = \left\| (v^c - (v^c)^2) \circ (2v^c - e)^{-1} \right\|_F$.

Proof *Using (23) and the second equation of the scaled predictor system (15) we have*

$$\sum_{i \in I_+} \left\langle d_x^{p\,(i)},\, d_s^{p\,(i)} \right\rangle \le \frac{1}{4} \|d_x^p + d_s^p\|_F^2 = \frac{\|v^c\|_F^2}{4}, \tag{25}$$

From (22) and (25) we obtain

$$\begin{aligned} \|v^c\|_F^2 &= \|d_x^p + d_s^p\|_F^2 \\ &= \|d_x^p\|_F^2 + \|d_s^p\|_F^2 + 2\langle d_x^p, d_s^p \rangle \\ &\ge \|d_x^p\|_F^2 + \|d_s^p\|_F^2 - 8\kappa \sum_{i \in I_+} \langle d_x^{p\,(i)}, d_s^{p\,(i)} \rangle \\ &\ge \|d_x^p\|_F^2 + \|d_s^p\|_F^2 - 2\kappa \|v^c\|_F^2. \end{aligned}$$

Hence, $\|d_x^p\|_F^2 + \|d_s^p\|_F^2 \le (1 + 2\kappa)\|v^c\|_F^2$. *We give an upper bound for* $\|v^c\|_F$ *depending on* δ^c *and* r. *For this, consider* $\sigma^c = \|e - v^c\|_F$. *Then,*

$$\|v^c\|_F \le \|v^c - e\|_F + \|e\|_F = \sigma^c + \sqrt{r} \le \sqrt{r}(\sigma^c + 1). \tag{26}$$

Consider $\bar{f}(t) = \frac{t}{2t-1} > \frac{1}{2}$. *Then, using Lemma 6 with* $\xi = \frac{1}{2}$ *and* $\bar{d} = \frac{1}{2}$ *we have*

$$\begin{aligned} \delta^c &= \left\| (v^c - (v^c)^2) \circ (2v^c - e)^{-1} \right\|_F \\ &= \left\| v^c \circ (2v^c - e)^{-1} \circ (e - v^c) \right\|_F \ge \frac{1}{2} \|e - v^c\|_F = \frac{\sigma^c}{2}, \end{aligned} \tag{27}$$

hence $\sigma^c \le 2\delta^c$. *From (26) and (27) we get*

$$\|v^c\| \le \sqrt{r}(1+2\delta^c). \tag{28}$$

In this way, we have

$$\begin{aligned}\|d_x^p \circ d_s^p\|_F \le \|d_x^p\|_F \|d_s^p\|_F &\le \frac{1}{2}\left(\|d_x^p\|_F^2 + \|d_s^p\|_F^2\right)\\ &\le \frac{1}{2}(1+2\kappa)\,\|v^c\|_F^2 \le \frac{r(1+2\kappa)(1+2\delta^c)^2}{2},\end{aligned} \tag{29}$$

which proves the lemma.

5.3 The predictor step

The first lemma of this subsection is a generalization of Lemma 5.5 given in [13] to Cartesian SCHLCP. It provides a sufficient condition for the strict feasibility of the predictor step.

Lemma 9 *Let* $x^c \succ_K 0$ *and* $s^c \succ_K 0$, $\mu > 0$ *such that* $\delta^c := \delta(x^c, s^c, \mu) < \frac{1}{2}$. *Furthermore, let* $0 < \theta < 1$. *Let* $x^p = x^c + \theta \Delta^p x, \quad s^p = s^c + \theta \Delta^p s$ *be the iterates after a predictor step. Then,* $x^p \succ_K 0$ *and* $s^p \succ_K 0$ *if* $\bar{u}(\delta^c, \theta, r) > 0$, *where*

$$\bar{u}(\delta^c, \theta, r) := (1-2\delta^c)^2 - \frac{r(1+2\kappa)\theta^2(1+2\delta^c)^2}{2(1-\theta)}.$$

Proof *Let* $\alpha \in [0,1]$ *and* $x^p(\alpha) = x^c + \alpha\theta\Delta^p x$ *and* $s^p(\alpha) = s^c + \alpha\theta\Delta^p s$. *Hence,*

$$x^p(\alpha) = \sqrt{\mu} P(w^c)^{\frac{1}{2}}(v^c + \alpha\theta d_x^p), \quad s^p(\alpha) = \sqrt{\mu} P(w^c)^{-\frac{1}{2}}(v^c + \alpha\theta d_s^p). \tag{30}$$

From (30) and using that $P(w^c)^{\frac{1}{2}}$ *and* $P(w^c)^{-\frac{1}{2}}$ *are automorphisms of* $int\,\mathcal{K}$ *we obtain that* $x^p(\alpha) \in int\,\mathcal{K}$ *and* $s^p(\alpha) \in int\,\mathcal{K}$ *if and only if* $v^c + \alpha\theta d_x^p \in int\,\mathcal{K}$ *and* $v^c + \alpha\theta d_s^p \in int\,\mathcal{K}$. *Let* $v_x^p(\alpha) = v^c + \alpha\theta d_x^p$ *and* $v_s^p(\alpha) := v^c + \alpha\theta d_s^p$, *for* $0 \le \alpha \le 1$. *From the second equation of system (15) we obtain*

$$\begin{aligned}v_x^p(\alpha) \circ v_s^p(\alpha) &= (v^c + \alpha\theta d_x^p) \circ (v^c + \alpha\theta d_s^p)\\ &= \left((v^c)^2 + \alpha\theta v^c \circ (d_x^p + d_s^p) + \alpha^2\,\theta^2\, d_x^p \circ d_s^p\right)\\ &= \left((1-\alpha\theta)\,(v^c)^2 + \alpha^2\,\theta^2\, d_x^p \circ d_s^p\right).\end{aligned} \tag{31}$$

Using (31) and Lemma 16 from Appendix we get

$$\begin{aligned}\lambda_{\min}\left(\frac{v_x^p(\alpha)\circ v_s^p(\alpha)}{(1-\alpha\theta)}\right) &= \lambda_{\min}\left((v^c)^2 + \frac{\alpha^2\,\theta^2}{(1-\alpha\theta)} d_x^p \circ d_s^p\right)\\ &\ge \lambda_{\min}\,(v^c)^2 - \frac{\alpha^2\theta^2}{1-\alpha\theta}\|d_x^p \circ d_s^p\|_F.\end{aligned} \tag{32}$$

Using $\lambda_i(e - v^c) \le \|e - v^c\|_F$, $\forall i = 1, \dots, r$, *we have*

$$1 - \sigma^c \le \lambda_i(v^c) \le 1 + \sigma^c, \ \forall i = 1, \dots, r. \tag{33}$$

From (27), (33) and $\delta^c < \frac{1}{2}$ *we have*

$$\lambda_{\min}\,(v^c)^2 \ge (1-\sigma^c)^2 \ge (1-2\delta^c)^2. \tag{34}$$

Note that $f(\alpha) = \frac{\alpha^2\theta^2}{1-\alpha\theta}$ *is strictly increasing for* $0 \le \alpha \le 1$ *and each fixed* $0 < \theta < 1$. *Using this, (32), (34) and Lemma 8 we get*

$$\lambda_{\min}\left(\frac{v_x^p(\alpha)\circ v_s^p(\alpha)}{(1-\alpha\theta)}\right) \ge (1-2\delta^c)^2 - \frac{r(1+2\kappa)\theta^2(1+2\delta^c)^2}{2(1-\theta)}$$
$$= \bar{u}(\delta^c,\theta,r) > 0. \tag{35}$$

This means that $x^p(\alpha)\circ s^p(\alpha) \succ_K 0$ *for* $0 \le \alpha \le 1$. *Hence,* $x^p(\alpha)$ *and* $s^p(\alpha)$ *do not change sign on* $0 \le \alpha \le 1$. *From* $x^p(0) = x^c \succ_K 0$ *and* $s^p(0) = s^c \succ_K 0$, *we deduce that* $x^p(1) = x^p \succ_K 0$ *and* $s^p(1) = s^p \succ_K 0$, *which yields the result.*

Consider the following notations

$$v^p = \frac{P(w^p)^{-\frac{1}{2}}x^p}{\sqrt{\mu^p}} = \frac{P(w^p)^{\frac{1}{2}}s^p}{\sqrt{\mu^p}},$$

where $\mu^p = (1-\theta)\mu$ and w^p is the NT-scaling point of x^p and s^p. Substituting $\alpha = 1$ in (31) and (35) we get

$$(v^p)^2 = (v^c)^2 + \frac{\theta^2}{1-\theta}(d_x^p \circ d_s^p), \tag{36}$$

$$\lambda_{\min}(v^p)^2 \ge \bar{u}(\delta^c,\theta,r) > 0. \tag{37}$$

The next lemma investigates the effect of a predictor step and the update of μ on the proximity measure. This is the generalization of Lemma 5.6 proposed in [13] to Cartesian SCHLCP.

Lemma 10 *Consider* $\delta^c := \delta(x^c,s^c,\mu) < \frac{1}{2}$, $\delta := \delta(x,s,\mu)$, $\lambda_{min}(v) > \frac{1}{2}$, $\mu^p = (1-\theta)\mu$, *where* $0 < \theta < 1$, $\bar{u}(\delta^c,\theta,r) > \frac{1}{4}$ *and let* x^p *and* s^p *be the iterates after a predictor step. Then, we have*

$$\delta^p := \delta(x^p,s^p,\mu^p) \le \frac{\sqrt{\bar{u}(\delta^c,\theta,r)}\left((3+4\kappa)\delta^2 + (1-2\delta^c)^2 - \bar{u}(\delta^c,\theta,r)\right)}{2\bar{u}(\delta^c,\theta,r) + \sqrt{\bar{u}(\delta^c,\theta,r)} - 1},$$

and $\lambda_{\min}(v^p) > \frac{1}{2}$.

Proof *From* $\bar{u}(\delta^c,\theta,r) > \frac{1}{4} > 0$, *using Lemma 9 we obtain that* $x^p \succ_K 0$ *and* $s^p \succ_K 0$. *This means that the predictor step is strictly feasible. Furthermore, from (37) we get*

$$\lambda_{\min}(v^p) \ge \sqrt{\bar{u}(\delta^c,\theta,r)} > \frac{1}{2},$$

hence the first part of the lemma is proved. Moreover,

$$\delta^p = \left\|(v^p - (v^p)^2)\circ(2v^p - e)^{-1}\right\|_F$$
$$= \left\|v^p \circ \left(e - (v^p)^2\right)\circ((2v^p - e)\circ(e+v^p))^{-1}\right\|_F. \tag{38}$$

Consider $\widetilde{f} : \left(\frac{1}{2},\infty\right) \to \mathbb{R}$, $\widetilde{f}(t) = \frac{t}{(2t-1)(1+t)}$, *which is decreasing with respect to* t. *Using this, (36), (37) and (38) and Lemma 7 we get*

$$\delta^p = \left\|v^p \circ \left(e - (v^p)^2\right)\circ((2v^p - e)\circ(e+v^p))^{-1}\right\|_F$$
$$\le \frac{\lambda_{\min}(v^p)}{(2\lambda_{\min}(v^p) - 1)(1+\lambda_{\min}(v^p))}\left\|e - (v^p)^2\right\|_F$$
$$\le \frac{\sqrt{\bar{u}(\delta^c,\theta,r)}}{\left(2\sqrt{\bar{u}(\delta^c,\theta,r)} - 1\right)\left(1+\sqrt{\bar{u}(\delta^c,\theta,r)}\right)}\left\|e - (v^c)^2 - \frac{\theta^2}{1-\theta}(d_x^p \circ d_s^p)\right\|_F \tag{39}$$
$$\le \frac{\sqrt{\bar{u}(\delta^c,\theta,r)}}{\left(2\sqrt{\bar{u}(\delta^c,\theta,r)} - 1\right)\left(1+\sqrt{\bar{u}(\delta^c,\theta,r)}\right)}\left(\left\|e - (v^c)^2\right\|_F + \frac{\theta^2}{1-\theta}\|d_x^p \circ d_s^p\|_F\right).$$

Using the definition of v^c, (6), (18) and (19) we have

$$\begin{aligned}\left\|e-(v^c)^2\right\|_F &= \|(v+d_x^c)(v+d_s^c)-e\|_F\\ &= \|v^2+v\circ(d_x^c+d_s^c)-e+d_x^c\circ d_s^c\|_F\\ &\le \|v^2+v\circ p_v-e\|_F+\left\|\frac{p_v^2-q_v^2}{4}\right\|_F. \end{aligned} \tag{40}$$

Beside this, using $\lambda_{min}(v)>\frac{1}{2}$, $v^2\circ(2v-e)^{-1}\succeq_K 0$, we have

$$\begin{aligned} 0\preceq_K v^2+v\circ p_v-e &= v^2+2v^2\circ(e-v)\circ(2v-e)^{-1}-e\\ &= (v-e)^2\circ(2v-e)^{-1}\\ &\preceq_K ((v-e)^2\circ v^2)\circ(2v-e)^{-2}=\frac{p_v^2}{4}. \end{aligned} \tag{41}$$

Using (40), (41) and Lemma 2 we obtain

$$\begin{aligned}\left\|e-(v^c)^2\right\|_F &\le \|v^2+v\circ p_v-e\|_F+\left\|\frac{p_v^2-q_v^2}{4}\right\|_F\\ &\le \frac{\|p_v\|_F^2}{4}+\frac{\|p_v\|_F^2}{4}+\frac{\|q_v\|_F^2}{4}\\ &\le 2\delta^2+(1+4\kappa)\delta^4\le(3+4\kappa)\delta^2. \end{aligned} \tag{42}$$

From (39), (42) and Lemma 8 we obtain the second statement of the lemma.

The next lemma gives an upper bound for the complementarity gap after an iteration of the algorithm.

Lemma 11 *Let $x^c\succeq_K 0$, $s^c\succeq_K 0$ and $\mu>0$ such that $\delta^c:=\delta(x^c,s^c,\mu)<\frac{1}{2}$ and $0<\theta<1$. If $\delta<\frac{1}{2(1+4\kappa)}$ and x^p and s^p are the iterates obtained after the predictor step of the algorithm, then*

$$\langle x^p,s^p\rangle\le\left(1-\theta+\frac{\theta^2}{2}\right)\langle x^c,s^c\rangle\le\left(1-\frac{\theta}{2}\right)\langle x^c,s^c\rangle<\frac{3r\mu^p}{2(1-\theta)}.$$

Proof Using the definition of v^p and (36) we have

$$\begin{aligned}\langle x^p,s^p\rangle &= \mu^p\langle e,(v^p)^2\rangle=\mu\langle e,(1-\theta)(v^c)^2+\theta^2 d_x^p\circ d_s^p\rangle\\ &= (1-\theta)\langle x^c,s^c\rangle+\mu\theta^2\langle d_x^p,d_s^p\rangle \end{aligned} \tag{43}$$

From the second equation of (15) we obtain

$$\langle d_x^p,d_s^p\rangle=\frac{\langle x^c,s^c\rangle}{2\mu}-\frac{\|d_x^p\|_F^2+\|d_s^p\|_F^2}{2}\le\frac{\langle x^c,s^c\rangle}{2\mu}. \tag{44}$$

Moreover, using (43) and (44) we get

$$\langle x^p,s^p\rangle\le\left(1-\theta+\frac{\theta^2}{2}\right)\langle x^c,s^c\rangle.$$

Note that if $0<\theta<1$, then we have

$$1-\theta+\frac{\theta^2}{2}<1-\frac{\theta}{2}. \tag{45}$$

From (45), $\mu^p=(1-\theta)\mu$ and Lemma 5 we deduce

$$\langle x^p,s^p\rangle\le\left(1-\theta+\frac{\theta^2}{2}\right)\langle x^c,s^c\rangle<\left(1-\frac{\theta}{2}\right)\langle x^c,s^c\rangle<\frac{3r\mu^p}{2(1-\theta)},$$

which yields the result.

5.4 Complexity bound

Firstly, consider the following notation:

$$\Psi(\tau) = \frac{\left(1 - \frac{2}{3}\tau\right)^2 - \frac{1}{2}}{\left(1 + \frac{2}{3}\tau\right)^2}. \tag{46}$$

In the following lemma we give a condition related to the proximity and update parameters τ and θ for which the PC IPA is well defined. This result is one of the novelties of the paper.

Lemma 12 *Let $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, where $\bar{c} \geq 2$ and $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$, where $\bar{g} \geq 2$. If*

i. $\bar{c} \leq \frac{1}{2}\bar{g}$,
ii. $\frac{r(1+2\kappa)\theta^2}{2(1-\theta)} < \Psi(\tau)$,

then the PC IPA proposed in Algorithm 4.1 is well defined.

Proof *Let (x, s) be the iterate at the start of an iteration with $x \succeq_K 0$ and $s \succeq_K 0$ such that $\lambda_{\min}\left(\frac{x \circ s}{\mu}\right) > \frac{1}{4}$ and $\delta(x, s, \mu) \leq \tau$. It should be mentioned that $\tau = \frac{1}{\bar{c}(3+4\kappa)} < \frac{1}{2\sqrt{1+4\kappa}}$, where $\bar{c} \geq 2$. After a corrector step, applying Lemma 4 we have*

$$\delta^c := \delta(x^c, s^c, \mu) < \frac{3 - \sqrt{3}}{2}(3 + 4\kappa)\delta^2.$$

The right-hand side of the above inequality is monotonically increasing with respect to δ, which implies:

$$\delta^c \leq \frac{3 - \sqrt{3}}{2}(3 + 4\kappa)\tau^2 =: \omega(\tau). \tag{47}$$

Substituting $\tau = \frac{1}{\bar{c}(3+4\kappa)}$ in (47), using that $\frac{3-\sqrt{3}}{2} < \frac{2}{3}$, $\kappa \geq 0$ and $\bar{c} \geq 2$ we obtain

$$\omega(\tau) = \frac{(3 - \sqrt{3})\tau}{2\bar{c}} < \frac{2\tau}{3\bar{c}} \leq \frac{1}{3}\tau. \tag{48}$$

Using $\frac{r(1+2\kappa)\theta^2}{2(1-\theta)} < \Psi(\tau)$ and (48) we obtain

$$\begin{aligned}
\frac{1}{4} < \frac{1}{2} &< \left(1 - \frac{2}{3}\tau\right)^2 - \frac{r(1+2\kappa)\theta^2}{2(1-\theta)}\left(1 + \frac{2}{3}\tau\right)^2 \\
&< (1 - 2\omega(\tau))^2 - \frac{r(1+2\kappa)\theta^2(1 + 2\omega(\tau))^2}{2(1-\theta)}, \\
&\leq (1 - 2\delta^c)^2 - \frac{r(1+2\kappa)\theta^2(1 + 2\delta^c)^2}{2(1-\theta)} = \bar{u}(\delta^c, \theta, r),
\end{aligned} \tag{49}$$

hence condition $\bar{u}(\delta^c, \theta, r) > \frac{1}{4}$ from Lemma 10 is satisfied. Furthermore, using $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, $\bar{c} \geq 2$ and (48) we have $\delta^c \leq \omega(\tau) < \frac{1}{18} < \frac{1}{2}$. Following a predictor step and a μ-update, by Lemma 10 we have

$$\delta^p \leq \frac{\sqrt{\bar{u}(\delta^c, \theta, r)}\left((3 + 4\kappa)\delta^2 + (1 - 2\delta^c)^2 - \bar{u}(\delta^c, \theta, r)\right)}{2\bar{u}(\delta^c, \theta, r) + \sqrt{\bar{u}(\delta^c, \theta, r)} - 1}, \tag{50}$$

where $\delta := \delta(x, s, \mu)$. It can be verified that $\bar{u}(\delta^c, \theta, r)$ is decreasing with respect to δ^c. In this way, we obtain $\bar{u}(\delta^c, \theta, r) \geq \bar{u}(\omega(\tau), \theta, r)$. We have seen in Lemma 10 that the function $\tilde{f}$ is decreasing with respect to t, therefore

$$\tilde{f}(\sqrt{\bar{u}(\delta^c, \theta, r)}) \leq \tilde{f}(\sqrt{\bar{u}(\omega(\tau), \theta, r)}). \tag{51}$$

From (49) we have $\sqrt{\bar{u}(\omega(\tau),\theta,r)} > \frac{\sqrt{2}}{2}$*, hence using (51)*

$$\tilde{f}(\sqrt{\bar{u}(\delta^c,\theta,r)}) < \tilde{f}\left(\frac{\sqrt{2}}{2}\right) = 1. \tag{52}$$

Using that $\delta \leq \tau$*,* $\delta^c \leq \omega(\tau)$ *and*

$$(1-2\delta^c)^2 - \bar{u}(\delta^c,\theta,r) = \frac{r(1+2\kappa)\,\theta^2\,(1+2\delta^c)^2}{2(1-\theta)}, \tag{53}$$

which is increasing with respect to δ^c*, we obtain:*

$$\begin{aligned}
&\frac{\sqrt{\bar{u}(\delta^c,\theta,r)}\left((3+4\kappa)\delta^2 + (1-2\delta^c)^2 - \bar{u}(\delta^c,\theta,r)\right)}{2\bar{u}(\delta^c,\theta,r) + \sqrt{\bar{u}(\delta^c,\theta,r)} - 1} \leq \\
&\leq \frac{\sqrt{\bar{u}(\omega(\tau),\theta,r)}\left((3+4\kappa)\tau^2 + (1-2\omega(\tau))^2 - \bar{u}(\omega(\tau),\theta,r)\right)}{2\bar{u}(\omega(\tau),\theta,r) + \sqrt{\bar{u}(\omega(\tau),\theta,r)} - 1}.
\end{aligned} \tag{54}$$

From $\tau = \frac{1}{\bar{c}(3+4\kappa)}$ *and* $\delta \leq \tau$ *we have*

$$(3+4\kappa)\delta^2 \leq (3+4\kappa)\tau^2 = \frac{1}{\bar{c}^2(3+4\kappa)} = \frac{1}{\bar{c}}\tau \tag{55}$$

Moreover, we use $\theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$ *and* $\kappa \geq 0$*, hence we obtain*

$$\frac{1}{1-\theta} \leq \frac{3\bar{g}}{3\bar{g}-2}. \tag{56}$$

We also consider

$$\theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}} \leq \frac{1}{\bar{g}(1+2\kappa)\sqrt{r}}. \tag{57}$$

Using (56) and (57) we get

$$\begin{aligned}
\frac{r(1+2\kappa)\,\theta^2}{2(1-\theta)} &\leq \frac{r(1+2\kappa)}{2}\cdot\frac{3\bar{g}}{3\bar{g}-2}\cdot\frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}\cdot\frac{1}{\bar{g}(1+2\kappa)\sqrt{r}} \\
&= \frac{3}{\bar{g}(3\bar{g}-2)}\cdot\frac{1}{3+4\kappa} = \frac{3\bar{c}}{\bar{g}(3\bar{g}-2)}\tau.
\end{aligned} \tag{58}$$

We use (48) and $3+4\kappa \geq 3$*, hence*

$$\omega(\tau) < \frac{2}{3\bar{c}^2(3+4\kappa)} \leq \frac{2}{9\bar{c}^2}. \tag{59}$$

Furthermore, from (53), (58) and (59) we have

$$\begin{aligned}
(1-2\omega(\tau))^2 - \bar{u}(\omega(\tau),\theta,r) &= \frac{r(1+2\kappa)\,\theta^2\,(1+2\omega(\tau))^2}{2(1-\theta)} \\
&\leq \frac{3\bar{c}}{\bar{g}(3\bar{g}-2)}\left(1+\frac{4}{9\bar{c}^2}\right)^2\tau.
\end{aligned} \tag{60}$$

Conditions $\bar{c} \geq 2$*,* $\bar{g} \geq 2$ *and* $\bar{c} \leq \frac{1}{2}\bar{g}$ *yield*

$$\begin{aligned}
\frac{3\bar{c}}{\bar{g}(3\bar{g}-2)}\left(1+\frac{4}{9\bar{c}^2}\right)^2 &= 3\frac{\bar{c}}{\bar{g}}\frac{1}{3\bar{g}-2}\left(1+\frac{4}{9\bar{c}^2}\right)^2 \\
&\leq 3\cdot\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{100}{81} = \frac{25}{54} < \frac{1}{2}.
\end{aligned} \tag{61}$$

From (55), (60) and (61), using $\bar{c} \geq 2$ we get

$$(3+4\kappa)\tau^2 + (1-2\omega(\tau))^2 - \bar{u}(\omega(\tau),\theta,r) \leq \left(\frac{1}{\bar{c}} + \frac{3\bar{c}}{\bar{g}(3\bar{g}-2)}\left(1+\frac{4}{9\bar{c}^2}\right)^2\right)\tau$$
$$< \left(\frac{1}{2}+\frac{1}{2}\right)\tau = \tau. \tag{62}$$

Using (50), (52) and (62) we have

$$\delta^p < \tau, \tag{63}$$

hence the PC IPA is well defined.

The following lemma gives a sufficient condition for satisfying Condition ii. from Lemma 12.

Lemma 13 *Let $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, where $\bar{c} \geq 2$ and $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$, where $\bar{g} \geq 2$. A sufficient condition for satisfying ii. from Lemma 12 is*

$$\frac{1}{\bar{g}^2} < \Psi\left(\frac{1}{3\bar{c}}\right), \tag{64}$$

where Ψ is given in (46).

Proof *Using $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$ and $\bar{g} \geq 2$ we have $\theta \leq \frac{1}{2}$. From this we obtain*

$$\frac{1}{2(1-\theta)} \leq 1. \tag{65}$$

Furthermore, using (57) and $\kappa \geq 0$ we get

$$r(1+2\kappa)\theta^2 \leq r(1+2\kappa)\frac{1}{\bar{g}^2(1+2\kappa)^2 r} = \frac{1}{\bar{g}^2(1+2\kappa)} \leq \frac{1}{\bar{g}^2}. \tag{66}$$

Beside this, from $\tau = \frac{1}{\bar{c}(3+4\kappa)}$ and $\kappa \geq 0$ we obtain

$$\tau \leq \frac{1}{3\bar{c}}. \tag{67}$$

It should be mentioned that the function $\Psi(\tau)$ is strictly decreasing with respect to τ, hence using (67) we obtain

$$\Psi(\tau) \geq \Psi\left(\frac{1}{3\bar{c}}\right). \tag{68}$$

In this way, using (64), (65), (66) and (68) we obtain the

$$\frac{r(1+2\kappa)\theta^2}{2(1-\theta)} \leq \frac{1}{\bar{g}^2} < \Psi\left(\frac{1}{3\bar{c}}\right) \leq \Psi(\tau), \tag{69}$$

which yields the result.

Lemma 14 *Let $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, where $\bar{c} \geq 2$ and $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$, where $\bar{g} \geq 2$. If Condition i. from Lemma 12 is satisfied, then Condition ii. from Lemma 12 also holds.*

Proof Note that (64) gives the following lower bound on $\bar{g}$:

$$\bar{g} > \frac{1}{\sqrt{\frac{\left(1-\frac{2}{9\bar{c}}\right)^2-\frac{1}{2}}{\left(1+\frac{2}{9\bar{c}}\right)^2}}}. \tag{70}$$

Condition i. from Lemma 12 yields another lower bound on $\bar{g}$, i.e.

$$\bar{g} \geq 2\bar{c}. \tag{71}$$

Hence, in order to satisfy Conditions i. and ii. from Lemma 12, it is enough to satisfy

$$\bar{g} > \max\left\{\frac{1}{\sqrt{\frac{\left(1-\frac{2}{9\bar{c}}\right)^2-\frac{1}{2}}{\left(1+\frac{2}{9\bar{c}}\right)^2}}}, 2\bar{c}\right\}. \tag{72}$$

Consider the following function

$$m(\bar{c}) = \frac{1}{2\bar{c}\sqrt{\frac{\left(1-\frac{2}{9\bar{c}}\right)^2-\frac{1}{2}}{\left(1+\frac{2}{9\bar{c}}\right)^2}}}, \tag{73}$$

which is decreasing with respect to $\bar{c}$. Thus, for $\bar{c} \geq 2$ we have

$$m(\bar{c}) \leq m(2) < 1. \tag{74}$$

Using (74) we obtain that

$$\max\left\{\frac{1}{\sqrt{\frac{\left(1-\frac{2}{9\bar{c}}\right)^2-\frac{1}{2}}{\left(1+\frac{2}{9\bar{c}}\right)^2}}}, 2\bar{c}\right\} = 2\bar{c}. \tag{75}$$

This means that if Condition i. from Lemma 12 is satisfied, then Condition ii. from Lemma 12 also holds.

Corollary 1 *Let $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, where $\bar{c} \geq 2$ and $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$. If $\bar{g} \geq 2\bar{c}$, then the PC IPA proposed in Algorithm 4.1 is well defined.*

Lemma 15 *Let $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, where $\bar{c} \geq 2$ and $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$, where $\bar{g} \geq 2\bar{c}$. Moreover, let x^0 and s^0 be strictly feasible, $\mu^0 = \frac{\langle x^0, s^0\rangle}{r}$ and $\delta(x^0, s^0, \mu^0) \leq \tau$. Let x^k and s^k be the iterates obtained after k iterations. Then, $\langle x^k, s^k\rangle \leq \epsilon$ for*

$$k \geq 1 + \left\lceil \frac{1}{\theta} \log \frac{3\langle x^0, s^0\rangle}{2\epsilon} \right\rceil.$$

Proof Using Lemma 11 we have

$$\langle x^k, s^k\rangle < \frac{3r\mu^k}{2(1-\theta)} = \frac{3r(1-\theta)^{k-1}\mu^0}{2} = \frac{3(1-\theta)^{k-1}\langle x^0, s^0\rangle}{2}.$$

Hence, if

$$\frac{3(1-\theta)^{k-1}\langle x^0, s^0\rangle}{2} \leq \epsilon,$$

then the inequality $\langle x^k, s^k\rangle \leq \epsilon$ holds. We take logarithms, thus

$$(k-1)\log(1-\theta) + \log\frac{3\langle x^0, s^0\rangle}{2} \leq \log\epsilon. \tag{76}$$

From $\log(1+\theta) \leq \theta$, $\theta \geq -1$, we deduce that (76) is satisfied if

$$-\theta(k-1) + \log\frac{3\langle x^0, s^0\rangle}{2} \leq \log\epsilon,$$

hence the lemma is proved.

Theorem 1 *Let $\tau = \frac{1}{\bar{c}(3+4\kappa)}$, where $\bar{c} \geq 2$ and $0 \leq \theta \leq \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}$, where $\bar{g} \geq 2\bar{c}$. Then, the PC IPA proposed in Algorithm 4.1 is well defined and it performes at most*

$$\mathcal{O}\left((3+4\kappa)\sqrt{r}\log\frac{3r\mu^0}{2\epsilon}\right)$$

iterations. The output is a pair (x, s) satisfying $\langle x, s\rangle \leq \epsilon$.

Proof The result follows from Corollary 1 and Lemma 15.

Corollary 2 *Consider $0 < \tau \leq \frac{1}{6+8\kappa}$ and $0 < \theta \leq \frac{1}{\sqrt{r}}\tau$. Then, the PC IPA proposed in Algorithm 4.1 is well defined and it performes at most*

$$\mathcal{O}\left((3+4\kappa)\sqrt{r}\log\frac{3r\mu^0}{2\epsilon}\right)$$

iterations.

Proof If $\tau \leq \frac{1}{6+8\kappa}$, then we can find $\bar{c} \geq 2$ such that

$$\tau = \frac{1}{\bar{c}(3+4\kappa)}. \tag{77}$$

Using $\theta \leq \frac{1}{\sqrt{r}}\tau$ and $\tau \leq \frac{1}{6+8\kappa}$ we have

$$\theta \leq \frac{1}{\sqrt{r}}\tau \leq \frac{1}{(6+8\kappa)\sqrt{r}}. \tag{78}$$

Hence, we can find $\bar{g} \geq 4$ such that

$$\theta = \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}}. \tag{79}$$

Moreover, from (77), (79) and $\theta \leq \frac{1}{\sqrt{r}}\tau$ we have

$$\theta = \frac{2}{\bar{g}(3+4\kappa)\sqrt{r}} = \frac{2\bar{c}}{\bar{g}\sqrt{r}}\tau \leq \frac{1}{\sqrt{r}}\tau, \tag{80}$$

hence $\bar{g} \geq 2\bar{c}$ holds. All the conditions from Lemma 12 are satisfied, hence from Corollary 1 and Lemma 15 we obtain the desired result.

6 Concluding remarks

In this paper we extended the PC IPA proposed in [13] to $P_*(\kappa)$ horizontal linear complementarity problem defined over the Cartesian product of symmetric cones. For the determination of the search directions we used the function $\varphi(t) = t - \sqrt{t}$ in the AET technique proposed by Darvay [9]. We showed that the introduced PC IPA has polynomial iteration complexity in the special parameter κ, the size of the problem, the bitsize of the data and the deviation from the complementarity gap. Hence, we proved that the proposed PC IPA has the same complexity iteration bound as the best available one for PC IPAs given in the literature. We also provided a condition related to the proximity and update parameters for which the PC IPA is well defined. This is the first PC IPA using the function $\varphi(t) = t - \sqrt{t}$ in the AET technique which is well defined for a set of parameters.

Acknowledgements

This research has been partially supported by the Hungarian Research Fund, OTKA (grant no. NKFIH 125700). The research of P.R. Rigó was supported by the TKP2020, Institutional Excellence Program of the National Research Development and Innovation Office in the field of Artificial Intelligence (BME IE-MI-FM TKP2020). Zs. Darvay is grateful for the grant AGC32921/30.07.2019 of Babes-Bolyai University.

Appendix

We present some results related to the theory of Euclidean Jordan algebras and symmetric cones [16, 17, 35, 37].

Let $\mathcal{V}$ be an n-dimensional vector space over $\mathbb{R}$ with the bilinear map $\circ : (x, y) \to x \circ y \in \mathcal{V}$. Then, we say that $(\mathcal{V}, \circ)$ is a Jordan algebra if for all $x, y \in \mathcal{V}$, we have $x \circ y = y \circ x$ and $x \circ (x^2 \circ y) = x^2 \circ (x \circ y)$, where $x^2 = x \circ x$. We say that $e \in \mathcal{V}$ is the identity element of $\mathcal{V}$ if and only if $e \circ x = x \circ e = x$, for all $x \in \mathcal{V}$. We call the element x invertible if there exists a unique element $\bar{x}$, such that $x \circ \bar{x} = e$. The inverse of x is denoted by x^{-1}.
The $\mathcal{V}$ with an identity element is called Euclidean Jordan algebra if there exists a symmetric positive definite quadratic form $\bar{Q}$ on $\mathcal{V}$, which satisfies $\bar{Q}(x \circ y,\, z) = \bar{Q}(x,\, y \circ z)$. For any $x \in \mathcal{V}$, we define the Lyapunov transformation $L(x)$ as $L(x)y := x \circ y$, for all $y \in \mathcal{V}$. The quadratic representation $P(x)$ of x is given as $P(x) := 2L(x)^2 - L(x^2)$, where $L(x)^2 = L(x)L(x)$. The degree of an element x is the smallest integer r such that the set $\{e, x, \cdots, x^r\}$ is linearly dependent. This is denoted by $deg(x)$. The rank of $\mathcal{V}$, is the largest $deg(x)$ for all $x \in \mathcal{V}$ and we denote it by $\operatorname{rank}(\mathcal{V})$. A subset $\{c_1, c_2, \cdots, c_r\}$ of $\mathcal{V}$ is called a Jordan frame if it is a complete system of orthogonal primitive idempotents.

Theorem 2 (Theorem III.1.2 of [16]) *Suppose* $\operatorname{rank}(\mathcal{V}) = r$. *Then, for any x in $\mathcal{V}$ there exists a Jordan frame $c_1, \cdots, c_r$ and real numbers $\lambda_1, \cdots, \lambda_r$ such that $x = \sum_{i=1}^{r} \lambda_i c_i$.*

Note that the numbers λ_i are named eigenvalues. Consider $tr(x) = \sum_{i=1}^{r} \lambda_i$ and $\det(x) = \prod_{i=1}^{r} \lambda_i$. For any Euclidean Jordan algebra $\mathcal{V}$, consider the corresponding cone of squares $\mathcal{K}(\mathcal{V}) := \{x^2 \;:\; x \in \mathcal{V}\}$. This cone is symmetric, i.e. it is self-dual and homogeneous, see [16]. We use the following notations:

$$x \succeq_K 0 \quad \Leftrightarrow \quad x \in K \qquad \text{and} \qquad x \succ_K 0 \quad \Leftrightarrow \quad x \in \operatorname{int} K,$$

and

$$x \succeq_K s \quad \Leftrightarrow \quad x - s \succeq_K 0 \qquad \text{and} \qquad x \succ_K s \quad \Leftrightarrow \quad x - s \succ_K 0.$$

The inner product is defined as $\langle x, y \rangle = tr(x \circ y)$. The induced norm is the Frobenius norm:

$$\|x\|_F = \langle x, x \rangle^{1/2} = \sqrt{tr(x^2)} = \sqrt{\sum_{i=1}^{r} \lambda_i^2(x)}. \tag{81}$$

The following lemmas are used in the analysis of the IPA.

Proposition 1 *The following statements hold:*
(i) $x \in \mathcal{V}$ is invertible if and only if $P(x)$ is invertible, in which case $P(x)^{-1} = P(x^{-1})$.
(ii) If $x \in \mathcal{V}$ is invertible, then $P(x)\mathcal{K} = \mathcal{K}$ and $P(x) int\, \mathcal{K} = int\, \mathcal{K}$.
(iii) If $x \in \mathcal{K}$, then $P(x)^{1/2} = P(x^{1/2})$.
(iv) If $x \in \mathcal{V}$, then $x \in \mathcal{K}$ ($x \succeq_{\mathcal{K}} 0$) if and only if $\lambda_i(x) \geq 0$ and $x \in int\, \mathcal{K}$ ($x \succ_{\mathcal{K}} 0$) if and only if $\lambda_i(x) > 0$, for all $i = 1, \cdots, r$.

Lemma 16 (Lemma 14 of [34]) *If $x, s \in \mathcal{V}$, then*

$$\lambda_{\min}(x + s) \geq \lambda_{\min}(x) + \lambda_{\min}(s) \geq \lambda_{\min}(x) - \|s\|_F .$$

References

1. Achache, M.: A new primal-dual path-following method for convex quadratic programming, Comput. Appl. Math. 25, 97-110 (2006)
2. Asadi, S., Mahdavi-Amiri, N., Darvay, Zs., Rigó, P.R.: Full-NT step feasible interior-point algorithm for symmetric cone horizontal linear complementarity problem based on a positive-asymptotic barrier function, Optim. Methods Sofw., DOI:10.1080/10556788.2020.1734803 (2020)
3. Asadi, S., Mansouri, H., Darvay, Zs.: An infeasible full-NT step IPM for $P_*(\kappa)$ horizontal linear complementarity problem over Cartesian product of symmetric cones, Optimization 66, 225-250 (2017)
4. Asadi, S., Mansouri, H., Darvay, Zs., Zangiabadi, M.: On the $P_*(\kappa)$ horizontal linear complementarity problems over Cartesian product of symmetric cones, Optim. Methods Softw. 31, 233-257 (2016)
5. Asadi, S., Zangiabadi, M., Mansouri, H.: A predictor-corrector interior-point algorithm for $P_*(\kappa)$-HLCPs over Cartesian product of symmetric cones, Numer. Func. Anal. Optim. 38, 20-38 (2017)
6. Asadi, S., Mansouri, H., Lesaja, G., Zangiabadi, M.: A long-step interior-point algorithm for symmetric cone Cartesian $P_*(\kappa)$-HLCP, Optimization 67, 2031-2060 (2018)
7. Bai, Y.Q., El Ghami, M., Roos, C.: A comparative study of kernel functions for primal-dual interior-point algorithms in linear optimization. SIAM J. Optim. 15, 101128 (2004)
8. Chung, S.-J.: Np-completeness of the linear complementarity problem, J. Optim. Theory Appl., 60, 393-399 (1989)
9. Darvay, Zs.: New interior-point algorithms in linear programming, Adv. Model Optim. 5, 51-92 (2003)
10. Darvay, Zs.: A new predictor-corrector algorithm for linear programming, Alk. Mat. Lapok, 22, 135-161 (2005), in Hungarian.
11. Darvay, Zs.: A predictor-corrector algorithm for linearly constrained convex optimization, Studia Univ. Babeş-Bolyai, Ser. Informatica, 54, 121-138 (2009)
12. Darvay, Zs., Illés, T., Majoros, Cs.: Interior-point algorithm for sufficient LCPs based on the technique of algebraically equivalent transformation, Optim. Lett., 15, 357-376 (2021)
13. Darvay, Zs., Illés, T., Povh, J., Rigó, P.R.: Predictor-corrector interior-point algorithm for sufficient linear complementarity problems based on a new search direction, SIAM J. Optim., 30, 2628-2658 (2020)
14. Darvay, Zs., Papp, I.-M., Takács, P.-R.: Complexity analysis of a full-Newton Step interior-point method for linear optimization, Period. Math. Hung., 73, 27-42 (2016)
15. Darvay, Zs., Rigó, P.R.: New interior-point algorithm for symmetric optimization based on a positive-asymptotic barrier function, Numer. Funct. Anal. Optim. 39, 1705-1726 (2018)
16. Faraut, J., Korányi, A.: Analysis on Symmetric Cones, Oxford Mathematical Monographs. The Clarendon Press Oxford University Press, New York, Oxford Science Publications (1994)
17. Faybusovich, L.: Linear systems in Jordan algebras and primal-dual interior-point algorithms, J. Comput. Appl. Math. 86, 149-175 (1997)
18. Güler, O.: Barrier functions in interior point methods, Math. Oper. Res. 21, 860-885 (1996)
19. Karimi, M., Luo, S., Tunçel, L.: Primal-dual entropy-based interior-point algorithms for linear optimization, RAIRO-Oper. Res. 51, 299-328 (2017)
20. Karmarkar, N.K.: A new polynomial-time algorithm for linear programming, Combinatorica 4, 373-395 (1984)
21. Kheirfam, B., Haghighi, M.: A full-newton step feasible interior-point algorithm for $P_*(\kappa)$-LCP based on a new search direction, Croat. Oper. Res. Rev. 7, 277-290 (2016)
22. Kojima, M., Megiddo, N., Noma, T. , Yoshise, A.: A unified approach to interior point algorithms for linear complementarity problems, Lecture Notes in Comput. Sci. 538, Springer-Verlag, New York (1991)
23. Lesaja, G., Roos, C.: Unified analysis of kernel-based interior-point methods for $P_*(\kappa)$-LCP, SIAM J. Optim. 21, 2994-3013 (2011)
24. Luo, Z., Xiu, N.: Path-following interior point algorithms for the Cartesian $P_*(\kappa)$-LCP over symmetric cones, Science in China Series A: Mathematics 52, 1769-1784 (2009)
25. Mohammadi, N., Mansouri, H., Zangiabadi, M., Asadi, S.: A full Nesterov-Todd step infeasible-interior-point algorithm for Cartesian $P_*(\kappa)$ horizontal linear complementarity problem over symmetric cones, Optimization 65, 539-565 (2015)
26. Nesterov, Y.E., Todd, M.J.: Self-scaled barriers and interior-point methods for convex programming, Math. Oper. Res. 22, 1-42 (1997)
27. Nesterov, Y.E., Todd, M.J.: Primal-dual interior-point methods for self-scaled cones, SIAM J. Optim. 8, 324-364 (1998)
28. Pan, S., Li, X., He, S.: An infeasible primal-dual interior point algorithm for linear programs based on logarithmic equivalent transformation, J. Math. Anal. Appl. 314, 644-660 (2006)
29. Rigó, P.R.: New trends in algebraic equivalent transformation of the central path and its applications, PhD thesis, Budapest University of Technology and Economics, Institute of Mathematics, Hungary (2020)
30. Rigó, P.R.: New trends in interior-point algorithms, In F. Friedler ed., Proceeding of the 8th VOCAL Optimization Conference: Advanced Algorithms (short papers), Esztergom, Hungary, 85-90 (2018)
31. Rigó, P.R., Darvay, Zs.: Infeasible interior-point method for symmetric optimization using a positive-asymptotic barrier, Comput. Optim. Appl. 71, 483-508 (2018)
32. Peng, J., Roos, C., Terlaky, T.: Self-Regularity, A New Paradigm for Primal-Dual Interior-Point Algorithms, Princeton University Press, Princeton, NJ (2002)
33. Roos, C., Terlaky, T., Vial, J.Ph.: Theory and Algorithms for Linear Optimization, Springer, New York (2005)

34. Schmieta, S.H., Alizadeh, F.: Extension of primal-dual interior point algorithms to symmetric cones, Math. Program. 96, 409-438 (2003)
35. Sturm, J.F.: Similarity and other spectral relations for symmetric cones, Linear Algebra Appl. 312, 135-154 (2000)
36. Tunçel, L., Todd, M.J.: On the interplay among entropy, variable metrics and potential functions in interior-point algorithms, Comput. Optim. Appl. 8, 5-19 (1997)
37. Vieira, M.V.C.: Jordan algebraic approach to symmetric optimization, PhD thesis, Delf University of Technology, The Netherlands (2007)
38. Wang, G.Q., Bai, Y.Q.: A new full Nesterov-Todd step primal-dual path-following interior-point algorithm for symmetric optimization, J. Optim. Theory Appl. 154, 966-985 (2012)
39. Wright, S.: Primal-Dual Interior-Point Methods, SIAM, Philadelphia (1997)
40. Ye, Y.: Interior Point Algorithms, Theory and Analysis. Wiley, Chichester (1997)