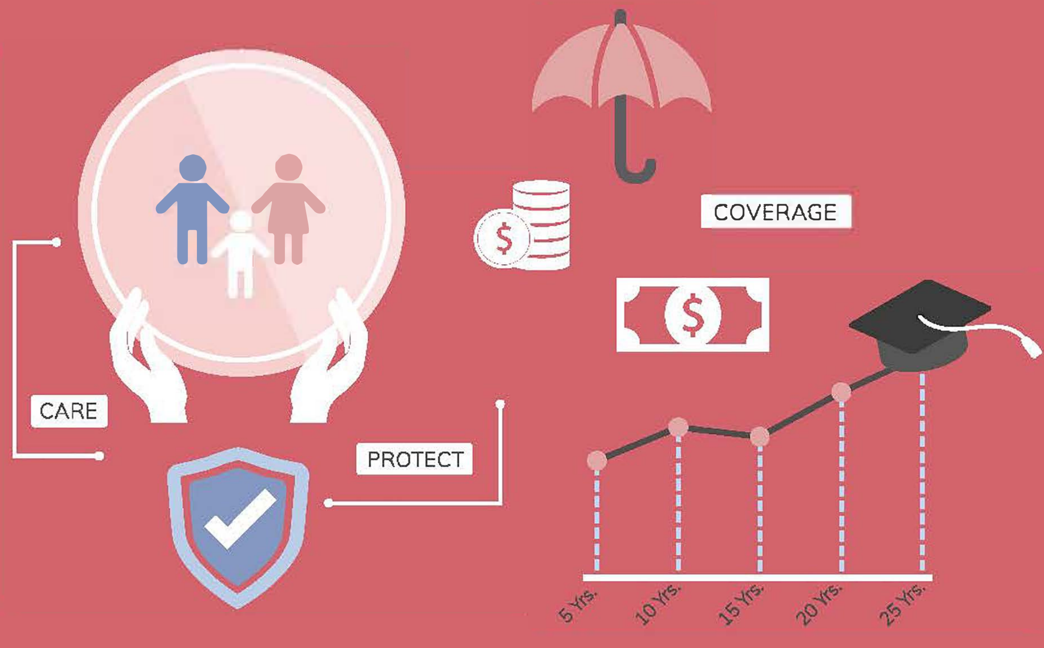
LIFE INSURANCE



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BUDAPESTI CORVINUS EGYETEM
MAGYAR NEMZETI BANK

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Life insurance

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INTRODUCTION TO THE EDITION IN 2003

The goal of the book is to give a general introduction to life, accident and health insurance (and some other areas that functionally belong here, e.g. the world of pension and health funds), and contains a possible discussion of the concept of life insurance. The concrete material of knowledge concerning the other mentioned insurance areas are contained in other textbooks, but the common basis can be found here.

> One of the classical figures of sociology, Max Weber declared a hundred years ago that a tendency of capitalism compared to the former social order, feudalism is that it makes all social relations rationally calculable. Insurance is typically "capitalistic" in this respect, or at least a modern phenomenon, since it makes events with uncertain financial outcome calculable with certainty.

Regarding life, accident and health insurance, first of all, we have to say that they are instruments to ward off events that can be anticipated in the life cycle and that occur in a standard way, but that cannot be anticipated on the individual's level and have financial effects (pension, death, accident, sickness), and to eliminate uncertainty in the financial sense. Altogether we can say that life, accident and health insurances are instruments of the financial planning of the human life cycle. This way the first part of the book (chapters 1-3) discusses the financial planning of the life cycle.

Insurance fundamentally handles risks in two ways, that are connected to each other:

1. Converting uncertain large losses into small, but certain ones.
2. By creating reserves to cover future needs.

The general principles of both methods will be discussed later on.

The book, that is the revised, re-edited and enlarged edition in a uniform structure of the author's former book, primarily follows the material of the "Life Insurance" course of the Actuary specialization at Corvinus University of Budapest. It complements the theoretical basis in the former edition with "practical" material. In these parts – just as in the university course – students can practice the terms and relations introduced in the sections discussing theory.

In the book the author uses the first person of plural, but where the author found it important to emphasize his own opinion on a subject, that differs from the opinion of other experts, the first person of singular is used.

INTRODUCTION TO THE HUNGARIAN (IMPROVED, EXPANDED) EDITION IN 2016 AND THE ENGLISH EDITION IN 2020

The time of publishing the new Hungarian version of this book had come in 2016, and now the English version. The book remained basically the same as the previous one, so the structure of the two editions are almost the same, but I have changed a few things. The main differences between the two Hungarian editions are (this English edition is following the latest Hungarian one):

- I have replaced the previous Chapter 3 with a new one into which I have put some important, general insurance concepts I am using later in the book.
- in some chapters I have incorporated into the text the outcomes of my research connecting to that topic I have reached in the meantime.
- I have also incorporated some minor additional topics that I have supplemented my regular teachings of that theme with.
- I have made some minor changes to the notations, so that it became more consistent and more similar to the intartationally recognised standards.
- I have continued to keep myself to the principle (similarly to the first edition), that I try to avoid any direct reference to legislation, or I refer to them in very general terms, because:
    - » the law is always changing (at least in Hungary, the regulation of many other countries are much more stable). For example, on a term shorter than two decades, already the third Incurance Act came into effect, but also the whole Civil Code is renewed (in which the whole insurance chapter was replaced by a new one),
    - » I am positive that insurance is what it is not because of the law, but on the contrary, the law (on insurance) is what it is because of the characteristics and logic of insurance, and here I would like to demonstrate this general logic from which the certain solutions of the regulation resulted,
    - » therefore the (relatively) new insurance “silver bullet” of the European Union, the Solvency II (shortly SII) was not reviewed in detail (it is not the topic of this book), I have only referred to some of its important considerations.

I. LIFE INSURANCE BASICS

1. BASICS OF DEMOGRAPHY

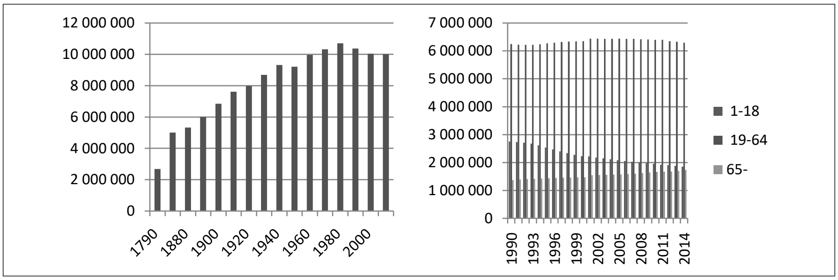
KEY WORDS

Average age	Size of population
Generation mortality table	Selection table
Mortality table	Life expectancy at birth
Probability of death	Probability of survival
Life table	Disability adjusted life expectancy
Age structure	Life expectancy

In order for insurance to be of help in the planning of the life cycle, the insurer must have concrete ideas and specific models in mind regarding the path of human life cycles and their most important parameters. (For example, their average length, the ratio of active and inactive stages, their distribution, the probability of death, illness, accidents, the expected extent of illness or injury from accidents, etc.) This information is usually obtained from public sources, which are mostly collected as a part of a separate social science, demography (the science of populations.) In the following sections we will get to know some demographical concepts and implications that are important with respect to insurance.

1.1. Total Population

In general, demography – similarly as insurance – deals with the patterns involving the „movements" of large population masses. One of the most important such indicator-systems is one that refers to the changes and composition of the population size of a regional unit (usually a country).



Source: Hungarian Central Statistical Office (HCSO)

Figure 1.1.: Population size in the current area of Hungary, its composition and change by age-groups

If we are only interested in the total population and its changes, then we can see right away that at any point in time this can be expressed as the resultant of two opposite factor-pairs. This is the balance of:

- births – deaths
- immigration – emigration.

If there is no immigration and emigration (as was virtually the case in Hungary in the 1970s and '80s) then the size of the population will increase if there is a greater number of births than deaths (as is the case today in most of the so-called developing countries – in Asia, Africa and Latin-America), and it will decrease if the number of births is smaller (for example in Hungary in the last two decades).

One might think that the equilibrium between births and deaths can be achieved if every single person has one offspring (or every couple has two), because this would reproduce the population. This is true in the very long term for a population that is in equilibrium in other regards as well, but this principle cannot be used to explain the seemingly paradoxical phenomena we were able to observe, for example, in China at the end of the 20th century and the beginning of the 21st century. Here, for decades, every married couple in urban areas was allowed only one child, while couples from rural areas – if the first child is female – were allowed up to two children. This meant that for decades the number of children for every couple was well below two, and yet during this time the population increased by several hundred million.

The solution to the Chinese mystery is simple: partly because in China, as well as around the world, life expectancy at birth increased significantly, and partly because in the second half of the 20th century a high proportion of the population was young, and therefore, there were many women of childbearing age among them. As a result of these, despite the fact that there were relatively few births, there were even fewer deaths, and both the average age and the size of the population were increasing. Although on a smaller scale, we can observe a similar phenomenon on the right side of Figure 1.1. Note that the number of children is decreasing, but the pace of the population decrease is smaller than that, because the increase in life expectancy generates an ever growing number of elderly people.

The population's size and average age, and their changes over time have a very important role in certain long-run macro-level planning – for example in the planning of the welfare system (health insurance and pension system), – and through these in the opportunities of private insurance as well.

1.2. Composition of the Population by Age and Gender

We can also gain more detailed information about the population than just its total size and its changes. It is important to know, for example, how the total population is distributed among the sexes and age groups.

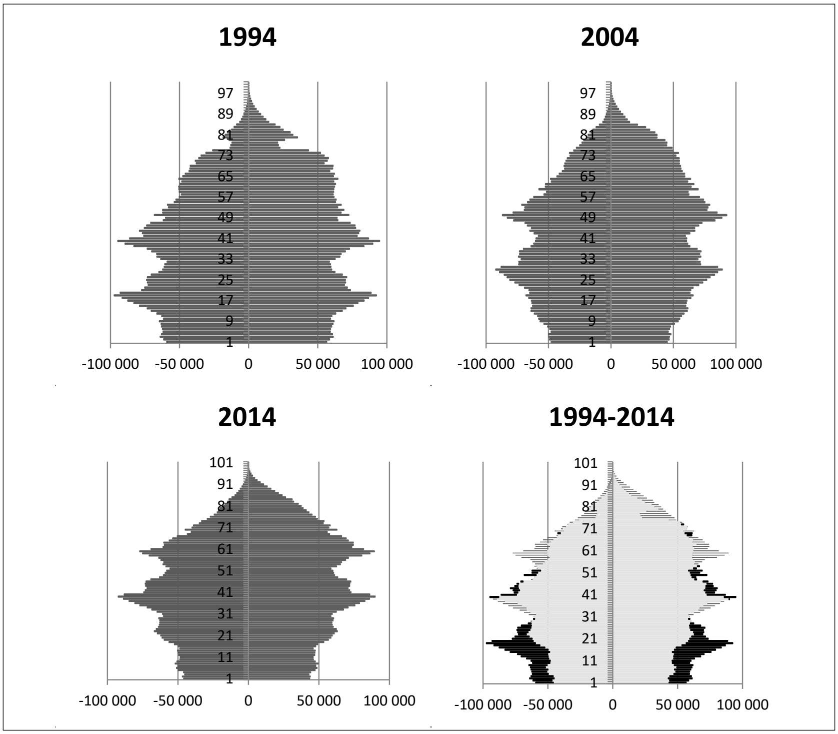


Figure 1.2.: The Hungarian age pyramid in different years, and in 1994 and 2014 comparing to each-other

The above figures show the age composition of the Hungarian population by gender (on the left is the male[1], on the right the female population). These figures give a much more detailed view of the population than what the simple total population, the average age figures and their changes do, because it shows its composition in detail, along with

[1] Whose number is, of course, non-negative, but the values are simply shown to the left of the y-axis!

certain important parameters (age and gender). It is true though that considering a single figure, this view is static, since we do not know exactly how we got to this point, or how this figure will look in a few years' time. Of course, we can read many things about the past and future out of such a static figure, but this can be made more dynamic by placing several figures pertaining to different time periods next to each other.

Looking at these figures (and the numbers behind them) we can draw many important deductions even from such a static figure, but we can make it dynamic by drawing successive figures representing different times or we can superimpose two figures.

Considering these figures (and the numbers behind them) we can conclude to numerous important statements. We can observe that the number of males at birth is significantly higher (in 1994, 2004 and 2014, for example, there were only 56 456, 45 008, and 43 454 females for 59 320, 47 936, and 45 890 males in Hungary, respectively[2])), but later this difference gradually decreases, and around age 40 the population of the two sexes more or less evens out[3]. After this age the ratio of women gradually increases (or decreases by less than that of men), and at age 84 there are more than twice as many women than men[4]. Since we can see from the figures (except 2014) that there are more 1-year-old children than newborns, and more 2-year-olds than 1-year-olds etc., we can observe that in the last decades there were fewer and fewer children born every year. On the figure relating to 1994 we can observe a peak around age 20 (which is wandering upward by 10 years per decade) and we can also observe that there was a peak in the number of births in the '70s in Hungary. Since then, the number of births has decreased year after year. This is also related to the fact that in 1994 there was also a peak at around age 40-45 (those born in the middle of the '50s), made up of people who are probably the parents of babies born in the middle of the '70s. Even from a simple figure as this one we can draw far-reaching conclusions about the necessity of certain macro-level political steps. For example, if there are fewer children, we need fewer nursery schools, kindergartens, and elementary school classrooms, teachers, etc. If however the number of twenty year olds reaches a peak, then the need for university capacity will be higher, etc. If the number of parents are higher than that of children, then within a few decades the ratio of the old- and middle-aged will change significantly (it is visible on the right side of the Figure 1.1.), etc.

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[2] The ratio of the two gender here 1,05, 1,065, and 1,056. It is said, that as an average, the number of newly born boys is 6% higher than the newly born girls. The Hungarian data supports this, that is here there is no selective abortion, as in many countries in Asia, where – because of this – the proportion of the boys is much higher than this.

[3] In Hungary in 1994, 2004 and 2014 the number of 40-year-old men was 86 055, 59 912, and 74 205 as long as the number of women was 86 735, 60 595, and 72 754, respectively. The ratio at this age is: 0,99, 0,99, and 1,02, that is much more even, than at birth.

[4] The ratio at this age, in these years in Hungary: 8762/19892 =0,44, 9224/21160=0,44, and 11590/26261=0,44.

The above figures resemble a tree very closely. The shapes of these figures have gone through significant change throughout history. Earlier it was generally more the case that a greater number of children were born, but the infant and child mortality rate was also high, and the average lifespan was also very low. These factors together create an age structure figure in the shape of a pyramid (that is why the name of the figure is age pyramid in English) or a pine tree, which can be seen schematically in the following figure.

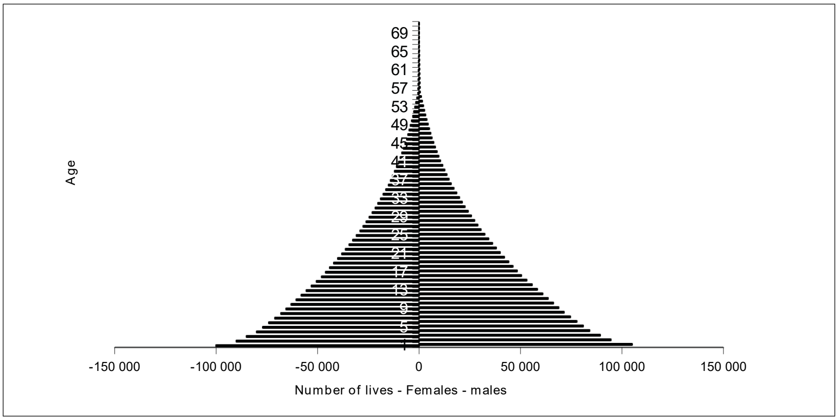

Figure 1.3.: Traditional pyramid-shaped age structure (age pyramid)

Hungary was described by such a „traditional" age structure figure at the beginning of the 20th century, and it is still typical today in some so-called „developing" countries. In the developing countries we can see a new trend along with the high number of children typical in the western countries 100 years ago. This – similarly as in the western countries' current situation – is the increase in the life expectancy at birth. This is mostly due to the disappearance of the earlier great epidemics, the radical decrease of the infant mortality and some improvement in the standard of living. These two factors together result in the phenomena called the „demographic boom", which resulted in an unprecedented increase in the Earth's population in the 20th century, especially in the second half, and this growth is expected to continue at least until the middle of the 21st century.

Of course, it is difficult to predict the total size of the Earth's population ahead of time. Even nowadays the estimates change year by year, and this is also true for the individual countries as well. In the '70s, for example, no one could foresee the appearance of a new, previously unknown, deadly epidemic in the southern half of Africa, which resulted in a decreasing population for a while in some countries that previously showed increasing tendencies. Even further, AIDS – since it mostly has affected young adults –has change the age pyramid in a very unique way[5]; a UN study described it as a „chimney" shape, which is shown in a forecast in 2000 for 2020 in Figure 1.4.

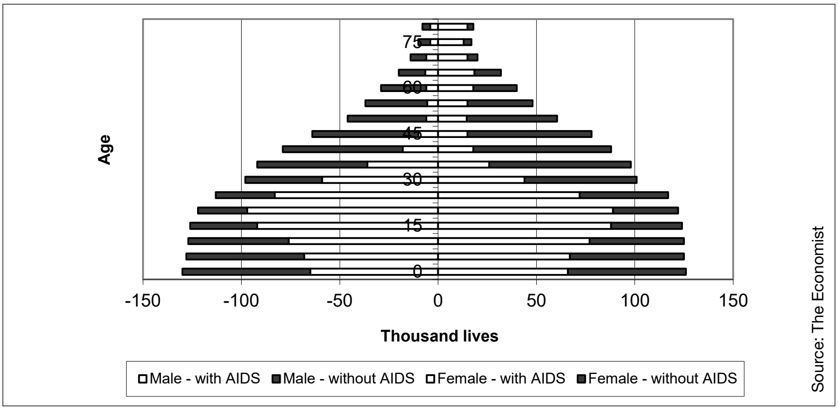

Figure 1.4.: The predicted population of Botswana in 2020 with AIDS and without

The scenario on the Figure 1.4. turned out to be too gloomy, because – according to the data of World Bank – the population of Botswana was growing continuously before 2013 (see http://data.worldbank.org/indicator/SP.POP.TOTL), and the life expectancy – according to the WHO, see Table 1.2.! – though decreased significantly by 2000, by 2013 it has almost restored to the level of 1990.

In the long-run the age structure figure changes from a pyramid shape to a tree that has an increasingly wide crown around the middle and then top, whose trunk gets gradually thinner and taller (so the number of children decreases, but the earlier generations of larger numbers live for a longer time), and finally the crown disappears completely. The number of children born will probably not decrease infinitely either, and thus in the even longer term (in a hundred years!) developed countries will have an age structure figure resembling a column, so in every year the same number of children will be born, and all who have been born will more or less live to 80 + x years old.

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[5] see: The Economist: 2000. July 15. issue 28. pp.91-93.: A turning-point for AIDS? In another article – 2001. February 10. issue 6. p.75.: Business and AIDS – The worst way to lose talent (South African firms are struggling to cope as AIDS spreads) – the figure in the article shows the life expectancy at birth in South-Africa in 1996 was above 60, but by 2000 this decreased to about 50, and by 2010 it is expected (in 2000) to decrease well below 40 (!) years. According to the data of WHO the situation has not became such a severe. The life expectancy at birth at men was 58,8 years in 1990 and it is really decreased significantly by 2000, but only for 54,4 years, and after that it started to increase. It is true, that the 57,1 years in 2003 still lower than the level of 1990. The same number at women 66,2, 61,8, and 63,6, that is the tendency is the same. (see http://apps.who.int/gho/data/view.main.61540?lang=en)

International organisations and the individual countires are continuously making long term population projections. The realisation of these naturally differ to the earlier expectations but the gradual aging of developed countires is a very strong tendency. And this basically endangers the old age care systems and their financing in the present framework.

While the number of children mainly in Islamic countries and Africa is still very high (often 7 children born for every woman on average, which can be regarded as the theoretical maximum), in developed western countries – and in Hungary – it has decreased well below the reproductive level (about 2 children per woman) in recent times.[6]

1.3. Life Expectancy, Probability of Death

Statistics pertaining to the size and composition of the entire population are very important for the purpose of politics, and the observations made from these serve as a framework for observations about the single individuals. In the planning of the life cycle (and later insurance) we are mainly interested in the statistics pertaining to the individual – and not those pertaining to the entire population. Naturally, the two are related: we take the social average as a representation of the individual.

From the above statistics of population movements we can first of all deduct the probability of death and the life expectancy of the individual.

Statistics regarding the size and the composition by gender and age of the population are collected during censuses. A census including the full population takes place relatively rarely (on average every ten years). Between two such censuses changes in the population are traced by the gathering of statistics from a representative sample of the population (micro census), so we have a more or less reliable estimate of the major statistics of the population every year.

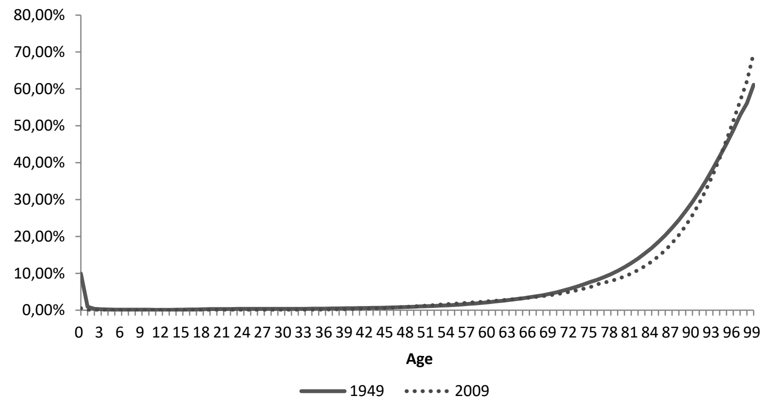
Based on the census statistics, we can compare by years of age and gender the number of those living at the beginning of the year with the number of those who died during the year, and thus calculate the raw probability of death. If these are then arranged (for example on a graph where the horizontal axis shows age), then the resulting figure can be divided into a theoretical trend and a random deviation from this trend. The raw data,

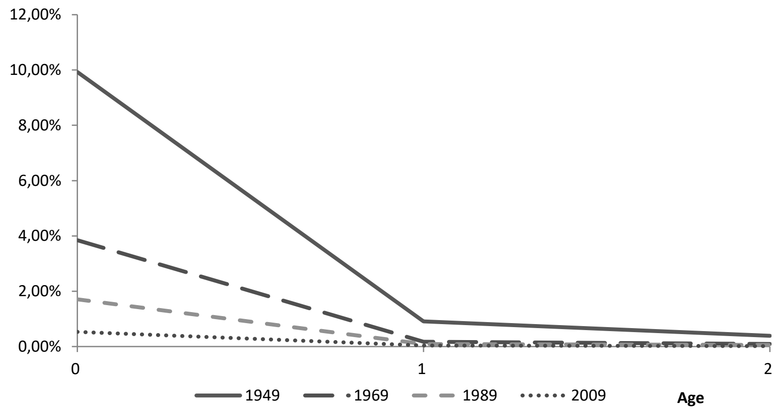
[6] It is important to note that there is nothing wrong with a low and stable birth rate. It is not a tragedy that the population of a country is decreasing especially if we know that one of the greatest problems of the world today is over-population. If the number of children is stable – no matter at how low a rate – then sooner or later the aging of the population will also stop (stabilize). Of course, this can be a big problem if the functioning of certain institutions (such as a pay-as-you-go pension system) were explicitly made to depend on a high birth rate, but there is no reason for this to be the only possible system of institutions. For example a national pension system can be not only a Samuelson-type pay-as-you-go system – see Banyár [2014].

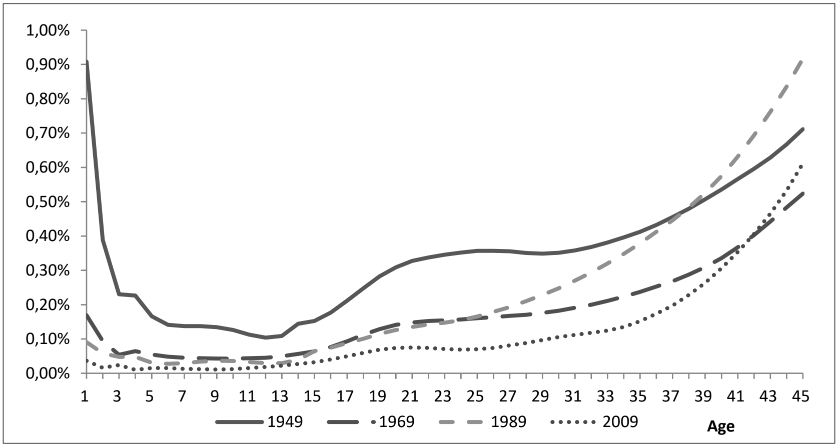
once it is cleaned of these random deviations, are nothing else than the theoretical death probabilities (q_x) pertaining to the given year, the meanings of which are:

q_x = the probability of someone dying before reaching age x+1, given that they survived to age x

Since these statistics come from a „momentary" survey of the population, every q_x marks a probability pertaining to generations born in different years living simultaneously, yet in the given moment these do give a snapshot of the mortality conditions of the entire population.







Source: HCSO

Figure 1.5.: The Hungarian male death probabilities in 1949, 1969, 1989 and 2009 in different segments and in different comparisons

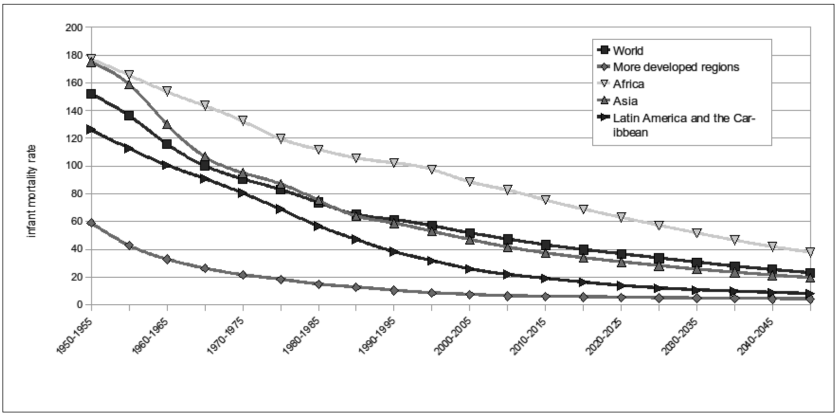
From the Figures we can see that in Hungary, starting from 1949, infant mortality decreased from almost 10% to much below 1% in 60 years. The improvement is going on, as we can see on the Table 1.1.

Year	1960	1970	1980	1990	2000	2006
Infant mortality/1000 born alive	47,6	35,9	23,2	14,8	9,1	≈5

Source: HCSO

Table 1.1.: The values of the Hungarian Infant mortality (for both gender together) for 1000 babies born alive

Furthermore this is an international trend as it is shown in Figure 1.6.



Source: UN World Population Prospects, 2008.

Figure 1.6.: Infant mortality in the different parts of the World in 1950-2050

At the same time we can also see on the Hungarian data that in different age groups the improvement in death probability is not linear. From 1949 until 1969 it improved practically for all ages, but until 1989 the mortality of those above 38 years had worsened to the level of the situation in 20 years earlier. After a subsequent 20 years this worsening had turned back, but until 2009 this improvement did not reach the level in 1969 in all age groups. We can see from the figures, that – except from the infant mortality – the curve of q_x is increasing monontonous with age (there is some uncertainty in the first half of the 20s), and this increase is accelerating („exponential-type"!).

Some other indicators can also be derived from q_x. Its complementary is the probability of survival, defined as:

px = 1 – qx = probability of survival = the probability of someone surviving to age x+1, given that they survived to age x

It can be seen quite easily that the product of the p_x-s gives the probability of someone surviving t more years given that they lived to age x, so:

$${}_{t|}\boldsymbol{p}_x = \boldsymbol{p}_x \cdot \boldsymbol{p}_{x+1} \cdots \boldsymbol{p}_{x+t-1}$$ **= the probability of someone surviving to age x+t, given that they survived to age x**

It is obvious that: $${}_{1|}\boldsymbol{p}_x = \boldsymbol{p}_x$$

It is customary in statistics to mark the highest shown age level[7] with the ω symbol. This level differs from country to country (in Sweden for example ω was 110 in 2015). In Hungary it is usually set at age 100, and even though we know about the existence of a few Hungarian citizens above that age, the number of such individuals is very small and their „appearance” is highly variable. If we sum the ${}_{t|}\boldsymbol{p_x}$-s over t from 1 to (ω-x), (and we correct the sum by 0.5)[8] the sum is given a new meaning: the life expectancy at age x.

$$\boldsymbol{{}_{1|}p_x + {}_{2|}p_x + {}_{3|}p_x + \cdots + {}_{\omega-x|}p_x + 0,5 = e_x} = \textbf{life expectancy at age x}$$

For x=0 this gives an especially significant statistic – the life expectancy at birth. This can be seen for a few countries below for the years 1990 and 2013. The table clearly shows the significant difference between the life expectancy of women and men, which can be observed in every country, and is especially high in Hungary. We can see that the life expectancy at birth has increased significantly during a quarter century almost everywhere all over the Earth.

Table 1.2.: Life expectancy at birth for some countries in 1999

Country	Year	Both sexes	Females	Males	Country	Year	Both sexes	Females	Males
Afganistan	2013	61	62	61	India	2013	66	68	65
	1990	49	50	49		1990	58	58	57
Australia	2013	83	85	80	Italy	2013	83	85	80
	1990	77	80	74		1990	77	80	74
Austria	2013	81	84	79	Japan	2013	84	87	80
	1990	76	79	72		1990	79	82	76
Botswana	2013	64	65	63	Norway	2013	82	84	80
	2012	62	63	61		1990	77	80	74
	2000	47	47	48	Poland	2013	77	81	73
	1990	65	66	65		1990	71	76	67
Chad	2013	52	53	51	Portugal	2013	81	84	78
	1990	45	47	43		1990	74	78	71
China	2013	75	77	74	Romania	2013	74	78	71
	1990	69	71	67		1990	70	73	66

[7] So, not the highest observed age but the age at which there are still a „statistically significant” number of people.
[8] The correction marks the half year average lifespan beyond age x+t of those who died at age x+t.

Country	Year	Both sexes	Females	Males	Country	Year	Both sexes	Females	Males
Czecn Rep.	2013	78	81	75	Russia	2013	69	75	63
	1990	71	75	68		1990	69	74	63
Denmark	2013	80	82	78	Slovakia	2013	76	80	72
	1990	75	78	72		1990	71	75	66
Finnland	2013	81	84	78	Spain	2013	83	86	80
	1990	75	79	71		1990	77	81	73
France	2013	82	85	79	Sweden	2013	82	84	80
	1990	78	82	73		1990	78	81	75
Germany	2013	81	83	79	Switzer-land	2013	83	85	81
	1990	76	79	72		1990	78	81	74
Hungary	2013	75	79	71	United Kingdom	2013	81	83	79
	1990	69	74	65		1990	76	79	73

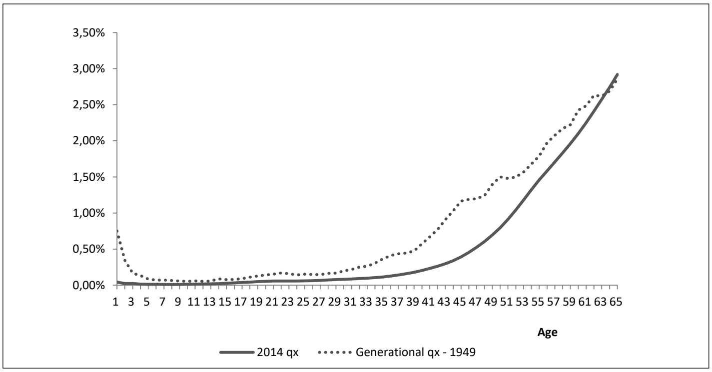
Source: WHO – http://apps.who.int/gho/data/node.main.688?lang=en

All these statistics (mortality and survival probabilities, life expectancy) are published organized by age in the so-called "mortality table" by the Hungarian Central Statistical Office. The mortality table includes a very useful technical row, the "number of survivors". This row of numbers, that is created from the cleaned probabilities of mortality, can be regarded as a basic indicator, which shows how many people out of a starting population (usually 100,000 people) will be alive at age x if the current mortality rates of each age group apply to them at every age. The symbol of the values of the number of survivors, or the "life table" is l_x, and so – based on the above description –, $l_0 = 100{,}000$.

Almost all the statistics necessary in life insurance can be constructed very simply from the life table. In the following, this constructed chain of values will be used in the majority of the calculations (see in more detail in chapter 1.4.!).

According to what was previously said, the mortality table does not apply to a single generation, but rather it is a snapshot of several generations living simultaneously. This is also true for the life table, even though it very strongly suggests that its statistics pertain to a single generation, as if they followed life paths of 100,000 infants born at the same time until they reach age 100.

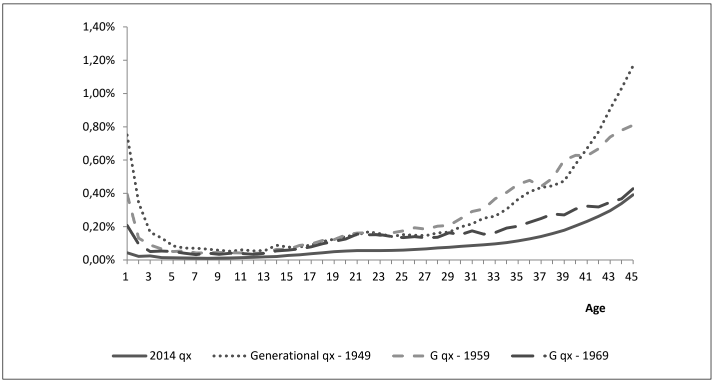
Of course, this statistic could also be constructed, but this would require data from 100 years, and would not truly reflect very flexibly the current mortality trends of a time. For the purposes of analysis, it is still best to construct the mortality table of a generation, the so-called „generational mortality table". This can be done most easily by taking the statistics of those born in the same year from all the different years of the survey. Since the Hungarian men's mortality tables are available starting from 1949, we can take from every year's table the mortality rates of those born in 1949 (so from the 1949 survey that of the 0-year-olds, from the 1950 the one-year-olds, ... from the 2014 the 65-year-olds), and thus construct the generation mortality rates for the people born in 1949. (Infant mortality was omitted from figure 1.7 as it was almost 10% in 1949, so with it the figure would have been distorted and the much smaller differences would not have been visible.)



Source: HCSO, own calculation

Figure 1.7.: 1949 generational and 2014 Hungarian male mortality rates from age of 1

Out of the data we can also construct other generational mortality rates. Naturally, these will be shorter than that of the generation born in 1949. In the Figure 1.8. I have presented 3 others – for the sake of comparability only until the age of 45.



Source: HCSO, own calculation

Figure 1.8.: 1949, 1959 and 1969 generational, and the 2014 „normal" death probabilities from the age of 1 year

One can see right away from the above figures that infant mortality has drastically, and child mortality has significantly decreased since ’49, but those of the young and middle-aged only in the recent age groups. It is also apparent that the curve of the 2014 mortality rates is much „smoother”, since the raw data was previously smoothed out using statistical techniques (which also means that our constructed generational mortality rates are not perfectly accurate).

It should also be noted that mortality tables can be made not only for the entire population but for certain segments of it as well, as some of these segments have very different mortality characteristics. For example, we would surely have different life expectancies among:

- The VIII. and XII. districts of Budapest
- In Győr-Moson-Sopron and Szabolcs-Szatmár-Bereg counties
- Those working in the finance sector and miners
- The divorced, married, widowed and singles
- Smokers and non-smokers
- Those who finished only elementary school and those with university degrees
- Etc.

So we can differentiate among the groups of a population based on place of residence, education level, occupation, income level, marital status, habits, etc. At the same time, these are not such permanent characteristics as is gender (since the place of residence, marital status, etc. may change frequently, while gender cannot[9]). From time to time complete analyses are reported based on these characteristics.

Insurers also create their own mortality tables based on their specific points of view – and usually, their own data. It is especially common in the Anglo-Saxon countries to differentiate between smoking and non-smoking insured, whose rates are calculated from separate mortality tables.

Even more common is the use of so-called selection tables. This is where they observe how the mortality profiles of those purchasing different insurance compare to each other[10]. For example, they can differentiate between the selection tables of purchasers of annuity or term insurance, since people with lower life expectancies are more likely to buy term insurance than the average, and vice versa, those with high life expectancies would rather purchase annuities. The selection tables show this difference very clearly. (Unfortunately, in Hungary insurance companies have not collected sufficient data for these, though some calculate mortality based on their own

[9] Nowadays one must be careful about this statement as well. It is possible that the time is near when surveys will have a category for „original gender” instead of simply „gender”.

[10] In a broader sense, the selection table can pertain to any kind of selection, for example, the differentiation between smokers and non-smokers as well!

data). In table 1.3. we can see the ratio of annuity mortality rates to the population mortality rates in the USA. As we can see, there is a significant difference between the different phases of the life cycle.

Table 1.3.: U.S. annuity qx-s compared to the population mortality tables (1990-1996)

Age	Male	Female	Age	Male	Female	Age	Male	Female
0	23%	22%						
1	123%	117%	41	35%	48%	81	60%	70%
2	101%	93%	42	38%	48%	82	60%	71%
3	99%	92%	43	41%	49%	83	61%	72%
4	111%	98%	44	44%	49%	84	62%	73%
5	110%	89%	45	46%	49%	85	63%	74%
6	107%	78%	46	49%	49%	86	64%	76%
7	106%	70%	47	50%	49%	87	64%	77%
8	132%	76%	48	52%	49%	88	65%	78%
9	163%	84%	49	52%	49%	89	66%	79%
10	195%	95%	50	52%	49%	90	66%	80%
11	198%	99%	51	52%	48%	91	67%	81%
12	156%	92%	52	52%	48%	92	67%	81%
13	107%	77%	53	52%	49%	93	68%	81%
14	75%	63%	54	51%	49%	94	68%	80%
15	57%	54%	55	51%	49%	95	69%	79%
16	47%	48%	56	50%	49%	96	69%	79%
17	41%	46%	57	49%	49%	97	70%	78%
18	38%	47%	58	48%	48%	98	70%	78%
19	37%	50%	59	47%	48%	99	72%	78%
20	36%	53%	60	46%	48%	100	73%	78%
21	35%	55%	61	45%	48%	101	75%	78%
22	35%	58%	62	45%	48%	102	77%	79%
23	36%	60%	63	44%	49%	103	79%	81%
24	37%	61%	64	44%	50%	104	82%	83%
25	39%	62%	65	44%	50%	105	85%	85%
26	40%	63%	66	45%	51%	106	89%	88%
27	41%	63%	67	46%	51%	107	93%	92%

Age	Male	Female	Age	Male	Female	Age	Male	Female
28	40%	62%	68	47%	51%	108	98%	96%
29	39%	61%	69	49%	52%	109	103%	101%
30	37%	59%	70	50%	53%	110	108%	106%
31	35%	57%	71	52%	53%	111	114%	111%
32	34%	56%	72	53%	55%	112	120%	117%
33	32%	54%	73	54%	56%	113	127%	125%
34	31%	53%	74	54%	58%	114	134%	132%
35	30%	51%	75	55%	60%	115	141%	141%
36	29%	50%	76	55%	62%	116	134%	134%
37	29%	49%	77	56%	63%	117	128%	128%
38	29%	48%	78	57%	65%	118	121%	121%
39	31%	48%	79	58%	67%	119	116%	116%
40	33%	48%	80	59%	69%			

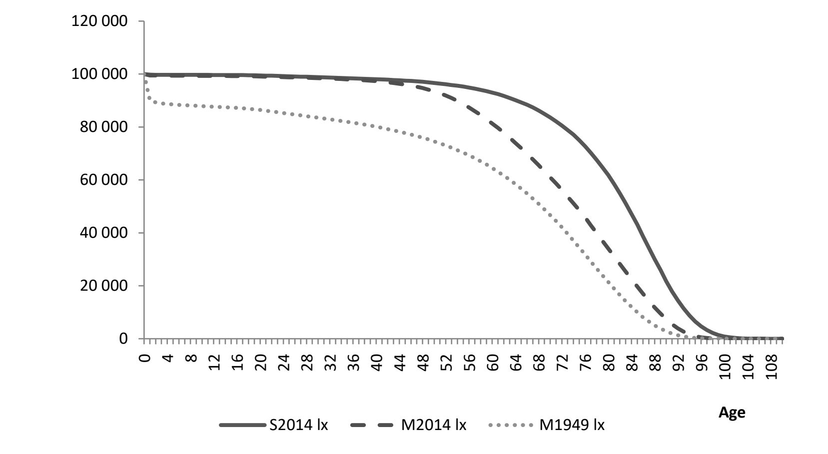
Source: SOA, and own calculation – http://mort.soa.org/

The insurers during their premium and reserve calculation are using the best available mortality tables, but they also have to take into consideration the current regulation. The national mortality tables for the whole population (generally separately for male and female and nowadays also an unisex) are available, at least to a certain degree, all over the developed countries. While some countries produce different tables for black and white (e.g. in the USA) or for different ethnic groups, in other countries this kind of differentiation is practically illegal or used for only the purpose of research. Many insurers are using these national mortality tables, often with some modifications, because it is known, that the clientele of insurers is selected compare to the whole population. At the same time, in some countries it is not allowed to make differences in the premiums based on gender (like e.g. in the European Union), so the need emerged for the so-called “unisex” mortality table for premium calculation. This contains – in the basic case – the weighted average of the male and female death probabilities, where the weights represent their proportion in the population. As for the calculation of reserves (for the details see Chapters 11-12.) it is not prohibited and, what is more, it is expedient to use mortality tables diffentiated based on gender.

As long as an insurer has a big and sufficiently old clientele, it can make its own, much more accurate mortality table than the population table. These are generally selection tables, that is, they make different tables for different types of life insurance, taking adverse selection into consideration. In some countries, insurers are merging their data and making such specific mortality tables together.

1.4. The Analysis of the Life Table

In figure 1.9. we can see an older and a current Hungarian and Swedish life table.



Source: HCSO, SCB – http://www.scb.se/

Figure 1.9.: Hungarian (1949 and 2014) and Swedish (2014) male life tables

The differences are obvious. The already mentioned infant mortality has improved a lot in Hungary in 65 years. This is shown by the fact that the curve starts with a steep fall immediately in 1949, while in 2014 this decrease is very small in the first year. The Hungarian male population was significantly higher in 2014 (for 100.000 newborn) than in 1949 in virtually all age groups. But comparing to the Swedish males the picture is not so bright. Approximately from the age of 40 years the Hungarian curve significantly deviates from the Swedish one. It seems as if the mortality would cease until the age of 60 years in Sweden. It is also spectacular that whilst in Hungary it is enough to make the mortality table until the age of 100 years, in Sweden it is made until the age of 110 years, however, over the age of 100, the absolute numbers are not significant in either countries.

It can be stated that:

- The later the historical era, or the more developed the country whose statistics are shown in the figure of the life table, the "fuller" it is, or the bigger the area under the curve is
- The more developed the country, the smaller is the initial fall of the curve
- The more developed the country, the flatter the curve is for the young and middle ages, and the later it begins to fall.

The probabilities of mortality and survival, and the life expectancy can be calculated from the life tables in the following way:

$$q_x = \frac{l_x - l_{x+1}}{l_x}$$

and

$$p_x = \frac{l_{x+1}}{l_x}$$

and

$$e_x = 0{,}5 + \frac{\sum_{k=1}^{\omega-x} l_{x+k}}{l_x} = 0{,}5 + \frac{l_{x+1}}{l_x} + \frac{l_{x+2}}{l_x} + \cdots + \frac{l_\omega}{l_x} =$$

$$= 0{,}5 + {}_{1|}p_x + {}_{2|}p_x + \cdots + {}_{\omega-x|}p_x$$

It can easily be seen that in figure 1.9. the area under the curve of the number of lives from age x represents the number of years to be lived by the x year olds (l_x people), thus the life expectancy at age x is simply the quotient of this area and the number of people who are x years old (l_x).

* * *

Due to its simplicity and ease of use, the mortality table is one of the most important statistical *inputs in insurance.*

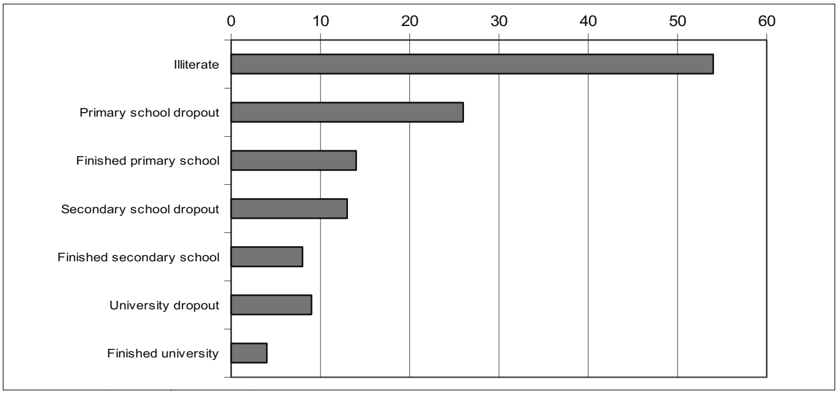
1.5. Statuses: Marital, Health, Economic

Of course, insurers are often interested in the population statistics in a more detailed breakdown. Such dimensions:

- the different states of health
- marital status
- the number and composition of households
- the distribution of the economically active and passive
- social, economic situation,
- etc.

From these numerous dimensions, we will only deal with two here, namely social situation (and its effect on infant mortality) and health, due to reasons mentioned earlier pertaining to the mortality table. There is a sort of "race" going on among the countries regarding health status and infant mortality, and those countries are viewed as healthier (or more exactly one with a better healthcare system) where the life expectancy at birth is higher, and the infant mortality rate is lower. (Hungary – among the more developed countries – is quite behind in this respect.)

With regards to infant mortality, a survey taken by the Argentinean Health Ministry[11] is quite interesting (their observations can probably be applied to other countries as well, and the relative rates are likely to be still valid even now). There are significant differences based on the mothers' educational levels (which are probably positively correlated with their economic and social status and situation), as can be seen in figure 1.10.:



Source: The economist

Figure 1.10.: Infant mortality per 1000 infants born as a function of mother's education – Argentina, 1998.

Nowadays, more and more people are pointing out that the life expectancy at birth is not the correct indicator when comparing countries, since health status is not qualified based on the life expectancy at birth, but rather the number of years of life spent in good health. For this reason the UN institution that deals with these issues, the WHO has introduced a new indicator, the "disability adjusted life expectancy", and on the basis of this, the data of Table 1.2 can be modified according to the followings (the data for 1990 is missing, because the indicator was introduced by the WHO after this date):

[11] Quoted by:: The Economist, 2000. May 6. issue 18. „Argentina Survey" p. 15. table 7.

Table 1.4.: Disability adjusted life expectancy in some countries

Healthy life expectancy (HALE) at birth (years)				
Country	Year	Both sexes	Female	Male
Afghanistan	2013	50	50	50
Australia	2013	73	74	71
Austria	2013	71	73	68
Botswana	2013	54	55	53
	2012	52	53	52
	2000	41	41	42
Chad	2013	44	45	44
China	2013	68	69	67
Czech Republic	2013	69	71	66
Denmark	2013	70	71	69
Finland	2013	71	73	68
France	2013	72	74	69
Germany	2013	71	73	69
Hungary	2013	65	68	61
India	2013	58	59	56
Italy	2013	73	74	71
Japan	2013	75	78	72
Norway	2013	71	72	69
Poland	2013	67	71	63
Portugal	2013	71	73	68
Romania	2013	66	69	63
Russian Federation	2013	61	66	55
Slovakia	2013	67	70	63
Spain	2013	73	75	71
Sweden	2013	72	73	70
Switzerland	2013	72	74	71
United Kingdom of Great Britain and Northern Ireland	2013	71	72	69

Source: WHO

2. THE INDIVIDUAL LIFE CYCLE

KEY WORDS

Inbound cash flow	Credit
Insurance	Income
Cash flow	Computability
Structure of cash flow	Outbound cash flow
Marital status	Risk transfer
GDP per capita	Private insurance
Foresight	Wage income
Life insurance	Self-insurance
Life cycle	Standardized life cycle
Cash flow of life cycle	Life, accident and health insurance
Phases of life cycle	Social security
Financial planning of life cycle	Reserve
Consumption	Capital income
Wealth	Risk
Active phase of life	Risk community
Inactive phase of life	

2.1. Foresight

How long ahead do people plan their future? If we search our own thoughts, or examine the actions of people we know, that allow for such deductions, we may get very different answers. There are some people who know in the autumn exactly when and where they are going on vacation next year, and some who do not know even at the beginning of their vacation. There are some who are continually increasing their wealth, and some whose wages never last them through the month, etc. In general: the longer ahead a people plan their future, and the more of a strategy they follow, the better off they are in life. Since almost every action a person undertakes requires some form of financial-type resources,[12] the basis of long-run future planning of any sort is long-run financial foresight (or long-run financial planning).

[12] Even if not in direct form, but as an „opportunity cost", for example, if someone does not make any money while he is occupied with certain activities (a hobby, studying, having children, taking care of social relations, etc.).

Of course, how far one can see ahead depends on many factors. The most important are:

Subjective factors: there are simply some who have better foresight, and are careful planners, and some who are more careless or less organized, and this has a great effect on the nature of planning for the future. [13]

Age: a child can objectively foresee a much shorter time span than an adult. A child is taken care of, so the trouble of foresight is lifted from his shoulders.[14] As he grows older, he must take care of himself and others more and more, which requires him to foresee his situation for a longer period of time.

Education (intelligence) level: The effect of intelligence can also be seen very clearly, which can be measured most closely by level of education, as they show a very strong correlation.[15] A more intelligent person is better at finding his way in the world, can see the connections between things more easily, and can distinguish between the important and less important factors, which all aid in better foresight. It can be observed quite frequently that social classes with low income but high education levels (as are doctors and teachers in Hungary, unfortunately) are able to do more in the long term (for example, educate their children, save for their old age rather than get caught up in the fashionable consumption trends) with the same income as classes with similar means but lower education levels.

Marital status: a single adult can much more readily live from day to day, and let their life "go with the flow" of things, than a parent with children, who cannot allow for great ups and downs in their financial situation, and must strive for stability – which can be achieved mostly using foresight.

Economy – individual economic situation: it is also true as a tendency (so it is not true in every concrete example) that wealthier people have more foresight than poorer people. Of course, wealth can be exchanged or compensated for by intelligence, or frequently wealth does not compensate for lack of intelligence. We could also say that

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[13] There are probably differences to be found between genders as well, for example, women are typically more careful than men, who, in turn, are usually – even in a financial sense – more driven to succeed, and more likely to think along the lines of a long term strategy in terms of their career.

[14] In many ways, and in certain countries, the relationship between the state and its citizens resembles that of a parent and child. The state „takes care" of the citizen's long-term safety (pension, healthcare) very similarly to a parent taking care of their child's ("paternalism"), and the citizen expects this from the state. In such cases – to which the Hungarian practice is very similar – the citizen remains a „child" in some sense regarding his own state of affairs. However, at the same time, in the case of modern states this is already a kind of expectation from the part of the citizens, who should pay attention to too many things. So, even such a supporter of the free markets and the self-care as the American Richard Thaler takes the stand for a kind of "enlightened paternalism": It means, that he asks the state not to apply force, but to give clever default options in the case of important, long term human aims, such as pension and health care, which are not compulsory, but automatically become valid if the citizens do not decide otherwise. (Thaler-Sunstein [2011])

[15] At the same time, the relationship between intelligence and education level is complicated, we cannot simply say that a person with higher education always has better foresight than someone with lower education, or that a more educated person is more intelligent than a lesser educated one, but in terms of general tendencies (so with many exceptions) this is the case.

wealth allows a person to be free of the burden of worrying about day-to-day survival, and this makes it possible to foresee a longer time period. This also makes it possible for someone faced with a choice between an alternative that pays well in the short-run but has long-run drawbacks, and one that pays off in the long-run only, to choose the latter if he is wealthier, while a poor person will be forced to choose the first (for example, certain occupations that are harmful to their health, but pay well are more likely to be filled by poorer people rather than the rich, etc.).[16]

Historical situation: It is very important, and a strong determinant of the possibility for foresight whether a person is living in a consolidated society, or one that is changing rapidly or perhaps at war? The basis of all foresight is a civilization that is predictable and consolidated, the presence of rule of law, etc. During the siege of Budapest, in the bomb shelters, the time horizon of foresight was probably only a few hours (Can I eat today? Will I have a place to sleep? Will I be alive at all tonight? etc.), not a few years. In a lesser degree, but it is also equally important whether an economy is in an economic boom or a recession, or perhaps undergoing basic institutional reform (as was Hungary in the first half of the '90s)?

Degree of civilization: Despite the fact that within every country there are careful and careless, single or married, richer and poorer people etc., the inhabitants of specific countries are much more similar to each other than to inhabitants of certain other countries. The reason for this is that as in the other areas of social life, people follow the examples they see, so they do things similarly as others, as they learned from their parents, at school, etc. These examples reflect the collected knowledge of the given country, which we could also call civilization. Civilization will appear in this book, in terms of foresight, as a summarizing category, or as a category that, along with foresight, mutually determine each other. In other words: the degree of civilization is higher if the time horizon of foresight of its members is longer, and vice versa: people with better foresight are at a higher degree of civilization. A very simple example: in the United States, saving for retirement is a very common and obvious activity, as is life insurance, etc. On a societal level, the USA has the ability to think about a defence for such far-away, unlikely risks as being hit by an asteroid. As a contrast, in numerouos (but far from all) African countries, the provision of everyday sustenance for the widest social classes presents the longest time horizon that they can foresee.

* * *

[16] A profane example: in Hungary it is typical that the maintenance of buildings is not thought of as something that must be done continually, but rather people live in them until they almost collapse, then they are renovated. This can be seen in the public buildings or at restaurants. These places are worth visiting for one or two years after their renovation, because the bathrooms are still clean, the paint is not peeling off the wall yet, but later one should look for a new hangout! This – in my opinion – is closely related to the fact that Hungary is not yet a rich enough country!

In the following, we will examine more closely some of the above factors (for example material wealth, age, marital status) involved in the planning of the life cycle. But first let's find out how come we are dealing with the issue of life cycle planning at this particular time in our history.

2.2. The Human Life Cycle Throughout History

If we take a look at the changes in life expectancy in the western world in the last few decades, the gradual and, in the 20th century, highly accelerated growth of life expectancy is very apparent. This tendency is shown in the following table as well.

Country	1750-1759	1800-1809	1850-1859	1880	1900	1930	1950	1987	2006	2015
England	36,9	37,3	40,0	43,3	48,2	60,8	69,2	74,5	79,2*	81,2*
France	27,9	33,9	39,8	42,1	47,4	56,7	66,5	76,1	80,7	82,4
Sweden	37,3	36,5	43,3	48,5	54,0	63,3	72,3	77,2	80,9	82,4
Germany	-	-	-	37,9	44,4	61,3	66,6	74,8	79,8	81,0
Italy	-	-	-	35,4	42,8	54,9	65,5	75,9	81,3	82,7
Netherlands	-	32,2	36,8	41,7	49,9	64,6	71,8	76,8	79,9	81,9
Soviet Union	-	-	-	27,7	32,4	42,9	64,0	69,4	66,4**	70,5**
USA (white population)	-	-	41,7	47,2	50,8	61,7	69,4	74,8	78,0***	79,3***
Australia	-	-	-	49,0	55,0	65,3	69,6	76,0	81,6	82,8
Japan	-	-	-	35,1	37,7	45,9	59,1	78,5	82,6	83,7

Source: 1987-ig: Livi-Bacci [1999], 138.o. WHO in case of 2006 and 2015 data.
* UK, ** Russia, *** The whole population of USA

Table 2.1.: Life expectancy in various western countries (1750-2015)

Among some other things, one of the main reasons behind the increase in life expectancy is the decline in the previously high rates of infant and child mortality. The historian Imhof says the following about this issue based on a graph of XVIII-XIX. century statistics (also outlined in his book):

„The computer graph[17] shows those who died each year in three dimensions, based on their absolute age. The first thing that jumps out at the observer is the large black wall in the background, which stands for the huge infant and child mortality of the time. Out of

[17] Unfortunately this cannot be shown here.

the 39251 people who died, no less than 12193, or almost a third were infants below the age of one. If we add to this the number of deaths of children below the age of eight, we've already reached half the total number of deaths (50,6%). The remainder is then pretty much evenly distributed among the 9 to 90 year olds.

At no other point in time in the later life was there a grouping of deaths even nearly as large. One person lived to twenty years of age, another to forty, the third to sixty, eighty, or even ninety or older." (Imhof [1992], p. 214-16)[18]

The observable increase in life expectancy beginning in the middle of the 18th century fits into a more general trend, according to Livi-Bacci: „from chaos towards order". From this time on „the order of mortality dictated by age became more stable. As opposed to the chaos of earlier times, which were characterized by random and unforeseeable death, the life processes started to become more orderly." (Livi-Bacci [1999], 134.o.)

What do we mean by „orderly life processes"? First of all: children generally die later than their parents – as opposed to the tendencies experienced up until the beginning of the 19th century. Secondly: death has its own place, it does not occur randomly during a lifetime due to war, epidemics, starvation, etc., but rather at the end of a „normal", we could say „standardized" life cycle. By standardized life cycle we mean that a person experiences all the main stages of a life cycle (child, adult, and elderly stages) within their own lifetime.

The formation of the standardized life cycle was made possible by the increase in the lifespan. The life expectancy at birth became sufficiently long during the 20th century such that it became highly probable (in Hungary the probability that a newborn will live to age sixty five was 72% for men and 86% for women in 2014) that an infant born would reach elderly age, or 65 years. (Moreover, today the start of old age is considered at a higher age than some decades ago, and this tendency will probably continue.)

The evolution of a standardized life cycle had a major role in the inception of the planning of individual life cycles. It is impossible to plan an individual life cycle without some certainty about its basic frame. As long as death was uncertain, the „planning" of the survival of the community (the extended family, earlier the village community, even earlier the clan) was more characteristic than individual planning.

Accordingly, the planning of the life cycle as a separate financial service only appeared in the last few decades in the western countries, and in Hungary it is just beginning to gain ground nowadays.

[18] The quoted graph: The distribution of deaths by age in the area of Berlin-Dorotheenstadt between 1715-1875.

2.3. Life Planning and Wealth

From the previous points, it is obvious that the ability for foresight is dependent, among other things, on the degree of wealth. Experience shows two conclusions which can be seen in large numbers:

1. The wealthier a country, the better foresight its citizens have
2. Within a country – regardless of its economic situation –, the higher the status (and thus in general the wealthier[19]) the social class we look at, the more signs of foresight (and future planning) we can observe.

There is a strong correlation between the amount spent on insurance and the level of the GDP, as we can see on the Figure 2.1. and the following Table 2.2.

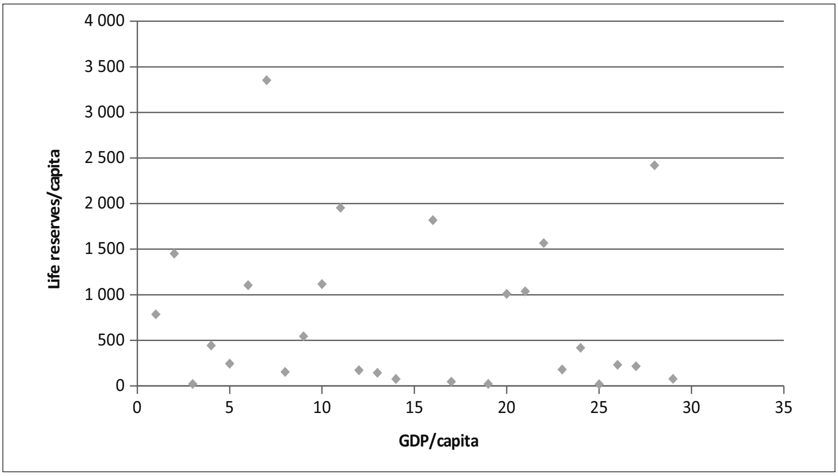


Figure 2.1.: The relationship between GDP per capita and the amount of life insurance per person in the Eu member countries in 2014

[19] Status and wealth can be temporarily separated from each other, like e.g. in Hungary in the case of teachers or physicians (majority of them), where the high status is not coupled with prominent economic situation. But after some time this kind of inconsequences are corrected by either descending status or improving economic situation.

Country	GDP –	Life premium per capita	Country	GDP	Life premium per capita
BG	5 900	21,13	CY	20 400	444,06
RO	7 500	18,31	ES	22 400	544,39
HR	10 200	77,63	IT	26 500	1 818,25
HU	10 600	144,18	FR	32 200	1 953,85
PL	10 700	180,04	UK	34 900	2 419,81
LV	11 800	22,07	BE	35 900	1 451,71
LT	12 400	46,41	DE	36 000	1 104,47
SK	13 900	215,43	FI	37 600	1 116,99
CZ	14 700	244,28	AT	38 500	785,83
EE	15 200	153,44	NL	39 300	1 037,48
GR	16 300	172,42	IS	39 500	77,10
PT	16 700	418,13	DK	46 200	3 352,99
SI	18 100	231,77	NO	73 500	1 567,22
MT	18 900	1 009,35	LU	87 600	42 590,76
			IE	41 000	8 039,73

Source:Eurostat, EIOPA

Table 2.2.: The GDP/capita and the life insurance expenditure/capita in the EU countires in 2014 (GDP in Current prices, euro per capita – 2014)

Sweden is not on the Figure, because it has not published life insurance premium data. Also missing Ireland and Luxembourg, because their data would totally distort the Figure. The reason that both of them can be considered as "hub" for life insurers, so high part of their life insurance premium in the reality comes from other countries (which also means that these premiums are missing from the data of the other countries). Permit me to remark, that there are other distortions in the data, which are hard to filter, because the savings for pension, or a part of them, is life insurance premium in some countries, but belong to a totally different financial institution(s) in other countries. In figure 2.1 we can clearly see that:

- The wealthier a country (so the higher its GDP per capita), the greater its expenditure on life insurance per capitaThe higher the GDP, the higher the fraction of it spent on life insurance[20].

[20] On the Figure we can see 3 not totally "regular" countries: Iceland, Austria and Norway. In case of all three the life insurance premium is lower, than in the countries with similar GDP, or in the case of Norway, what can be justifiable on the basis of the GDP. The relationship between GDP and the insurance premium first was analysed by Kovács [1999], later by Banyár (ed.) [2011]. This latter has stated that some kind of curves can be fitted well to the data (which contained more countries than presented here). This kind of curve is the logistic one, so if we were examining the under- or overinsured status of a country, then we have to see not only the simple insurance premium/GDP quotient compering to other countries, because in such a simple comparison the loser always will be the countries with lower GDP (like Hungary). Instead of this we have to see whether the premium/capita value is on, under or over the trend curve. In such a comparison, Hungary, compared to its GDP, seems just properly insured.

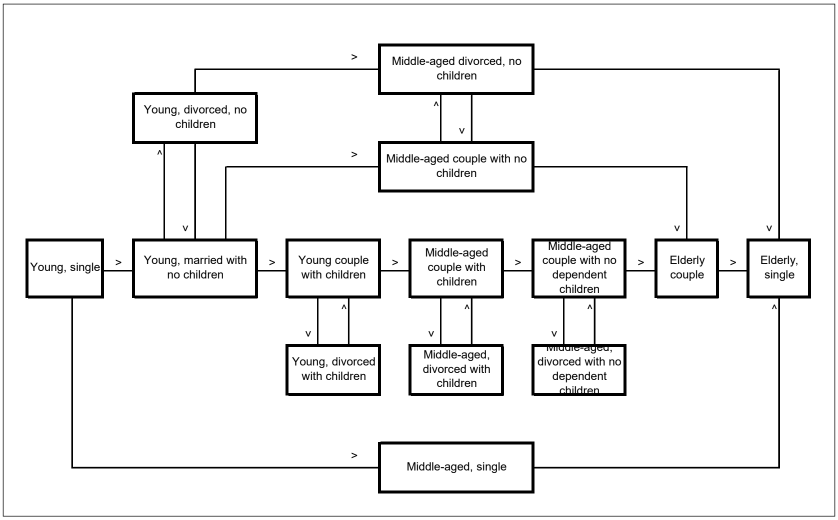
· We can regard the amount of life insurance as something that is positively correlated with the degree of foresight.

In addressing the low ratio of insurance in the countries with low GDP, we can say that while a country is relatively poor, the pressure to fulfil the primary needs (sustenance, clothing, shelter) is so strong, that there are no resources left for higher-level needs, and we can regard the security of older age as such a higher need. So, the opportunity for foresight/planning decreases – at an increasing rate – as wealth decreases. In the poorest countries and social classes we can simply see people living from one day to another, without any kind of foresight.

2.4. Variations of the Life Cycle

Based on what was stated above: the basis for the planning of the life cycle is the evolution of a standardized life cycle that is expected to be rather long, comprised of all the possible life stages, as well as a population with a significant ratio of sufficiently wealthy people – or the development of a sizeable middle class.

In the planning of a concrete life cycle we must consider other important factors beyond the lifespan and sufficient wealth, namely the family situation, which is related to age as well. The following figureshows the main variations schematically:



Source: Bauer-Berács [1999], 54. o. (After Murphy-Saples [1979], 12-22. o.!)

Figure 2.2.: Variations of the life cycle

A typical life cycle can be regarded as the life cycle line on the horizontal axis in the figure:

Young, single → young couple with no children → young couple with children → middle-aged couple with children → middle-aged couple without dependent children → elderly couple→ elderly, single (probably widowed)

If we are speaking of a life cycle in general, this is usually what we imagine, and this will be the basic concept in the rest of the book as well. At the same time, it is important to note that there are exceptions, which appear in the graph as deviations from the main axis. It is possible to get a divorce while one is still young, with or without children. It is possible to get married more than once. Perhaps a couple – by choice or not – will not have any children, or someone may not get married. Furthermore, the figure does not include some increasingly popular (or at least more common) cases, such as a lifelong partnership without marriage, or that of having children without marriage or a lifelong partner, or the more rare cases of long-lasting relationships between two people of the same gender, or communes[21].

In terms of financial planning, it is very important to be aware of these possible variations and, in concrete cases, one must seriously consider them. As we will see, in the financing of the life cycle it matters how many children one raises, and whether one does so alone or with a husband or wife (or lifelong partner), or whether the children come from a single or multiple marriages. From a social perspective: if a married couple raises two children, then they pretty much pay back to society what had been spent on raising them themselves. More than two children[22] can be regarded as a net contribution to the welfare of society. At the same time, single people or married couples without children have a lesser burden, they can achieve a higher standard of living, but they do this by failing to fulfil their obligation to society.[23]

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[21] After all, the figure originally was made in 1979, and it reflects the beliefs of the time period.

[22] An important constraint! Only if they did not just give birth to the children, but also raised them in a satisfactory way, and they received proper socialization so that they can become useful members of society. Someone who has a child or children without being able to fulfil these criteria is actually not paying back, but rather increasing their debt to society!

[23] A possible solution to this could be a childlessness tax, which is really a way of getting back the cost of raising and schooling from those who are trying to avoid paying back their debt. The amount would be equivalent to the cost of raising a child.

2.5. The Cash Flow of the Life Cycle

The life cycle can be divided into the following economic stages:

In the active phase of the life cycle, a person makes enough income for current needs, and usually more than that. In the economically inactive phase, there is no current income from work, so either one depends on financial support from others (parents, relatives, goodwill organizations, the state) or lives from their own (inherited or saved) assets and its interest. Since we are also the parents, relatives and tax payers who support the state, a country does well overall if its citizens individually make enough income not only for their current, but also for their lifetime consumption, or even somewhat more than that. If some people make less than that, then there are reasons and consequences:

- Living off of other people (or "being a parasite") or
- Living up the inherited assets, or
- An inherited or received physical or mental disability (being handicapped, or mentally retarded) in the case of which society accepts the consumption of the product of others' work without contribution (or with a lesser contribution) to the creation of goods out of solidarity[24], or
- Early death, which keeps one from beginning the active phase or from completely covering one's consumption up to that point.

If it becomes the norm in a society for active people to generate less income than what they consume during their lives, then that society:

- Uses up the assets compiled by earlier generations,[25] or
- Systematically robs other countries,[26] or

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[24] We will not list here among the cases of solidarity those who are theoretically able to work, but unable to support themselves due to social (lack of training) or economic (regional differences between industry and work force, an unsatisfactory structure of existing and needed education, general economic recession) reasons. In these cases the need for solidarity is temporary, while those able to work but inactive can re-train themselves, or move to the appropriate location. Then our general observation will apply to them as well. If such a person still stays jobless permanently and society is forced into solidarity, then there is something wrong with the socio-economic apparatus – but we will refrain from dissecting this issue further here.

[25] Perhaps Spain could be brought up as an example here at the beginning of the New Age. Although it is true that those compiled assets came from robbery (of America).

[26] Behaviour that is typical of the colonial times. See, for example, the one-sided stream of income from India to England between the middle of the 18th and 20th centuries.

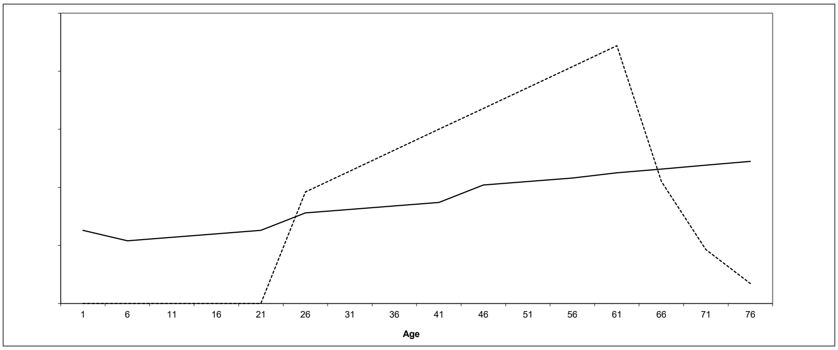
- Has compiled assets[27] that it can live off of without losing the assets (and without currently making an effort).

None of the above cases are typical, so the original concept can be upheld – with some clarification:

A society is all right financially if its members who are capable of work and have average lifespan make as much income as they consume during their entire life cycle and, beyond that, enough so they can contribute to society (in the amount needed) to support the members who are unable to work.

Further clarification: it is implicitly assumed in the above that the reserves that are freed up when the people who produced them die at the end of their active phase before they could consume them compensates for the shortage caused by those who die earlier. If this is not the case, so if more people die too early, this shortage will increase the solidarity burden weighing on those who had average-length life spans. If, on the other hand, there are more reserves freed up by those who died at the end of their active phase than what is needed for the above compensation, then the next generation of the society will start out better off than the previous one.[28]

The following figures represent the above relations:



Source: own drawing

Figure 2.3.: The relationship between income and consumption

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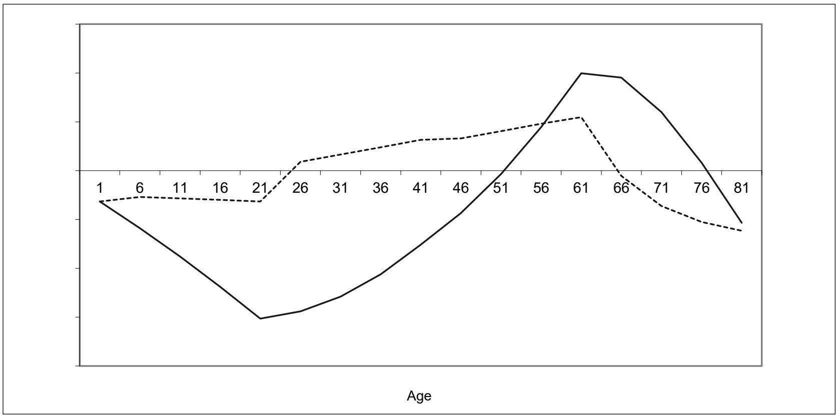
[27] Certain „oil countries" came close to being in this situation. For example, Kuwait (especially before the Iraqi invasion) but even Norway accumulated certain reserves. At the same time, members of a society that are prone to being passive and living off the interest of assets will sooner or later be left out of the „main flow of events" and be left behind, their assets losing their value.

[28] It is advisable then to avoid compensating the excess resources freed up in this way by consumption without work, perhaps due to a mistaken state solidarity policy (for example, long-term unemployment benefits, without necessary re-training programs). – Important! We are not talking about the Hungarian situation here, or valuing it, but about general relations!



Source: own drawing

Figure 2.4.: The cash flow of the life cycle (difference between income and consumption – and the cumulated difference)

Figure 2.3. shows the general relationship between income and consumption during an average-length lifespan. The above-mentioned relations can be seen clearly, namely that in the active phase the income is greater than consumption,[29] and in the inactive age either there is no income from work, or it is well below the consumption.

Figure 2.4 shows the cumulated amounts and the long-term equilibrium of income and consumption.

According to the figure, during our childhood (our first inactive phase) we surmount a huge and ever-increasing debt to "society",[30] which we begin to pay back in our active phase, and which disappears around the end of our middle age, and turns into an ever-increasing surplus, which in turn is gradually used up in our second inactive phase. (In the figure showing the relationship of income and consumption, we still assume a significant, though strongly decreasing income during the retired years, which goes along with the tendency that today's retired people are healthier, and thus more "active"[31].)

The usual form of paying back the debt accumulated during our childhood is the paying of the social security contribution. In line with this, we can create our (social security) pension by raising children (although the present pension system does not

[29] Not counting the indebtedness due to the long-term „investments" (mainly buying a home)!

[30] We could also say that to our parents. We can amortize this practically by paying social security pension contribution. We should consider bringing up our children as an investment (so we could correct the figure such a way, that the accumulation started earlier), because our pension (significatn proportion of it) will came from the social security contribution will be paid by them. The present social security based pension system obscures this relationship – see Banyár [2014].

[31] This does break down somewhat the theoretical basis for the separation of active-inactive phases, but that's life: we can only more-or-less speak of active and inactive phases, not with clarity!

reflect this, still, this is its point). By raising (properly) two children per couple we can make a suitable level of "human capital investment", to secure our pension.

The figure suggests that the incurred debt is paid off around the time when our children reach their active phase, and then the net accumulation begins. It is important to call this accumulation a "net" accumulation, because the above figure does not reflect the structure of debt and surplus. So, while the children are raised, the creation of the surplus already begins, but due to the existing large debt at the time the accumulated outcome seen on the figure is still negative, the paying back of the debt is still going on (so this is during the young adult stages of our children's lives – for example, during their university years, and afterwards) when overall the surplus has surpassed the debt.

According to the figure, our "lifetime wealth" starts from 0 at the beginning of our lives, and goes back to 0 at the end. This – and the assumption of a three-phased life cycle[32] itself – is a simplification, behind which there stand a few assumptions, or from which there are some deviations.

Assumptions:

- Since the length of the life cycle cannot be foreseen for each individual, only as an average for a group, we assume a mechanism that equalizes over the society, which redistributes the money required for an average-length lifespan according to the needs (this book deals with such mechanisms regarding the life insurance and social security)
- People are born without any assets and die without leaving behind any assets
- The life cycle has three phases and those phases are of normal length, so – most importantly – no one dies during their active phase, or loses their ability to work (so the active phase is not too short).

Possible deviations:

- There are some people who do not start out with zero, but rather a significant amount of inherited wealth, and during their life cycle they either increase this further, or use it up, so they leave more or less to their children[33] (or: they may even leave a debt behind)
- It is possible – in fact, highly probable – that consumption fluctuates much more during the life cycle than suggested by figure 2.3. The timing of children may be at very different stages of our lives as well.
- The life cycle consists only of an incomplete inactive, or a full young inactive and an incomplete active phase.

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[32] So the inactive-active-inactive phases follow each other!

[33] It should be noted that leaving an inheritance is not necessarily a voluntary decision. If, for example, the state taxes its citizens significantly, and from that builds roads etc., then someone can leave a significant amount to the next generation even if he dies seemingly without a penny to his name.

Of the possible deviations, the last one is most significant from the point of view of this book. The next chapter deals with this deviation and its consequences, as do the sections dealing with different personal insurance forms in detail. About these for the time being we can say only, that in the active phase of the life cycle everybody has to produce the cover of the consumption for the whole life cycle. It also has to be take into account that some unexpected events can shorten this active phase compare to the originally planned one and that shortening will obstruct the individual in paying back his/her debt, in producing the material goods necessary for present consumption and in the accumulation (including the child raising).

2.6. The Structure of Cash Flow During the Life Cycle

The above aggregated cash flow must be examined in its composition, in particular what the sources of our revenues are, and exactly what our expenditures are for. This structure depends on two factors:

- age and
- socio-economic situation.

The **structure of outgoing cash flow** depends strongly on the current phase of the individual's life cycle (which can be best represented by age)[34], their social situation, and also on which version of the possible life cycles the individual is living. First we will concentrate on the differing structure of expenditures during the different phases of the life cycle assuming a typical middle-class *life cycle.*

Life cycle phase (age)	Description of phase	Typical expenditures	Financial decision maker
1-6	Small child age	Basic necessary goods (food, clothing, shelter) (=necessary)	Parent (guardian)
7-18	Elementary and high school	Necessary + educational + recreation	parent + independently regarding pocket money
19-23	University	Necessary + educational + recreation + travel	parent, state, independently

[34] But is should be known that at the same age – especially in the adult ages – different people are at different stages of their life cycles. Some are already parents at age 18, some only after 30. Some reach the pinnacle of their career by age 25, some move forward gradually, etc.

Life cycle phase (age)	Description of phase	Typical expenditures	Financial decision maker
24-27	Young entry-level worker, single	Necessary + educational + recreation + travel (with different ratios!)	Independently, state
28-30	Young married without children, beginning of career	Necessary + educational + recreation + home buying (also different ratios!)	same
31-40	Young married, with small child,	Necessary + educational + recreation + travel + child-raising + child's education + home(payment, renovation, trade) car + precautionary saving for self and children	same
41-50	Middle-aged married with bigger children, pinnacle of career	Necessary + self-training + recreation + travel + child raising + child's education + home(payment, renovation, trade) car + precautionary saving for self and children (different ratios!)	same
51-65	Middle-aged married, no dependent children, stable, high income	Necessary + self-training + recreation + travel + occasional support of child + home(renovation, trade) car trade + precautionary saving for self (different ratios!)	Increasing financial independence
66-75	Elderly married, active retired	Necessary + self-training + recreation + travel + home renovation + car trade	Financially independent
76-85	Elderly widowed, retired	Necessary + recreation + travel + healthcare	same
86-	Elderly widowed, in need of care	Necessary + healthcare + personal services/care	Decreasing independence

Table 2.3.: The structure of outgoing cash flow

Figure 2.5. summarizes schematically the most important expenditures:

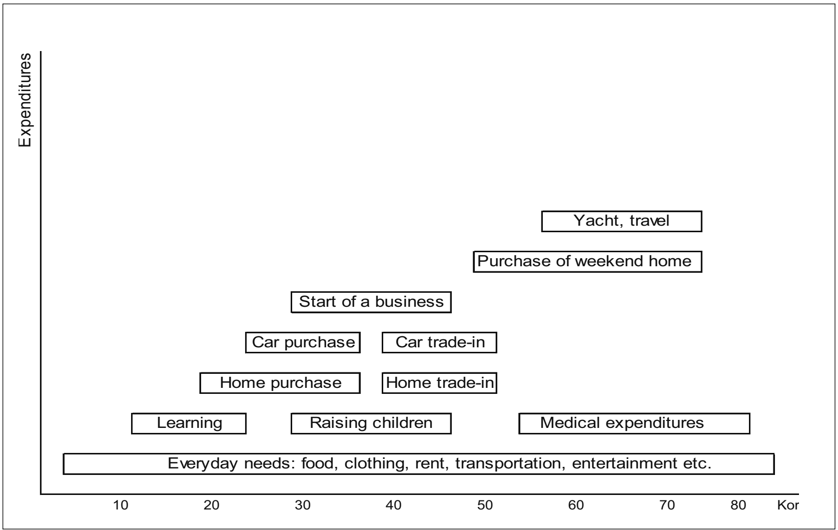

Figure 2.5.: The structure of expenditures as a function of age

The **incoming cash flow** can also be considered in a break-down similar to table 2.3:

Phase of life cycle (age)	Description of phase	Typical incoming cash flow
1–6	Small child	none (accumulation of debt)
7–18	Elementary and high school	Basically none, or some pocket money, later some part-time jobs (debt accumulation continues)
19–23	University	Regular part-time work, pocket money and further accumulation of debt
24–27	Young entry-level worker, single	Regular wage income
28–30	Young married without child, beginning of career	Regular wage income
31–40	Young married, with small child	Regular wage income (paying back of debt, and human capital investment)

Phase of life cycle (age)	Description of phase	Typical incoming cash flow
41–50	Middle-aged married with older children, pinnacle of career	Regular wage income, beginning of income from capital (paying back of debt, human capital investment)
51–65	Middle-aged married without dependent children, stable, high income	Regular wage income, significant income from capital (paying back of debt)
66–75	Elderly married, active retired	Basically income from capital, perhaps pension from pay as you go social security system, irregular wage income
76–85	Elderly widowed, retired	Basically income from capital, perhaps pension from pay as you go social security system
86–	Elderly widowed, in need of care	Basically income from capital, perhaps pension from pay as you go social security system, perhaps income from the activated LTC[35] private insurance

Table 2.4.: The structure of incoming cash flow

The main tendency of incoming cash flow as age increases:

credit → wage income → capital income (+pension) → (perhaps) income from insurance

Naturally this varies depending on social situation. In social classes with higher income, the capital income is dominant, possibly even the only form of income during the life cycle.

Figure 2.6 contains a summary of the most important cash flows and reserves. The horizontal axis shows the phases of the life cycle, on the vertical axes (since we put two graph on top of each other!) are the Forint amounts.

[35] Long Term Care: see in the Explanation of terms appendix!

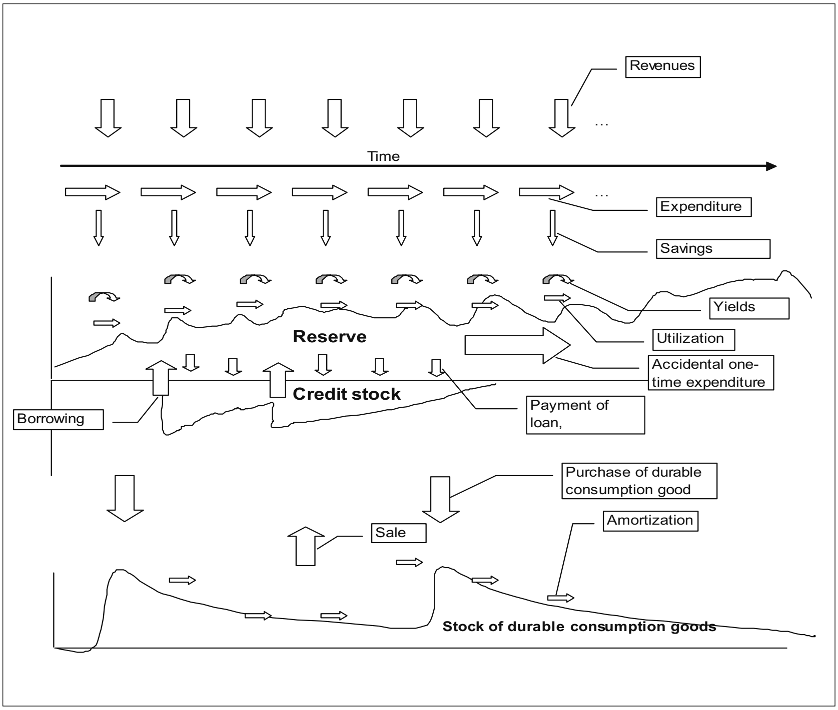

Figure 2.6.: The structure of cash flow during the life cycle

The "debt to society (to parents)" that is accumulated during childhood is not marked on the figure, it concentrates on the stocks formally showing up as debt. The stock of debt can be regarded as negative surplus accumulation, so it is shown along with the stock of reserves, as its mirror image. The net reserves are the difference of the reserves and the stock of credit.

2.7. Risks Threatening the Cash Flow and the Methods of Defence

The following requirements must be met by the cash flow of the life cycle:

- Liquidity should be assured at every moment, so the needs of the individual must be financed
- There should be sufficient funds for the achievement of goals reaching beyond the individual (care, leaving a legacy, other obligations to society)

- Great fluctuations of the standard of living should be avoided if possible (especially large drops in it[36])

The standard of living is also expected to grow continually[37]

Let us systematically examine what risks threaten the financial life program reflected in the figure of the last chapter, or the attainment of a three-phased life cycle, and how these can be defended against – using financial-type instruments?

The most important threats:

- Death
- Becoming unable to work
- Inability to produce income
- The devaluation of reserves, the falling apart of the system of institutions that serve as the frame of our life cycle

Of the above risks we will only deal with the first two here, and to a lesser extent the third one. The description of the last threat is not the subject of this material, but it is important to be aware of the fact that objectively thinking there exists the possibility of a war, a large scale economic crisis, regional or world-wide catastrophe, which the country or the suitable institutions must prepare for, and which must be avoided along with its consequences.

Death is unavoidable, so it is not its occurrence in itself that causes problems (at least not from the point of view of long-run financial planning), but rather when death doesn't occur at the "appropriate time", or not at the social average. In this respect, two kinds of deviations are possible:

- Earlier than average, or
- Later than average death.

Later than average death causes a problem if we want to end our lives with 0 wealth (or if we don't want to leave anything behind). In this case there is a need for a risk community, which redistributes our accumulated wealth among those still living. The solution to this (as we will see!) is the annuity, which is dealt with by life insurance.

---

[36] Large increases are also better avoided, because they may cause a fallback later on. When someone's actual income rises, the temptation to immediately increase their standard of consumption, the reference point, is strong. Before anyone does so, he should ask the question: „Will my increase in income be permanent, will it make this high consumption level sustainable in the long-run?" It is also wise to include: „Did I raise the level of my savings by enough to ensure that I will not have to lower my consumption after I retire?"

[37] Historically, this is a relatively new expectation, but nowadays it has completely assimilated into the expectations of the western man, and seems natural. More precisely: our expectation is that our standard of living should not grow more slowly than that of the groups relevant to us (for example „neighbours"). Overall, it is important that a person should feel that they are „constantly moving forward".

If someone wants to leave some wealth behind, then he can live off of its yield for any length of time.

The picture becomes more differentiated in the case of earlier than average death. The problem is different if death occurs within the following intervals:

- 0 – about 30 years old (from birth to the birth of the first child). We do not deal with this case here, we assume that at this point the person has not seriously considered the planning of the life cycle. From a social point of view, we can say that we should save up for such cases as well, so couples should raise more than two children on average for this reason as well.
- about 30 years old – about 50 years old (from the birth of the first child to the end of raising the last). In this case, death occurs when the individual has not yet finished the childraising. For this purpose, a risk community with this specific aim should be created (see below!). (We'll discuss this later under term insurance when introducing the types of life insurance! Of course this problem does not come up in the case of sufficiently large inherited or later – with small probability – attained wealth.
- about 50 – about 60 (after raising the last child, before retirement). In this case the human capital investment has been done, and the individual just started accumulating supplemental capital for retirement. Death is a problem here if there is still an obligation to support someone (for example a non-working spouse). In this case as well, term insurance is the solution. If there is no such obligation, then a risk community based on pure endowment insurance is ideal, since then the money accumulated for ourselves will go to those who may still need it (and, of course, this is worth doing because we do not know if we will be the ones to receive the money).
- about 60 – about 75 (after retirement, before the average lifespan). This case is the opposite of the case of a long lifespan discussed earlier, so its solution is also the annuity.

People may become temporarily unable to work (and thus make a living) during some time of their active life due to illness or accident. During an average life cycle we can defend against this with the forming of a risk community (social security, accident, medical or disability insurance). The costs of forming a risk community are added to the costs of a standardized life cycle beyond what we have discussed (since anyone can have an accident ...!)

It is very important to consider what expenses these risk communities should provide coverage for at these times:

- for the current treatment of the consequences of the illness or accident
- to compensate for the living costs of the income earner (as long as the inability to work lasts)

- for the current consumption of dependents (the foregone payment of the „debt”)
- for the foregone savings for retirement age

There may be other reasons for inability to work beyond accidents or illness – basically in the case of unemployment. This is sometimes related to the previous (so someone cannot perform his earlier job duties due to illness or accident, but is not in general unable to work), but mostly it is due to socio-economic reasons, and so its discussion is beyond the scope of this book. What we should note regarding the planning of the life cycle is that the individual can do the following to avoid or defend against the effect of threats:

- does not rely solely on one occupation, but trains himself to perform multiple jobs, and educates himself continually
- „keeps his eyes open” for new opportunities and for signs of problems, and tries to take advantage of these or defend against these ahead of time
- tries to become independent of employers, and start his own business
- always has a suitable size of reserves for transitional situations

The most important tool in our defence against the financial consequences of various threats is the accumulation of **reserves**. This is such an important instrument, that it can be substituted for all others[38] – which is why, in the above, we always treat the suitable size of wealth[39] as an exception.

The reserves can be temporary („precautionary reserves”) or for the longer term. Here we must primarily deal with long-term reserves, but the various temporary reserves are also important, which smooth out the occasional fluctuations of the cash flow. The various financial-type fields deal specifically with the issue of reserves.

The form of reserves can be of many different types, and this also depends on the aim of the reserves. The temporary reserves must be liquid (cash, demand deposits), or easily turned into liquid form. The form of long-run reserves is more likely higher interest bearing, less liquid pension funds, life insurance, investment fund, treasury securities, bonds, stocks, property and possibly (only in special cases) durable consumption goods (automobile, furniture, etc.). The following figure summarizes the major forms of saving, according to three important points of view: liquidity, interest, and risk.

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[38] As a reminder: it can not be substituted for threats to the reserves themselves (catastrophe, war, social decay etc) but this is not the subject of this book.

[39] Naturally what this “suitable size” is requires further investigation, and depends on many factors, and also varies by country and social class. In any case, even relatively large wealth can only partially substitute for other solutions, so the spectrum of transitional solutions between wealth and others is very broad.

Form of savings	Liquidity	Interest (and its possible fluctuations)	Investment risk
Cash	immediately usable, the most liquid	none	none
Bank deposit	quickly mobilized	little	negligible
Savings deposit	can be mobilized quickly with a loss	medium	negligible
Treasury bill	mobilized in a few months	medium	none
Long term government securities	sold in the market – mobile after the sales procedure	medium-high	none
Bond	same	medium-high	low – variable, depending on issue date
Stock	same	high	high
Voluntary pension fund	can only be mobilized after ten years	medium-high	medium
Traditional life insurance	before maturity mobilizing is a several day long procedure	medium – high	low
Unit Linked type life insurance	as the duration moves forward with decreasing loss, but mobilizing is a several day long procedure	low – very high	depends on us: we can choose from low or very high risk
Property	the length of the sales process is unpredictable	negative – very high	very high
Own business	same	depends on us	mostly depends on us

Table 2.5.: Major forms of saving

The form of reserves also varies by social class. Starting from the lower-middle class, the ratio of long run reserves starts to shift from pension funds first to life insurance, then to securities (investment fund, government securities, bonds, stocks in this order) and property. The reason for this is that the fluctuations of the value of these differs (it increases in the order listed), and that the defence against these (portfolio-generation) is only possible with relatively high reserves on one's own, with smaller amounts it must be left to professionals (pension fund, life insurance, investment fund).

The entitlement to pension in the pay-as-you-go pension system can also be seen as a specialized form of reserves[40], which is guaranteed by the stability of the state's institution system (and its payment depends on this stability). The risk community discussed below can also not function without reserves.

The **risk communities** are a substitute for traditional communities, and, along with the larger communities (local governments, the state) can even take over their roles. The traditional communities (clan, extended family, village community, multi-generational family) have basically disappeared nowadays (in the developed countries),[41] and their earlier roles of protection-defence were taken over by the society's specialized system of institutions. The main elements of this system of institutions:

- risk communities,
- wealth generation,
- charity organizations (for example churches, foundations),
- local governments – state.

We have already mentioned the creation of wealth (reserves). Perhaps it is surprising to call this an „institution", but thinking about it: could a symbol on the computer function as wealth in the less developed parts of the Earth without modern communication channels? For this to be possible it is necessary to have a system of institutions that guarantees that everyone should view that symbol as a stock expressing ownership in MOL, etc.

The goodwill institutions and the state cannot be regarded as an instrument of self-protection – as we have already mentioned earlier – and thus will not be discussed here.

The risk communities have in their name the fact that they have taken the place of earlier „natural" communities. It is a crucial difference that compared to those:

- the risk community only fulfils a single, specialized task,
- the members of the risk community do not usually form a community otherwise, and don't even know each other,
- the risk community has a formalized set of rules of operation.

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[40] The form of this is the human capital investment, that is child raising, because this create the basis of the contribution payment later. But this is not reflected in the entitlements acquired in the pay-as-you-go system.

[41] With the exception of the grouping called the single "family", but what happens here can be thought of as a game of "who can define it smaller?". Does a single man with an adopted child constitute a family? etc.

In its simplest form, a risk community is a community of solidarity (they jointly help those in trouble – this is more or less built on a natural community, for example that of a village), in its more evolved form (which is what we are speaking of here) it is the creation of a common reserve and its operation. The basic principle of the common reserve is that the unexpected events do not hit everyone at the same time, so it is not necessary for everyone to generate and maintain the total amount of funds individually, it is enough to do so in common. Thus, the amount of necessary reserves can be lowered. This is especially important when the financial situation of the members of the risk community would not make it possible to generate the total amount of reserves otherwise.[42]

- The formation of a risk community also has the following advantages:
- The entire amount of the reserves is always available, even if the members only contributed a small amount to its creation (so if the member had tried to create the reserves on his own, it might not even have been available to a sufficient degree at the time of the risk's occurrence).
- The reserves don't run out even after the repeated occurrence of the risk (so even if someone has enough reserves for a negative event, luck may have it that it occurs twice in a row, before the sufficient reserves for a second occurrence has accumulated, etc.).
- It makes planning the expenditures on random events possible (see below!).

The accumulation of reserves by the individual and by the risk community are substitutes to some degree. It can be clearly seen from the previous that the larger someone's wealth, the fewer risk communities he will have interest in, and vice versa. It is important to note, however, that with the increase of wealth the character of the risks and the risk communities that concern the individual change. The risk communities formed for the purpose of protecting physical wealth, for example, change with the type of wealth. In the case of the middle-class, a risk community against the burning down of their home is important, but the risk of their yacht sinking is probably not relevant. And a sufficiently expensive car is probably protected by its owner using more unique methods (for example, hiring a chauffeur), rather then via a risk community.

The risk community as a virtual community is nowadays mostly organized as an economic (so self-sufficient, profit-oriented) entrepreneurship, or perhaps as the specialized institution (social security) of an even larger community (local government, state).

[42] This, of course, is not a contradiction only if it can be assumed that the event the reserves were created for does not occur in the majority of cases!

The relationship of the risk community and its member – not forgetting that the risk community means the management of common reserves – can be thought of as a **transfer of risk** (from the member to the risk community, or the economic agent representing it). The main goal of the transfer: in exchange for a fee, the economic agent accepts a risk from the individual that he could not handle on his own (so its handling surpasses his own reserves). This fee is composed of three parts:

- contribution to the creation of common reserves (from which compensation takes place in the event of a risk's occurrence),
- the part of the fee needed to handle the administrative tasks of the risk community,
- a risk premium paid to the economic agent who runs the risk community in exchange for taking over the risk.

The transfer of risk simultaneously represents trade in several directions (realized in several projections):

1. **Risk transfer as trade in the market**: in this projection we can say that the transfer of risk is realized via trade in the market, so: the risk is sold by one who cannot undertake it to someone who can. In this case, someone whose potential loss surpasses his financial resources shifts its responsibility to some agent in the market (private individual, more likely a business) for whom it is not so large relative to their own assets. The acceptor of the risk is motivated by the risk premium he receives in exchange.

2. **Risk transfer as an evening out in „time" and „space**: the individual does not necessarily avoid the financing of the consequences of the risk that occurred because of the transfer of risk, in fact, in the majority of cases he has to pay more, than without the risk transfer. Still, it is a very useful and important thing, because it results in the evening out of the cash flow, so via the risk transfer he trades the uncertain, large loss (including the catastrophe – the collapse of the cash flow!) for a series of certain, small losses (payment of the fee).

This idea is equivalent to regarding the risk transfer as a way of evening out in „space", so something which allows a one-time large loss to be distributed „in space" among the current members of the risk community, since the large risks are unpredictable with respect to each individual, but with respect to the risk community as a whole it is a rather regular occurrence.

The usability of the risk transfer depends on the frequency of each given risk. Three different cases should be distinguished:

- occur frequently during a person's life (for example, a cold)
- occur rarely, but probably a few times during a person's life (for example, illness that lasts a few weeks)
- occurs only once or never during a person's life (catastrophic) large loss (for example, complete loss of ability to work due to a car accident).

In each case a different strategy should be followed, and there are different expectations of what the risk transfer should provide.

In the **first case**, it is not appropriate to use any kind of risk transfer, this can be solved individually (with the use of short-run reserves).

In the **second case**, it all depends on the extent of the risk. If the loss that occurs infrequently is significant, it is worth using the risk transfer, if not, then the solution via individual means is best here as well. At the same time, if risk transfer occurs, another requirement appears regarding the evening out in time, namely: for each person the amount paid during the life cycle (subtracting the risk premium and the administrative costs of the risk community) should be in balance with the amount used.[43] Then the main purpose of the risk transfer is the evening out of the cash flow.

In the **third case**, risk transfer is pretty much mandatory, and, naturally, the addendum above regarding the evening out in time does not apply.

Insurance is the most important private method of forming a risk community, and gives the best examples of risk transfer. We can also say that the

Insurance = risk transfer realised via a virtual (risk)community

The remaining chapters will deal with insurance in greater detail.

One of the most important functions of both the risk community (or of the risk transfer and insurance) and the accumulation of reserves is that they allow the cash flow to be evened out in the case of unexpected events, so the person does not have to diverge significantly from his original life plan. So, they make it possible to plan our life cycle ahead of time, and to stick with this plan. In a financial sense they make **computability** possible.

There remains one important question, which has to be made clear, namely: **who has interests in the individual's life**, and its suitable financial planning, how do these appear, and how can these be asserted? In the following we will summarize the most important observations in table form, specifically noting the most important private solution from the point of view of this material, personal insurance.

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[43] Some authors, mostly dealing with policy issues, have a tendency to only look for state solutions to risks of this nature, and they cover up the question of the balance between contribution and usage with an ungrounded and not well-defined notion of "solidarity". As it will be seen later on, I try to define the operational space of solidarity in a much smaller space and more precisely.

Who has an interest regarding the life of the individual?	what are the interests?	How are these realized?	The role of personal insurance
The individual himself	Maintaining the living standard attained in the active phase, ensuring its safety; protection against the inability to work; ensuring the re-establishment of the ability to work	Precautionary saving; annuity; accidental and disability insurance, medical insurance; leaning on the state; leaning on acquaintances, relatives, altruist institutions	saving-type products, annuities, accidental and medical insurance on a voluntary basis
The state	- social stability - management of obligation to provide for citizens - maintenance of international competitiveness via the improvement of the state of the population	- redistribution, mandatory precautionary saving and planning These can be achieved in a separate state system or via market agents Left to goodwill organizations	Can do everything except for redistribution – if allowed to
Dependents of the individual (children, spouse)	Financial security in the event of a fallout in earned income	The individual takes care himself leaves it to the state, acquaintances, relatives, altruist institutions	Either in its entirety, or as a complement to the state
Those effected by the individual's life (creditor, employer)	So they get their money even in the individual's absence, or to substitute for him quickly	Via insurance on the individual's life, health etc.	Almost entirely on a private insurance basis – often with group insurance

Who has an interest regarding the life of the individual?	what are the interests?	How are these realized?	The role of personal insurance
Voluntary social organizations	Lowering of social inequality	Voluntary redistribution	Not much – can also support these
Business partners	Taking over business ownership, organization of other business advantage via the provision of personal insurance (for example, gift of accidental insurance built into a card)	Private insurance	Entirely his own business

Table 2.6.: The interests related to the life of the individual and their methods of realization

It can be seen in the above that the individual does not always assert his interests himself (although this would be ideal, and that is the message of this material, that everyone should realize these consciously and as independently as possible), and that in some cases, the individual is not the only one with an interest in his life. If someone else acts in the place of the individual, in an optimal case it is for the following reasons:

- They see the interests of the individual better than he himself (for example the state recognizes earlier the need for saving for retirement and makes it mandatory).
- If the individual does not directly have interest in his life (health, etc.), or if his interest in this respect is less, than others' (for example he is the key worker of his employer, or the creditor's interest).
- The individual is in a dependent position (e.g. child, dependent, disabled).
- The individual is sometimes (or in some cases always!) irresponsible.

We also assumed the ideal of the self-reliant middle-class person. Self-reliance of other classes (as we have seen earlier) may differ from this. For example:

- Upper classes: have no need for the risk transfer realized via risk communities
- Lowest classes: cannot support themselves. Some of their care is taken over by the middle classes (through goodwill organizations and individual donation), since it is also in their interest to assure they do not spread epidemics, and that they do not worsen the public situation.

3. BASICS OF INSURANCE

KEY WORDS

Antiselection
Autoselection
Captive insurer
Claims prevention
Co-insurance
Composite insurer
Intergenerational contract
Loss sharing
Moral hazard
Mutual insurer
Pay-as-you-go system
Pool
Primary insurer
Prospect theory
Reinsurance
Reserving
Risk aversion
Risk equalization
Risk management
Risk sharing
Risk spreading
Safety
Self-care
Solidarity

Before diving into our topic of interest, life insurance, we need to clarify some key concepts that apply to all sorts of insurance. First and foremost, we would like to establish the meaning of the word „insurance". However, the best way to grasp the essence of insurance and understand its functionality in our lives, is to see how this definition is nested into the system of other known concepts. In the following sections we will go through the definitions out of which our point of study will emerge, in the meantime introducing more and more complex definitions for insurance, until a satisfactory final version is reached. After that we will give a quick insight into actual forms of insurance, its methods and inner interrelations. We assume no prior knowledge in insurance, so we start our study with a more general concept, "safety".

3.1. Safety

One of the age-old driving forces of all human actions (as usually said, "Even the ancient Greeks...") is the search for safety. This is why the caveman was not satisfied with his own hair, and borrowed the hide of killed animals as a garment, and later this extended "garment" served as a fly of a tent, in order to handle the vicissitudes of climate in **safety** – or at least on a higher level of safety as before. Staying **safe** in the threat of wild animals or the attacks of other human groups was also a reason for constructing a variety of self-defence weapons. Jumping forward in time, we take note of people who set aside money on their bank accounts in order to stay **safe** from the effects of some feasible threats (sickness, unemployment, etc.). Clearly, we can notice the search for safety in almost all human actions – think of carefully stepping off the pavement or taking an umbrella with ourselves.

We are about to draw the conclusion that a paramount aim of ours is to stay safe in all aspects and under any circumstances, but we must also state that living in perfect safety is terribly dull. After all, how could we encounter the love our life by chance if we avoid stepping out of the house, or win a bat on a horse race without risking some money? On the flip side, **hazards** and **assumption of risks** are at least as dominant aspirations as the search for safety – think of thirty-year-old youngsters joining the military chasing for adventure, subordinates talking back to their bosses, or textbook writers joking around while writing.

Well, is it safety or risk then, what man aspire for? It depends, one might say, but of course this is not a satisfactory answer. We should rather say that man aspire for both safety and risk at the same time, that is, searching for perils in some aspects of life, and striving for safety in others. While the adventurous youngster joining the legion takes the chances of either dying or receiving the Medal of Honour, in no way he would risk getting punished for his dirty boots, so – striving for **safety** – he would rather clean them.

It is apparent from the above examples that all human activities involve multiple risks, in other words, carry multiple hazards. To define what we mean by risk and hazard a bit more precisely, let's look at how it is usually defined. Here are some definitions:

1. Hungarian Explanatory Dictionary:

Risk is a danger associated with an action or an enterprise, [...], with the possibility of material loss or damage.

2. English Explanatory Dictionary:

Risk is the chance of loss, injury or other adverse consequences.

3. Investment textbook:

Risk is the possibility that an investment's actual gains will differ from the expected return.

4. Risk theory textbook:

Risk is an objective doubt about the outcome of a given situation. Risk is the tendency of an investigated process resulting in an outcome that is different from the expected outcome.

Unfortunately, the above definitions mean different things by risk. If we take a closer look, the first two and the last two of the definitions are roughly the same. For the sake of simplicity, therefore, only the following two (contradictory) definitions are analysed:

I. Risk is the hazard associated with an action or enterprise, with the possibility of material loss or damage.

II. Risk is the tendency of an investigated process resulting in an outcome that is different from the expected outcome.

In general, we can say that the two definitions differ in that the first considers risk to be asymmetric and the second to be symmetrical. Considering the vulgar tongue, we tend to use the concept of risk in both above senses, even within a single train of thought. For example, in the first half of the chapter, we talked about risk in the II. sense, but in insurance, interpretation I. is more common.

Therefore, in the following, by **hazard** or **risk** we mean that uncertainty threatens with adverse consequences or loss. By loss, we mean an economically onerous effect that results in the creation of an unforeseen need.

The chances of winning, so risk in the II. sense – to some degree, varying from individual to individual – is sought by everyone. However, situations that do not promise a chance to win (and whether a situation offers someone a chance to win varies from individual to individual), i.e. risk in the I. sense, are the ones that everyone tries to avoid, or at least defend against, that is, man try to be **safe** from these negative consequences.

Those who take risks (in the II. sense) far beyond the normal level in the hope of gaining far above the normal return are called gamblers. Even before we know what **insurance** is, we need to state that gambling is not part of it. The purpose of insurance is only to mitigate or prevent the negative consequences of hazards or risks.

Accordingly, **in the first approach**, we can say that insurance is a method of providing security, or in other words, a kind of **strategy for risk management**.

3.2. Methods of risk management

Thus, everyone strives to manage their risks and keep them in check.

One way to do this is through insurance. However, before defining what insurance actually is, we will take note of some other methods and strategies for managing risk, so that we can better see the peculiarities of insurance.

3.2.1. Risk avoidance

The simplest strategy of managing risks is to completely get out of their way, that is, trying not to even create situations or carry out activities that are risky, i.e. whereby certain adverse consequences and losses may occur. The strategy of risk avoidance can be applied to almost any situation – just it may not be worth it.

To bring an example from insurance: if someone builds their house on a hilltop opposite the floodplain of a river, they are virtually certain that it will not be washed away by floods, and so on.

Risk avoidance strategy is the best or, in certain situations, the only possible way to manage risks, as in the example above. The risk-averse man „goes for sure." Yet, in general, we can say that in many situations this strategy is only a rough and approximate solution that requires great sacrifices. That is, in many cases, small (or low-probability) expected damage is avoided by sacrificing high (or high-probability) profits. For example, one doesn't get on a plane because it might crash and therefore loses a lot of time, and so on. Therefore, in most cases, it is advisable not to "go for sure" but to take a "bit of risk", while trying to mitigate the extent and likelihood of possible negative consequences. This consideration is common to all other risk management strategies.

3.2.2. Loss prevention

Man knows from his own and others' experience that a result can be achieved by creating certain situations, constellations, or performing activities that almost attract certain losses, and the same result can also be achieved by creating or performing others that only rarely involve financial losses. In many cases, the kind of situation one is getting into, lies in one's conscious decision about it, how one is arranging the world around oneself, and what activities one is doing to achieve the same goal. A person's actions are only considered **preventive** regarding losses if they always consciously strive to achieve their goal through constellations and activities where the likelihood of damage occurring is minimal, even if the preventive action requires extra effort.

We demonstrate the meaning of the somewhat vague term „constellation" through an example: if a strong lock is installed on the door of an apartment, and perhaps even an alarm system, the owner has made an extra effort to prevent burglary, i.e. a loss. That in itself does not avoid the possibility of material damage, but significantly reduces its likelihood. A strong lock and the presence of an alarm system is a less favourable „constellation" for a burglar than any unprotected entrance.

Loss prevention therefore means that one consciously tries to reduce the probability of damage occurring by performing appropriate activities or creating a suitable situation.

A loss prevention strategy significantly reduces the likelihood of damage occurring but does not rule it out. If damage does occur in one area, it can have spill-over effects in other areas. Therefore, if we strive for safety, we need to have some strategy in place to prevent the spill-over of damage that has already occurred. For example, we cannot prevent for sure that we get a serious illness. However, if we are unable to get the necessary treatment, we may lose our jobs, that is, the source of our livelihood, as a consequence of the disease, become impoverished, and so on. And, complete recovery is often a simple matter of money. One such method could be self-insurance, which could help to put a check on spillover damage.

3.2.3. Self-insurance

In the case of self-insurance, we try to counterbalance the negative results that may occur in one of our activities or situations, with the positive results of another one.

Reserving is a generally applicable and widely used method of self-insurance. Internal risk equalization, on the other hand, is the preferred method only in certain situations. Now we will briefly examine both methods.

3.2.3.1. Reserving

In order to quickly neutralize the effects of a damage that has already occurred or to prevent its further negative consequences, it is advisable to have reserves for „unforeseen events". Such an unforeseen event is an accident, illness, fire, etc. As a result of these, one may temporarily or permanently lose some of their assets, work equipment, comfort equipment, or an ability. The lack of lost items causes inconvenience and possibly additional financial losses. If, on the other hand, sufficient reserves are available, the lost items can easily be replaced or made up for.

Reserves can be set up in cash or in kind. It is always good to have a few spare types of spare buttons, the right colours of thread, flour, etc. at home, or it is always advisable for a company to have certain spare capacities, e.g. buildings that seem to be redundant for the normal production process. A country needs adequate grain and oil reserves, camp hospitals that can be set up quickly if necessary, and so on. These are examples of reserves in kind. In these cases, the cash reserve is not a sufficient substitute for in-kind reserves. However, in general, cash reserves can quickly bridge most difficulties.

Self-insurance by reserving is one of the most important risk management strategies. We find examples of it in all areas of life. In several situations, this is the only right method, but in several others, it fails to bring the desired results. For example, if someone is afraid that their house will burn down, it is advisable to have another house in reserve for them. If the first one burns down, they can immediately move on to the second one, and the negative effects of the house fire have already been prevented. However, this second house can also burn down, so on a sure-what-sure basis, it is good for a person to have a third, and for the same reason a fourth, fifth, and so on, home. This strategy, in addition to not being available to most people, is very wasteful in this and many other cases, but we can clearly notice an element of self-insurance. We can therefore state that self-insurance is not an appropriate risk management strategy here. In such cases, the method of self-insurance must be replaced by insurance.

3.2.3.2. Risk spreading, internal risk equalization

The possibility of using this form of self-insurance is rather limited and can only take place in certain special situations. The essence of internal risk equalization is that the entrepreneur does not invest all his assets in risky companies but splits them between different companies.

For example, an entrepreneur is interested in two lines of business: he operates a beach and grows vegetables on a relatively large plot of land. These two businesses are affected differently by the early summer rain. It hurts the beach shop but it's good for the vegetable. The same is true in the case of sunny weather without prolonged rain, just in the opposite direction. Overall, this entrepreneur does not need to care what the weather is like, since in each scenarios, he will gain a more or less average profit.

The essence of the strategy: our entrepreneur neutralized the risk of weather with the right combination of activities.

* * *

None of the above strategies are considered to be insurance. The common feature of the risk management methods mentioned above is that the individual who is exposed to the risk acts against it on their own and handles it with individual strategies. Insurance, on the other hand, is the **cooperative strategy** of risk management, and hence we can give its definition in a second sense.

3.3. The risk community

By simply stating this fundamental feature of insurance, we still have not defined it exactly. For example, employing a group of armed guards to secure the path of a money shipment can also be considered as a cooperative strategy of risk management, still this is not what we will call insurance. Certain special solutions and institutional systems are also of paramount importance in the insurance business, which will be discussed later.

So, in insurance, the person exposed to risk does not face the perils alone, but as a member of a community. We call this group of individuals a **risk community**.

Before beginning to discuss the concept of a risk community, let's take a look at some of the historical examples, as risk communities were already organized very early in human history. Based on a very old story, ancient Chinese merchants sailing on a river also used the method of organizing a risk community. The problem for the merchants was that there were robbers ranching along the river who regularly looted the merchants' barges. If one of the barges was captured, its entire stock of goods perished, and the owner suffered a great loss. However, the other barges arrived at their destination intact, without any loss to the owner. The worst thing about it was not the fact of the loss, but its precarious and catastrophic nature. In all likelihood, the trader did not suffer any loss, but if he did, everything was gone. Traders would therefore have been happy to replace the risk of possible total collapse with a certain but small loss. They acted accordingly: they formed a group, and each of them divided their entire stock of goods into as many parts as were the merchants in the group. Each barge was made up of only a portion of each merchant's stock, so each barge had goods from each merchant. If one of them was robbed, the entire barge was lost, but each merchant lost only a small portion of his stock, so none of them went bankrupt.

Another example is the case of medieval guilds. These guilds also functioned as risk communities in many aspects. Each guild had a so-called „guild box", in which the contributions of guild members were kept. When a guild member met with some serious disaster (his house burned down, a relative died, became incapable of work etc.), his material damage was completely or partially covered from that guild box.

For a group of people to form a risk community, certain conditions must be met. These:

1. Group members form a risk community only in the face of the **same hazard**. So, if every member of the group is exposed to some hazard — for example, one that their house is burning down, another that is having a stroke, and a third that they cannot take care of their child after their death — and come together to discuss those hazards, they certainly form a group, but still they are not considered a risk community. A risk community is only formed by those who all fear that their house is burning down, or that they all fear that they will have a stroke, or that they all fear that they will not be able to care for their child after their death. Of course, the same individuals may belong to several different risk communities, but these must be clearly distinguished from each other.

2. The risk community must be **homogeneous**. That being said, it is not enough that the **nature** of potential harm is the same for members of a risk community, so that everyone is afraid of house fire or everyone is afraid of cancer. The **magnitude** of the potential damage must also be roughly the same, or at least within certain limits. To set an example: the 20-year-old Suzuki is as much a car as the brand-new Mercedes. Any of them can be stolen. Yet the character of the risk of car theft is not the same in the two cases, so the owner of the Suzuki

and the owner of the Mercedes are not in the same risk community. Above all, the Suzuki owner forms a risk community with the other Suzuki owners, possibly even with the Skoda and Dacia owners, and the Mercedes owner with the owners of Lexus, BMWs, etc.

3. The number of members in a risk community must be **large** enough for **risk equalization** to take place. Risk equalization means that it is unlikely that many – or even all – of the members of the risk community will be harmed at once, that is, a one-off, „point-like” damage at the level of individual members will be „evenly spread” over time at the level of the risk community.

Thus, we can conclude (in a third approach) that insurance is the cooperative strategy of risk management, which is implemented by organizing risk communities.

3.4. Loss sharing

Among the risk management methods used by risk communities, the oldest, simplest method is splitting losses. The operating principle of risk communities (possibly insurers) running in a **loss sharing system** is the so-called **”sharing and charging” system**. To illustrate the essence of loss sharing, let’s look at an example!

Dwellers of a village came together to form a funeral association. The problem they were facing, for which the association was founded, was the following: when one of the villagers passed away, a worthy funeral often caused financial problems for the relatives, since the funeral expenses had to be paid immediately after the – more or less unexpected – death, and it was not certain that the extra money needed for the obsequies was there in the household affected.

This problem was prevented in such a way that the association footed the bill for the funeral of its members. Specifically, if someone died, the funeral expenses were **shared** among the members of the association – everyone was **charged** their share, and the costs were covered from the payments received. The principle was that covering a fraction of the funeral expenses would not cause intolerable levels of financial hardship, even if it was unexpected.

The above example demonstrates an important feature of the loss sharing system, which can also be seen as its weakness compared to the **risk sharing** system described later. This is namely the subsequent nature of the pay-as-you-go system, i.e. the amount of money to cover the losses is collected from the members of the risk community only after they have occurred. This method goes hand in hand with a number of subjective uncertainties that can only be satisfactorily addressed in smaller risk communities, with members personally knowing each other. One such factor of uncertainty is, for instance, that after the losses have occurred, a member of the risk community is unwilling or unable to make the payment subsequently imposed on them. A for-profit company cannot expose itself to such uncertainties, so the loss-sharing system can only be considered by non-profit insurers – basically mutual insurance companies or insurance associations. However, even there such alternatives are usually limited. The risk-sharing system is therefore a modern method employed exclusively by private insurers and widely by mutual insurers.

In the context of modern insurance, there is no talk of loss sharing. Instead the method of risk sharing is usually mentioned. In a sense, we could also say that the borderline between the former and modern forms of insurance is drawn by the concepts of loss sharing and risk sharing. Therefore, loss sharing will not even be included in our definition.

3.5. Risk sharing

So far, what we know about insurance is that it is a cooperative strategy for risk management, whereby risk is addressed *in* or – more precisely – *by means of* risk communities.

The loss sharing system is not suitable for a modern insurance company, so private insurers operate exclusively in a **risk sharing** system. Perhaps the most important feature of a risk sharing system compared to a loss sharing system is that the former is anticipatory in nature, as opposed to the subsequent nature of the latter. Here, the organizer of the risk community (the insurer) assesses the expected amount of loss in advance (risk) and collects the countervalue of that risk, the insurance premium (the „membership fee" of the risk community), from its members (insured) beforehand.

In assessing its risks, the insurer basically relies on previous experience and calculates the appropriate insurance premium from the data using mathematical-statistical methods. This is such an important activity for the insurer that the use of mathematical-statistical methods is typically included in the definition of insurance itself. This is also how we will proceed, and this gives us the final definition of insurance:

Insurance is the accumulation of capital based on statistical methods of risk sharing, with the aim of meeting contingent and quantifiable future financial needs of the contributing members of a risk communities.

The basis of the application of mathematical-statistical methods is the **law of large numbers**. According to the law of large numbers, the more participants there are in a risk community, the less likely it is that many of them will be damaged at the same time. Of course, this is just a rough formulation of the law of large numbers, applying only to a very special case. A more precise formulation of the law of large numbers reads as follows:

„If we perform n independent experiments for the occurrence of an event with probability p, and out of these, the event of interest occurs m times, then the larger the n we choose – i.e. the more experiments we take –, the closer m/n, the so-called >>relative frequency<< of the event, will be to the theoretical probability p."

By probability we mean the proportion of cases in which the event of interest is **expected** to occur. Therefore, the probability p can also be viewed as the „**expected value**" of the relative frequency m / n. For this reason, the law of large numbers can also be formulated as follows: in the case of n independent experiments for the occurrence of an event, the higher we choose n, the more accurately the relative frequency of the event approaches the expected proportion of successful outcomes.

Let's look at an example! Let the experiment be a roll of the dice, and the event is that an odd number comes out. Since the total number of possible outcomes is 6 (either 1, 2, 3, 4, 5 or 6) and that of the odd ones is 3 (either 1, 3 or 5), i.e. half of the outcomes, the probability of throwing an odd number is 3/6 = 50%.

Now let's start rolling the dice, always calculating what percentage of the rolls so far was odd (relative frequency). Suppose the first throw is 2. Since this is not odd, the relative frequency is 0/1 = 0%. Let the second throw be a 6, the third a 3, the fourth a 4, the fifth again a 3. Then the relative frequencies proceed as follows: 0/2 = 0%, 1/3 = 33%, 1/4 = 25%, 2/5 = 40%.

It is easy to see that as long as we performed only a small number of experiments (dice rolls), the relative frequency can deviate very much from the probability, i.e. in this case from 50%. But let's keep experimenting! Suppose that out of 100 throws, we had 46 odd outcomes, which corresponds to a relative frequency of 46%, and out of 1000 throws, we had it 511 times, which is 51.1%. Thus, we find that as the number of throws increases, the relative frequency gets closer to the probability of an odd throw.

For the law of large numbers to work in relation to a phenomenon, it must meet certain conditions. To put it precisely, the law of large numbers is the law of independent, random, homogeneous mass phenomena. That being said, in order for a risk to be insurable, the event in which the loss occurs (insured event), must be random, independent, homogeneous, and occurring on a large scale.

We have already mentioned the need and importance of homogeneity and mass. Now let's take a look at the meaning of randomness and the requirement of independence.

Two events are said to be **independent** if the occurrence of one does not affect the probability of the occurrence of the other. For example, two (potential) house fires are independent of each other if the two affected houses are far off from each other and from all other houses. Then, the ignition of one does not increase or decrease the likelihood of the other igniting. Independence in insurance is a very important requirement. For example, if a large number of people take out accident insurance that offers a service in the event of, say, a drowning, the requirement of the law of large numbers to perform many „experiments" seems to be met. But if all the insured are on the same ship, the safety of all of them probably depends on whether the ship sinks or not, so their contingent drownings are not independent of each other, and so this is not a large number of events, but a single one. In this case, what we mean by mass, is the clientele of several ships.

The meaning of **randomness** can be approached in many different ways, and the nature of randomness can also be subject of heated philosophical debates. We want to avoid this by all means, so here we highlight only one – not too precisely formulated – feature of randomness that is important in practice. Namely, that an event is considered random if its occurrence is not known in advance for the affected. An insured event must be random in any case, and it is essential for the insurer to ensure this condition. Specific technical words were therefore born for the various shortcomings in randomness. The most important of these are adverse selection, autoselection, and moral hazard. Let's look at these in turn.

In insurance practice, **adverse selection** means that one of the contracting parties, the insured, taking advantage of the information asymmetry, conceals the real extent of their risk from the other contracting party, the insurer. Information asymmetry means that the knowledge about the magnitude of the risk is not the same for the insurer and the insured, so their information is not symmetric. In general, the insured person is better aware of the concrete circumstances and therefore the magnitude of the risk. For example, in case of a health insurance policy, the insurer's initial assumption is that the health status of each client is average. When a seemingly average client applies to the insurer, they might know something that the insurer does not, such as the fact that their most recent medical records indicate a serious illness, which they withhold from the insurer. In such a case, it is particularly advantageous for the client to take out the insurance contract on average terms, as their risk is much higher than average. Adverse selection is a very dangerous phenomenon for the insurer and therefore it should definitely try to sift out such cases.

Autoselection, as opposed to adverse selection, occurs when the reason for the insured to enter into an insurance contract is not their risk being higher than average, but them being excessively afraid of the occurrence of a certain damage. As a result of autoselection, the proportion of those who are particularly afraid of the realization of a certain peril is higher in the insured population than that of the total population. The effect of autoselection is not necessarily negative for the insurer, but there is often a greater risk behind the greater fear. As a typical example of autoselection, the life expectancy of annuity policyholders is significantly longer than that of life assurance policyholders – of course, ceteris paribus.[44]

Moral hazard is a very insidious phenomenon from the insurer's perspective. It does not occur in all branches of insurance or in the same magnitude everywhere. Where it does occur, it more or less hinders the insurer from making a long-term calculation. The essence of moral hazard is that the fact of being covered by insurance has an effect on the likelihood of making a claim. To illustrate this with an example: many of those who do not have a comprehensive car insurance drive more carefully, at a lower speed than they would do to their heart's content, since they are afraid of losing their car in the event of an accident. However, after taking out a comprehensive car insurance, this fear evaporates, as the owner does not lose the vehicle (or, to be precise, its value) in the event of a car crash. So, they drive less carefully, which increases the likelihood of a loss. That is, it is the existence of insurance that works in the direction of increasing the chance of damage.

3.6. Classification of insurance

Insurance can be classified according to many different aspects. Based on the subject of insurance, we distinguish between personal and property insurance. Based on the nature of loss, we distinguish between fixed benefit and indemnity-based insurance, and finally, in terms of provisioning, a distinction is made between life and non-life insurance. The different subdivisions have different perspectives, so in principle they can live side by side in peace and we can use the terms indemnity plan, non-life insurance, and personal insurance within the same text. However, regulation may highlight some of these, and this is precisely the situation in the EU: it basically draws a borderline between life and non-life insurance.

3.6.1. Classification of insurance based on the subject of insurance

According to its subject, insurance can be divided into two main groups: **personal insurance**[45] to protect individuals against the financial consequences of damage to their life, physical integrity and health, and **property insurance** to compensate for damage to property. We can further break down personal insurance:

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[44] The distinction between adverse selection and autoselection is a characteristic only of the Hungarian insurance literature. In English, the two are treated together as adverse selection.

[45] The Hungarian and English insurance terminology differ here as well. Here we described the Hungarian version. In the English literature, personal insurance includes e.g. home and motor insurance, so it is used much more in the sense of a "retail" insurance.

- life,
- accident, and
- sickness insurance.

The aspect of personal/property classification was reflected, for example, in the structure of the old State Insurance Company in Hungary, where the various property insurance departments (motor vehicle, corporate property, retail property) operated alongside the unified personal insurance department. This subdivision, however, is now somewhat outdated and also reflects the rudimentarily structured insurance supply of the socialist period, namely the insurance monopoly. This is because more modern types of insurance, such as the wide range of liability insurance or legal protection insurance, cannot be implicitly classified as property insurance (although liability insurance originally appeared as a rider attached to such policies).

3.6.2. Classification of insurance based on the nature of loss

The term „nature of loss" refers to a single property of the loss itself, namely whether or not its magnitude **can be evaluated**, at least in theory. Material damage that can be evaluated both theoretically and practically, may be covered by **indemnity-based** policies. On the flip side, policies with a **fixed benefit** are taken out to cover losses of theoretically or simply practically unquantifiable magnitude.

Indemnity plans are typically applied in property insurance, such as insurance against house fire. If a damage occurs, i.e. a fire breaks out in an apartment, the event is first reported to the insurer, then a claims adjuster investigates the scene and assesses the amount of damage, from which the insurer eventually calculates the compensation to be paid. Thus, in the case of non-life insurance, the claim settled by the insurer depends on the actual amount of the damage.

Note that the claim amount *depends on* the actual amount of damage, so it is not certain that the compensation will correspond with the actual damage. For the sake of clarity, in the following we will distinguish between the concepts below:

Sum insured:	The maximum amount of payment to be made by the insurer as set out in the insurance contract.
Insurable value:	The value of the insured asset.
Amount of damage:	The actual value of the damage or loss incurred.
Amount of loss:	The amount determined by the insurer's claims settlement policy in the specific case based on the amount of damage and the insurable value.
Claim amount:	The amount actually paid out of the amount of loss.

The claim amount is capped by a very important rule that "damage profiteering is not allowed", i.e., the compensation can be at most the amount of the damage. This rule is important because it keeps the insurer and the client "in one camp," which ensures that not only the insurer but also the client tries to avoid the occurrence of damage. If, on the contrary, profiteering was allowed from the damage, its occurrence and not its prevention would be in the client's interest. And the client can do much more for the event of damage happening than the insurer can do to avoid it.

Secondly, the insurer can possibly establish an amount of damage of HUF 1 million, whereas the claim amount is only HUF 500,000. This is what happens in the case of „underinsurance". **Underinsurance** means that the sum insured is less than the insurable value, i.e. the magnitude of the actual value of the insured property. In such cases, the claims adjuster not only determines how much the damage was, but also how much it could have been in total, i.e. how much the property was originally worth. If he finds that the value of the property (the apartment in our example) was HUF 10 million, but it was only insured for HUF 5 million, the insurer considers the property worth 10 million being co-insured by them and the client in 50-50% proportion. In other words, the damage incurred is also borne in 50-50% proportion, so in the case of a HUF 1 million damage, the insurer covers HUF 500,000 and the client also covers HUF 500,000. This method provides an incentive to avoid underinsurance, as it is detrimental to the insurer (in a more valuable asset, a fire, burglar, etc. can inherently do more damage than in a less valuable one). In this example, the compensation is paid on the so-called pro rata basis. However, this compensation principle is not always used (it cannot be used in all cases).

In the case of „premier risk" insurance, the claim amount shall be equal to the amount of damage up to the sum insured. If the amount of damage exceeds the sum insured, the claim amount is equal to the sum insured. The premier risk principle is applied, for example, in certain forms of medical expenses insurance where the insurable value cannot be predetermined.

According to the „total value" principle, there is no upper limit on the claim amount, as opposed to the premier risk principle. Here, the claim amount is always equal to the amount of damage. This principle is widely applied in liability insurance.

The amount of compensation may differ from the amount of damage due to several factors. Insurers tend to exclude minor claims, as these are usually not a particular burden for the policyholder, but their claim settlement procedure is as costly as for large claims. In addition, the number of such minor claims is much higher than that of larger damages.

Refusing to pay a compensation below the deductible is another principle, similar to that of the exclusion of minor claims. Since the amount of the deductible is usually chosen by the policyholder, it essentially means that they can decide on their own what they consider a minor damage. In practice, however, this is often not the case, as policies with high deductibles, i.e. the relatively cheaper ones, are taken out by those in a less well-off financial situation, while policies with lower deductibles, i.e. the more expensive ones, are taken out by the more affluent, for whom the minor nature of a loss ends at a much higher amount.

In some cases, by paying less than the amount of the loss, the insurer encourages policyholders to take loss prevention activities and also tries to offset the effect of moral hazard. Therefore, if the insurer finds the lack of measures to prevent losses or to reduce the amount of the damage occurred, it will set the compensation at less than the amount of the damage.

However, there are types of insurance where the claims adjustment process described above is inherently hopeless. For example, imagine the situation that a claims adjuster is applied in life insurance! The relatives report the death of the insured, i.e. the loss, and the claims adjuster goes out to the scene and estimates how much the insured has died, how much the relatives are missing them, and so on. Obviously, this is absurd, and therefore only the occurrence of the insured event needs to be proven here. The compensation will be a predetermined amount in the insurance contract, and that is why we call such policies fixed benefit insurance – actually, all forms of life insurance are of this type.

It is also worth noting that some types of insurance form a transition between fixed benefit and indemnity-based insurance, i.e. it is feasible under both principles. A typical example for this is accident insurance, which is a fixed benefit insurance if the insurer pays a certain percentage of a predetermined amount based on the extent of the resulting permanent injury, and an indemnity-based insurance if the insurer reimburses the medical expenses incurred as a result of the accident.

3.6.3. Classification of insurance based on provisioning

In terms of provisioning, **life insurance** is completely different from other types of insurance, i.e. all forms of non-life insurance. The reason for this will be discussed later and will not be explained in detail now.

Property insurance is a typical example of non-life insurance. (If we recall the distinction between personal insurance and property insurance, the difference between this and the life/non-life distinction is that from personal insurance we take accident and health insurance and add it to property insurance as they are technically similar to them.) The term of a typical property insurance policy (unlike long-term life insurance contracts) is one year (although they are usually automatically renewed the following year). Apart from some special factors, the risk is usually the same in consecutive years. It is therefore not necessary to pile up reserves from premiums on an ongoing basis (or at least not to the same extent as in the case of life insurance). In the case of property insurance, the premium for a given year basically covers the losses incurred that year. However, the claims volatility generally observed in property insurance, as opposed to life insurance, is very whimsical. Therefore, property insurance is profitable in some years and unprofitable in others.

The division between life and non-life insurance is tied to an organizational consequence: insurance companies operating in EU Member States cannot deal with both life and non-life insurance.[46] This is exactly because of the oftentimes hectic loss patterns of property insurance, which might entice company executives in onerous years to cover the loss from the stable reserves of their life insurance business. To avoid it happening, a life/non-life separation measure had to be taken.

The two main groups of insurance (life and non-life) are also called insurance **branches**, and the subgroups under these main groups are the insurance **classes**. Classes of non-life insurance include various types of motor insurance (there are several of them), accident and sickness insurance, etc. The classes of insurance that insurers may be licensed to pursue are covered by the EU insurance directives and (based on them) the Act on the Business of Insurance in Hungary.

3.7. Risk spreading

The safe conduct of business in insurance often requires the cooperation of insurers in assuming certain risks. It is the magnitude of the risk that makes it necessary to cooperate. For example, insuring some particularly high value objects (a tower block, a nuclear power plant, etc.) against elemental damage (fire, earthquake, etc.) may be too risky for a single insurer, as upon the occurrence of the damage – however low-probability it is –, the indemnity obligation can bankrupt the company. This is particularly undesirable since that way the insurer cannot fulfil its contractual obligation. The insurer defends itself against this by **risk spreading**, that is, by retaining only the part of the risk that its risk-bearing capacity allows, i.e. the extent to which any indemnity does not yet jeopardize the company's business continuity.

There are several methods of spreading risks.

Through **reinsurance**, the primary insurer, that originally assumed an excessively large risk, transfers the excess over its risk capacity to another insurer (the reinsurer), together with a proportional share of the policy premium. The reinsurer may, of course, find that the risk transferred is too large to handle, and at another reinsurer it may also re-reinsure (retrocede) part of it, and so on. This results in the potential loss being borne by more than one insurer, and even if this loss is large, each insurer will only be compelled to cover as much of it as its risk-bearing capacity can withstand.

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[46] To be more precise, as an „acquired right", companies already operating as a „composite" insurer before the directive came into effect may continue to pursue both branches but have to separate their business operations on a life/non-life basis.

Of course, the fact that several insurers combine their resources to pay the compensation does not necessarily have to be known to the client (although the insolvency of a reinsurer poses a certain risk to them if as a consequence their own insurer also becomes insolvent), as it is handled „over their head" by the primary insurer, i.e. with whom they originally concluded the insurance contract. This is not the case with co-insurance, another method of risk spreading.

In the case of **co-insurance**, insurers spread their overly large risks by their joint assumption. That is, they conclude an insurance contract where there is not only one contractual party to the client, but say 10, of which the first assumes, 20%, the second 15%, the third 13% of the risk, and so on. The client pays the premium to the various insurers in this proportion and, upon the occurrence of damage, the same proportions apply when receiving compensation from them, with neither insurance partner guaranteeing that the other will pay its share. Thus, in terms of content, the same happens as in the case of reinsurance, but the form of the contract and the way it is conducted differ significantly.

Risk pooling, the third form of spreading risks, is suitable for solving slightly different problems than what we have dealt with so far. In the case of a **pool**, the problem is not that, for example, the value of an asset to be insured is too high, since it is usually mass insurance that is handled in the pool. In the case of mass insurance, the main risk lies with the portfolio being too small (for example, in the case of home or motor insurance, a few hundred contracts are considered very small), and therefore the law of large numbers does not work, i.e. a single loss that is greater in magnitude can cause unbalance in claims and premiums.

This problem can be eliminated by merging the holdings of similar, but individually too small portfolios of contracts of several insurers, to form one large portfolio. That is, the premium income is added together in one place, and the claims are paid from this same place, and the profit is shared, say, in proportion to the premiums written. The aggregated portfolio is already quite large, so the degree of claims volatility is smaller than in the case of the constituting individual portfolios.

3.8. Types of insurance companies

Insurers can also be classified according to several aspects. Based on the level of the insurance business in which the insurer operates, we distinguish between **primary insurers** and **reinsurers**. Primary insurers, also known as ceding companies, are the ones in the insurance market who initially undertake to insure various things (property, life, health, etc.). Reinsurers, as mentioned earlier, are insurers of insurers, i.e. their customers are insurers themselves, and they take over some of the risks borne by primary insurers. Of course, a company can be a primary insurer and a reinsurer at the same time. Generally, the largest primary insurers also deal with reinsurance, but it is more common that the reinsurance activity is organized within the framework of a separate professional enterprise of the concern, which only deals with reinsurance. The largest reinsurers are usually professional insurers only pursuing reinsurance activity.

In the following, we will only deal with the classification of primary insurers. According to the branches and types of insurance, these can be divided into two main groups – specialized and composite.

A specialized insurer only deals with one branch of insurance (such as life insurance) or only one type of insurance (such as motor insurance, fire insurance, accident insurance, etc.). Under EU rules, only such type of new insurers can be established.

A composite insurer – as opposed to a specialized insurer – operates in multiple branches of insurance (i.e. it is a life and a non-life insurer at the same time). The advantage of a composite insurer over a specialized one is its ability to diversify, and the ability to serve customers in a complex way, but the drawback is that its strength can be fragmented between different insurance branches and ultimately it cannot pursue any of them at a high level of efficiency. Of course, the composite insurer also manages the premium income and reserves of each insurance branch separately, prepares separate balance sheets for them and does not mix them. The composite nature therefore refers to the complexity of the service rather than the fact that the losses of one branch are covered from the reserves of the other branch by the insurer (certainly, this is prohibited).

The ban on composite insurers resulted in the creation of insurance groups (or at least „twin insurers") where the life and non-life businesses of an insurer are run under separate corporations, i.e. this means setting up de jure specialized, but de facto composite companies.

In terms of the ownership of insurers, we can distinguish between a mutual insurer and an insurance joint stock company (private insurer).

In the case of a **mutual insurance company** (often in the form of an association, society or a co-operative), the insured and the ownership positions are not separated, i.e. the individual insured by the mutual insurance company is also the owner of the insurer. Hence one of the most important features of the operation of a mutual insurance company, which distinguishes it from a private insurer: its non-profit nature, i.e. the purpose of its operation is not to make a profit, but to satisfy the insurance needs of the owners in the most expedient way possible.

A **private insurer** usually operates in the form of a joint stock company, and here the status of the insured and the owner have nothing to do with each other. The goal of a private insurer, like of any other joint stock company, is to attain profit.

Both the private insurer and the mutual insurer are directed by the management chosen by the owners. However, while private insurers typically have only a few (or only a few

important) owners, a mutual insurer can have hundreds of thousands. An individual insured by a private insurer has to pay the expected value of the damage, the operating costs and also the profit of the insurer, while an individual insured by a mutual insurer (i.e., an owner) only has to pay the first two. So, at first glance the client is “better off” if they turn to a mutual insurer instead of a private one but this is not necessarily the case. This is partly due to the fact that, under market conditions, competition between mutual and private insurers can lead to the equalization of premiums, and partly due to the large number of owners, mutual insurers are less able to exercise management control than private insurers and may therefore work with lower efficiency and higher costs than private insurers.

The mutual insurer distributes the difference between the premiums and claims (which is the profit or loss for the private insurer) among the members. If the premiums do not cover the claims paid that year, additional fees may be charged to members. (This cannot be done by the private insurer. One might also say that while private insurers operate under a strict risk-sharing system, mutual insurers also have elements of loss-sharing.) If, on the other hand, there is an excess of premiums over claims, it is distributed among members. Of course, there may be several ways to do this, one of the most popular being that the surplus is included in the next year’s premium, so that it decreases as a result.

The above divisions do not show the technicalities of some special types of insurance. I would mention one of these special types, the so-called **„captive**” companies. These are mainly set up by very large corporations (many of them are transnational), possibly by the state, which have so many assets or employees that it is worth organizing risk equalization for themselves, because the law of large numbers already applies. In other words, the owner of the „captive” company, and its only client, is the large company itself or the state.

3.9. Social security

3.9.1. Personal insurance and social security

So far, we have found that personal insurance, that is, life, accident, and sickness insurance, are useful and indispensable tools for the financial planning of a life path and will be discussed in detail later in this book, especially life insurance. We also mentioned that it is by no means self-evident how these tools should be used, and that therefore life path planning is an ever-expanding business for consultants. This is a clear consequence of the voluntary basis of personal insurance, as we deal with it here, since only those who meet at least these two conditions take out such policies:

1. they have money for it
2. they have motivation to buy it (e.g. at least it comes to their mind).

If both are met and the individual receives exactly the right level of protection, then we are talking about 'self-care'. The level of self-care itself depends on the customs and other qualities of the given country, e.g. from the "degree of civilization" already mentioned above. However, even in the most conscious countries, politicians are up against broad classes of people for whom the degree of self-care is inadequate. The greatest problem is that if self-care is not started in time, it will be more and more difficult to catch up on, which inevitably results in old-age poverty. Moreover, unprovided seniors, as voters, put pressure on politics. Research suggests that the highest proportion of voters are found among the elderly, compared to other age groups. In most developed countries, therefore, the state does not fully entrust people with the care of their old age or of their health, and instead organizes it in one (or more) mandatory state systems. The essence of these state schemes in general is that people are required to pay contributions, and, in exchange, they receive health care or pension benefits. The whole system is based on general insurance principles, but it also differs from private insurance in many respects, so it is given a separate name – "social security". A strongly distinctive characteristic of social security, which clearly separates it from voluntary private insurance, is redistribution, i.e. the transfer of income from the wealthy to the less well-off to a certain extent. (Sometimes it works the other way around, which is called "perverted" redistribution.) Redistribution is also described by the term "solidarity". Voluntary schemes – at least where the risk community is not pre-selected, i.e. they do not necessarily include only those who know each other or belong together "naturally", e.g. workers in an industry – inherently lack the element of solidarity, as competition brings differentiation in customers, which is the exact opposite of redistribution. [47] Other differences will be outlined in more detail in the description of each system.

The organizational system of private insurance and social security is necessarily separate, as both have a different type of legal relationship between clients and institutions. The former has competing service providers, while the latter typically has a state monopoly, and so on. At the same time, social security is very important for private insurance in that it acts as a kind of "competitor", i.e. it limits its possible scope. Where there is a state pension system, the pension products offered by life insurers can only be supplements, not the main protection. Public regulation is also important for private insurance in that it fundamentally determines what eventually belongs to, say, life insurance. In Hungary, for example, a special, voluntary pension savings

[47] Some sanguineous proponents of social security draw far-reaching conclusions from this about the moral superiority of social security over private insurance and, uselessly, try to establish solidarity in voluntary schemes such as the health fund in Hungary. Such "experts" understand everything but the point.

institution, the pension fund, was established, which was organisationally separated from life insurers and life insurance. Decision-makers could have decided differently, as they have done in other countries, so it is somewhat casual to simply compare the scale of life insurance sectors in different countries: it is important to know what is and what is not considered life insurance in a certain country.

In the following, we briefly discuss the two main branches of social security, health insurance and pension insurance.

3.9.2. State health insurance

Throughout human history, health-related costs have generally not appeared as a separate large item in the personal budget. If someone became ill, family members cared for them, possibly bought some herbs or medicines from the nurse or the pharmacy, or, ultimately, called the doctor once or twice, who was paid for his visits. The model is called "fee for service" (FFS) in English, and this is how things work in the poorer half of the world today, but in the more developed world this has now become virtually impossible. As a result of health technology and medicine developments in the XX. century (and has been going on ever since), thanks to which there is now some kind of – often costly – cure for almost every disease. For this reason, financing also required a new model, as the old FFS model cannot be applied, as for the majority of people there is usually not enough disposable financial resources available to treat an unexpectedly diagnosed illness right on time. The new method of financing has become the two forms of insurance in general, private and social insurance, the proportions varying from country to country, despite the fact that most illnesses do not meet the abstract criteria of "insurability" that almost all insurance events in the non-health insurance sector meet.

Hungary is one of those countries where social security has become dominant in health care financing, and the majority of non-social security health expenditures is spent on an FFS basis instead of private insurance. The general model for social security financing is that the economically active pay a certain proportion of their wages as a health contribution to the social security fund, from which everyone entitled (the vast majority of the population) receives healthcare more or less free of charge, or at least for a fraction of the actual cost. This model worked well as long as the number of active people kept expanding. Today, however, there is already a largely opposite demographic trend in the developed world, making this funding model increasingly problematic. Moreover, with new medical discoveries, there is a widening gap between what is medically possible and what is financially affordable, making the reform of the model increasingly urgent. In broad outlines, the following aspects of the reform seem logical (different countries have implemented these steps to varying degrees). The essence of almost all of them is thrift, and we can group them as follows:

- *Ensuring a better match between revenue and expenditure.* One of the main concerns today is that the range of health care users (mainly the elderly who no longer pay contributions) and those who pay for it (the active who use relatively few health services) are extremely different, which makes funding vulnerable. It would therefore be appropriate to extend the payment of contributions to the elderly, which would essentially mean that part of their pension would have to be reclassified as a health contribution. In order to maintain their standard of living, pension savings (and thus health care savings) would have to be incentivized, so this solution would essentially be one of the possible concrete realizations of health savings.
- *Reducing the use of social security by „transferring"* certain healthcare treatments to private financing.
    - » To do this, the services provided by social security and the risks it covers should be specified precisely. This is also called the "*basic package*", so the task would be to *define* it. This is quite a politically sensitive issue because it would mean that the "full care" introduced decades ago would have to be formally withdrawn. Instead, in many countries, politicians choose de facto retreat instead of a formal one. However, this makes it difficult for private insurers to be able to define exactly what they are providing a competitive service for, as de facto withdrawal means that certain benefits can only be obtained through the bribery of leading doctors, which could be done at a lower cost than in the free market.
    - » Therefore, this can only be achieved in conjunction with a strong curb of health corruption, which in any case can be seen as a cost-saving measure in itself.
- *Reducing the use of social security by encouraging clients to save.*
    - » One of the most effective tools for this is the introduction of various *deductibles*. Such was the short-lived doctor visit fee introduced in 2007, which significantly reduced the number of visits to doctors. Unfortunately, its initial unpopularity was transformed into a political gain and was abolished by a referendum[48]. The visit fee is an example of a small deductible, which does not mean a particular financial burden. However, a higher deductible may also be introduced for more expensive healthcare treatments, which should be linked to a health savings scheme to finance this.
    - » The strain on the healthcare system would also be reduced if people did more for their health, i.e. we strengthen prevention. One way to do this is to

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48 The idea of the referendum was raised by Zsuzsa Hegedűs (http://nol.hu/belfold/20110802-_azt_nezem__hany_ehes_gyerek_van_-1155331). Hereby congratulations to her!

encourage healthy behaviour and to punish the unhealthy ones, for example, by partially linking the social security contribution to certain continuously monitored health parameters (weight, blood pressure, etc.). Unless we consider the excise duty on cigarettes as a punishment (which is only de facto, since the proceedings are not transferred to the budget), the current funding system does not reward prevention, nor does it penalize misconduct.

3.9.3. State pension

The state-organized pension system, typically in the form of social security, is a common phenomenon in the developed world today. However, pension itself, especially in this extent, is historically a brand-new phenomenon and suffers from such serious structural flaws that it is unsustainable in this form in the long run.

Before establishing modern state pension systems – typically after World War II –, pension itself was only available to small groups of people, such as:

- former employees of kings and lords
- later: civil servants (e.g. military officers, teachers)
- for employees in certain key occupations, a pension plan was organized by the employer.

For most people, however, especially for the vast majority of the population working in agriculture, these solutions were unattainable. Their careers were characterized by lifelong work that began as early as childhood. The minority who lived for so long that they could no longer do any work, was supported by their children living in the same household as them. It is customary to talk about a "traditional pension system" in this regard, which is defined precisely by the phenomenon of elderly parents being taken care of by their children. The basis of the traditional pension system was the traditional family business (e.g. and mainly: farm), whose prime mover was the cohabiting, multi-generational family. However, this broke apart in the XX. century, and therefore a different solution was needed. An alternative solution was first organized by the state on an ad-hoc basis, which was later justified by the ideology of the renowned economist Samuelson (Samuelson [1958]). In English, this system is called the pay-as-you-go (PAYG) pension system. Samuelson's ideology, also known as the intergenerational social contract, has become so popular that it is now seen everywhere as the operating philosophy of the PAYG system, and it indeed works accordingly everywhere in the world.

According to Samuelson, the "traditional pension system", where children support their elderly parents has gone out of fashion, and has instead been replaced by a new Hobbes-Rousseau social contract between generations, whereby the young will today support the aged, in exchange for the promise of their retirement subsistence, guaranteed by the yet-unborn, and so on. This idea, assuming a static population, can be illustrated as in the figure below:

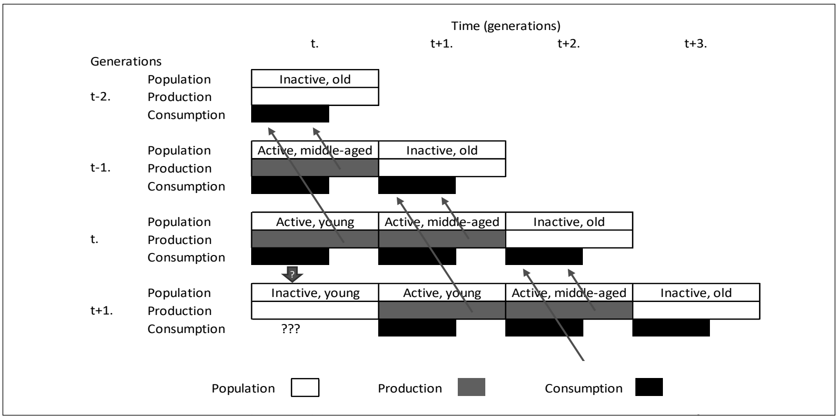

Figure 3.1.: Samuelson's idea of the social contract behind the PAYG pension system (assuming a static population)

Samuelson does not reckon with the cost of raising children and allocates all the surplus of production to the elderly. This also means that the elderly, in case of growing population, receive a much higher pension than their active-age consumption. This is what Samuelson calls "biological interest". In the days of Samuelson and for decades after the emergence of PAYG pension systems, population growth was a self-evident phenomenon. Today, however, this demographic trend has been reversed, not least due to the introduction of the PAYG pension system itself, and it is becoming increasingly difficult to maintain this pension system. Moreover, today, in contrast to the period of introduction, the vast majority of pensioners have paid contributions throughout their lives. The own logic of the system implies that the payment of contributions creates the right to pension benefits, namely in proportion to the payment of contributions. But if there is no contributor, that right is not worth much.

Nowadays, the fault of the system is becoming more and more obvious: it links the pension (entitlement) to a factor (the contribution paid by the individual) that has nothing to do with the actual income (means). For this reason, the above ideology also needs to be reviewed (see Banyár [2014]). According to this, Samuelson incorrectly claimed that the traditional pension system was out of fashion. He should have said that its circumstances (families living together) had changed, and today children can evade maintenance responsibilities towards their parents simply by moving far away from them. Yet the essence of the traditional pension system was that the parents, by raising their children, with the resources and time spent on it, made a kind of investment in human capital, the benefits of which were reaped in their old age, as pensions. That is, by supporting their elderly parents, children essentially reimbursed the costs of their own upbringing. So, it is not the figure above that indicates the actual situation, but the following:

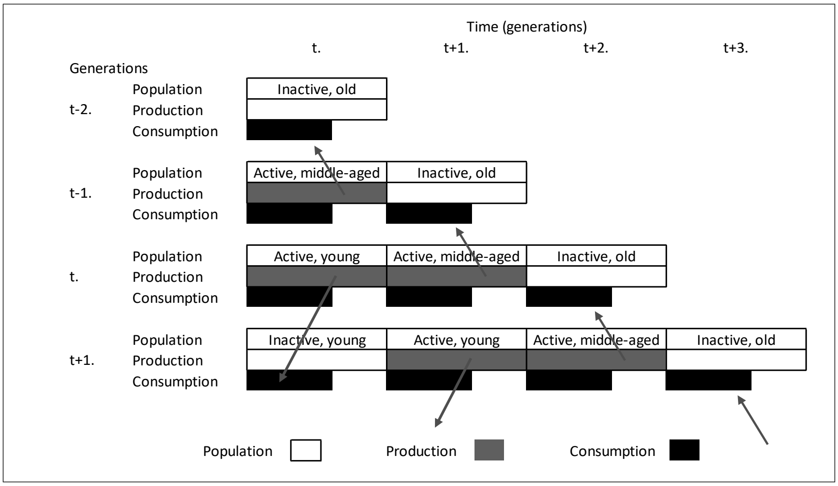

Figure 3.2.: A sustainable social contract behind the PAYG pension system (assuming a static population)

Accordingly, a PAYG pension system would only be sustainable if it explicitly recognized the investment in human capital, i.e. the contributions of the active were distributed among those who had contributed to their upbringing. Contributions of the active is nothing more than reimbursing the cost of raising them, for which no reward should be expected. If one wants a pension, one need to accumulate funds, which can happen in two ways (possibly as a combination of these):

- as an investment in human capital, i.e. by raising contributors
- as actual savings.

Those who do not raise a child will save on the costs involved, so they have coverage for these savings. In such a pension system, entitlement and financial means are not separated, so it would be sustainable in any demographic situation.

3.10. The psychology and microeconomics of insurance

The development of the concept of expected value was a vital step in the theoretical grounding of the operation of insurance and its important concepts – although at that time it was not used in the underdeveloped modern insurance. This can be attributed to the Dutch physician Christiaan Huygens, who proposed in 1657 that the value of a game should be determined as a weighted average of its possible outcomes (Moss

[2004]). Thus, if there is a chance to win 100 HUF with probability 1% and win nothing with probability 99%, then its value will be 1 HUF. The expected value was then linked to the notion of "fair price". Using this for insurance: if someone wants to insure themselves against a loss of HUF 100 with probability 1%, its fair price will be HUF 1. It sounds logical, but we know that an insurer cannot operate under such conditions, and it also needs to be explained why most people are willing to pay more for insurance than the expected value of the loss.

The problem was solved by Daniel Bernoulli in 1738 (Bernoulli [1738], cited by Moss [2004]) in connection with a tricky question posed by his cousin Nicolas Bernoulli to a famous mathematician, Pierre Rémond de Montmort in 1713. This later became known as the St. Petersburg paradox. The question was, how much money would you give for a game that promises the following payouts: if you toss coins and toss a head, you get HUF 1. If you only throw a head for the second time, then 2, if only for the third time, 4, or if only for the n-th time, then 2^{n-1}. The problem is that every person would give a maximum of a few forints, while the expected value of the game payout is infinite, as the expected value

$$\frac{1}{2}\cdot 1+\frac{1}{4}\cdot 2+\cdots+\frac{1}{2^n}\cdot 2^{n-1}+\cdots=\frac{1}{2}+\frac{1}{2}+\cdots+\frac{1}{2}+\cdots=\infty \qquad (3.1.)$$

Therefore, no one would be willing to pay the price that was previously considered "fair".

Bernoulli solved the problem by introducing the concepts of utility and expected utility. In this connection, he essentially set up the law of "diminishing utility", i.e., in his view, the greater our wealth is, the less additional utility is gained with the same increase in wealth. Namely, according to him, this changes logarithmically, that is, in essence, it is not the absolute growth, but the growth rate that matters. For example (this is my own example!), if we calculate the utility of the above with the function $U(x)=1+\log_2 x$, then the expected utiliy of the game will not be infinite, but the following:

$$\begin{aligned}\frac{1}{2}\cdot(1+\log_2 1)+\frac{1}{4}\cdot(1+\log_2 2)+\frac{1}{8}\cdot(1+\log_2 4)+\cdots+\frac{1}{2^n}\cdot(1+\log_2 2^{n-1})+\cdots=\\ =\frac{1}{2}\cdot(1+0)+\frac{1}{4}\cdot(1+1)+\frac{1}{8}\cdot(1+2)+\cdots+\frac{1}{2^n}\cdot(1+n-1)+\cdots=\\ =\frac{1}{2^1}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots+\frac{n}{2^n}+\cdots\approx 2\end{aligned} \qquad (3.2.)$$

Diminishing utility, which is still one of the basic ideas of microeconomics, is also accepted by modern psychology as a special case of the stimulus intensity rule. According to this, whenever the intensity of a stimulus increases (in a multiplicative way), it is accompanied by the same (additive) increase on the psychological scale. If e.g. the voice increases from 10 to 100 and this is taken as 4, then an increase from 100 to 1000 also adds 4 in psychological intensity (Kahneman [2013]).

Bernoulli's suggestion can be translated so that insurance gets an explanation. This is because diminishing utility means that people prefer secure wealth to an uncertain one. Decreasing utility otherwise means risk-averse behaviour. Consider the following figure, which shows the values of a logarithmic utility function on the vertical and the value of our wealth (W) on the horizontal axis.

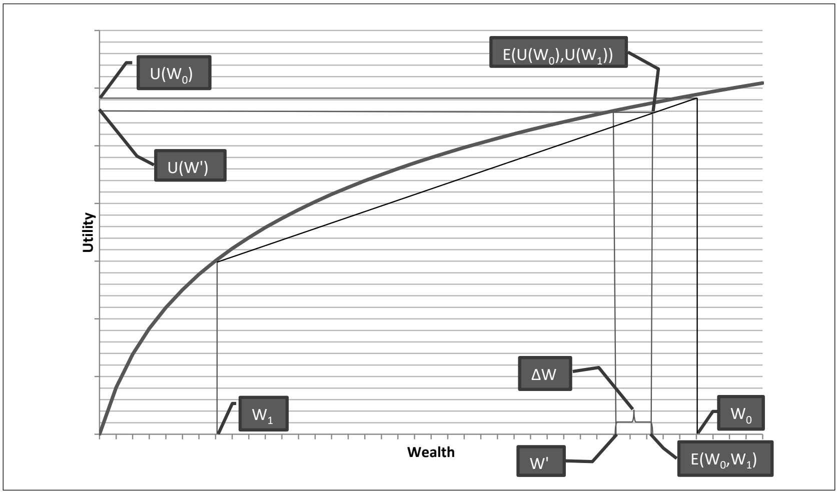

Figure 3.3.: An illustration of the Bernoulli utility function

According to this, if the two possible values of our wealth are W_0 and W_1 such that their expected value is $E(W_0,W_1)$, then the expected utility of the uncertain wealth will be $E(U(W_0),U(W_1))$, which is equal to the utility of a certain amount of wealth, W'. According to this, preference is given to all smaller but certain amount of wealth of size at least W', over wealth is are only expected to be $E(W_0,W_1)$. The difference between the two is the ΔW wealth "band".

This can be used directly to explain insurance, as the insurance premium can be seen as a waiver of part of our wealth in exchange for making our uncertain wealth certain. After all, the above situation can also be interpreted as meaning that when a loss occurs, our current wealth (W_0) may decrease to W_1, so that our wealth, which we believed to be certain, is actually only an expected value with the same utility as the certain wealth W'. According to this, we are willing to pay a higher insurance premium than the expected loss (W_0-$E(W_0,W_1)$), but maximum W_0-W'.

The above theory, which can be attributed to Bernoulli, explains insurance well, and this explanation can still be accepted today. It is important to note, however, that 250 years after Bernoulli, Daniel Kahneman and Amos Tversky, described this theory as flawed at one important point and replaced it with prospect theory. The main realization of this was that we value gains and losses differently, we hate losses much more than we love gains. For this reason, it is not enough to look at the magnitude of our wealth in general, but it is important to look at what we compare it to, that is, Kahneman and Tversky have brought the point of reference point into the picture. This is usually the status quo. (Kahneman [2013])

With the example of Kahneman:

Problem 1: Which one would we choose: we get $ 900 for sure, or are we get $ 1,000 with probability 90%?

Problem 2: Which would we choose: we lose $ 900 for sure, or we lose $ 1,000 with probability 90%?

According to him, we are probably giving a risk-averse answer to problem 1 – that would not have surprised Bernoulli either. In the case of problem 2, on the other hand, we prefer to take risks, because then there is a chance that we will not lose anything. In other words, the negative value of a $ 900 loss is much greater than 90% of the value of a $ 1,000 loss. We have strong resentments against certain losses, which compels us to take risks.

Our choice is shown in the figure below, where the reference point is roughly the inflection point of the figure. To the left of this are the losses, and the fact that the curve here is concave and, in addition, assigns greater psychological value to the same change in wealth shows our strong aversion to loss and our tendency to take risky behaviors in the face of large losses.

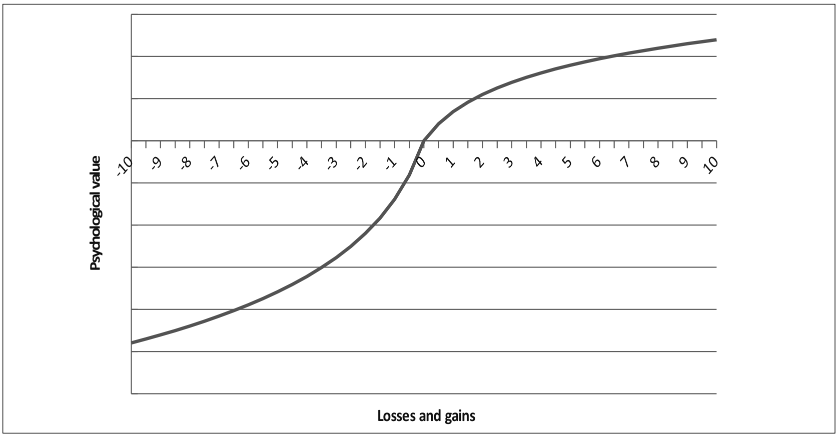

Figure 3.4.: The illustration of Prospect Theory

II. THE STUDY OF LIFE INSURANCE PRODUCTS

4. THE ROLE, CONCEPT AND MAIN TYPES OF LIFE INSURANCE

KEY WORDS

Transfer between funds	Asset Fund
Unit Linked Life Insurance	Accumulated Units
Accident Insurance	Conditional Annuity
Accidental Death	Term Insurance
Accidental Disability	Annuity
Sickness Insurance	Beneficiary
Maturity	Initial Sum Insured
Insurance Term	Initial Units
Insured Event	Term Insurance
Sum Insured	Hospitalisation Daily Allowance
Insurance Benefit	Critical Illness
Insured	Term Fix Insurance
Age of Insured	Surgery Benefit
Table of injuries	Pension Insurance
Pure Endowment Insurance with Premium Refund at Death	Sum Insurance
Whole Life Insurance	Critical Illness, Dread Disease
Health insurance	Disability Waiver of Premium
Unit	Disability Annuity
Offer Price	Policyholder
Life Insurance Policy	Endowment Insurance
Top-up Payment	Bid Price

In the following sections of the book we will take the life insurance from the earlier mentioned solutions that facilitate the planning of the life cycle, and discuss it in more detail. First we will try to find the concrete situations of life and the concrete types of life insurance that can be used to achieve individual (in some rarer cases organisational) goals. The following discussion focuses primarily on the product, briefly mentioning the most important institutional specialities, and at the same time referring to the changes that both are currently undergoing.

4.1. Financial Needs

The change in demand for financial – and within this life insurance – products has its own logic, that the insurance companies have to follow both on the sales and on the product development side. The most important elements of this logic:

- As people are becoming wealthier, new financial needs arise, and these needs are differentiating – parallel to the differentiating society. (It has to mention, that an opposite tendency also appears: because of the economic development some earlier financial needs – e.g. the need for cheque-book – are ceasing, respectively certain groups of the society are merging.)
- Parallel to the enrichment of people, their financial literacy is developing (because they have more and more possibility to deal with finance and to observe the problems), and consequently:
  - » They also understand more the functioning of life insurance, and they have a growing need for more comprehensible products (so in a certain sense demand is gradually shifting from traditional to modern life insurance).
  - » Needs are becoming more differentiated, which requires more and more individually fit products[49].
- Because of enrichment and competition the certain consumers' comfort and demand level is increasing, and this way they are less satisfied by only the product itself, they expect a complex solution to problems. They respect much less the argument that "my competence as an insurer ends here, seek the advice of other kinds of institutions with your further problems".
- As a counter tendency, other consumers – parallel with they rising financial knowledge – would like more and more simple and basic financial products out of which they are able to mix the product mixture they need.
- With the integration of financial areas it is much more difficult to define what exactly life insurance is. Due to integration, the competition of other financial institutions (banks, brokers, mutual funds) is becoming more definite, but at the same time new opportunities open up for insurance companies, they won't be locked up as much into a relatively narrow field of action.

Behind the change in consumer demands – e.g. the change in demand for financial products – we can discover a kind of order. We see the same order in the specialisation of institutions on different financial areas. Retail banks are specialised in handling mostly daily, short term financial affairs, that is, in handling cash flows, deposit collection (also short term), that is logically connected to these, and consumer loans (again short term),

[49] Which has the sales and network-organisational effect that they have a growing need for quality counselling.

and the capital market firms are specialised to invest large scale, “superfluous” capital. Life insurance supports the realisation of “strategic” goals of the life cycle requiring a greater volume of money, and it neutralises the dangers threatening the realisation of these goals. So, the areas accessible by life insurance can be defined by two dimensions, the financial need and its term.

The order behind the demand for financial products (especially those that are long term and require the consumption of greater volume of money) is: First the most pressing need is satisfied, after that the most pressing among the remaining, etc… If we want to order – by main points – the “strategic” (thus above the elementary, that is eating, drinking, sleeping, etc.) needs by “how pressing they are”, then we get the following:

1. “Some kind of” housing (sublease; a room in the parental house; a house gradually built and modernized in a lifetime’s work) – its special feature is that any “excess” of money is immediately “built in”.
2. Precautionary reserves for “general use”– the smaller, the more liquid (so the order is: under the pillow, sight deposit, short term saving account, time deposit, life insurance). Because the reserve is not satisfying, the goals are not really differentiated: It is spent on whichever comes first, and they hope that there won’t be too many needs “coming in” at once.
3. Complementing the general reserve: A low premium, low sum combined insurance covering a number of life and related risks (the best example for this was the Group Life and Accident Insurance – CSÉB – which was sold en masse in Hungary from the ‘60s until the beginning of ‘90s).
4. From the “general goals” the one that first becomes independent, “providing for children” – that is, saving up money dedicatedly for them. It has two phase: the undifferentiated (money saving in general) and the differentiated (for different aims: e.g. for the death of the parent, for university costs, etc.)
5. Buying a car, or a gradual change in quality.
6. Immediate solution of quality housing – on the debit of discounted future excess income –, and the periodic change of housing satisfying higher and higher level needs
7. Saving for the case of sickness
8. Pension complement, or saving up the necessary capital for an adequate level of pension, for ourselves and for our spouse – later the gradual depletion of this (or other assets, like home).
9. Leaving wealth to the dependants
10. Advance provision of our helpless elderly selves, or organising the concrete care

It is characteristic of the market of a country which needs of the list above its financial institutions have started to satisfy. It can happen, that only – let us say – needs

2-3, although the insurers widely advertise their "pension insurance" products, but that products are bought by the consumers only as a "general purpose" reserve products. Not especially rational that the car buying is coming before the quality housing, but – knowing the preferences of the modernt consumers – it can be considered as a fact. Actually, in case of a rational consumer, the pension could follow the children immediately in the rank, before the car and the home.

4.2. The Relation of Life Insurance with Other Insurances, the Nature of Life Insurance Risk, The Characters of a Life Insurance Contract

Let's look at what life insurance is, and what distinguishes it from other types of insurance.

We gain the definition of life insurance[50] from the general definition of insurance. We get its specialities if we delimit it from other insurance types. The delimitation can be of several aspects. In the following we will analyse the relation and differences of life insurance and other insurances in the following aspects:

1. the insured event,
2. the character of claim and reimbursement,
3. the specialities of reserving.

The term "life"-insurance itself is partly correct, but partly euphemistic, since primarily those insurances are called life insurance, where the insured event is related to the death of the insured. This – given by the nature of the matter – can be exactly of two kinds:

1. the incurrence of death,
2. the non-incurrence of death.

More precisely the possible life insurance events can be phrased as

1. death as an insured event, if the death of the insured happens during a pre-determined term,
2. living through a term as an insured event means that death doesn't happen during a certain pre-determined time-period.

[50] We have to pin down right at the beginning that here and further on, under life insurance we primarily mean the insurance of profit-oriented joint stock companies, or private life insurances. On the other hand most of this discussion holds for the plans of mutual insurers, co-operative insurance societies.

Consequently we get the two elemental insurances that are most important in many respects:

1. term insurance (for death) and
2. pure endowment insurance (for living through).

In the following sections this book is mostly persistent in regarding as life insurance only the insurances that can be characterised by these two insured events, but we have to add that this conceptual clarity is difficult to hold due to two reasons. These are:

1. **Tradition,** which has several layers. First of all, life insurances traditionally tend to contain accidental and health elements, too. Conceptually these cannot be regarded as life insurance, but practically they are accounted under life insurance[51]. On the other hand there are two risks that do not really fit into the definition of insurable risk, but from time to time they appear in life insurance policies all the same. These are marriage and childbirth. Even European Union guidelines recognise this tradition and allow the cultivation of life insurance policies containing these risks under the life insurance branch. Anyhow, we do not handle these two risks here.
2. **The development of life insurance products** takes us in the direction that the former complementary function of life insurance, long term saving is becoming the main function in many products, and in these products we do not find an element that can be pointed out as insured "event". The book will handle these products as life insurance with full rights, and doesn't require a life insurance to contain one of the above insured events. [52]

We can say about the claim and the reimbursement, that life insurance principally cannot be a reimbursing insurance, which would mean that if the insured event occurs, the level of reimbursement paid by the insurer is determined based on assessment of damage, depending on the size of actual damage occurred, as for example in property-casualty insurances. Even the usage of the term "claim" itself can be questioned in

[51] There is a kind of "dominance" rule in the Hungarian life insurance practice (but also in the majority of other countries'), according to which if a product covering several types of risks contains also life insurance risk, then the whole product is regarded as life insurance, regardless of whether the premium part of the life insurance risk is the major part of the premium or not. Later we will see that within a plan life insurance risks can only appear together with accident and health risks, and no other types of risks.

[52] According to the EU regulation an insurer can sell only insurance and insurance can be sold only by insurer – at least as an own product (that is we do not speak about the situation when an insurer acts as intermediary of a different type of institution). But what is the difference between a modern, single premium, savings-type life insurance and a capital market product? According to the European practice the difference is, that the life insurance product also consist of at least a minimal death element, that is in case of death not the 100, but – let us say – 101% of the accumulated capital is paid. This is – from a certain angle – is a ridiculous solution, but – at least so far – nobody has dared to "query" this practice, because it can impugn the principle quoted at the beginning of this footnote, and this would be pregnant with unforeseeable consequences and it is not clear, that which other principle could substitute it.

relation with a life insurance insured event, since this term has been developed mainly to describe material, or materialised losses. Of course the parties of a life insurance contract also have losses if the insured event occurs, but on the one hand this is not really tangible (if the insured event is living through the insurance term), and on the other hand it appears through more transpositions (if the insured event is death during the insurance term) than in case of the typical reimbursing insurance.

If possible, we do not use de term “claim” in life insurance, or only in an analogous way with other types of insurance, to refer to the occurred insured event.

Because of the above, insurance can only be so called “**sum insurance**”, which means that the benefit paid by the insurer is not determined by the size of damage, but a level, a “sum” defined in advance in the policy.

Regarding reserving, life insurances (at least 99% of them) differ totally from other insurance types, from all non-life insurance (at least from 99% of these). The cause of this lies in the typical length of insurance terms, and in whether there is a characteristically different, definite change in the probability of claim during the term. So, let’s look at the difference between the two mentioned insurance categories in these respects.

A typical life insurance policy has a term of several years, or rather several decades. The mortality rate continuously increases throughout the term, and the actual death rate is usually – with a little fluctuation – around the theoretically expected value concerning the whole portfolio. From this, and from the fact that – contrary to e.g. property-casualty insurance – in case of life insurance maximum one claim can occur during the term, follows that the insurer uses the premium received during the whole term to pay claims – especially if it is a savings-type (that is it pays not only in case of death) -, so most part of the premiums have to be put aside, reserved. This has the effect that a life insurance company has long term, stabile reserves.

We can take for example property-casualty insurance as typical non-life insurance. A property-casualty insurance policy – contrary to a life insurance policy – is typically signed for a year even if it is generally automatically prolonged in the next year when the risk is generally – disregarding some special factors – the same as in the previous year. So, there is no need to gradually save up from received premiums in a reserve (at least not to the degree as in case of life insurance). In case of property-casualty insurance the claims of a given year are generally covered by the premium of the same year. On the other hand the fluctuation of claims is – contrary to life insurance – very erratic. This way property-casualty insurance can be profitable in one year, and results losses in the other. This is caused by – among other factors – that we can question property-casualty claims in three dimensions, unlike the 1 dimension of life insurance. In case of life insurance the only question that can be asked regarding a claim is “when?”. In case of property-casualty insurance beside this we can also ask “how many times?” and “how big?”. The defence against the erratic claim history of the property and casualty insurance is, that the insurer can modify the premium of the contract at insurance anniversary. In case of the life insurance, this practice – because of the fixed long term – is not possible. The long term of the life insurances is not only possible, but also the natural need of the clients.

The abovementioned differences are the causes of the practice evolved in some insurance markets that life insurance and other classes of insurance ("non-life insurance") were sharply separated from each other. These also became the names of the two "branches" of insurance. When the uniform European Union insurance regulation was worked out, this distinction was also took over, so today in the whole European Union insurances have to be separated into this two branches.[53]

The organisational consequence is, that in the European Union a newly established insurer is not allowed to make operations on both insurance brances in the same time – with some exemptions.[54]

Finally it is worth to shortly review the main characters of the life insurance contract. Let's start the introduction of it by defining one of the most well known life insurances, the term insurance:

In a *term insurance* contract the *insurer* undertakes the liability – against the premium payment of the contractor (*policyholder*) – of paying a sum specified in advance (*sum insured*) to the person specified in advance (*beneficiary*) if the person specified in advance dies (*insured*) during a certain period of time (*insurance term*). If the insured lives at the end of the insurance term, the insurance policy is terminated without benefit payment.

It is worthwhile to analyse this definition a little bit, because it contains a number of key words that we can use later on. First of all the characters (or subjects) of a life insurance policy. As we can see from the definition, there are four of these:

1. the insurer

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[53] Theoretically it would be possible to distinguish more than two brances. Practically it would mean the further partitioning of the non-life insurance which could be considered as a big "other" category. For example it is possible to argue, that because one of the possible service of the legal expences insurance is the protection against a not appropriate service of an insurer, that is a potential conflict of interest is possible between the legal expenses insurer and the other insurers, therefore these type of insurers also have to separate to each other.

[54] Actually, the reserving characteristics belong less to the life – non-life feature, but to the long- or short run feature. However, it would be difficult in the daily practice to separate insurance types according to this feature. The distinction between life and non-life in turn covers the long- and short run separation very well, and easily feasible in the practice (although it has to be referred here the uncertainty already quoted in connection with accident- and sickness insurances). In practice, we can see examples for long run non-life insurance contracts at the sickness insurance. The emerging problems are often solved by the lawmakers by reckoning such a sickness insurances among life insurances. Nevertheless the term "long term insurance" is often used for life insurances, which is not totally correct but expresses the point quite well.

2. the policyholder
3. the insured and
4. the beneficiary.

Taking the insurer as obvious, we have to know the following about the others:

The ***policyholder*** is who signs the insurance policy and pays premiums. He is the "owner" of the insurance policy (he "holds the policy"), he can make legal statements relating to the policy, he names the beneficiary, and he has the right to terminate the policy (surrender). The policyholder can be a natural person or a legal entity. As a natural person, he is usually – but not necessarily – at the same time the insured, too, and can naturally be also the beneficiary. Nevertheless it is worth distinguishing whether the same person is mentioned in the role of the policyholder, the insured or the beneficiary. When this role is not important, or the same statement can be made in the case of several roles, then the place in the policy is only important compared to the insurer, and we can use the term **client** (as we will in this book). From a legal aspect the main character of the life insurance contract is the policyholder (naturally beside the insurer).

The insurance policy is about an event related to the ***insured*** person's life. From an insurance technical perspective he is the main character, and can only be a natural person. If the policyholder and the insured is different, then the policy requires the written consent of the insured to become effective. But from the legal aspect this is almost the only right he has. Apart from this, if the policyholder wants to surrender the policy, he has the right to take his place as the policyholder. We have to point out, that the fact that from the insurance technical and from the legal aspect the main character of the insurance policy is different, sometimes leads to problems, mainly when interpreting the subject of tax allowances, if the state incites life insurance through tax allowance.

The ***beneficiary*** is the one to receive the benefits paid by the insurer if the insured event occurs. In many respects he is in the most favourable position, since he has no obligations, only rights but, on the other hand, his position is the least stabile. The policyholder can change the beneficiary any time (unless in the clause of the contract – like in the case of the credit insurance – the change of the beneficiary is prohibited), even without the former beneficiary having knowledge of this. Of course, the beneficiary doesn't have to be only one person or a natural person.

If we further analyse the above definition, the next keyword is **term**. Life insurance policies most often have a definite term, but it might also have an indefinite term. Policies of definite term have usually a length of whole years. These whole years usually do not overlap calendar years, but start with the inception of the policy, and the so-called insurance anniversary is every year on this same day.[55] The time period between two insurance anniversaries is an insurance year[56]. The minimum possible term of definite term insurances is generally 5 years, and the maximum is – varying from company to company – between 25 and 40 years. A maximum relative to the age of the insured is also generally used, e.g. the insured cannot be older than 75 years at the maturity of the policy.

The more developed a country, the longer the average term. However, in case of average term we have to distinguish between proposed and materialized term. The contract contains the proposed term, but the materialized term can be shorter than this if the policyholder quits the contract (surrender it). In certain markets – thus also in Hungary – quitting the contract is quite widespread, so the materialized term can be much shorter than the proposed term. It can be said also in the case of materialized term that the more developed a country, the longer it is. There is no statistics about this, but a good lower estimate of it the life reserve/premium income ratios, which were in Europe in 2013 the following:

LV	IS	BG	PL	SK	RO	HU	CZ	LT	EE
3,1	1,0	2,7	2,7	3,5	3,6	3,8	4,0	4,0	4,5
IT	GR	PT	MT	IE	CY	HR	LU	ES	FI
5,4	4,6	5,1	5,4	5,7	5,7	5,8	6,0	6,3	7,7
LI	DE	UK	AT	NO	BE	FR	NL	DK	
11,7	9,8	10,8	10,8	11,8	12,3	12,5	15,2	15,3	

Source: EIOPA

* Data of Sweden and Slovenia is missing, the Danish data is from2014

The ratios above are always smaller than the average materialized term: the bigger the ratio, the bigger the difference between the two. It is distinct, that – except in Iceland (where probably there is some oddity in the life insurance market) – in the beginning of the list there are only ex-communist countries, in turn at the end of it the most developed counties of Europe.

Life insurance policies with indefinite term usually end with death or surrender. In certain types the expected ending event is rather death (e.g. in whole life insurance policies), in other types it is rather surrender (basically in unit linked policies in which the term is not marked).

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[55] At least nowadays internationally this is the most prevalent solution. But previously in Hungary the start of the insurance was adjusted to the beginning of the calendar year (or to the end), because in case of manual data processing this was the most practical solution. It could be thinkable also to adjust to calendar year, but in Hungary this practice was used in case of mandatory third party liability insurance, not in the field of life insurance.

[56] The generalisation of the insurance year is an insurance period, which is usually defined by the time-period between two premium payments, in other words the period covered by the premium. The most common periods are insurance month, quarter, half-year and year.

The last important term is the **sum insured**. Since life insurance is sum insurance (it principally cannot be anything else), this way declaring it in the policy is essential. A life insurance policy can have more than one marked sum insured (although most of them have only one) depending on how many insured events are allowed. According to this, we can speak of term, pure endowment (and accidental death, accidental disability, etc…) sums insured. These might be the same, but might also be different.

4.3. Introducing the Most Important Life Insurances

In the following sections – without any kind of logical order – we introduce the most common life insurance products (plans) and how they are used (function).

4.3.1. Term Insurance

We have already seen the definition of term insurance above. Based on this it seems that the functioning of the insurance means that the money collected by the risk community will go to only a few, to those (or persons connected to those) who die during the insurance term. Because of this, it is possible to receive relatively high levels of benefits with low levels of contribution, but this has the price, namely that in case of living through the term nothing is received (since the collected premium has been distributed to the beneficiaries related to the deceased).

When using a term insurance, the most important thing to consider is that a death in the life of the family – if it is the death of a wage-earner – causes great difficulties, or it may even lead to total bankruptcy. The more the life of the family depended on the wage-earner, the greater the bankruptcy.

Example: A 30 year old woman raises two children on her own, and at the same time is building a house. She thinks that if everything goes well, the building operations will be finished in 5 years. If, on the other hand something happened to her, a term insurance sum would save her children from bankruptcy. This way she takes out a 5 year term insurance policy.

Every family can be regarded as an enterprise. A distant example is: a small factory works with two high-capacity machines on 100% utilization, with constant over-ordering and constantly renewed loans necessary for further development. If one of the machines suddenly has a break-down, this small enterprise can be auctioned. (Not to mention the stress that this constantly threatening possibility causes.) This way in all such enterprises the machines are insured against such “outage”. In the family the wage-earners are such “high-capacity machines”, and the term insurance corresponds to the outage insurance.

It logically follows that from this analogy we immediately move on to the area of enterprises. Here the entrepreneur himself is a “machine” of even higher capacity, so losing him would cause an even greater problem to the family.

During the normal operation of the enterprise, death (due to physical wear and tear) or accident can happen to anyone. Enterprises are legally responsible for their employees, and the management of many enterprises feel that it has to provide for the family of its employees, that is, the enterprise has to compensate the negative financial consequences of such an event. The government often helps it by allowing enterprises to account the premium of the term insurance policy he pays in favour of the employee as expense (this is the situation also in Hungary). According to the above thoughts we recite a few concrete situations when it is useful to take out a term insurance:

- Term insurance is the “cheapest”[57] insurance in the sense that the benefit received is greatest compared to the premium paid. This way this type of insurance can especially be recommended to people who at present are not in a financial situation to have savings of greater volume. These can be for example young householders, who are currently trying to build the bases of their living (building a house, starting a business, etc…). They don’t have much money that could be saved up, but are afraid that their family could be deprived of a promising possibility of financial prosperity due to their sudden death.
- In relation to the above example we can also mention using term insurance as a credit life or loan insurance. If the guarantee of repaying a loan on an enterprise or simply on building a house is the entrepreneur or the householder himself, his family is in a difficult situation if he dies. This situation should be parried by a loan cover, or credit life insurance.

Some insurers provide the option to the client of taking out a risk insurance rider or riders with shorter term or terms beside a term insurance main policy. This option might be useful in both of the above two cases. This way the policyholder can achieve a higher sum insured in the first part of the term. In the first example this could be useful, because if he dies earlier, raising his children requires more time and also more money than if he dies later. In the second case its reason would be that the capital of loan to be

[57] To use the phrase „cheap” in a casual, “sloppy” meaning. Actually this is a serious mistake, because it is supposed implicitly, that insurance premium = the price of insurance, but this is not the case – see Banyár-Vékás [2016].

repaid decreases in time, and this way later on a smaller sum insured provides sufficient cover.[58]

A further – more general – area of use of this plan is to level out earnings inequalities within the family. In the consuming structure of a family husband and wife both spend about half of their total joint income. So, if the income of the wife is higher, she consumes less than her actual income, and her husband consumes more than his actual income. In case of such asymmetric earnings the loss of the partner is particularly threatening for the party with lower income. Here the life insurance on the partner with higher income serves to evade the financial consequences of his death.

4.3.2. Pure Endowment Insurance

In case of the **pure endowment insurance** the insurer undertakes the liability – against the premium payment of the policyholder – of paying a sum specified in advance (*sum insured*) to the insured (or to a beneficiary specified in advance) if the insured is alive at the end of a certain period (*insurance term*). If the insured dies before the end of the term, the insurance policy ends without benefit payment.

If we think about it, this design works in a way, that the sum gradually accumulated over a long period of time by the risk community is distributed exclusively among the survivors. In term insurance the risk is early death (and consequently leaving dependents who need to be cared for), against which the insurance gave protection. Here, on the other hand, it is life that means a “threat”, that someone lives through the period in which he can live on his own earnings. This is not a threat to those, who don’t live for a too long time

The pure endowment insurance is an important theoretical design, and is a building stone of many other insurances, but it practically doesn’t exist on its own (although after thorough research undoubtedly many would find concrete examples of it throughout the world – myself know only a Duch practical example, where the informal name of this policy is „bachelor insurance”). We can roughly imagine why this is so, even based on the above logic of the pure endowment insurance. Anyway, one of the most important reasons is that – as we will see later on – pure endowment insurance cannot be surrendered, that is, if someone cannot go on with premium payment any more, he will lose a relatively large amount of money.[59]

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[58] Such a role of the rider makes it clear the deficiency of the term insurance from the angle of the user, since almost all the concrete examples show, that the demand for insurance protection, and the degree of this protection gradually decreasing by passing the time and not ceases suddenly. Actually it would demand for such a term insurances, where the benefit is an annuity for the remaining term, that is the sum assured is changing, the present value of the annuity for the remaining term. In fact this would be a conditional annuity insurance.

[59] In an even more taut situation: the insured will be death-sick and the accumulated reserve would be enough for his/her therapy, but the insurer has to refuse the surrender claim. There would be few newspaper editor who would not put on the front page the complaint’s letter of the insured against the practice of the insurer.

This defect is corrected by the design of the so called pure endowment insurance with premium return guarantee (in other words this has a surrender value), which has a death benefit also beside the living benefit, which is the value of the premiums paid so far, calculated in a certain way. Because of its name it is often regarded as a pure endowment insurance, although it would be more correct to account it as an endowment insurance with a special death sum insured.

In case of the pure endowment insurance with premium return guarantee we are talking about the calculated value of premiums paid, because when determining the death benefit, the premium additives or loadings applied upon premium payment are not taken into account, but the effect of inflation handling is calculated.

4.3.3. Endowment Insurance

Instead of the pure endowment insurance, in practice insurers recommend to clients the combination of a term insurance and a pure endowment insurance, the so called **endowment insurance**. Technically the endowment insurance is simply the sum of these two plans. Since in this design the insurer pays in all events, it is accepted by clients a lot easier than a pure endowment insurance.

Some people by life insurance because they are afraid of death, others because they are afraid of living. The two goals naturally require a different kind of insurance. The endowment insurance unites in itself the two life insurance types that satisfy these goals, i.e. the term and the pure endowment insurance. The risk part of the endowment insurance serves the same purposes as a term insurance, basically providing for those who are left behind. The pure endowment part on the other hand aims mostly at the insured providing for himself.

Naturally the pure endowment part can also serve as providing for others. E.g. the insured wants to create the starting basis of the living of a child through an endowment insurance. This way the term insurance part provides for a child in case of death, and the pure endowment part has he same purpose in case the parent is alive at the end of the term. In these cases it is thoughtful to choose a term such that the maturity date falls together with a special age of the child (18 years, when he graduates from high-school, or 23, when he receives a diploma, etc…).

We have to add, that the endowment insurance is not as good a design as it appears at first sight. The term of the two main goals (providing for others and providing for myself) is generally not the same, this way the exact purpose of the insurance in a particular case can hardly be defined. Probably in Hungary, where endowment insurance was the most popular traditional life insurance[60] this popularity rather has shown the

[60] Until the beginning of the 2000s this was the most popular life insurance, but 10 years later the new contracts decreased to a minimal level, it was crowded out by the modern UL insurance.

under-developed state if the insurance culture, since it is not so much the insurance benefit, i.e. a conscious foresight that has sold this type of policy, but it is rather used as a kind of general reserve.

The primary form of an endowment insurance is the design, when the death sum insured is equal to the maturity sum insured. In a more general sense we can accept as endowment insurance every life insurance that has both a death sum insured and a maturity sum insured, even if these two are not the same. E.g. in the beginning of the 90s in Hungary several companies experimented with plans where the death sum was half or double of the maturity sum insured. (They never really became popular.) Some companies have built more than one maturity into the design (these are so-called "stepped" designs) – e.g. the insurance with a term of 20 years has a partial maturity and a corresponding partial maturity sum insured at 10 and at 15 years. As we have already mentioned, the pure endowment insurance with premium return guarantee can also be regarded as an endowment insurance rather than a pure endowment insurance, and from the aspect of surrender it also works like an endowment insurance. Finally the term fix life insurance that will be discussed later on can also be regarded as a special endowment insurance (although not from the angle of the cash-flow towards beneficiary), similarly to a pure endowment insurance with premium return guarantee, with a varying death sum insured.

From the side of product design we have to add that the (primary) endowment insurance can not only be handled as a term + pure endowment insurance, but also as a simple "savings account" complemented by a term insurance with varying death sum insured. This varying death sum is in every moment such, that it could supplement the current account value to a certain sum defined in advance (the sum insured).

4.3.4. Whole Life Insurance

Since the possible age of humans is not infinite, if we start to stretch the insurance term of the above definitions, the term insurance and the endowment insurance "meet". We can view this as a new type of insurance, the so-called whole life insurance. Whole life insurance differs from others in insurance term, which is so long, that the remaining life of the insured fits into it, this way in any case it will end with the insured's death, and consequently with benefit payment. But generally the premium term is limited e.g. until the insured reaches age 85, and after this the policy is in effect without premium payment.

Since the term of such an insurance is very long, the question, whether it will surely be longer than a certain period, e.g. 10 years is meaningless. Yes, it is longer, since the term is not the same as the period in which the insured is alive. If it were, a policy of 10 years would have to be subsequently redefined to 5 years if the insured would die at the end of the 5th year. But we never do this, so the term of the whole life insurance cannot be identified with the period until the insured dies, only with a longer, but not necessarily precisely declared period.[61]

The features of a whole life insurance are also between a term and an endowment insurance, e.g. it has a surrender value (see later on – contrary to the regular premium term insurance which generally does not have surrender value -, but the relaitive size of this is smaller than the endowment insurances').

The whole life insurance has special purposes.

- Coverage of funeral expenses (ceremony, shrine, etc…). If it is important to the insured that his funeral should be of appropriately high level, then he can collect the money this requires through such an insurance in small fractions.
- To pay for legacy duty. If the testator doesn't want the inheritors to sell his property in order to be able to pay the legacy duty, then it is useful to take out a whole life policy of a significant sum insured, that will pay exactly when the legacy duty has to be paid.

Relating to this it is important to mention a very favourable feature (in the Hungarian law[62] and order) of life insurance, namely that it is not part of the legacy, this way the beneficiary receives it before the – sometimes very long – legacy procedure[63]. Moreover the life insurance benefit (as most of the insurance benefits) is generally free of duty.

4.3.5. Unit Linked Insurance

Unit linked insurance that has been introduced in the United Kingdom in the 50-s at first has been nothing else but the combination of a traditional term insurance and a few investment funds.[64] The client regularly paid a premium to the insurer, which had two components of fixed size, that the client could also see:

- the premium of the term insurance
- the premium part filling the investment funds

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61 If we still have to determine the term of the whole life insurance, then we could say that it is at least ω-x years, where ω is the statistically still relevant highest possible age, and x the entry age of the insured. We can consider all the term not shorter than this as the term of the whole life insurance.

62 Although probably it is a general feature, not a Hungarian speciality.

63 Of course only if a beneficiary was named in the policy, and it is not stated as "the inheritor of the insured" – in which case naturally the benefit can only be paid after the legacy procedure has been ended, so that the insured knows whom to pay to.

64 The British Unit Linked insurance is practically the same as the American Variable Universal Life Insurance, that has been developed as the generalisation of a whole life insurance. In the first step they made the death sum assured variable during the term (Universal Life), and after that they made the investment options also variable during the term (Variable Life).

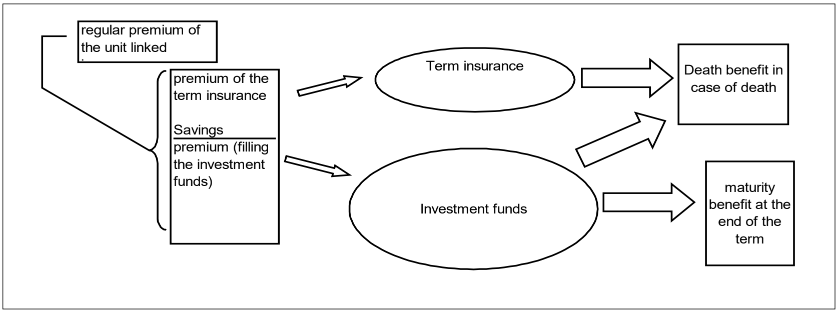

Figure 4.1.: The premium and benefit structure of the first Unit Linked Insurances

The benefit paid in the event of the insured's death had two components:

1. the death sum assured
2. the current value of investment funds

At maturity the term insurance – the usual way – ended without benefit payment, and the beneficiary received the current value of investment funds as insurance benefit.

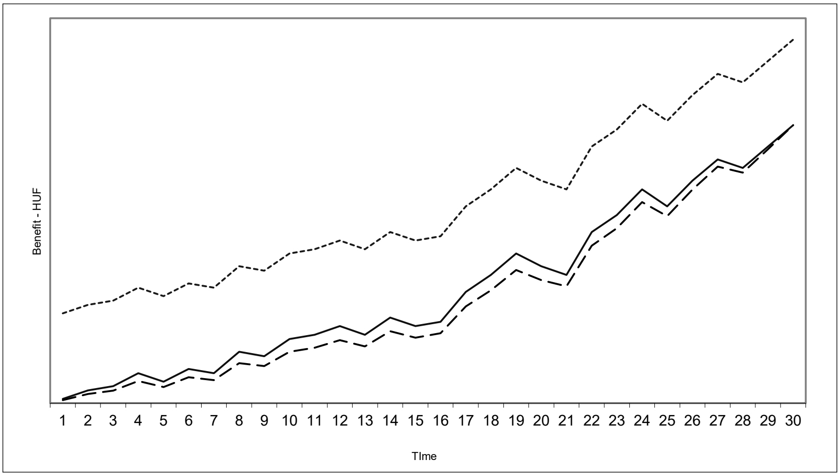

Figure 4.2.: The benefits of the first Unit Linked Insurances (sequentially: death benefit; the sum of the value of the funds; surrender value)

Similar product can also be found nowadays, but the basic construction of unit linked insurance has changed significantly. First of all, regulation separates investment funds and the funds of unit linked insurance. The former is rather referred to as "unit fund" or “asset fund”. Secondly: the total premium currently goes to the asset funds – with the following restriction:

1. a certain part of the premium is subtracted right at payment, to cover expenses of the insurer,
2. certain other types of premiums are also immediately taken – for other types of expenses.

If we disregard these second types of expenses, then the premium and benefit structure of modern unit linked insurance is the following:

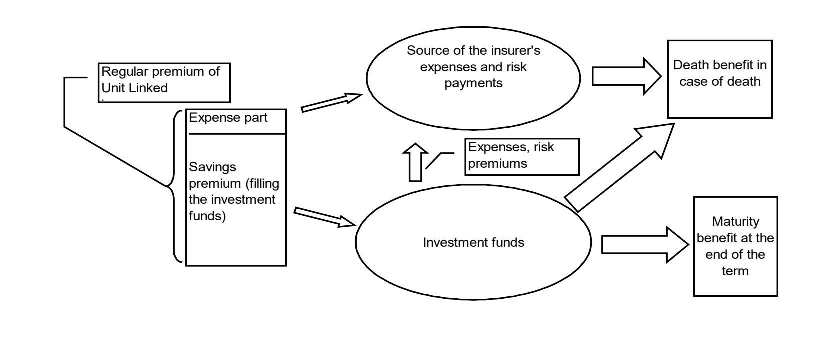

Figure 4.3.: Premium and benefit structure of modern Unit Linked insurance

In the event of death, the benefit received from the insurer is a fixed sum declared in advance, or the current value of unit funds, if it exceeds the death sum insured. At maturity the benefit paid is the current value of investment unit funds once again, as shown by the following figure:

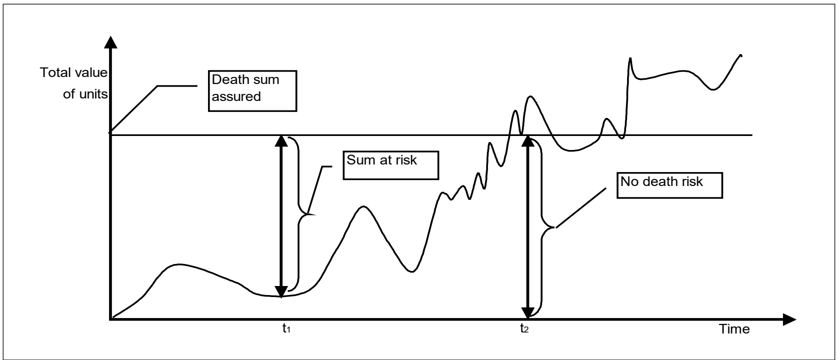

Figure 4.4.: The value of unit funds in a Unit Linked Insurance

In the name of the insurance “unit” means that the clients money is accounted in the asset funds – the same way as in investment funds – in units. Investments are usually evaluated daily, this way the value of units can change daily, which is brought to the inquiring client’s knowledge through the internet, or an automatic telephone line (as it is required by the regulation in force). Units are accounted on the client’s “account”. The units of the different offered asset funds are accounted within the client’s account on separate sub-accounts. We get the current value of all money in an account by multiplying the number of units with their current price.

The insurance company evaluates units on two prices:

- the offer price
- and the bid price.

Buying and selling are viewed from the insurer’s side, so we can look at it the way that when the policyholder pays the premium, the insurer sells (offers) him units, so this is made on the offer price, and when the insurer pays the benefit, he buys units from the client, so he uses the bid price in this case. The same happens when he subtracts units from the client’s account during the term upon different grounds.

Naturally, the offer price is higher than the bid price, usually by 5-6%. The difference is immediately taken at premium payment by the insurer to cover expenses. On the sub-accounts units are practically accounted on bid price, since after changing the premium to units all accounting is performed only on this price.

The expenses and profits of the insurer have 4 sources in the unit linked insurance:

1. the abovementioned bid-offer spread
2. subtraction of certain types of units
3. regular subtraction of units from the fund
4. fund management fee

Two types of units are distinguished:

1. accumulation („ordinary”) units
2. initial units

What has been said so far regarding units concerns mostly the accumulation units. The insurer uses the initial units technique to cover initial (mainly) acquisition expenses. The essence of this is that part of the premiums of the first (or the first two) years (e.g. the part of the first year’s premium not exceeding 100,000 Forints) are marked, they are not turned into accumulation units, but to initial units. Formally these initial units work the same way as the accumulation units, but with one significant difference: A certain percentage (generally 5%) is subtracted during a certain period (usually 10 years) at the beginning of each year. After a definite period the remaining initial units are converted to accumulation units.

The continuous subtraction of initial units is only a way of dressing, since the initial units to be subtracted during the whole term are in reality subtracted at the payment of the premium, so they are changed to units only formally.[65] We can easily calculate how much needs to be subtracted (supposing 10 years and 5% yearly): $1\text{-}(1\text{-}0{,}05)^{10} = 0{,}4013$, i.e. 40,13% of the initial units. The surrender value of the insurance is determined in such a way that it only contains the initial units remaining until the end of the term.

The above 4 expense sources cover characteristically different expenses:

1. primarily renewal commission,
2. primarily acquisition commission,
3. continuous administration expenses,
4. fund management and the insurer's profit.

The insurer usually subtracts for covering administration expenses a monthly fixed portion, determined in a Ft sum from the policyholder's funds, so that he mobilises as many units as are necessary on the current bid price to cover administration expenses. The insurer generally increases this fixed expense part yearly by the inflation rate.

The premium of the death risk and the possible rider risks is usually collected similarly to the administration expenses, by subtracting the adequate number of units monthly. The premium of insurance riders is typically fixed, but the premium of the death risk varies depending on the total value of all units in the asset funds. If this exceeds the death sum insured, then the insurer doesn't have any death risk in that particular month, this way it doesn't collect a separate death premium. If, on the other hand, the death sum is greater, then the difference, the sum at risk, is the risk of the insurer, and a premium is collected to cover it. This premium part is determined simply by multiplying the sum at risk with a multiple corresponding to the age and gender of the insured (which is basically derived from q_x)[66], and subtracts the number of units from the client's account having the value of this Ft sum.

An important feature of unit linked insurance is flexibility. Both the premium, the death sum and the sum insured of riders can be relatively freely modified during the term. But if this is not the consequence of a regular, inflation-handling indexation on policy anniversaries, then – in case of a larger sum insured – it requires a new underwriting procedure.

Flexibility can be detected in other forms, too. Unlike traditional insurance, unit linked can handle top-up premium payments above the regular premium payment of the client. Since the insurer changes premiums to units when those arrive, this way the insurer can easily handle the situation if the client is not paying premiums accurately when due, or maybe leaves a few premiums out. The exact declaration of maturity is also not so important, it can be smudged together with the surrender of the policy.

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65 Because of this it is questionable the raison d'étre of the whole technique, since its aim – implicitly – the delusion of the client. It is presented to him/her as if his/her money was still on his/her account, while the situation already totally different. That is why the prohibition of this technique can be raised on the basis of consumer protection considerations.

66 See the subsection of 12.7.!

A policy paid up (that is the premium payment of a regular premium policy is suspended and simultaneously its sum assured is detracted – for more details see subsection 11.4!) practically doesn't require any special procedure, and surrender can have many directions. It might be partial surrender (simply withdrawing some money from the investment funds) or a regular, annuity-like withdrawal of money.

The insurer offers several kinds of asset funds to clients, and they can freely choose which ones and in what ratio the insurer should place their money in. The asset funds can be bond funds (Hungarian bonds, international bonds, government or corporate bonds), equity funds (specialised to a country, to an industry or an index etc.), Real Estate or other funds.

The client decides whether to place regular premiums only in one fund, or to divide them in a certain ratio between all or some of these. The dividing ratio can be changed at any time concerning future premiums, and can apply a separate, one-time dividing ratio to the occasional top-up payments.

He can also regroup existing units into other funds – but the insurer usually charges a separate changing fee for this.

By choosing funds, the client gets – compared to traditional insurance – a greater degree of freedom, but consequently he also has to take over the investment risk from the insurer. But if he follows an aggressive investment strategy (if he chooses stock funds with higher risk), then in the long term he can achieve higher returns than on a traditional insurance.

However many clients dislike the high investment risk, that is why from the '90s, first in USA, and later in Europa (lagging behind by 10-15 years) has started to gradually spread the investment guaranties linked to UL insurances. This kind of guaranteed UL insurances in some markets (e.g. in USA) almost have crowded out the non-guaranteed versions. The guarantee is different from the guaranteed yield at the traditional life insurances, and it is produced a different way as well. The guarantee is produced by derivatives, for which the client pays by a part of the yield (it is generally quite high, annual 1-2% of the reserve). The name of these new UL insurances is quite misleading: „variable annuity".[67] The term „annuity" got into the name (practically instead of the term "universal life"), because these kind of insurances contains an "annuability option", that is a promise for the client. According to this promise the client can switch

[67] The name „variable annuity" would not be misleading if it would have been applied to an other product: to such kind of life annuity, of which benefit would have been defined by units, so it its current benefit would depend on the current value of the units. To make the confusion total the term is also used sometimes in this sense, but it seems, that this kind of product not really widespread (yet?!). The „Variable Annuity Model Regulation" (http://www.naic.org/store/free/MDL-250.pdf), issued by the National Assotiation of Insurance Commissioners (NAIC) in 2007 contains, that „A variable annuity providing benefits payable in variable amounts delivered …".

the benefit of the insurance at expiry. This is not especially important feature of this kind of product, partly because only a tiny fraction of the clients use it, and partly because that the client would have been able to buy a life annuity without this option.

However this issue calls the attention to the fact, that a „variable annuity" can contain other options too, mainly the followings (EIOPA [2011]):

- **GMWB** (guaranteed minimum withdrawal benefits): deferred or immediate, temporary or lifelong income stream even if the account value has fallen to zero; this is practically a kind of guaranteed surrender value;
- **GMAB** (guaranteed minimum accumulation benefit): Minimum guaranteed capital after a predefined period; (for example after 5 years at least the 100% of the premium paid, or 120% ater 10 years, etc. – this is practically the abovementioned investment guarantee);
- **GMIB** (guaranteed minimum income benefit): Minimum guaranteed lifetime or term annuity starting at a predefined age on a defined benefit base; practically this is the origin of its name;
- **GMDB** (guaranteed minimum death benefit): Minimum benefit in case of death. (This is not different from the death benefit discussed at the modern UL policies.)

An important feature of unit linked compared with traditional insurance is that the expense structure of the former is transparent to clients (with the exception of the expenses hidden in initial units). The goal of unit linked is in the long run the same as that of the traditional insurance, only here the savings motif is especially emphasized. Almost all traditional insurances can be "simulated" with unit linked insurance. For Example:

- Unit linked itself can be viewed as a generalised endowment, or whole life insurance.
- If the death sum is chosen high enough, so that at the end of the term the client's account is practically emptied, then we get a traditional term insurance.
- If we complement the unit linked insurance with a conditional annuity, that in the event of the insured's death pays an annuity equal to the regular premium to the client's account until the end of the term, we practically get a term fix insurance.

4.3.6. Term Fix Insurance (à terme fix)

Term fix insurance (in Hungary also referred to as à terme fix after the French name) is in Hungary the most popular type of life insurance after the endowment and the unit linked insurance. In this case the insurer undertakes the liability – against the premium payment of the policyholder – of paying a sum specified in advance at the end of the premium term under all circumstances (and not conditionally, as in a pure endowment insurance) to the beneficiary (or if the beneficiary dies during the term, the insurer pays back the premium reserve or the premiums paid so-far – which is practically a preferential surrender[68]). The period of premium payment is until the end of the term, or the death of the insured, if this happens earlier (from this point on the insurance is paid up).

A typical area of using this insurance is saving in advance for a child, e.g. for a starting life support (starting a business, contribution to housing), or the initial expenses of founding a family (as a trousseau insurance). The insurance is more than a savings account in the way that the child receives the required sum even if the parent – because of an earlier death – cannot save up the total initially planned sum.

If we think about it, this is – technically – really an endowment insurance with a varying death sum, since when the insured dies, the claim has happened from the insurer's point of view, even if he doesn't have to pay at once. But from this point on he doesn't receive premiums to fill the reserve, so the insurer has to fill it up at once to the level that will reach the maturity sum at the end of the term, compounded by the guaranteed (technical) interest rate. This way we can look at the term fix insurance as an endowment insurance with a death sum insured always equal to the value of the maturity benefit discounted to the time of death. To be more precise, we have to add that the term fix insurance would only be an endowment insurance with varying sum insured if in the event of death the benefit would immediately be paid. Since it is not, we can suppose that this endowment insurance with varying death sum implicitly contains another policy, according to which the policyholder can change the death benefit to a single premium term fix insurance, that has the same maturity date as the original policy (and its sum assured is the original maturity benefit).

This single premium term fix insurance is rather a theoretical design, since the insured event is missing. But as a theoretical design – similar to the pure endowment insurance – it has an important role in other insurances, e.g. in the case of investment-profit sharing of the term fix insurance itself.

The term fix insurance can not only be regarded as an endowment insurance with varying death sum, but also as a simple savings "account", with a conditional annuity rider. The condition that starts the annuity payment is the death of the insured, and the annuity benefit is that it performs further payments of the savings part of the premium to the "account".

Which one of the above theoretical constructions it's up to us, the two solutions are equivalent (later we will prove it exactly). The insurer does the same in booth cases when the insured dies: it fills up the reserve until the level of a single premium term fix policy.

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[68] The surrender value in this case can be preferential, because it is needless to be afraid of the adverse selection during the surrender, because there is no correlation between the death of the child and the health status of the insured, which is the main cause of the adverse selection during the surrender. See more details at surrender later!

In case of the term fix insurance we have to mention that it is usually signed by parents as insured persons for children as beneficiaries, this way here it has particular significance who the insured parent is. If it is mostly one parent who makes a living for the family, then he has to be named as insured, because it is his death that causes economic difficulties to those left behind. A typical sales mistake is, when instead of the householder of the family it is the parent who is at home who is named as insured, because it is easier this way, or because the insurance is cheaper this way.

4.3.7. Annuities

Annuities have many forms, and often they are also combined with other insurances. Not every form of annuity can be regarded as life insurance, only those that have a beginning, an end and a term depending on the insured's death. There are single premium and regular premium, immediate and deferred, paid in advance or in arrears, temporary or lifetime annuities. We can also make distinctions between single life and joint life annuities.

It is expedient to start the presentation of the *Single Life Annuities* by its most common version, the *single premium, immediate, lifetime annuity paid in advance*. Here the insurer undertakes the liability against the single payment of a greater sum, of paying at the beginning of every (insurance) year (or as this happens in the practice, in monthly instalments), starting from that moment, a certain sum to the insured while he is alive. When the insured dies, the annuity ends.

It can be clearly seen that this insurance in reality is a series of pure endowment insurances, that have the same sum insured, but different terms, the successive ones have a term one month longer than the preceding one.

This type of insurance serves us as a good starting point, since we can easily place the other types in relation to it.

Whether an annuity is paid *in advance, or in arrears* is simply a question of technique. The difference is whether the insurer pays the annuity at the beginning, or at the end of each year. Of course there only is a difference in the first (and – in case of the temporary annuities – the last) year. In case of an annuity paid in advance the insured receives the first instalment right at premium payment, while in case of payment in arrears only a year later. (The insured receives the last instalment in case of lifetime annuities – or at temporary annuities if the insured dies during the term – at the same time in both cases. However, in the case of temporary annuities, – if the insured is still alive at expiry – the outcome is different, the last instalment is arriving one year before the expiry in one of the cases and exactly at the expiry in the other case).

The difference between *immediate and deferred annuities* is that in case of the immediate annuity benefit payment starts in the first year (in advance or arrears), while in case of deferred annuities only a few years later (which is naturally defined at contracting). During those few years the sum of the singe premium works like an interest deposit, an investment, or a single premium pure endowment (or maybe endowment) insurance (the default is the single premium pure endowment insurance).

This way we can also say that the deferred annuity is the combination of an interest deposit or a pure endowment (or maybe endowment) insurance and an immediate annuity.

The difference between *single premium and regular premium* annuities is when the insurer receives the capital providing the base of the annuity. If the client pays in a lump sum, all at once, then we are dealing with a single premium annuity. If it is collected gradually, in instalments, then it is a regular premium annuity. Actually, life annuities a priori could be considered as single premium insrances, because we can separate the accumulation phase from the "decumulation" (annuity) phase. And in the former phase we speak generally not as an annuity, but as an interest deposit or a regular premium insurance, as in the deferred case. This way the regular premium annuity can be regarded as the combination of an interest deposit (or a regular premium pure endowment insurance) and a single premium immediate annuity.

Deferred and regular premium annuities are connected, since premium payment has to end before the annuity starts, this way regular premium payment is only possible during the deferred phase. A special case can be an exception of this, when the policyholder and the insured is not the same person, and the insurer starts to pay the annuity benefit to the insured, while the policyholder gets the allowance of paying the premium in instalments.

The difference between a *lifetime annuity and a temporary annuity* is that the (simple) lifetime annuity is paid certainly until the death of the insured, while the temporary annuity is paid until the death only if the insured dies in a predefinied term. If he/she was alive at the end of this term, the annuity would also cease. The temporary life annuity differs from an annuity certain (which cannot be considered as life insurance), that the latter one is paid until the end of the term, regardless of what happens.

It has to mention the *conditional annuity*, which is paid generally until a date (e.g. until somebody's age of 18), but its set-out is linked to the fulfilment of a condition. This condition is the death of (one of) the insured(s) (e.g. parent, spouse, etc.). The *guarantee period* is also an important concept. The life annuity with guarantee period is practically a combination of a certain and a life annuity, and it means that during the guarantee period the annuity is certainly paid, even after the death of the insured. The guarantee period can be put both, at the beginning and the end of the annuity.

From time to time, certain concrete annuity plans appear in life insurance practice that are named "Pension insurance", "Widow/widower's annuity", "Orphan's annuity". These names can cover several kinds of constructions, or combinations, and often these are joint life annuities.

A pension insurance plan is usually a lifetime annuity or a deferred lifetime annuity with or without a guarantee period. The temporary annuity is decidedly unsuitable for pension insurance, since this would mean that the insured receives the annuity until alive, but only until reaching a certain age, e.g. 80 years. The premium of a temporary annuity is naturally smaller (but it does not necessarily mean that it is also cheaper) than a lifetime annuity, since it generally provides less benefits. This way it can be attractive to those, who think that they won't reach age 80. If this happens all the same, the annuity benefit will end just when this might be the main income of the insured, when he is too old to make a living from his own work.

A typical joint life annuity is the orphan's annuity, which, on the other hand, should only be non-temporary in very special cases (e.g. physically or mentally disabled child). Here mostly the different types of temporary (and conditional) annuities can have a role, since usually the orphan child only needs the orphan's annuity until reaching the working age, or until his/her possible death. Of course many orphan's annuity constructions can be imagined. A possible design is, e.g. a two- or three-person temporary, conditional annuity, where one insured is the child, who is also the beneficiary of the potential annuity, and the other (two) insured(s) is (are) the parent(s). The annuity can be single or regular premium. The term of the annuity is the period until the child reaches a certain age (say, 18 years). The payment of the annuity only starts after the death of the parent (or in the three-person annuity after the death of one of the parents) as a condition, and lasts up to the end of the term, but only until the earlier death of the beneficiary (the child). (Since the probability of death during infancy is very low, the annuity after the parent's death can also be a certain annuity, so the child isn't necessarily insured.) In the regular premium version premiums are paid until the end of the premium term, or the earlier death of the parent. But this regular premium version has problems connected to the premium reserve, that we will mention in relation to the discussion of the negative premium reserve.

The widow/widower's annuity can also have several types. The annuities with guarantee period, that will be discussed later on, can be regarded also as widow/widower's annuity, but the widow/widower's annuity is most often a two-person joint life annuity.

These joint life widow/widower's annuities can be divided into two large groups. Using my own terms we can say that there are "symmetric" and "asymmetric" annuities. In case of the symmetric annuities either one of the insured persons can be regarded as widow/widower, so we do not declare beforehand who the widow/widower will be, but simply say that it is the survivor. So, here there will always be a survivor, with the low-probability exception of the two deaths occurring at the same time.

In case of the asymmetric annuities it is not certain, that there will be a survivor, because we declare beforehand that the widow/widower can only be the insured that we declare in advance, and only if this insured lives out the other insured (which is of course evident).

Let's look at a few possible widow/widower's annuity designs! Within the symmetric annuity type a version is possible in which case after the death of only one insured a yearly annuity of 1 Ft is paid to the other insured as a beneficiary, until his/her death.

An example of an asymmetric annuity is:

Let there be a primary insured (whose death makes the other insured a widow/widower) and a secondary insured (the potential widow/widower). The construction is single premium. After the death of the primary insured the secondary insured receives an annuity of 1 Ft until his/her death, if he/she is alive when the primary insured dies. If the secondary insured dies before the primary insured, the insurance ends without benefit payment.

Naturally, a number of other pension-, widow/widower's- and orphan's constructions can be imagined beside the above.

4.3.8. Pension Insurance

Pension insurance is usually mentioned separately, although logically it belongs to life insurance. The name "pension insurance" typically refers to the goal of the life insurance policy rather than a special type of product design.[69] This goal can be achieved through many kinds of plans. Basically two types of plans are used as pension insurance, and their function is different:

- savings type (capital accumulating) life insurance
- annuity insurance

According to the logic of pension insurance everyone saves during the active part of the life cycle the required level of capital, that he changes to annuity at retirement, and uses up by the end of his life. Naturally these two insurance types can be combined into one plan. Then the term of the combined insurance can be divided into two parts: a capital accumulating phase and an annuity payment phase.

During the capital accumulating phase the savings type insurance might be an endowment insurance, a pure endowment insurance with premium return guarantee, a unit linked insurance, or a simple "deposit account" type saving (although this latter one – on the basis of quite formal reasons – it is not considered as a life insurance, unless it is consist of a minimal size death cover (a "lig-leaf"), which is "converting" it into a life insurance. This is also the logic of the definition of the pension insurance of the Hungarian personal income tax act.[70]). Naturally capital accumulation can be

[69] Though in Hungary the term "pension insurance" was defined from the beginning of 2014 by the personal income tax act, and only the type 1 fits into this definition. The reason why the income tax act has definied the term is that the insurers have fought it out – after a long lobbiing – a tax benefit for some kind of life insurance policies.

[70] About the Hungarian pension insurance see . Banyár et al. [2014].

performed outside of the insurance sector, and the capital saved up this way can also be changed to an annuity.

During the annuity payment phase a single premium annuity starts to function. This phase usually begins with retirement.

Pension insurance generally receives outstanding government attention because of its outstanding social significance. This outstanding attention can have different forms in different countries. Tax allowance is generally used, the definition of a special pension product (“account”, or investment fund) is also frequent, or even the creation of a separate institution system (e.g. pension fund). Where government regulation has created a separate institution system for pension insurance, it seems that this is a separate sector, but, in reality, pension is an organic part life insurance (although not necessarily private life insurance).

4.3.9. Complementary Risks – Insurance Riders

Life insurance is very often sold with coverage provided also for the following risks:

- accidental death,
- accidental disability,
- certain “critical illnesses” or “dread diseases”,
- disability,
- surgery,
- hospitalisation.

The death of the householder always causes financial difficulties. These difficulties can appear in a cumulative way if the death was unexpected, due to an accident. This way in these cases the insurer offers a supplemental cover beside the “normal” death sum insured, if the cause of death was accident. **Accidental death** usually means a sudden, outside effect[71] independent of the insured’s intention, that causes the insured to die within a year. The accidental death sum insured is unusually 1-2-3-times the “normal” death sum insured, with the restriction that the insured sets an upper limit on this sum.

Accidental disability insurance is usually sold independently and also as a rider to life insurance. In insurance companies offering independent accidental death coverage, the accidental death risk is not included in accidental disability, while the insurer that doesn’t sell accidental death as independent coverage, includes this risk in the accidental disability coverage. If accidental death and disability are separate insurances, the insurer has to pay attention to the following rules:

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[71] Usually not including death caused by overstrain from lifting, sprain, freezing, sunstroke or heat-stroke. These are border-line cases from the angle of intended-unintended, and that is why they potentially make it possible to abuse the policy, so the insurers prefers to exclude them explicitly making the situation unambiguous.

- Accidental disability cannot be offered without an accidental death coverage (but it can be offered the other way around).[72]
- The accidental death sum insured cannot be lower than the benefit payable in case of 100% accidental disability (it would not be too easy to sew the insured for part of the sum paid for 100% disability if he dies within a year after the accident).
- The covered term of accidental death cannot be shorter than the term of the accidental disability cover, and the accidental death cover cannot be ended sooner than the accidental disability cover.

The definition of accidental disability is quite similar to that of the accidental death: accidental disability means that the insured suffers severe and permanent deterioration of health caused by a sudden, outside effect independent from the insured's intention, within one year. (sunstroke, etc. is usually excluded from the definition here also.)

The level of permanent health deterioration is determined by the insurer's doctor. The "table of injuries" or – after the common German terminology – "gliedertaxe", that is part of the insurance terms and conditions helps him in this. This enumerates the most common losses of body parts and functions. Individual insurance companies might use different percentages, but they usually work with very similar tables. The table of injuries measures a kind of general health deterioration. It doesn't take into account that the body-parts, capabilities (e.g. because of his occupation) of the given insured might be more important than generally for most of the people (e.g. the fingers for a surgeon, a piano player). If someone requires coverage for such special risks, he has to take out an individual accident insurance.

[72] It is interesting that when I teach this rule to people actively practicing in the insurance business, there are always a few who argue on this rule, saying that the accidental disability benefit is payable while the insured is alive, and has the function of facilitating the insured's further life, while the accidental death benefit has a principally different function, as it goes to the dependents. It can easily be imagined that someone doesn't have dependants to provide for, so it is unnecessary for him to purchase the accidental death coverage. This is an acceptable train of thought, but it doesn't count with the fact that after an accident the formation of the final level of disability or death is a gradual process. According to this, there might be several benefit payments (because of the disability becoming more severe), and it is not too purposeful to sue for repayment. The problem doesn't have a clear solution, but it is wiser to take the points of the insurer into account here.

An example of the gliedertaxe[73]:

Injuries of body-parts	Degree of disability
Total loss or loss of function of one of the upper limbs from the shoulder joint	70%
Total loss or loss of function of one of the upper limbs above the elbow joint	65%
Total loss or loss of function of one of the upper limbs below the elbow joint or the total loss or loss of function of a hand	60%
Total loss or loss of function of a thumb	20%
Total loss or loss of function of an index finger	10%
Total loss or loss of function of any other finger	5%
Total loss or loss of function of one of the lower limbs above the middle of the thigh	70%
Total loss or loss of function of one of the lower limbs up to the middle of the thigh	70%
Total loss or loss of function of one of the upper limbs below the elbow joint or the total loss or loss of function of a hand	50%
Total loss or loss of function of a foot at the level of the ankle	30%
Total loss or loss of function of a big toe	5%
Total loss or loss of function of any other toe	2%
Loss of the sight of both eyes	100%
Loss of the sight of one eye	35%
Loss of the sight of one eye if the insured has lost the sight of the other eye previously to the insured event	65%
Total loss of ability to talk	60%
Total loss of ability to smell	10%
Total loss of ability to taste	5%

Table 4.1.: Table of accidental injuries

Under the term health insurance we usually mean accident or sickness insurance. It primarily has insurance technical causes that the uniform health insurance is divided into two categories. Accident insurance – as we have seen – is under all circumstance an insurable risk. This cannot be stated of most of sicknesses, the auto-selection and moral

[73] This was used by the former ABN-AMRO Insurance Company.

risk is too strong. But this is not true for some sicknesses, that – we can say – behave in an accident-like way. Insurers should offer cover against the sicknesses that satisfy all of the following criteria:

- they appear in relatively rare instances,
- people know them well and are afraid of them,
- they would do anything to avoid them,
- if they happen, they cause significant financial consequences.

These illnesses are called by an overall term "**Dread Diseases**". (Instead of this name the term "**critical illness**" is becoming more frequent nowadays.)

Dread diseases are defined differently from insurer to insurer, but the following usually belong to the insured circle everywhere:

- heart attack,
- stroke,
- cancer,
- the need for artery-bypass operation,
- kidney failure.

With such a cover, if one of the sicknesses in the policy terms and conditions is diagnosed in the insured, then the insurer pays a sum (which serves the purpose that the insured can finance the treatment of the given illness), independent from the other benefits, or the benefit payment (or a part of it) of the life insurance – that was the main policy of the dread disease rider – is brought forward.

Severe disability is a case, when the insured (if he is at the same time the policyholder) cannot necessarily continue the premium payment of the life insurance, while he still needs the coverage it provides. In case of traditional insurances the **disability waiver of premium** cover or rider solves this problem. According to this insurance, if the insured becomes severely disabled during the (premium) term of the main policy, then the risk community takes over further premium payments (so the main policy becomes paid up for him). The paid up term is usually the remaining term of the main policy (or, naturally the period until the earlier possible death of the insured), but sometimes the insurer declares that if the state of the insured should get better, the premium payment might be restored.

Disability is usually considered severe by the policy terms and conditions if it is 67% caused by accident, or 100% caused by illness. In Hungary there is no coverage offered for disability of lower degree.[74] The social security distinguishes two types of 100%

[74] At the beginning of the '90s 67% disability caused by illness (in other words – using the categories was used then – category III. disability of the social security system – the first two denoted the two degree of the 100% disability) was also part of the coverage, but it has been left out due to the numerous cases of frauds. 67% disability in Hungary seems to be too subjective and can be manipulated too much.

disabilities caused by illness, and this practice is generally taken over by the private insurers, moreover they tie the benefit payment to the declaration of the disability by the social security institute. The difference between these two types is that while the lower degree fokú 100% disabled is able to take basic care of himself, the upper degre 100% disabled needs care in the everyday life.

Premium payment in modern insurances (primarily the unit linked insurance) is not as well defined as in case of traditional insurances, this way the benefit of the disability waiver of premium also cannot be well defined. This problem can be solved if we realize that the disability waiver of premium is implicitly an annuity insurance, namely a conditional annuity that begins with the disability and has an annuity payment equal to the premium of the life insurance. This implicit annuity can be made explicit and then we get a **disability annuity rider** that is practically the same as the disability waiver of premium rider. This can serve primarily in case of unit linked the same function (naturally it can be taken out as a rider of traditional insurance, also) as the disability waiver of premium rider by traditional insurances, but it can also be made an independent benefit. But insurers don't really like this, because they are still afraid of the disability risk "tamed" this way.

Beside the disability annuity insurance the most common sickness insurance ride in Hungary is:

- surgery benefit and
- hospitalisation daily allowance rider.

In case of the surgery benefit the insurer groups possible surgeries into categories of "severity" (generally 5 categories), and if a surgery is performed on the insured, then a part of the sum insured of the surgery benefit rider is paid as a lump sum payment that corresponds to the category of severity of the surgery performed (e.g. 20% of the sum insured in case of category 1 and 100% of the sum insured in case of category 5). The goal of the insurance is to cover expenses arising in relation to the surgery.

If the insured is hospitalised, this also has costs, and are mainly proportional to the number of days spent in the hospital (loss of income, supplementary expenses, "gratitude money"). The hospitalisation insurance rider satisfies the needs arising due to these, which has a benefit of a certain daily allowance (e.g. 5,000 Forints a day). This is usually not paid for short (3-5 days) hospital stays (saying that the financial needs are not as demanding, and the costs of handling the claim would exceed the degree of benefit payment), only if it is longer than this lower limit – usually with an upper limit also applied (e.g. 60, 180 or 365 days).

5. CATEGORISATION OF LIFE INSURANCE

KEY WORDS

Main policy
Group insurance
Individual insurance
Single premium
Elemental life insurance
Main policy
Traditional life insurance
Long term care insurance
Insurance rider
Modern life insurance
Regular premium payment
Reduced premium payment
Multiple life insurance

5.1. Usual Classification of Life Insurances

The study of life insurance is an applied science. Its subject, life insurance has been created based on practical considerations centuries ago (or thousands of years ago – according to other opinions) and it has developed by practical challenges, not theoretical discoveries. Because of this, the theory also tried to follow and reflect these challenges and did not aim at the axiomatic structure of the "classical" sciences (primarily mathematics and physics) that serve as examples to other sciences. This way – although it would seem as a necessary first step and basic requirement – there is no widely accepted common categorisation of life insurance, but it changes from author to author, is strongly inferior to the later matter, and usually gives an ad-hoc impression. Now let's look at a few examples from the Hungarian and the English literature.

5.1.1. Categorisation of Life Insurance in the Hungarian Literature

In the Hungarian market, considering questions of insurance theory it was Dr. Dezső Csabay in the last decades, who has created the most enduring foundations, and his writings serve as a standard up to our days (although in many respects it would be useful to rethink them). Considering life insurance he (Csabay [1971] 333-366.p.) states the following under the title "The Classification of Life Insurance" (ibid. 351.p.):

"There are several usual classifications of life insurance according to several aspects:

I. According to the conditional or unconditional liability of the insurer. …

II. According to conditions depending on the health status of the insured: normal or abnormal (high risk) life insurance.

III. According to the insurance benefits: capital and annuity insurance.

IV. According to the insured event: term endowment, pure endowment insurance and annuity.

V. According to business management, mostly based on the administration and transaction method of policies:

a) major life (or regular life),

b) minor life (or popular, or industrial, workman)

c) group insurance”

The individual (life) insurance products themselves can be further categorised according to several aspects (ibid. 356.p.):

By the type of premium payment:

1. single premium

2. annual or monthly premium (recurring premium)

a) fixed (level) premium

b) variable premium

As per the number of insured:

1. single life insurance

2. double or multiple life (mutual) insurance

„The most important categorisation of insurance products is categorising based on the *purpose of the insurance* – or the insurance term. Basically all products of this aspect can be derived from two basic products or their different combinations. We distinguish four main types:

1. Term insurance, ….

2. Pure endowment insurance, … annuities belong to this type, …

3. Endowment insurance, which is the combination of the two above …

4. Fix term insurance …” (ibid. 357.p.)

dr. Csabay makes similar statements 10 years later (Csabay [1980], 107.p. life insurance entry-word): „LIFE INSURANCE is the most important type of the personal (sum)insurance branch. It is an insurance where the declared sum assured is to be paid when the insured lives to a fixed date or a fixed age (pure endowment insurance), or upon the death of the insured (term insurance), or in both cases (endowment insurance). Annuities also belong here (recurring, periodically payable pure endowment insurance).”

“The different types of life insurances are usually divided into several groups. These are the major life (or regular) insurances of greater sum assured, and the minor life (or “popular”) insurances of smaller sum assured; in the western world: industrial or workers’ life insurance, short term life insurance (risk insurance) type credit insurance, group insurance, etc.

The insurance types of the individual groups are divided to insurance products (or tariffs) – according to their purpose. The three major products are: term, pure endowment and endowment insurance. All of these exist in a number of forms.”

The few remarks on the next couple of pages are also very interesting and typical, although they cannot be considered totally accurate in an actuarial sense:

"With the exception of the pure risk insurances (term and pure endowment) the net premium of the other life insurance types consists of two main elements: a risk premium and a savings premium. (Premium reserve.)

In the western insurance literature life insurance with a fixed sum assured – in order to differentiate from annuities – is sometimes also called "capital insurance".

A lot of insurance theoreticians and lawyers since the 19th century consider the endowment (and the pure endowment with premium refund) insurance according to its economic purpose not as insurance, but as a savings deposit, because it contains only a minor risk element."

László György Asztalos in 1995– basically agreeing to the above classifications in the chapter "the categorisation of life insurances" (Asztalos [1995], 330.p.), that "An individual life insurance policy can only be categorised, professionally defined by applying at the same time (using a combination of) several, at least the following 6 aspects (groups)." These are:

A. the purpose of the insurance policy,

B. the date(s) related to the insurance policy, or the term,

C. the mode of benefit payment,

D. the number of insured lives,

E. the technique of premium payment,

F. the return of yields.

Variations of the first aspect are:

„The purpose of the insurance is in every case that the insurer takes over the risk caused by the uncertainty of the duration of life. All insurance products can be listed under one of the 4 basic types.

1. In case of the term insurance the sum assured is to be paid in the event of death of the insured, that can occur at any time (Whole-life Policy). If, on the other hand, the insured doesn't die during the term of the policy, the insurance contract expires – without any benefit payments received from the insurer.
2. In case of the pure endowment insurance (Versatile Endowment Policy) the insurer pays only if the insured survives a fixed age (survival age).
3. In the combination of the above two methods, in case of endowment products the insurer pays the benefits if the insured

(a) dies before reaching a specified age, or

(b) reaches the specified age alive.

4. In case of the so called term fix (fix term, "a terme fix") insurance it is not the fact of death or being alive that is important, but (the time of) some other event that may not even happen. At this time the sum assured is paid under all circumstances,

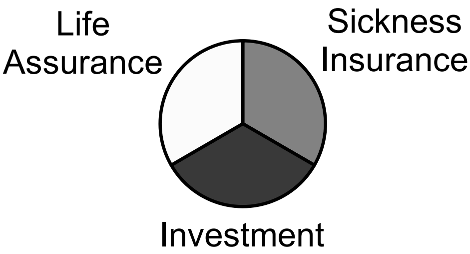
(a) either to the living insured person, or

(b) the beneficiary that the insured has declared in advance (e.g. inheritor)."

Interesting in the above categorisation is that – uniquely in the literature on the subject – it identifies term and whole life insurance, and the author only mentions annuities in the subgroups of aspect C).

5.1.2. Life Insurance Classification in the English[75] Literature

The English literature handles categorisation in a much more practical way even compared to the above discussion. The volume CII (Chartered Insurance Institute), that – in a certain sense – serves as an official curriculum in England, discusses life insurance under the title "Basic types of long-term insurance policy" (Popplewell [1992] 3/2.p.):



"..., as a brief revision and introduction to those policies, you might find it useful to think about the vast range of policies as falling in one of the sections of the pie chart below.

Some of the policies you will encounter will fall easily into one of these sections, being entirely either life insurance, sickness insurance, or investment-based, but others may fall under two or even all three sections.

Consider the following examples, and decide where each of them could most appropriately be placed in that chart.

- term assurance
- endowment assurance
- whole life policies
- permanent health insurance
- annuities."

[75] Unfortunately, I'm not proficient in the German professional literature, but I might not be too far from reality in supposing that the German categorisation has fundamentally influenced Csabay's grouping.

The book gives the following solutions:

- term assurance = pure life insurance,
- endowment assurance = minor part life assurance, major par investment,
- whole life = major part life assurance, minor part investment,
- permanent health insurance = totally health insurance,
- annuity = important type of assurance, but cannot place it in the diagram.

He also remarks that: “Finally, you will probably be aware of a number of other types of contracts not mentioned above, such as “critical illness policies” (being partly sickness insurance, partly savings, and also usually partly life insurance) and “long term care” (sickness insurance).”

The volume „Life Insurance” (Black-Skpper [1994]) of imposing size, that has a history of a century doesn’t deal with the categorisation of life assurance in a comprehensive way. As an introductory remark it says that “The simplest form of life insurance protection is yearly renewable term (YRT) insurance.” (ibid. 24.p.) It gives the following definition of this type of insurance: “Yearly renewable term life insurance provides coverage for a period of one year only, but guarantees the policyowner the right to renew (i.e., continue) the policy even if the insured suffers poor health or otherwise becomes uninsurable. Each year’s premium pays the policy’s share of mortality costs for the year. The renewal premium rate increases each year to reflect the annual rise in death rates as age advances.” After this the author considers the discussion of premium payment types to be most important.

He only returns to the categorisation in a later chapter (“Overview of types of life insurance”) (ibid. 82.p.):

“As suggested in Chapter 2, life insurance policies can be constructed and priced to fit a myriad of benefit and premium-payment patterns. Historically, however, life insurance benefit patterns have fit into one or a combination of three classes:

- Term Life Insurance
- Endowment Insurance
- Whole Life Insurance”

He also mentions that (ibid. 83.p.): “Another class of insurance issued by life insurers is annuities. […] Most annuities are savings instruments designed to first accumulate funds and then systematically to liquidate the funds, usually during one’s retirement years.

The above life insurance classification scheme remains valid today, although it is not always possible to determine the exact class into which some types of policies fall at issuance. As discussed in Chapter 6, some policies permit the policyowner’s flexibility, in effect, to alter the type of insurance during the policy term, thus allowing the policy to be classified as to form only at a particular point. For presentation purposes, these flexible forms of life insurance are discussed as if they were an additional classification, even though all can properly be placed (at a given point in time) into one or a combination of the three traditional classes."

The following discussion is also interesting. Term assurance and endowment assurance appear in the same chapter, as subchapters, but the Whole Life, the "Flexible-premium Life Insurance Policies" (the 6th chapter) and the "Annuity and Special-Purpose Policies and Benefits" received separate main chapters.

Concerning endowment assurance it declares that (ibid. 95-96.p.) "There are two ways of viewing endowment insurance: in terms of (1) the mathematical concept, and (2) the economic concept.

Mathematical Concept. The insurer makes two promises under endowment insurance: (1) to pay the face amount if the insured dies during the endowment period, and (2) to pay the face amount if the insured survives to the end of the endowment period. The first promise is identical with that made under a level term policy for an equivalent amount and period. The second introduces a new concept, the pure endowment. A pure endowment promises to pay the face amount only if the insured is living at the end of a specified period; nothing is paid in case of prior death. Pure endowment insurance is not sold as a separate contract in the United States. It is said that few people are willing to risk the apparent loss of all premiums paid in the event of death before the end of the endowment period. …

Economic Concept. Another analysis of endowment insurance, the economic concept, divides endowment insurance into two parts: decreasing term insurance and increasing savings. The savings part of the contract is available to the policyowner through surrender of or loan against the policy."

Chapter 6. mostly discusses the Universal and the Variable Universal Life type assurances.

It is interesting that Life Insurance, Theory and Practice (Mehr-Gustavson [1987]), that was also published in America a few years earlier, has also lived many publications and is also a thick book – although splits assurances into similar groups – doesn't define the same categories as Black and Skipper. In the chapter "Basic Types of Life Insurance Policies" (Mehr-Gustavson [1987] 51.p.) it writes the following: "Life insurers issue numerous types of life insurance contracts. Many of the policies are special combinations or variations of what are often considered to be the basic forms of life insurance: term, whole life, and universal life." The authors insert here in a footnote that: "Life insurers also write annuities … Some persons like to argue semantically that annuities are the only form of true life insurance as they insure persons against outliving their income. These persons argue that what is called life insurance should be called death insurance as it insures a person against loss caused by death." The very beginning of the chapter emphasizes that two important variations must be remembered: the insurances called variable life and variable universal life.

The authors handle “other” life assurances in a separate chapter (“Product Diversification and Special Purpose Policies”). They find that many types of new insurances have been introduced in the ‘70s and ‘80s that give the customer many new options. This had the effect that „In recent years, the diversification trend has resulted in some blurring of the demarcation lines that traditionally distinguished the various types of financial institutions. Most observers predict that further breakdowns in institutional distinctions will occur as diversification continues.” (ibid. 93.p.)

The first special purpose insurance it discusses is the endowment insurance. „Although once considered to be one of the basic forms of life insurance, endowment coverage has declined drastically in popularity in recent years, partly due to the development of more flexible products, such as universal life. During 1984, less than half of 1 percent of all new ordinary insurance purchased was endowment coverage.” (ibid. 96.p.)

Funnily, although both books have the title “Life Assurance”, both discuss health insurance and different kinds of welfare plans in full detail without considering them conceptually as life assurance.

5.1.3. General Features of the Categorisations

Based on the above, one can make the following statements concerning the usual life assurance categorisations.

1. There is no unambiguous tradition concerning the classification that could be continued or that has to be followed.
2. There is no general aspect that gives a basis to everyone’s categorisation. There probably is no single aspect by which all products can be satisfyingly categorised.
3. The authors do no aim at comprehensiveness, they handle exceptions quite easily.
4. The authors tend to regard the most common products as basic products (e.g. the handling of endowment assurance by English language authors, and also the absence of fix term insurance in their writings).
5. It is uncertain and has always been so what they regard as life insurance and what not, but this also doesn’t really bother the authors.

5.2. Practical Classification of Life Insurance by Different Aspects

In the following we will outline a multi-aspect, practical classification – mostly following Csabay’s track, but also deviating from it at many of points. The aspects taken into account are:

1. the historical order of development,
2. the logic of the internal structure, and the type of benefit,
3. administrative and legal considerations,
4. the relation of the status of the policyholder and the insured,
5. the number of insured persons,
6. the term and frequency of premium payment.

5.2.1. From the Aspect of the Historical Order of Development – Traditional and Modern Life Insurances

There is a classic form of life insurance developed in the 19th century, that has changed very little during the 20th century. But the challenges (e.g. inflation, competition with other financial products) and potentials (primarily information technology – IT) of the 20th century gradually began to undermine the well established and well-thought-out traditional product design and new types of products, characteristically different from the former ones have evolved. Because of this, it is useful today to distinguish the classical type insurances from the new types, so the terms "traditional" and "modern" life insurance are widely used.[76]

The term traditional insurance usually means the products designed by the combination of term insurance and pure endowment insurance, that is: term insurance itself, endowment insurance, fix term, pure endowment with premium refund, whole life and annuities. Their characteristics are fixed technical interest rate, and that all major parameters of different points of the insurance term can be well foreseen.

The modern life insurances differ from the conception of traditional insurances in two respects:

1. product design and
2. insured events.

It was the Unit Linked Insurance that has brought innovation in the product design by integrating the main features of investment funds into life insurance, and by practically eliminating the pure endowment part as a building stone through the uncertain development of the reserve.

From the insured event side it was the newly emerging risks at the boundary line of life insurance and health insurance that widened the field of life insurance (and at the same time smudged the boundary line between the two insurance branches). Some well defined, serious diseases (Dread Diseases or Critical Illnesses) and the need for care have been introduced into life insurance plans as insured events. The new insurances

[76] For example the Hungarian insurance act went into force in 2003 (extinct from 2016) consisted of the term "traditional life insurances" (and it denoted the sector I of the life insurance branch), although it does not consisted of the term "modern life insuranes". The new insurance act which replaced it contains none of them.

corresponding to these events appeared either as insurance riders (mostly Critical Illness), or as independent policies (typically Long Term Care Insurance).

Modern life insurances first appeared in the ‘50s, but their world-wide spreading can be placed in the ‘80s and ‘90s.

5.2.2. By the Logic of the Internal Structure, and the Type of Benefit

Traditional life insurances form a well constructed system, they can be built from certain building blocks – as we have mentioned earlier. They can be built from two basic building blocks, this way we can also call these atomic life insurances. The two atomic life insurances are

- the Term Insurance and
- the Pure Endowment Insurance.

A few brief examples: endowment is the combination of these two, whole life is the border-line case of term Insurance, annuities are basically series of pure endowment insurances, etc.

We can conclude that traditional life insurances are the combinations of atomic insurances. The most common combinations are usually called **basic life insurance**. It changes from country to country what the basic life insurances are. In Hungary it is primarily the:

- endowment,
- fix term,
- term and
- immediate, single premium annuity

that can be considered basic life insurance.

5.2.3. From the Administrative and Legal Aspect – Main Policies and Riders

The insurance companies form “commercial packages” from the above life insurances, and sell these packages. When the policyholder signs an insurance contract, he actually buys such a commercial package. This commercial package can be constructed in two different ways:

1. The insurance company builds into a single design, the so-called “insurance product” one or a few of the above life insurance types, and maybe ads on – also built into the plan – a few non-life (accident or health insurance) elements, too.
2. A policy is built from several optional elements. In this case one element, that has to be a life insurance is dominant, this is what we call the main policy, and the other elements – that can also be non-life elements – are additional, supplementary elements, called riders to the main policy.

On the relation of life- and non-life insurances we have to mention that according to European Union guidelines (that the Hungarian insurance legislation naturally took over) a life insurance company can only sell life insurance, with the exception of accident and sickness insurance, if the life insurance contains these as “supplementary risks”. (In this case these non-life insurances are accounted under the life branch.) Nor the guideline, nor the Hungarian insurance legislation defines precisely just when a risk can be considered supplementary, but one point is certain: a life insurance company cannot sell accident or sickness insurance separately. In Hungary at the time of the insurance monopoly (i.e. until 1986) separate insurance riders didn’t exist, these were added to the set of insurance designs only with the development of market economy. On the other hand, built-in non-life risks have been part of life insurance policies even then, mostly health risks, where the insurer provided a waiver of premium benefit if the insured became disabled[77].

There are two separate types of insurance riders, providing:

1. independent benefits, or
2. options.

The **rider providing independent benefits** has two subtypes, depending on whether the benefit of the rider is:

- independent from the main policy, or
- depends on the benefit of the main policy.

The former subtype can be life, accident, or health insurance. Typical examples are: term assurance rider (life assurance), accidental death or accidental disability insurance rider (accident insurance), and hospitalisation daily allowance insurance rider (sickness insurance).

An example of the later is the waiver of premium disability insurance rider (sickness insurance).

An example of **options** is the premium increase option that allows the policyholder to increase the premium at certain periods without repeated underwriting.

The technical solutions of insurance riders (inflation handling, premium frequency etc.) usually follow that of the main policy, and expire when the main policy expires.

5.2.4. By the Relation of the Status of the Policyholder and the Insured – Individual and Group Insurance

Insurances can also be categorised as individual and group insurances.

Individual life insurance is usually signed by individuals as policyholders with individuals as insured. In case of group insurance, the policyholder either signs the insurance contract for a total collective (e.g. members of an association, employees of

[77] See subchapter 4.3.9.!

a company, etc.) at once, or a prerequisite of becoming member of a certain group (e.g. customers of a bank taking out a loan) is signing the insurance contract.

Sometimes certain standardized individual insurances can be also considered as group one, if it was contracted by many policy holders at once, and, because of this, the insurer gives some discount. But the "real" group insurances are differing from the individual ones also in technical terms. Group insurances are rather similar to the non-life insurances, namely their term is one year with a renewal option. The type of these are almost exclusively term insurance, possibly with some accident or sickness riders. These kind of group insurances are cheaper, which is one of the main reason of their sales. Since in this case the underwriting, and possibly also the administration, becomes simpler and therefore cost-saving, this saving can be the base of a premium discount. On the other hand, the so-called "group calculation" (what was applied in case of CSÉB) is also possible, which means that the members of the group pay a level premium for a unit of benefit, independent of the differences in their own personal risk (e.g. age). In this case premium calculation is based on certain "average" characteristics of the group.

The marketing of group insurance, entering the risk community in a simplified way and offering a level premium is based on the fact that the group was not organised for signing the insurance, but for some other goal, but the members automatically have the insurance. This is the only way that anti-selection – otherwise undoubtedly appearing – can be eliminated.

Regarding group insurance we cannot go by without mentioning one of the most important group insurances of the Hungarian insurance history, the Group Life and Accident Insurance (CSÉB). This group of policies, that was started in the '60s and sold up to the beginning of the '90s was held by more than 4 million Hungarians in its days of glory (the '70s and '80s). It basically was a simple, standardised insurance providing cover (at a low level) for all significant personal insurance risks, that was only called group insurance because of the method of premium calculation and marketing (since in other respects it was an individual type insurance). The premium calculation didn't distinguish insured persons by age and gender, this way it was particularly advantageous for older males to sign the policy and particularly disadvantageous for young females. Since this tendency, if only older males had signed the CSÉB contract would have caused a problem for the insurer, a special marketing technique was used to avoid this situation. It was only sold at places of work. The socialist mammoth companies typical of that period were subdivided among agents, and an agent was allowed to move on to the next company with selling CSÉB only if he had reached a certain level of saturation – projected on the total number of employees – (which meant that he didn't only sell the insurance to older males, by "cherrypicking" the risk community). It is obvious that this policy was the product of the period of insurance monopoly and group insurance with this principle cannot be sold on the competitive market.

Today group insurance is usually short-term, without premium reserve, so in this respect it can be regarded as non-life insurance.

5.2.5. By the Number of Insured Persons – Single Life or Joint Life Insurance

Individual life assurance can be single life or joint life insurance, or there may be one or several insured persons declared in the policy. The default is one insured, this way we made this implicit supposition in all the above cases. On the other hand, joint life insurance policies are also relatively common. This is usually taken out by married couples, which gives an explanation to why this type of policy is so rare in Hungary. Because of the high divorce rate, a major part of these policies would be surrendered, even if the need that it satisfies is still there.

Clearly, a joint life insurance is not the same as two – otherwise similar – single life insurances. The difference lies in the definition of the insured event. In joint life insurances death usually means the first death, or (in certain kinds of annuities) we differentiate benefits according to which insured died. Practically all traditional insurances can be designed as joint life insurances. E.g. the joint life term insurance pays if either insured dies during the insurance term (and the insurance ends with this death, so the possible death of the other insured is irrelevant in regarding the insurance), and if both insured persons are alive at the end of the term, the contract expires without benefit payment, etc.

The need for joint life fix term insurance is quite logical if the family has two wage-earners with similar salary levels. In this situation, if either one falls out, the education and generally starting a career for the children could present a problem.

Another type of multiple life insurance – that is at present unknown in Hungary – is the so-called *co-partner insurance*. This is usually signed by the owners of small enterprises, so that if one of them dies, the legacy can be paid from the insurance benefit, and the enterprise doesn't have to be reorganised or terminated.

5.2.6. By Premium Term and Premium Frequency

Life insurances can also be categorised according to the *frequency* of premium payment. Most often the premium of life insurance is paid at *regular intervals* (monthly, quarterly, semi-annually or annually) throughout the term of the contract. But policies also exist with premium payment term shorter than the total insurance term. In these cases we are dealing with *limited premium term* insurances. One extreme case, usually mentioned separately, is when premium is paid once, at the beginning of the insurance term. This is the so-called *single premium* insurance. The limited premium term insurance behaves mostly like a regular payment insurance during the premium term, but after that – until the end of the insurance term – behaves basically like a single premium insurance.

Modern insurances – mostly the unit linked life insurance – allow premium payment frequencies that are not specified in advance, and allow also so-called single premium top-up payments, additional premium payments supplementary to the regular payment. In case of traditional insurances such additional payments are not allowed.

6. COMPARING MODERN AND TRADITIONAL LIFE INSURANCES

In the following section from the modern group we will only analyse the unit linked insurance in detail, since the Critical Illness and Long Term Care type insurances have not yet appeared, or at least are not really popular in the Hungarian market, and, on the other hand, because apart from extending the concept of insured event – that is, by the way, also important – in other respects they follow traditional insurances quite well.

6.1. Modern Life Insurance in General

The Unit Linked life insurances was the a great novelty of the Hungarian life insurance market in the '90s, that very quickly has gained a leading position on the life insurance market, and has been gradually crowding out the "traditional" life insurance products. These unit linked (or UL) products have significantly changed our views of life insurance and our expectations from regulation (that regulation has not always followed). Unit linked insurances have made a great step toward making life insurance products more transparent to clients, and gave them new kinds of options compared to the former ones, but in return also gave the clients more responsibility in handling their own financial affairs. Summarizing: the unit linked insurance expects a consumer who is more mature than the former one, and, in a certain sense (in the long run), it also "raises" a more mature consumer. However, this supposition often does not meet the reality, making a lot of conflicts, and so these product are also often blamed.[78]

[78] One of the most common complaints, that these products are very expensive, meaning that (theoretically correctly) the cost component of them is quite high. The curiosity of this complaint is, that it is true from many angles if we compare them to complementer products from other financial sectors (e.g. bonds or pension fund), but. it is false if we compare them to the earlier, traditional life insurances. I did the examination of the cost levels in 2013(see Banyár [2013]), and I concluded, that the cost level of the UL insurances is approximately the half of the traditional life insurances, still not the latter one, but the UL insurances are called as "expensive". The probable reason of this, that the costs of the traditional life insurances are much more hidden, and they are not really demonstrable with simple tools, contrary to the UL insurances. Namely the cost transparency is a risky (or if we turn to the angle of the consumers: very useful) thing: it induces per se a pressure to reduce costs.

6.2. Life Insurance Before the Appearance of Unit Linked

Traditional life insurance has a more than 100 year past of settled, refined format and product design, and a well constructed, closed, independent calculation methodology and notation system. The basic traditional life insurance policies were signed even in the 19th century practically (regarding the most important features) in the same form as nowadays. This product design reflected well the age and the environment in which it was created and formulated. The most important characteristics of this environment (from our point of view) were the following:

- the stability of money, low or no inflation, stable (usual) interest rates
- paper based, manual administration
- only a very small, selected part of the upper classes had access to the capital markets
- relatively low life expectancy and therefore large mortality risk – paired with typically high birth rates[79]

These characteristics had a fundamental impact on the construction of long term insurance of that time (that is – with a little uncertainty – almost the same as the presently available life insurances). The most important elements of this product design are the following:

- The incidences related to the life insurance policy starting from its inception until its termination are tied to a fixed, rigid "scenario" allowing only a few variations (that means that the premium term, premium frequency are specified in advance and cannot be changed, the reserve runs on a pre-calculated path, etc).
- The most important parameters of the insurance (premium, sum assured, yield of reserve, term) are specified in advance and fixed.
- They tried to avoid all in-between changes (raising or decreasing the premium, changing the insurance term, changing the face amount, changing the dates of premium payment), or place these changes outside of the product design (e.g. they regarded late premium payment – from the reserve aspect – as if it had arrived on time, but charged a late interest to compensate for the profit loss, etc.

Regarding the product design all traditional life insurances can be built from two basic building blocks, two final elements: the term insurance and the pure endowment

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79 The main motive of this was probably the impossibility of contraception and so the "family planning" before the discovery of the "pill" at the beginning of the '60s. It is noticeable that after this discovery the birth rate reduced radically.

insurance[80]. The most frequent "ready" insurance products that dominate the market of traditional life insurance up to this day are:

- Term insurance (including also whole life assurance)
- Endowment insurance
- Term fix insurance
- Annuities (primarily single premium, immediate, lifetime annuity paying in advance without any guarantee period)

The common talk on life insurance has been such (and is still determinant at present), that when the advantages and features of life insurance were described, it covered mainly the term insurance. This was further supported by the rhetoric of life insurers, who emphasized mainly the benefits provided by term insurance. On the other hand, the weight of these particular insurance products has been (and remained) very different on the individual markets. On Anglo-Saxon markets were dominated by the term insurance types (so here the rhetoric was appropriate), but the French, German and Hungarian markets were (and still are) dominated by endowment and term fix insurances, beside a negligible portion of term insurance. (It would be interesting to investigate in detail why this is so, but we won't do this now.)

Long term savings were the most important motivation of signing life insurances even on the Anglo-Saxon markets, where this need was primarily satisfied by whole life insurance. Furthermore, life insurance has been the only option for long term savings for those not particularly wealthy. Because of low life expectancy and large families the goal of this type of insurance was evident: to provide for the dependents in case of a life too short, which meant that uniting death risk with long term savings was quite self-evident.

6.3. The Development and Circumstances of Development of Unit Linked Insurance

The development of Unit Linked insurance started in the '50s in England, but its world-wide spreading only started in the '80s and '90s. Parallel to the English development (where the unit linked insurance was at first a traditional term insurance tied together with investment funds) an American evolution, somewhat independent from the English one ran its course, where they started to make the cover of whole life insurance flexible (Universal Life), and later allowed also free choice between investments (Variable Universal Life), and this way a life insurance plan similar to the English version was

[80] To be precise we have to add that the construction of certain plans requires a certain kind of pure endowment insurance, that calculates with certain survival, that we can consider as a third element, as a "funded contract", or single premium term fix "insurance".

created. Adding some yield guarantee to it has developed into the Variable Annuity product of our days.

If we look at those factors in the '80s and '90s that we considered the most important "environmental" factors having influenced the formation of traditional life insurances, we see a completely different picture:

- The comparable stability of money and low inflation rate in the developed world has more-or-less returned by the second half of the '90s, but after a long, uncertain period, and with the perspective that prices could become instable once again very easily. And in the significant part of the world inflation hasn't become low up to our days. The stability of interest rates is also history, not in the least due to the increased and diverse investment options, and naturally because of the instability of inflation and the typically large stock of state debts, or rather due to the practice of low interest rate of the central banks during recessions, with which they try to avoid to deepen it. From a different angle, this means, that the burden of the recession are put onto the shoulders of the savers (inclusive the life insurance policy holders).
- Paper based, manual administration is completely replaced by computer administration, and this way financial services have been freed from the stocks.
- Almost anyone has access to the capital markets directly and indirectly – not in the least through unit linked insurances (although the role of the mutual and pension funds here is even more important).
- The life expectancy has radically increased in the whole developed world (that means that the death risk has decreased – though Hungary somewhat lags behind this trend), and parallel to this, birth rates and the number of children per household have very strongly decreased (also in Hungary).

Another new, changed environmental condition can be fit in the above enumeration, namely that in the past 50-100 years the living standard and the individuals' life quality expectations in the developed world have increased dramatically, and not independent from this process the wealth of the upper, as well as the very wide middle classes has also increased. The new environmental conditions have formulated new needs concerning unit linked insurance, and at the same time created new potentials:

- Computer administration makes it possible to create flexible products, with certain parameters variable even daily. The face amount, the premium, its frequency, its arrival date may be varied, the insurer can handle single premium top-ups, and these changes can be handled within the product design. The extent of flexibility is only bounded by the insurance principles and not the feasibility of the computer administration (which is almost boundless).
- The increasing life expectancy and the increasing living standard expectations of the individuals have upgraded the significance of self-care (contrary to the care about others) in life insurance. This has been enforced by the decreasing need of

providing for others caused by the lower birth rates, and this way in life insurance the needs have shifted from death benefits toward living benefits.[81]

- The greater wealth increased the risk-bearing willingness of individuals, and with it their demand for investments carrying greater risk, but higher yield. This tendency increased the number of innovations not only in the insurance market, but generally on the capital markets.

The new environmental conditions have also increased the elbow-room of life insurance substitutes, and this way – primarily from the part of the newly developed investment funds – life insurance had to face new challenges. The creation of unit linked insurance is the answer to this challenge, since this type of insurance integrates many features of the investment funds into the traditional life insurance.

The change in environmental conditions meant, at the same time, that from the historical antecedents, or compared to them, the need for an independent (long-term) investment option has evolved that is independent from any specific risk, which raises certain "philosophical" questions about the nature of insurance.

6.4. Similarities with Endowment Insurance – Definition Arguments

Unit linked insurance is mostly similar to endowment insurance (and its special case, the whole life assurance). We can also say that endowment insurance is the ancestor of unit linked insurance, or that Unit Linked can be viewed as the generalisation of endowment insurance.

If we look at the premium-construction of the (regular premium) endowment insurance, it can be partitioned into more simple elements in two ways.

1. Traditional partitioning: the endowment insurance is the sum of a term insurance and a pure endowment insurance.
2. Modern and generalised partitioning: the endowment insurance consists of two elements, one is an instrument providing fixed interest on the net part of current payments (premiums) and the cumulated part of earlier payments, the second is a term insurance with variable face amount having the following properties:
   a. The face amount is a sum that can complement the current capital level of the instrument to a pre-defined level in the event of the insured's death.

[81] We have to mention that in Hungary this shift has happened without the earlier significant spread of term insurance. This way the overall insurance cover is significantly under the optimal level, which makes it theoretically possible for the Hungarian market – regarding insurance covers – to develop in a different way compared to the international tendencies.

b. This pre-defined level is not arbitrary, but the sum that the capital level of the instrument reaches at the end of the term.
c. The premium of the term insurance is net premium, and is always equal to the risk of the period, i.e. it changes from period to period.
d. The premium of the term insurance is deducted from the capital cumulated in the instrument at the beginning of the period.

For a very long time only the traditional partitioning was mentioned, but that cannot be generalised. The modern partitioning comprises the possibility of generalisation (if we leave the fixed nature of the interest and the strictly pre-defined sum of 2.b, and at the same time formulate 2.a more precisely, so that the face amount is zero if the pre-defined level is lower than the current level of the capital).

The first partitioning doesn't interfere with the traditional, general definition of insurance, which is the following in the Act LXXXVIII of 2014 on the Business of Insurance (but the earlier version of this Act contains similar – but not identical – definitions):

'insurance services' shall mean underwriting insurance in a commitment that is based on an insurance contract, whereby the insurance company undertakes to designate a group of persons deemed to be exposed to the same risk or similar perils (risk group[82]) in order to assess the risks that can be insured and measured by mathematical and statistical means, establish and collect a consideration (premium) for the commitment, set up specific reserves, assume the risks stipulated and provide services as contracted, including the pursuit of activities for the provision of annuity benefits carried out on a commercial basis, irrespective of whether consideration for the commitment represents payment of a specific sum of money (premium) or any other form of remuneration;

The phrasing does not contain as possibility or as element the "simple" capital accumulation.

The problem with the traditional definition of insurance is not that it was created before the appearance of unit linked insurance, when the partitioning type 1 of endowment insurance was self evident and the possibility of partitioning type 2 could easily be neglected, but that although the definition doesn't allow partitioning 2, it is still in use. This means that currently there is a kind of mismatch between the definition and the practice of life insurance.

This seems like a simple definition problem, but if we consider that the regulation and distinction of financial markets is institutional, we can almost be sure that these institutions are/were in permanent fight with each other over the "frontiers". These fights have mostly ended by now and have been closed with "peace" or "ceasefire".

[82] In this book „risk community".

Their results are definitions that precisely draw the boundary lines of the exclusive territories of each institution. Until the point when something happens. Then the fights start over again.

In case of the unit linked insurance the recurrence of the fight is always possible – and it happens over and over again – caused by the unclearness of the definition. In such a case (e.g.) some full-blooded representatives of the banking business can accuse insurance companies of "unlicensed deposit collection"[83] in relation to unit linked insurance, saying that according to the definition, unit linked cannot be considered insurance.

What can the solution be to these – somewhat scholastic – problems?

1. First it is important to point out that life insurance has always contained the independent (independent from the specific "insurance" risk) savings option ("simple capital accumulation") in a latent way, but this latency only became apparent with unit linked (this is exactly why these products are attacked). The principle of "pure" long term accumulation/saving is not exterior to insurance, moreover, for a long time insurance companies were the only institutions providing this type of service to wide classes of the population. This way when unit linked insurance is attacked, it is attacked on home territory.
2. We have to take into account that due to the – above described – change in need for safety the savings motive is becoming more and more independent, to which the insurance business has to respond.
3. The definition arguments also point out that the traditional institutional regulation and the institution-based separation of activities is currently totally outdated and mostly unjustified. Industry practice is putting a continuous strain on these borders set by the "old wars", and sooner or later it will enforce activity-based regulation, when phrasing the question as definition-based will have little significance. However, it is not going to happen overnight, because a lot of interests have been built onto the status-quo. So, the interested parties generally repulse the intentions of fundamental changes, saying that the practice has a priority to the theory (that is: because of the theoretical logic-choppering it is not allowed to disarrange the well evolved practice).

[83] This is the "cease fire" definition of the bank industry!

6.5. Major Changes Brought by Unit Linked Insurance

6.5.1. Changes Regarding the Client

The most important changes that unit linked brought to clients can be summarized in the following 4 categories.

Compared to traditional insurance – that is fundamentally inflexible, and almost impossible to change during the term – unit linked insurance is **flexible**, and allows the client a number of options and possibilities to change parameters that the traditional insurance lacked, namely:

- The client can choose – within the options provided by the insurer – the ratios by which the reserve of the client's insurance is divided between the different types of assets. The client can reallocate funds between the different types of assets as many times as he wants, naturally on payment of an extra charge.
- The client can – within certain limits but with no penalty – freely deviate from the date of premium payments, and may perform irregular top-up payments. (Besides the regular payment is remaining the default, because if the client does not pays regularly, then after a while he/she would not pay at all.)
- The client can freely choose – again, within a wide range – the minimum sum of all benefits payable in the event of death, and this sum can be changed during the term relatively easily.
- The term itself is also flexible, it can be adjusted to the changing needs of the client. This also means that the distinction between maturity and surrender will gradually disappear.

On the other hand, flexibility doesn't only bring the freedom of choice. It also means that the insurer can flexibly adjust to developments since the commencement of the policy, e.g. to changes incurred in mortality. Contrary to traditional insurance, where the invariable premium also meant that during the term the insurer didn't take into account necessary adjustments of the risk-premium due to changes in mortality, in unit linked insurance the insurer reserves the right to change the death-risk premium during the term. The direction of this change – naturally – is not known beforehand, so the client doesn't know if the change will be favourable or unfavourable, however since in developed countries mortality rates have decreased for decades, this type of flexibility is bringing rather the former than the latter result.

In exchange of flexibility the client has to pay a certain price. By having the option of selecting the assets of the investment of the reserve, the client **takes over the yield-risk from the insurer**. This frees the insurer from the responsibility of having to achieve a certain yield under all circumstances, which makes it possible to invest in instruments with higher risk, but also higher expected yield. Of course, the insurer – for an additional fee – may undertake a guarantee on the yield of individual investment forms (asset funds) offered to the clients, (cf. variable annuity!).

Transferring the greater part of investment responsibilities from the insurer to the client supposes in an implicit way the **financial maturity of the client** (as the investment practice of traditional insurance hidden from the eyes of the client supposes the financial immaturity of the client), since only a client informed in the capital markets at least on a basic level can make correct decisions, in accordance with his own risk-bearing "capacity", regarding the allocation of his reserve. This supposition on the maturity of clients is often not yet justified in the Hungarian practice, moreover it is arosing big conflicts from this from time to time.

The expense structure of traditional insurance and the magnitude of expenses is invisible to clients, and insurers consciously aimed at this invisibility. Unit linked insurance has brought a great change in this respect, the **expense structure** and the magnitude of expenses is **fundamentally visible** and can be planned by clients. The administration fee, the fund management fee and the bid-offer spread are all openly announced amounts.

On the other hand, insurers would like to hide certain expenses even in case of unit linked insurance – similarly to the traditional insurance. These are primarily acquisition costs and their coverage. The technique of Initial Units has been created to realise this, which hides the fact that the greater part of the premium of the first (two) years is not accumulated and invested for the client, but taken away to cover acquisition expenses. Since this technique can be applied to mislead clients, its use is forbidden in more and more countries.

The transparency of the expense structure at the same time makes possible a **more even expense loading.** On even loading we mean that expenses are charged on premiums and on the reserve according to the actual services, and not some other factors. In traditional life insurance expenses are usually charged as a certain percentage of the premium, this way loadings are proportional to the sum assured[84], while the actual incurring expenses of the insurer depend on a lot of other factors, so in reality it is not proportional to the sum assured. In unit linked insurance naming separately the different types of expenses and defining them as proportional to different factors closely related

[84] This is only partly eased by the practice of premium discount for a higher sum assured, that is often applied by traditional life insurance, or the corresponding, but reversed technique, when the premium tariff depends on the sum assured, relatively decreasing.

to their source makes fairness[85] of a greater degree possible. Maintenance commission and expenses of premium collection are premium income proportional, this way they are covered by the bid-offer spread, fund managing fee is proportional to the reserve, the administration fee is independent from the premium or the sum assured, so it is usually defined as an absolute sum.

Because of all these effects, in unit linked insurance – compared to traditional life insurance – there is much less cross-financing between different groups of clients.

6.5.2. Changes in the Relationship of the Insurer and the Client

The transparent construction, the visible magnitude of expenses at the same time puts pressure on insurers to **decrease expenses**. Visible expenses make it possible for clients to compare also from this point of view – either by themselves, or through an agent arguing beside the product of his company – the offers of individual insurance companies. Insurers automatically take into account this effect, and emphasize more strongly in their pricing that the product should not be much more expensive than competitor products, or that if possible, they should offer the same service cheaper.

Naturally, as we have indicated earlier, greater transparency has its limits (e.g. initial units), insurers try to resist the pressure of decreasing expenses, that they try to accomplish by hiding certain expenses of unit linked insurance, or by presenting them to clients in a special way.

Reducing the expenses is encouraged also by regulators. One of the most effective tools here internationally is the cost indicator. Its use has been spreading from the end of ‘2000s years and the EU has made it compulsory (see EU [2014]). Hungary has introduced it in the field of Unit Linked insurances as one of the first countries in 2009 (see MABISZ [2009]), and we have gained rich experiances. The products with extremely high cost indicators has disappeared from the market as the effect of this indicator, but the reason was not the unwillingness of the clients to buy very expensive products. Namely the clients are willing to buy – unfortunately – practically anything, if the insurance intermediary persuaded them. Instead of this, the mode of action is the following: the product defelopers themselves have recalled these products, averting the wrong position of their product in a public comparative list. The other important factor was, that the intermediaries have started to argue for the low cost indicator of their insurers’ product, so they have started to urge the reduction of the value of the cost indicator at their own insurer.

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[85] The term “fairness” are used in Hungary (and probably in many other countries as well) parallel in two often opposite meanings. The meaning of this here: everybody has to pay that cost, what has really arosen connecting to his/her concrete contract. Namely I consider fairness and correctness as synonyms. The often used different meaning of the term is: the cost charged is relatively higher for “richers” and smaller for “poorers”. In other words: some redistribution has to be realized.

The environmental factors promoting the appearance of unit linked, the – above analysed – product design characteristics of unit linked itself, and the shift in emphasis compared to traditional life insurance have inevitably **changed the systems of arguments** used by insurers – and their representatives – when selling unit linked.

One of the most emphasized elements in the unit linked design is the choice between different investment options. Correspondingly, this investment characteristic is the most emphasized when selling the product – and in many cases emphasized too strongly by insurance agents – generally on the account of the death coverage. We are witnesses of an interesting phenomenon in relation to this. One reason for the appearance of unit linked insurance throughout the world is the increasing demand for living benefits on the account of the death benefits. In Hungary we see strong under-insurance regarding term assurances – in an international comparison, of historical reasons – but at the same time – similarly to developed countries – unit linked insurance is spreading even on the account of term assurance. Furthermore, the Hungarian mortality rates do not give grounds for Hungarian consumers not to view this risk as important, either. Because of this, some insurance companies see a marketing chance, and incite their agent network through high commission rates to sell high death coverage. In spite of this, there is a certain shift in emphasis in life insurance "rhetoric" from providing for dependants toward self-care.

Because of the above, a number of stock market phrases and connections have been imported into the system of arguments, and agents have to help clients in recognising their own risk-bearing "capacity" and in forming an investment portfolio accordingly.

We have to mark that in Hungary unit linked insurance has been introduced at a very favourable historical moment, in the stock market boom, this way they could become popular, well-known and widespread in a short period of time. This was not the case in every country (Spain is usually mentioned internationally) where due to the bad timing these products were unpopular for long.

Traditional life assurance is traditionally sold by agents having an exclusive contract with the insurer, against payment by results (an acquisition commission depending on the term and annual premium of the insurance, payable at the inception of the policy). The cover of the commission is the greater part of the premium of the first (sometimes the first two) years, that is not accumulated in the client's reserve. From the clients point of view this commission of an unknown sum appears (or would appear if the client had precise information about it) as the fee of the guidance and services of the agent, who in a general sense plays the role of a financial advisor. This guidance and service includes a very general survey of needs and the (insurance) solution to these needs. Traditional insurance is largely standardized, so we can say that these are uniform solutions to uniform needs, and the guidance of the agent is quite simple.

This **traditional marketing technique faces a lot of challenges** – mainly from the new circumstances incorporated by unit linked insurance – to which the insurance industry has not necessarily found the adequate answer yet. The challenges can be characterised by the following contradictions:

- Because of the visible expenses (and the competition of alternative investment options) there is pressure to reduce the largest item, acquisition expenses, while the increasing complexity of needs and products increases the demand for quality counselling. On the other hand – due to the habitude reinforced by the foregoing practice of financial service providers – consumers tend to think of counselling as a free service.
- The diverging and more-and-more unique needs of clients would make it necessary to separate and make independent the costs of counselling and mediation, but these two expense elements are not separated in the current practice of financial providers.
- But the most important contradiction is that although wide classes of society have increasing possibilities and objective needs for long term investments and generally for the formation of long term individual financial strategy, only a minority recognises this fact by himself, the majority has to be persuaded by government instruments (e.g. compulsory pension funds) and through the marketing pressure of financial intermediaries (through agents).

Because of all these, there is a kind of "crisis", seeking ways and means in the marketing of life assurance throughout the world – in Hungary also –, of which the solution cannot yet be seen, but some factors will probably intensify:

- The marketing of very simple unit linked insurance with low expense ratio, not including counselling fee, only the cover of mediation is increasing through alternative intermediary channels such as the Internet, bank agents and direct mail. Mostly the financially well-informed clients can take advantage of this.
- The need for the counsel and life-cycle planning service of highly qualified advisors independent from insurance companies is probably increasing, and parallel to this the willingness to pay separately for these services will probably increase, too.

6.5.3. Changes in Insurance Technique

One of the most striking characteristic of unit linked insurance is **universality**, since a unit linked insurance can be viewed as a very general life insurance containing all possibilities. The name of the American version (Variable Universal Life) probably tries to imply this through the word "universal". From this respect we can say that – as we have hinted earlier in this chapter! – that unit linked can be viewed as a generalisation of traditional life insurance, where many restrictions have been resolved and made optional, variable.

Universality also means that the insurer is able to satisfy all insurance needs of an insured within one policy.[86] Internationally there are examples of insurance companies selling only one product, a unit linked insurance. When an insurance company is offering several Unit Linked products, this is probably because of marketing reasons, and is deliberately not exploiting all possibilities provided by the flexibility.

Universality necessarily means also that compared to traditional insurance it has much more features in common with non-insurance financial instruments, so the boundary line between life insurance and other financial areas is starting to fade, not in the least thanks to unit linked insurance.

A more even expense structure brings a kind of **stability** into the expense structure, since the expenses debited to the insurance follow more closely – compared to traditional insurance – the actual incurring costs. At the same time the flexibility of the design brings **a new type of instability** into the cash flow of the product, since the client has more options of premium payment, can choose the timing and sum of premium payment, this way the cash flow into the insurance company is – compared to traditional insurance – much less calculable. Naturally this depends strongly on the type of clientele of the insurance company, if it is middle class with stable income with a disciplined accumulation ethos, or recruited from other classes. Knowing the characteristics of the clientele is many times what motivates insurers not to use the flexible options provided by unit linked, and they deliberately set the premium timing and the sum to be inflexible, this way trying to make their cash flow more calculable. Calculability might be important regarding the commission, too. If the insurer pays large acquisition commission (i.e. uses a traditional commission regulation), that is covered by future expected premiums, then it is important that this cash flow should be as stable as possible.

The death-risk premium variable over the premium term (when calculation is based on a new, more precise mortality table) also brings the insurer closer to a more stabilised cash flow.

It is very important that unit linked insurance can **only** be managed with a modern, **large-capacity computer background** and sophisticated software. This also caused that (and not only because of the lack of a properly developed stock market) in an average Hungarian insurance company this type of insurance product could not be introduced at the beginning of the '90s. The most important requirement from this

[86] Moreover, even the insured person could be modified, which would principally make a unit linked insurance even inheritable, so it would be able to satisfy the life insurance needs of the insured's descendants.

point of view is the daily valuation of reserves broken down to the level of individual clients.

Insurance companies selling savings type insurance are important players of the financial intermediary system, since a significant part of those having excess money and those who want to use this money meet each other through life insurers. In case of the traditional (savings) life insurance the insurer itself as an institution raised a wall between these two groups, they could only meet with the active participation of the insurer. Although the insurer gains the interest credited to the premium reserve of the savings type life insurance from the investment yield of this reserve, he appears to the policyholder as an individual guarantee undertaker.

Through unit linked those offering and those seeking money meet almost directly, the insurer "only" brings them together, and (generally) doesn't guarantee anything on the performance of those demanding/using the money. In this respect the **financial intermediary position of insurers weakened** compared to the former. This seems to fit into a general tendency: the role of the stock market is increasing – that is the meeting point of the demand money and supply of money – and the role of traditional financial intermediaries is decreasing (banks, life insurance companies). If this tendency truly exists, unit linked insurance can be the forerunner of a life insurance sector with a completely changed function, where the new function of life insurance is much more counselling, individual financial planning and giving aid in the realisation of plans than financial mediation itself.

The weakened financial intermediary role of life insurers means at the same time a stronger and better asset-liability matching, since there is no better matching than when money owners and money borrowers are in direct contact without an institutional filter, since this institution may manage to invest in assets covering its liabilities, or it may not.

In case of traditional life insurance the insurer guarantees all benefit elements, so prospective reserve calculation is very important[87]. On the other hand, in case of unit linked insurance the future maturity benefit is not known, the death benefit is also uncertain (if in case of a policy the value of all funds is higher than the death sum insured), and there is no insurer guarantee on the sum of these elements (excluding the death sum as a minimum death benefit). Because of this, benefits can be calculated **only** based on "historical" data of the policy, that is based on the **retrospective method**.

[87] See section 12.! This is even stipulated by law.

6.5.4. Expected Further Changes

Naturally nobody can see the future, so regarding the expected changes we have to rely – more or less – on guesses. Based on international experience, existing but unsatisfied needs and simple logic, we can predict certain changes.

Before looking at expected changes in detail, it is important to state that unit linked insurance is not regarded as positively by all players of the insurance market (players in a narrow sense) as it is described in this study. Furthermore, some very developed life insurance markets exist (e.g. the German market), that do not like unit linked insurance at all, and tend to mean only traditional products under the term life insurance. In the Hungarian market it is also said sometimes that unit linked is "not insurance, only an investment", and it can be blamed for "people turning away from life insurance" (and this primarily means term insurance). The single premium unit linked insurance is especially blamed, which was truly pushing the envelope of the traditional concept and traditional "rhetoric" of life insurance.[88]

Unit Linked insurance is also attacked from other sectors (primarily from the side of investment funds), saying that through unit linked, insurance companies are in reality doing their business (since regulation doesn't handle unit linked funds as investment funds).

Supposing that the development of unit linked insurance will follow a more or less unbroken path, we can sketch a few demands and tendencies:

1. In the options of asset funds there will probably appear those with interest guarantee, or maybe the closed-end funds with fixed interest fixed term. These will especially fit those insurances, where dominant is the death benefit, not the investment characteristic.
2. Products might appear, where the benefits are also declared in units. We have to think about unit linked ("variable") annuities primarily.
3. The unit linked technique have appeared on the boundary lines of life insurance, in Hungary mainly in pension funds.

On the other hand, it is possible that the total life insurance market will change direction sometime in the future. A change in the direction of development doesn't have any significant signs yet, but we can already phrase justifiable questions that cannot be satisfyingly answered with the current organisation and product design of life insurance. It cannot be predicted yet if in reality these questions will have to be phrased or not. A few examples are:

- Will the current sector and product structure of the financial intermediary system stay in its present form, or will it change into totally new sectors and sector boundaries, and totally different product accordingly?

[88] The main point of this challenge is: Can an instrument of mainly short term, providing no death or other life, accident or health type cover be considered life insurance?

- Is it justified to hide risk type insurances into a savings product?
- Will the insurance sector have to detach the savings products from itself and return to risk type insurances?
- Is it justified that individual savings products are as complicated as some unit linked insurances?
- Is it socially and economically justified to maintain such an expensive insurance intermediary system for long term investment products, or could this be replaced by cheaper media (e.g. banks, other institutions, Internet)?
- Socially isn't it rather the compulsory regulation, savings legislation that should incite long term savings instead of the individual persuasion of agents?
- Shouldn't the role of agents be limited to risk (life and non-life) insurance, and a financial advisor class independent from insurers be built?
- Isn't it bad politics from the part of insurers to spread expenses primarily on saving products and not the risk-type insurances?

7. COMPARING LIFE INSURANCES TO EACH OTHER AND TO SUBSTITUTES FROM OTHER FINANCIAL SECTORS

KEW WORDS

Cost indicator	Differentiation of clients
Liquidity	Volatility

When a client faces the possibility of concluding a life insurance contract, he/she generally has to be able to answer the following questions:

1. What kind of life insurance should I conclude?
2. Which concrete product should I conclude out of many similar ones?
3. At all: should I conclude a life insurance or instead of it should I buy a different financial product?

7.1. Choice amongst life insurances with different purposes

In general, the following aims can be reached by different life insurances:

1. A protection against the financial consequences of some risk – primarily death. This is the main aim of term and whole life insurance, but – partly – of all other types of life insurance with death component.
2. Saving and accumulation for future aims. These can be achieved by savings type life insurances. Those life insurances can be called as "savings type", which pay benefits either way. It means that only term insurance is not a savings type life insurance. But these aims can also be achieved by other products than life insurance.
3. Phased (and safe) depletion of the savings. (Safe means here: it is surely not running out during my lifetime.) This aim can be best achieved by life annuities. I do not need a life annuity, if I do not want to deplete my savings, because the regular yield of it is enough for my living, so I want to pass the principal down to my heir.

The living conditions, financial circumstances, aims, etc. of somebody are determining which kind of life insurance contract – or life insurance at all – are expedient to conclude. Theoretically, everybody can analyse all of these self, but experience shows that the financial literacy of people in general is quite imperfect, so they need advice on this field. The adviser is often the agent of a life insurance company or an independent insurance intermediary. If he/she acts conscientiously, then he/she first assess the financial circumstances and needs ("needs analysis") of his/her potential client and on the basis of this he/she suggest him/her a personalized recommendation. The needs analysis is a legal obligation in many countries – e.g. also in Hungary. (see e.g. PSZÁF [2006])

7.2. Comparing life insurances to each other

After the clien is able to – on basis of the above – choose the type of life insurance, the question is arisen: which specific product? Even an insurer can have similar products, but the need for comparison mainly appears between similar products of different life insurers, according to the followings:

1. primarily the price,
2. secondly the differentiation amongst costumers,
3. thirdly the specific benefit mix.

Let us see these in reverse order! As we can later see even the second and third factors will conclude to the question of price in the long run.

7.2.1.Comparing Benefits – the potential grievances of the costumers

Insurers generally aspire to versatilely satisfy the needs of the costumers. To this they partly develop products with diverse benefits and options and partly they make it possible to conclude diverse riders to the main policies. Their aspiration has two important motives:

1. to satisfy as many needs of the costumer as possible. Partly to make him/her satisfied, partly to prevent costumer to apply to another insurer for service. This latter embodies a business risk for them because in this way they lose the costumer partly at least, but they can lose him/her entirely, if he/she deserts to the competitor he/she has met.
2. the more complex a product (it contains many components), the more difficult to exhibit its real cost and in this way to make it comparable to the products of the competitors. It also means, that an insurer can use the intensify of the complexity of the product to raise the expenses. The expenses – and the gains of the insurer – can be also raised by putting into the product unimportant elements as well.

The motive of the consumer is partly the opposite of this: he/she would like to buy as cheap as possible. That is why to him/her the comparability of the prices of the different products is fundamental, but it is also advantageous for him/her that more than one of his/her needs is satisfied simultaneously and for this he/she is ready to pay some extra charge.

As a resultant of these two motives – in optimal case – a mutually advantageous contract can be concluded, but the costumer loses quite often, namely he/she less prepared than the insurer. That is why institutional protection of the customers, the financial consumer protection is necessary. Two levels of it is possible:

1. handling concrete problems concerning insurance contracts. Many countries have established separate institutions – financial ombudsman or financial arbitration boards – for this, where the customers can make concrete complaints.
2. handling general problems by introducing consumer protection rules. This becames more and more important task of the insurance/financial supervisors, but in many countries a new, a so called "twin peaks" model appeared. According to the rationale of this the prudential and consumer protection considerations can confront, so it is better to organize the two institutionally separated. An important (and relevant) expample is the compulsary application of the universal cost indicator.

7.2.2. Differentiating Among Customers

The issue can arose as a problem or an opportunity mainly in the case of term insurances and life annuities, and less at savings type insurances and theoretically it is attached to the equivalence principle in the Chapter 9 which expresses equality in two meanings:

1. first: it expresses the equality of the incomes and expenditures by major consumer-groups;
2. second: it expresses the equality of the own risk of the insured and the premium paid by him/her. (Anyway, this latter implies also the first equivalence, namely if the second comes true, then also the first, but the opposite is not.)

In Chapter 9 we will discuss mainly the first meaning, but here second one.

To understand the significance of differentiating between customers, let's look at a fictitious example. Fictitious, because we start from an existing insurance product, but since we know the outcome of the example beforehand, the insurance companies are not in competition with each other in this form.

CSÉB (Group Life and Accident Insurance) was the popular personal insurance product of the '60s and '70s. This is an insurance combination that contained also a life element, and provided (and provides) the same benefits for the same premium for everyone. This is the perfectly non-differentiated state, since it doesn't differentiate between insured and insured in any respect (or rather doesn't differentiate between insured persons of active age, but this is not important at present).

The State Insurance Company could do this because on the Hungarian market it had a monopoly, and could suppose with perfect confidence that everyone will sign this insurance, so it is enough to calculate from averages of the whole population. This meant, e.g. that since the mortality rate of elderly is higher that that of the younger, they paid less than their own risk, while the younger insured paid more. This way only the first type of equivalence could be satisfied, and the second type was only satisfied in case of a small fraction of the population having exactly an "average" mortality rate.

The State Insurance Company – since it was alone in the market – wasn't forced to satisfy the second type equivalence, although in a competitive environment this is inevitable. Regarding the CSÉB, the transition from monopoly to competition could also have happened the following way.[89]

The fist competitive insurance company appears in the market, who observes that while the mortality of men and women differs significantly favouring women, women pay the same premium for CSÉB as men. So, if it creates a WOMEN'S CSÉB separately for women, then all women could be attracted by premium reduction to the first rival insurer. This will have the effect that only men remain at the old insurer. Men pay a premium smaller than their own risk, since the premium of CSÉB was calculated so that the premium deficiency of men is compensated by the excess premium of women. This way the old insurer is forced to raise premiums to keep its calculation in order, and loses half of its insured group.

This is when the second insurance company steps into the picture. It recognises that young men pay a premium significantly higher than their own mortality risk. It creates the CSÉB OF YOUNG MEN, and attracts all young men with a premium discount. All old men stay at the old insurance company who pay a premium lower than their own risk, since the premium was calculated so that the premium deficiency is compensated by the excess premium of young men. The old insurer once again lost half of its clients, and is forced to raise premiums.

The third rival insurer experiments with the CSÉB OF NOT TOO YOUNG, BUT NOT TOO OLD MEN, the fourth comes out with the CSÉB OF MIDDLE AGED MEN WITH FAMILIES, and so on... The result will be that the old insurance company gradually looses all of its clients (and those who would stay are frightened away by the constant raise in premiums), while the market only offers CSÉB's that provide the same benefits with premiums differentiated by age and gender.

Naturally – I have to underline once more – the above example is fictitious. CSÉB is a much more complex insurance than one that can be analysed exclusively from the premium side, and this way its extinction (that happened gradually after the termination of the insurance monopoly) went on a different path. A element of this is, for example, that the competition of more serious life insurances (has been called "major life insurance" by some insurance companies) is attracting the best clients from CSÉB.

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[89] This explains that the idea of CSÉB could not have appeared in a market economy.

The lesson of the above example is that in a competitive environment insurers are forced to differentiate in the second sense their portfolio in a deeper and deeper level.

Of course differentiating among clients can have its limits. In some countries regulation prohibits insurers from differentiating clients by some characteristics, e.g. religion, race, etc. These regulations are perfectly understandable, and there is no other way to avoid differentiation conflicting such general principles as legal prohibition, since without these prohibitions competition among insurance companies would objectively lead to this kind of differentiation and exclusion. This problem has not yet occurred in Hungary.

It is also important to mention, that from 2012 it is not allowed – on the basis of a very questionable ideology – to differentiate insureds according to their gender in the whole EU (see EU [2004])

Differentiation – in the case of insureds with "good risk" – can influence more the premium of the insurance than the expenses charged in it.7.2.3. The "price" of insurances – comparison of Expenses

People want to buy insurances – like anything else – as cheap as possible. Which insurance is cheaper sometimes can be detected by simple comparison of the premiums. In case of insurances with fixed sum assured (basically insurances without profit-sharing feature – it is quite common among term insurances and life annuities), lower premium means lower price. However, unlike term insurances and life annuities almost all savings type life insurances have this feature, so the simple comparison of the premiums is not a viable solution here. For example the premium of an endowment insurance calculated with a lower technical interest rate is inevitably is higher, than the premium of an endowment insurance with a higher technical interest rate, but it is not sure that the former one is cheaper than the latter one, because the profit sharing will be higher – ceteris paribus at least. Namely the price and the premium are not the same.

But what is the price of the insurance products? This question has not really been asked so far either for insurances or for other financial products. However, it can be realized that the price cannot be the premium, only its cost part. This is the part of the premium the costumer will not get back. The expected value of the benefits of the insurance is equal to the net premium, so this part of the (gross) premium will get back to the costumer. (Banyár [2013], Banyár-Vékás [2016])

The expression of all the expenses in a single indicator is a present intention from the end of 2000s years in very different financial sectors. The Hungarian life insurance sector was among the first in the application of the cost indicators. First, he Hungarian Financial Supervisory Authority (HFSA, in Hungarian: PSZÁF) recommended the use of this kind of indicator for the savings type life insurances (PSZÁF [2007]). On the basis of this (but using a different method than the suggested one by the HFSA) the MABISZ (Hungarian Insurance Alliance) has created such an indicator for the UL insurances (MABISZ [2009]). One year later the EU has introduced a compulsory cost indicator (TER) for the mutual funds and later a general cost indicator for all packaged investment and insurance products (PRIIPs) (EU [2014]). These cost indicators can also be interpreted as an effectuation of the price of the financial products, namely as annual reduction on yield. Another (equivalent) effectuations are also possible, for example what ratio of the premium paid by the costumer is cost – see Banyár [2016].

The cost indicator makes the answer of the following question simple: which insurance is cheaper? The answer is simple: which has smaller cost indicator.

At the same time we have to draw attention that the cost insicators is suitable only for comparing similar type insurances and their substitutes. Comparing directly the cost indicators of different insurance types is problematic (at least in case of these two kind of cost indicators, the reduction in yield and cost ratio of the premium). The statement, that cost component of the premium of an endowment policy is 50% can be shocking, but a term insurance with the same cost component is totally normal. At the same time we can find a common basis to compare directly the costs of different type of insurances. The sum assured can be a good candidate to this role.

7.3. Comparing life insurances and other savings instruments

If the aim of an insurance is the 1st or 3rd in the Subchapter 7.1., then these aims can be achieved only by life insurance products: term insurance and life annuities. These products do not have competitors outside the insurance sector. The 2nd aim however can be achieved not only by life insurances but by diverse substitute products from other financial sectors. In these cases we have to compare not only the supply of the life insurers to each other, but also mutual funds, pension funds and long deposits. In this comparison one of the most important aspects is the cost component, so an universal, cross-sector cost indicator can help us. This universal cost indicator was the aim of the abovementioned PRIIPs regulation.

Naturally there can be other angles, like liquidity, not only the cost. One of the (a little bit paradoxical) advantages of the life insurance is its relative illiquidity. This is generally interpreted as disadvantage, but if the aim of the costumer to defend his/her savings from himself/herself (namely from the effects of continuous buying impulses) on the long run, then the illiquidity can be positively advantageous for him/her. And such a costumer is not so rare, especially towards the bottom of the income ladder. It is worth to mention, that term insurances and life annuities do not have non-insurance alternatives only if their premium remain inside a certain interval. In case of term insurance, this interval is so large, that the exit from it is practically impossible (the premium has to be smaller than the sum assured). However, in the case of life annuity this exit is more easily feasible (but it is hard here as well). The borderline here, that the premium of a life annuity cannot be higher than the premium of a perpetuity, because in case of the latter we always deplete only the (real) interests, never the principal contrary to the former where we deplete both. At the same time, comparing the annual interest of a deposit or a bond to the annual sum assured of a life annuity is superficial, especially in the times of inflation, namely we have to consider the the average long run interest, and – if the life annuity itself has valorization feature – not the nominal, but the real interest.

It is worth to include into the comparison – from a practical angle – also the tax benefits, in case of competing products. From a very general angle it is hard to justify that the government provide a tax benefit one of the competing products while deny the others or it provide different tax benefits, but it happens from time to time. (And the handicappeds are lobbying for the liquidation of the handicap.)

8. THEORETICAL CONSTRUCTION OF LIFE INSURANCE

KEY WORDS

Elements of life insurance Bet

Duality

In the following we will introduce the most general "building blocks" of life insurance in order to be able to place all possible (traditional and modern) types of life insurance in a uniform frame.

8.1. The Most Important Elements of Life Insurance

If we look at the matter objectively, we see that life insurance, in the end, is the special case of a wider group of phenomena, namely the "bet". Since there are generally negative associations tied to betting, it is not surprising that insurance studies don't advertise this "relation", or if it is mentioned at all, they try to deny it with all power.

Despite this, the situation is that insurance is a special kind of bet, where the subject of the bet is undoubtedly the incurrence of an event having negative consequences, and where the insured – in a certain sense – makes a bet against himself. The first insurance (the one first regarded as an insurance) is supposed to have been formally a bet, where the owner of the ship took his bet on the ship not returning to the harbour with the carried goods. If he lost the bet, then in reality he won, because the ship returned safely, and if he lost his ship, then he won the bet. (Of course the stake of the bet from the part of the owner of the ship was much less than its value. For the other party, on the other hand, it was the ship – loaded with goods – that became the stake.)

In case of life insurance – following the pattern of the two possible life insurance events – two kinds of possible bets can be defined. The insured either makes his bet on that:

- he dies during a certain period, or that
- he doesn't die during a certain period.

We discover one or both of these bets in all possible types of life insurance[90], this way we can look at them as the elements of insurance that constitute all life insurance products.

[90] Life (and insurance!) sometimes seems to launch an attack against this categorical statement, because insurers sometimes put on the market almost purely savings products as life insurance. Because of this, we can modify our statement so that every life insurance contains one, two or all three elements – and no other element! – of the these two bets and the third element, that will be discussed shortly.

The same way as in the bet, there are two parties in an insurance, and the strategies of the two parties are exactly opposite. When the client bets on himself dying within a certain period, then the insurer, in contrast, takes the bet on him not dying within a certain period, so its strategy is the other possible outcome. This way, regarding the bet, the position of the insurer and the client is dual – the insurer takes exactly the opposite bet as the client. But it is important to add that the financial position of the insurer and the client is not the same, so only those bets are possible in insurance, where the insurer is the „bookmaker" at the same time, so the client pays his stack in advance to the insurer. Reversed case or third party (including a real bookmaker) doesn't exist.

From the two elemental bets we can build two very simple life insurances:

- A short term, single premium term insurance: I bet I'll die during this period.
- A short term, single premium pure endowment insurance: I bet I won't die within a certain, well defined period.

In the above the premise "short term" is important, because we can only disregard the interest return of the deposited money in the short term. If, on the other hand, we want to take a bet of a longer term, we have to ensure an interest. This way beside the above two elements we need a third element for the construction of life insurance that, for the sake of simplicity we'll call "account". From these three elements all life insurance types can be completely built.

With the introduction of the account our possible range of betting is significantly wider. For example the client can make a contract with the insurer that promises not only one bet, but a series of future bets. This bet-series can be standardised so that they result in the traditional, regular premium term- and pure endowment insurances:

- Regular premium term insurance: the client promises to bet on the same amount of money for a certain term, at certain periods (e.g. annually), and will manage the payment of the stack– that varies due to the changing age of the client – (the premium) from regular payments made to the account and the received interest.
- Regular premium pure endowment insurance: the client promises to make bets at regular intervals with the same expiration date, and always raises the amount at stake with this (for the sake of simplicity only the hoped final sum is highlighted, the continuous increase is not shown).

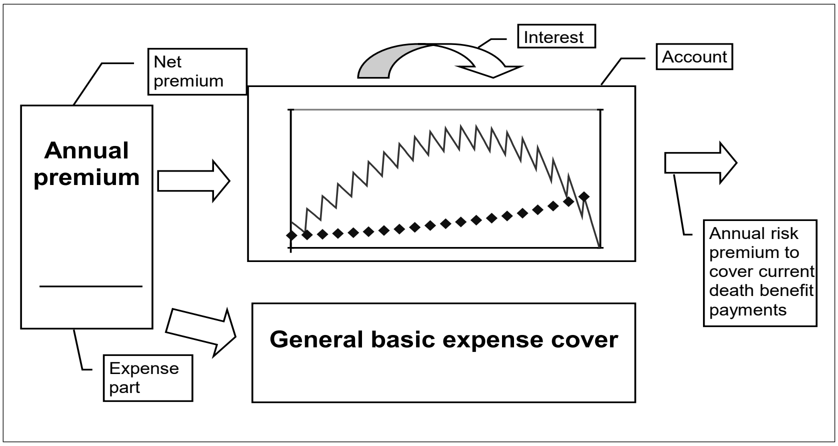

Figure 8.1.: The pattern of regular premium term insurance

Naturally the client can not only promise a series of bets, but can also make several different bets at the same time, e.g. pure endowment bets with different expiration, or different types at the same time. This takes us to the most diverse life insurance designs.

We do not see the above elements directly on the market, but all insurance types we know can be built from them. However, we find that until 1997 insurances have been built from these elements in a systematically different way than after this date (when the first UL insurance have appeared on the Hungarian market).

It is worth playing a little bit with the bet-analogy! We have mentioned that the term and pure endowment bets are dual, in other words opposite pairs of each other, one party of the bet plays one, the other plays the other. All the same, the two bets, or the two betting strategies cannot be exchanged between potential players. One should rather play one, the other should rather play the other strategy (but of course both might have reason to apply both strategies). For the client it is the term bet, for the insurer it is rather the pure endowment bet that is the better strategy. The difference lies in the fact that the client plays one bet, while the insurer plays bets with lots of clients at the same time. The client risks a relatively large stake with the pure endowment bet, for a sum barely higher. It is usually not worth playing this game. But the insurer can make a noticeable profit with the number of small bet profits on the pure endowment bet. Naturally this is closely connected to the different financial position of the two parties.

If we think of life insurance as a cash flow between the client and the insurer, and sometimes reverse this cash flow, then we get an other well known insurance (in this sense these are also each other's duals). For example the reverse of the single premium lifetime annuity is the regular premium whole life insurance. The reverse of the single premium temporary annuity is the regular premium endowment insurance.

8.2. The Construction of Traditional and Modern Life Insurance

In case of traditional life insurance we have to add further properties to the above introduced elemental bets and account, to be able to derive all traditional life insurance types from them.

A typical group of characteristics has been formed in case of traditional life insurance in the 19th century. If we try to find the cause, the root of these, we find it in the administration and computation potential of that time. Since everything had to be calculated and manipulated manually, they aimed at the widest standardisation and to have every possibility played beforehand, and then tried to force all procedures to move on pre-calculated paths.

According to this:

- they fixed the interest rate (this was not a problem, since it was typically stable for long periods of time, as a given parameter),
- they fixed the timing of regular bets (the periods of premium payment) – and tried to handle any deviation from these outside of the system, e.g. by late interests, to be able to handle the premium as if it were paid on time, also in case of late payment,
- the insurer promised the insured not to perform the whole procedure over again in case of the new bets (not to apply underwriting before premium payment, and with this, the insurer took on the risk of anti-selection), but in return demanded stable commitment from the client, in other words, limited the possibilities to quit the insurance,
- with the exclusion of a few rare cases, they never allowed even the slightest modification of the policy – e.g. increasing-decreasing of the premium, or occasional single premium payments during the term,
- by certain insurances the premium has been determined so that by the end of the term the sum of the account reached zero (term insurance), and by other insurances so that it reached the sum of the original bet,
- they promised clients to use the same mortality rates throughout the term, even if mortality changed significantly along the way.

Summarising the above, we can state that the two elemental bets have been combined with an account so that the elemental insurances, the term and the pure endowment insurances were created, and all traditional life insurances could be built from these two elements.

Building **modern insurance** from these three elements is even more easy. Here the account is an instrument returning interest in a wide range, handling premium payments received and benefits paid out flexibly. From the bets – in case of the modern insurances developed so far – we only need the term bet. This naturally doesn't rule out the possibility that later on say, the annuity insurances will be modernized, and insurers offer annuities with benefits defined in units. Then the pure endowment bet can also become in important building block of modern life insurances.

III. THE TECHNIQUE OF LIFE INSURANCE PRODUCTS

9. THE PREMIUM OF LIFE INSURANCE

KEW WORDS

Actuary
Provisions for adverse deviation
Insurance premium
Insured period
Gross premium
Frequency of premium payment
Equivalence principle
Risk premium part
Unearned premium
Net premium
Technical interest rate
Premium loading

9.1. Parts of the Premium

As we know, the premium of non-life insurance is composed of three parts:

1. risk premium,
2. provision for adverse deviation,
3. profit loading.

Provision for adverse deviation serves to cope with the fluctuation of claims. Since life insurance is probably the best "behaving" insurance regarding claims, that is, the volume of claims can be predicted with very little deviation, there usually is no separate provision for adverse deviation applied. According to the traditional approach, the premium of traditional life insurance consists of two parts:

1. risk premium,
2. premium loading.

The risk premium itself is usually called **net** premium, and the risk premium and the premium loading together are called **gross** premium.

Dividing the premium into two parts indicates that the insurer uses the collected premiums for two fundamentally different purposes. The greater part of the premium, the *risk premium* serves as the cover for the undertaken benefit liabilities. That is, if the insured event occurs, (death or living at the end of the term), then the insurer pays the benefits defined in the policy from the sum accumulated from the payments of this part. The smaller part of the premium, the premium loading serves as the cover of expenses (wages, office rent, profit, commission, etc.) of the insurer.

The premium serves the above two purposes also in the case of unit linked insurance, that is, to cover the expenses of the insurer and the undertaken risks, but dividing the premium into the above two parts is somewhat problematic. The problem comes from the construction of the insurance. The insurance is designed so that the total premium

– with the exception of two components – goes to the asset funds. The premium of the death risk and the rider insurances, the administration fee and the fund management fee are from time to time subtracted from the asset funds. The payment of the premium and its timing, the subtraction of the above factors and their timing is different as a main rule, and due to this different timing the value of units changes. This way it is theoretically impossible to define these subtractions as a percentage of the premium – only some kind of subsequent – approximate – calculation is possible after each period. The two above mentioned components, that don't go into the asset funds are the bid-offer spread, and the value of those initial units created from the given premium payment that will be certainly subtracted. These elements definitely belong to the expense part of the premium, but the expenses of the insurance are higher than these.

Despite the above, the premium of modern life insurance can be understood relatively easily due to its transparent structure, this way here we will focus on the introduction of the premium of traditional life insurance in the following.

9.2. Premium Calculation

The insurance premium is calculated from certain basic data by insurance mathematicians (actuaries). Premium calculation has a traditional and a modern method. This mostly follows the traditional insurance – modern insurance division with the addition that traditional insurance can be calculated by modern methods, but not vice versa. Here we will describe the principles and elements of classical premium calculation and will deal with modern premium calculation later. In a certain sense the risk part – premium loading division is a requirement of the classical premium calculation – this kind of division is not absolutely necessary in modern premium calculation methods (however for the calculation of reserves it is necessary to calculate the net premiums even in case of modern methods). But in the classical method the calculation begins with calculating the risk premium, and the calculation of the premium loading is based on this. The basic principle of the calculation of the risk premium is the simple so-called **equivalence principle**:

The present value of expected incomes = the present value of expected payouts.

The term "expected" in the equivalence principle refers to a mathematical (probability-theory) concept, the *"expected value"*, that we got to now earlier.

In life insurance both the income and the outgo items depend on chance (which in the present case means when the insured dies, since up to that point he pays, and after that the insurer), this way we can only predict their expected value, and even that only if we know the *probabilities of death* (mortality rates).

And where do we get these? A technical apparatus has been developed for this several centuries ago, the so-called **mortality table**, that we have also seen already.

In the equation reflecting the equivalence principle it was necessary to put the present value of incomes, because payments due at different times cannot be directly compared, and we know that income will occur continuously (which means that at different points in time), and payouts also don't occur all at the same time. Payments at different times will become comparable if we measure them with a uniform measuring unit. This uniformization can be achieved by calculating present values.

We also know that present value calculations require a discount factor, and this is calculated by aid of an interest rate. This interest rate is always the current interest rate of the market. However, we know that this changes all the time, while we might have to calculate the premium of a policy of 45 years based on the equivalence principle, so we can only do discounting with an interest rate that is certain on this time-frame. This can only be the long term real interest rate, that is in a consolidated economy between 2-5%. In the long term, namely, we have to count with the possibility that the inflation rate stops, and the (nominal) interest rate drops to the level of the real interest rate.

Because of this, all insurance companies choose a so-called ***technical interest*** rate, and calculate the premium of life insurance using this interest rate[91]. We use this interest rate in the equivalence principle to calculate present values.

The technical interest rate means at the same time also a *guaranteed* interest rate.[92] The insurer guarantees that the investment of the premium reserve will return an interest of at least this level, that will be given back to the policyholder. If the insurer doesn't achieve this yield by investing the premium reserve, he still has to return it to the clients from some other source. The yield of the technical interest rate doesn't appear to the clients in the form of the increase of the sum assured, since it is already calculated in the premium. The higher the technical interest rate, the lower the premium of the insurance will be, as shown on figures 9.1 and 9.2[93]:

---

[91] The possible maximal level of the technical interest rate was regulated earlier in Hungary (bun also in other countries) by a decree. In Hungary during a 28 years period gradually decreased from the initial 7% to 5,5, 4 and finally to the 2,9% level. In 2016, by introducing the Solvency II, this maximum was abolished. Instead of it is used a capital requirement dependent on the level of the technical interest rate, to keep it on a reasonable level. .

[92] This is practically a convention. Theoretically it is possible to distinguish technical interest rate as a calculation tool and the quaranteed yield, which could be lower or higher. But – as I know it – internationally there is no such a practice in the regulation.

[93] The calculations was made using Hungarian Male People Mortality table from 2014, supposing a 40 years old entry age insured.

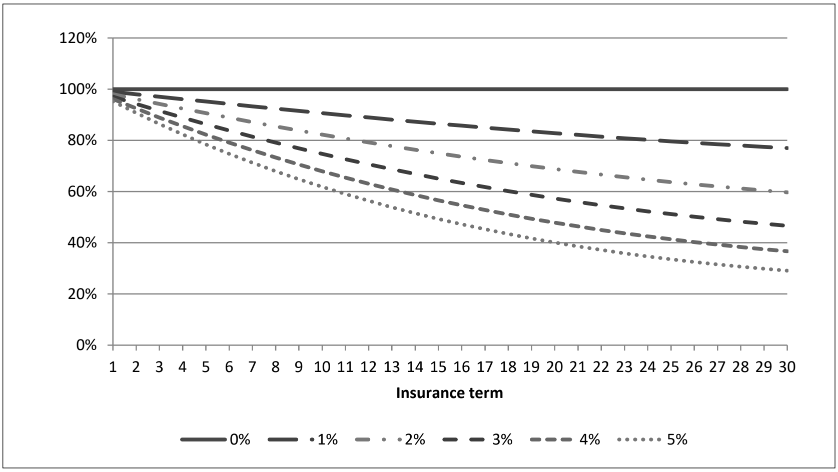

Figure 9.1.: The absolulute (for sum assured 1) and relative premiums of a single premium endowment insurance by different technical interest rates, compared to the 0% interest rate

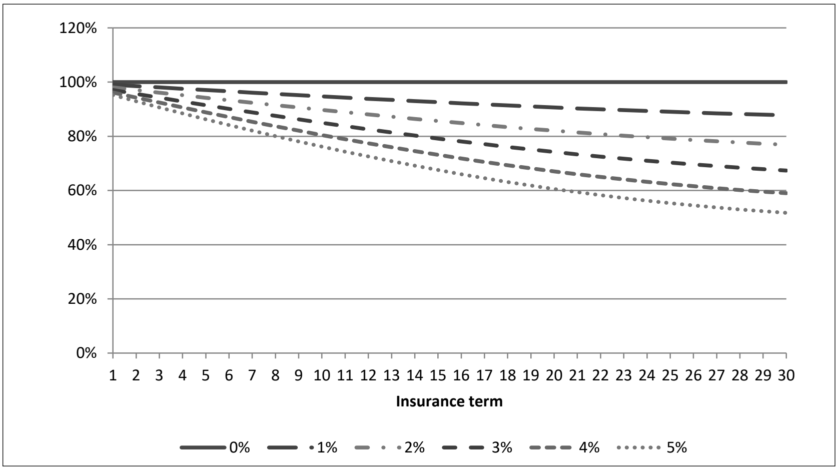

Figure 9.2.: The relative premiums of a regular premium endowment insurance at different technical interest rates, compared to the 0% interest rate

The guaranteed interest of the premium reserve calculated in the premium causes that e.g. in case of an endowment insurance with 1,000,000 Forints sum assured and a term of 30 years the yearly premium is say 25,000 Forints the client has to pay

25,000*30 = 750,000 at most, while the beneficiary receives 1,000,000 Forints in all cases. (Naturally only if the insured doesn't die during the term and doesn't use the less frequent premium payment options.)

The difference – that is a lot more than 1,000,000-750,000 = 250,000 because of the premium loadings – is covered by the return on the premium reserve corresponding to the technical interest rate.

What is this premium reserve that we have mentioned so often?

Before discussing it in detail in a later chapter, it is important to pin down what it is not.

The premium reserve is **not** all premiums paid until the point of time in question.

10. THE PREMIUM CALCULATION OF LIFE INSURANCE

In the following chapter we'll discuss again what the previous chapter said about the premium calculation of traditional life insurance, but now we phrase the relations with the aid of mathematical formulae.

In this chapter we will discuss not only the premium calculation of the traditional life assurances, but also the traditional method of premium calculation – in two variants. Modern premium calculation methods will be discussed later, in chapter 16.

The notations are also important, because there are (not considering some some rare exemptions) internationally unified, conventional symbols about what an actuarial conference made a decision in Paris at the end of the 19^{th} century.

The logic of the classical premium calculation is ensued from the principle of equivalence and we present it in the following 4 steps (the first four subchapters are in harmony with these steps):

Step 1: first we present the calculation of the single net premium of classical (non annuity) life assurances. These single net premiums are noted by A (further details are below!). The cause is, that in case of single net premiums the left side of the equivalence equation (the excepted value of the discounted incomes) will be simple: the single net premium itself, because it will come in surely and immediately, so there is no need any discounting.

Step 2: calculation of the single net net premium of life annuities. These net premiums are noted by a (or by ä – the details are below!). These premiums are naturally also single premiums. To distinguish them from the other single premiums is caused by expediency, which will be obvious in the step 3.

Step 3: calculation of the regular net premiums. Here we use two important things: (1) the regular premium itself is – technically – a life annuity, so we can use the premiuim of the annuity here, and that (2) the payouts side of the equation of equivalence for single and regular premium insurances are same (except an only one excemption discussed separately). And, since the equation of equivalence itself says that income and payout are equal, so we can write to the payouts side of the equivalence equation of the regular premium insurances the (already known) single premium.

Step 4: we convert the net premiums into gross one – by using loading factort.

Below we first discuss these four steps, and later we review the connections amongs these already known premium formulae. For the first four subchapters we introduce – also in line with the logic of classical premium calculation – auxiliary functions, the so called "commutation" functions. At one time their role was to decrease the necessary calculations (until almost the middle of the last century actuaries calculated premiums and reserves "by hand", so preparing the tariff of a new insurance product needed calculating some weeks. Today this aspect is not important any more, but appeared a new, positive feature of the classical premium calculation method: it gives a structure for the calculation, so it makes easier to review the steps and the accidental fault-finding, which could be hopeless without it.

10.1. The Single net Premiums of Single Premium Insurances

So, at first we present the first step of the classical logic of the premium calculation.

10.1.1. The Single Net Premium of Term Insurance

We present the calculation of single net premium of the Term insurance quite a detailed way, because we also present some very general considerations later we use regularly.

Let

$$A^{1}_{x:\overline{n|}} \tag{10.1.}$$

mean the following: The net premium of a single premium term insurance with 1 Forint sum assured and a term of *n* years, if the entry age f the insured is x years. E.g. the notation of the single premium of a term insurance with 1 Forint sum assured, 15 years term in case of an insured 45 years old when entering the policy is:

$$A^{1}_{45:\overline{15|}}$$

Out of this pile of notations I would stress the followings:

- "A" means the net single premium at any life assurance (except annuities) in a standardised way, for a sum assured of 1 (HUF, USD etc.).
- we put into the righ subscript the entry age of the insured and the term of the insurance, separated from each other by a colon. Sometimes we leave some of them if it is not relevant. For example, the age in case of single premium term fix, and the term in case of a whole life insurance, so in these cases only one value appears in the right subscript so the question appears: it is an age or term? To make it unambiguous we also put a "bend" above the term.
- the two "elementary" life assurances (term and pure endowment) have own, separate notations, the others not. If we put "1" above the age, it means Term, above the term it means Pure endowment.

Since the premium of the insurance is directly proportional to the sum assured (disregarding the potential premium discount from the premium loading depending on the level of the premium, that might be applied by some companies), it is enough to declare the premiums of 1 Forint sum assured, and the premium of the actual sum assured can be calculated very easily from this.

The base of the calculation is the life table. As we know, this shows the number of living at age *x* from a starting population (generally 100,000 lives), as a function of age. In the premium calculation we always start with the simplifying supposition that the number of insured persons of age x*,* having an insurance of *n* years term is l_x , according to the life table. (This supposition doesn't have any particular significant meaning, it only shows that the risk community doesn't consist of only one person.)

The basis of the calculation is naturally the equivalence principle, which here also means that:

The present value of expected income = the present value of expected payout

(The equivalence principle refers to the net premium now.)

In case of single premium insurances it is very easy to calculate the present value of income, since all premium flows in to the insurer at the beginning of the term. So, the expected value equals the actual premium paid, and we do not have to discount it. This way:

$$income = l_x \cdot A^1_{x:\overline{n|}}$$

(10.2.)

We have to introduce another simplifying assumption to be able to calculate the other side of the equation, the "present value of expected payout". This assumption states that all payouts of a certain insurance year are performed at the end of the insurance year. This assumption, as we'll see, makes our job a lot easier.[94]

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[94] **Remark:** This assumtion can be further refined the following way: we suppose that in the given insurance year everyone dies at exactly the end of the insurance year, and in the event of death the insurer pays the benefits at once. It is clear that both supposition are unreal. Deaths occur continuously during the year. Because of this, a different, more realistic assumption is sometimes used, that all deaths occur exactly in the middle of the insurance year. This, of course means that insured persons die "averagely" at this time, which is a more realistic supposition, but it somewhat complicates the calculation, so the above, more simple approach is often used in practice.

Supposing payments at the end of the year results a little lower premiums than if we put the payments to the middle of the year, since the liability of the insurer is due later. But the difference is not too big, and the effect is somewhat compensated by the fact that payment usually doesn't happen immediately at death, but generally 1-2 months later. The insurance company needs this time for the assessment of the circumstances of death and the claim, and until that time the sum assured gains interest on the accounts of the insurer, for the insurer.

To eliminate the above problem, usually a correction factor is inserted in the formulae, that we will not discuss in the following.

As already mentioned, the number of deaths at a given age can be defined based on the life table:

$$d_x = l_x - l_{x+1} \tag{10.3.}$$

So if the number of insured in the starting year is l_x, from which
the number of deaths during the first insurance year is d_x,
the number of deaths during the second insurance year is: d_{x+1},
...
the number of deaths during the n[th] insurance year is: d_{x+n-1}.

Supposing 1 Forint sum assured the insurer's liability at the end of year t is: $1 \cdot d_{x+n-1}$ Forints. If we discount the expected payouts of the individual years to the beginning of the insurance and add them up, we get the other side of the equivalence equation that we were looking for:

$$d_x \cdot v^1 + d_{x+1} \cdot v^2 + \cdots + d_{x+n-1} \cdot v^n \tag{10.4.}$$

where

$$v = \frac{1}{1+i} \tag{10.5.}$$

and
i: is the technical interest rate.

This way the equivalence equation is:

$$l_x \cdot A^1_{x:\overline{n|}} = d_x \cdot v^1 + d_{x+1} \cdot v^2 + \cdots + d_{x+n-1} \cdot v^n \tag{10.6.}$$

from this we can get the single premium simply dividing by l_x:

$$A^1_{x:\overline{n|}} = \frac{d_x \cdot v^1 + d_{x+1} \cdot v^2 + \cdots + d_{x+n-1} \cdot v^n}{l_x} \tag{10.7.}$$

The obtained result is totally adequate and satisfying, and it is easy to write a computer program based on this, that computes the adequate premiums using an arbitrarily chosen life table and arbitrary technical interest rate.

A few decades ago the option of a computer program was not available for actuaries, and because of this, the above formula was further simplified by introducing new, standard notations.[95] The basis of the simplification at that time was that insurance companies changed life tables and technical interest rates relatively rarely (only in the perspective of decades), so both factors could be considered given at every point of time. From the life tables some standard functions, the so-called commutation functions, or commutation numbers were created, and the values of these functions were given together with the values of the life tables, and calculated in advance.

First of all the "discounted value" of living and dead were introduced, D_x and C_x. These Were defined the following way:

$$D_x = l_x \cdot v^x \tag{10.8.}$$

$$C_x = d_x \cdot v^{x+1} \tag{10.9.}$$ [96]

With the aid of these commutation numbers, we can write equation 10.7 of the premium calculation in a different way, by broadening both the nominator and denominator:

$$A^1_{x:\overline{n}|} = \frac{d_x \cdot v^{x+1} + d_{x+1} \cdot v^{x+2} + \cdots + d_{x+n-1} \cdot v^{x+n}}{l_x \cdot v^x} \tag{10.10.}$$

If we substitute into 10.10. the new symbols 10.8 and 10.9.:

$$A^1_{x:\overline{n}|} = \frac{C_x + C_{x+1} + \cdots + C_{x+n-1}}{D_x} \tag{10.11.}$$

(We can see that the asymmetry of multiplying l_x with v^x but d_x with the form of a power one higher was necessary because of the assumption of "death at year end".)

The above equation is a little more simple than the original one, but not very much. Therefore, a new commutation number was introduced, M_x.

Let

$$M_x = C_x + C_{x+1} + \cdots + C_\omega \tag{10.12.}$$

where ω is the highest age regarded when constructing the life table (in Hungary this is 100 years).

We see that the relation of the above equation gives:

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95 On the other hand, nowadays, when premiums are calculated by computers, these standard symbols are very useful in programming, because they make it possible to design the premium calculation formulae in a "building block-like" manner, and this way the program becomes structured and clear-cut.

96 The aforesaid correction – putting the death benetif payment to the middle of the year from the end of the year – can happen the most simple way that we use instead of (10.9.) $C_x = d_x \cdot v^{x+\frac{1}{2}}$ form.

$$C_x + C_{x+1} + \cdots + C_{x+n-1} = M_x - M_{x+n}$$

(10.13.)

and so it can also be written in the form

$$A^1_{x:\overline{n}|} = \frac{M_x - M_{x+n}}{D_x} \tag{10.14.}$$

Let's look at other single premium insurances!

10.1.2. The Single Premium of Pure Endowment, Whole Life and Endowment Insurance

In respect of the **pure endowment insurance** our assumptions are similar as before. We suppose that l_x number of individuals of age *x* take out a single premium pure endowment policy of 1 Forint sum assured and *n* years term. We use similar symbols for the notation of the net single premium of the pure endowment insurance of 1 Forint sum assured and *n* years term as before:

$$A_{x:\overline{n}|}^{\;\;\;1}$$

The income side of the equivalence equation now is (because of similar considerations):

$$l_x \cdot A_{x:\overline{n}|}^{\;\;\;1}$$

(10.15.)

Since *n* years from now l_{x+n} individuals are assumed to be alive from the initial l_x, this way the expected pure endowment benefit payment will be:

$$l_{x+n} \cdot 1 = l_{x+n}$$

(10.16.)

Since this payment is due in exactly *n* years, the present value of the expected payout is:

$$l_{x+n} \cdot v^n$$

(10.17.)

So the equivalence equation is:

$$l_x \cdot A_{x:\overline{n}|}^{\;\;\;1} = l_{x+n} \cdot v^n$$

(10.18.)

From this:

$$A_{x:\overline{n}|}^{\;\;\;1} = \frac{l_{x+n} \cdot v^n}{l_x} \tag{10.19.}$$

From this, similarly as before, multiplying the numerator and the denominator by v^x we can obtain the premium defined with commutation numbers:

$$A_{x:\overline{n|}}^{\;\;1} = \frac{D_{x+n}}{D_x}$$

(10.20.)

As we know, from the technical point of view the **endowment insurance** is nothing but a term insurance plus a pure endowment insurance.

The single net premium of an endowment insurance with 1 Forint death and maturity sum assured, *n* years term, supposing an insured of age *x* is:

$$A_{x:\overline{n|}} = A_{x:\overline{n|}}^{1} + A_{x:\overline{n|}}^{\;\;1} \tag{10.21.}$$

The same formula using commutation numbers:

$$A_{x:\overline{n|}} = \frac{(M_x - M_{x+n}) + D_{x+n}}{D_x} \tag{10.22.}$$

$A_{x:\overline{n|}}$, contrary to $A_{x:\overline{n|}}^{1}$ and $A_{x:\overline{n|}}^{\;\;1}$ notations, notes not specifically the endowment insurance, so if we use it, we have to define in the text about which kind of insurance we are speaking.

The whole life insurance technically can be considered as a Term insurance with very long term. The term is so long, that during this the insured surely will die, so the insurance will end with benefit payment, namely death benefit payment. The "very long" term means, that it is at least ω-x+1 years, because ω the last possible age when the insured is still alive. That is why the single premium of the whole life insurance can be easily originated from the Term's single premium on the following way:

$$A_x = A_{x:\overline{\omega-x+1|}}^{1} = \frac{M_x - M_{\omega+1}}{D_x} = \frac{M_x}{D_x} \tag{10.23.}$$

In the case of whole life insurance, several appropriate terms are possible, so it is needless to note it separately. Therefore, the notation A_x surely notes a whole life insurance.

To the whole life insurance we can reach with a different train of thought. We can also say, that whole life is similar to endowment insurance, because both surely end up with benefit payment. Indeed, if we get an endowment insurance with a very long term, we get whole life insurance too, because the probability of maturity will be zero.

10.1.3. Single net premium of special insurances („staged" term, term fix, pure endowment with premium refund)

Above we have already discussed the single premiums of the most important classical life assurances, except term fix insurance. This is special in that sense, that the insurance protection here means – according to the definition of the product –, that the risk community will continue to pay the insurance premium after the death of the insured. In another words, this insurance is regular premium one already in the egg. At the same time, according to the logic of the premium calculation we originate the regular premium from the single premium, so we need the single premium of the term fix insurance too. This is the single premium, that in n years we surely get 1. The premium is obvious:

$$A_{\overline{n|}} = v^n \tag{10.24.}$$

simply the discounted value of 1. As this premium is independent from the age of the insured, it is needless to note it separately. But this is also the reason why the question, whether single premium term fix is really a life insurance does appear. Theoretically it is not, but technically the single premium of it is (10.24.), and we can use it for calculating regular premium.

Above this I would like to present the single premiums of two even more special, but quite familiar classical life assurances. The first is the "staged" Term insurance, the second is the pure endowment with premium refund.

The sum assured of the "staged" term insurance – which is also called as "credit" insurance (however the term "credit insurance" is a much broader category, than the "staged" Term insurance) – is decreasing in every year at the policy anniversary with the proportional part of the original term. That is, if it 1 at the beginning of the term, then it will change to the 1., 2. , … etc. anniversary according to the following:

$$\frac{n-1}{n}; \frac{n-2}{n}; \dots \frac{1}{n}$$

The change can be followed by the next graph:

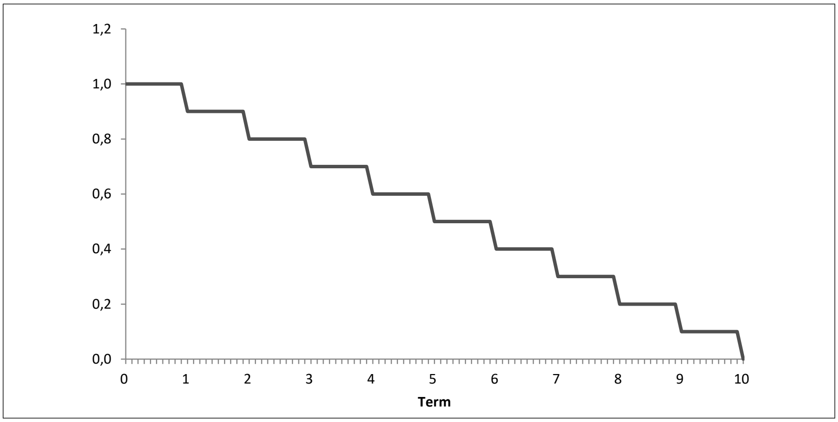

Figure 10.1.: The sum assured of the "staged" Term insurance and its change during the term (in case of n=10)

The insurance has earned the term “credit”, that it is mainly concluded together long term mortgages, and its initial sum assured is equal to the sum of the mortgage. In case of the death of the insured, the insurer pays the principal remained from the original mortgage, and it will cease. In another words it protects the relicts after the death of the “breadwinner” by making their flat unencumbered. The principal itself naturally decreases not by annual “staging”, because the amortization of mortgages almost exclusively monthly. And the decrease of the principal also depends on the interest rate as the following graph shows it:

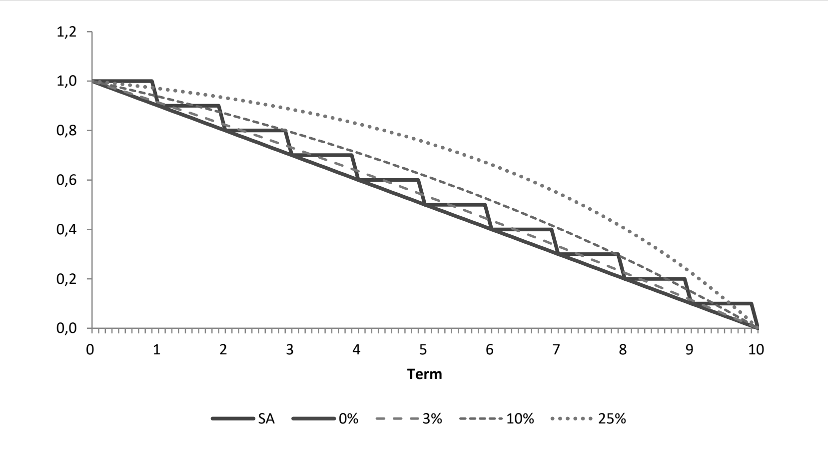

Figure 10.2.: Change of the principal of the mortgages during the term with different interest rates

From the graph we can see that in case of not too high (3-5%) interest rate, the principal mainly “fit” into the sum assured, but in case of high interest rates this kind of insurance is not answer well the purpose, however, in case of high interest rates morgages are not really offered, because in such circumstances it means incalculable burden for both lender and debtor on the long run.

On the basis of the abovementioneds we can write the equalence equation for the single premium of the “staged” term insurance on the following way:

$$l_x \cdot A_{x:\overline{n|}} = \frac{n}{n} \cdot d_x \cdot v^1 + \frac{n-1}{n} \cdot d_{x+1} \cdot v^2 + \cdots + \frac{1}{n} \cdot d_{x+n-1} \cdot v^n \quad (10.25.)$$

By making the reductions, and the rewriting the result onto the known commutation functions we get, that

$$A_{x:\overline{n|}} = \frac{\mathrm{n} \cdot C_x + (n-1) \cdot C_{x+1} + \cdots + 1 \cdot C_{x+n-1}}{n \cdot D_x} \quad (10.26.)$$

We can transform the numerator on the following way:

$$\mathrm{n} \cdot C_x + (n-1) \cdot C_{x+1} + \cdots + 1 \cdot C_{x+n-1} =$$
$$= (C_x + C_{x+1} + \cdots + C_{x+n-1}) + (C_x + C_{x+1} + \cdots + C_{x+n-2}) + \cdots + (C_x + C_{x+1}) + C_x =$$
$$= (M_x - M_{x+n}) + (M_x - M_{x+n-1}) + \cdots + (M_x - M_{x+1}) =$$
$$= \mathrm{n} \cdot M_x - (M_{x+1} + \cdots + M_{x+n})$$

(10.27.)

And here it is practical to introduce a new commutation function:

$$R_x = M_x + M_{x+1} + \cdots + M_\omega \tag{10.28.}$$

With the help of this we can write the final form of the single net premium:

$$A_{x:\overline{n|}} = \frac{\mathrm{n} \cdot M_x - (R_{x+1} - R_{x+n+1})}{n \cdot D_x} \tag{10.29.}$$

It is also worth to put the third line of the (10.27.) into (10.26.), and to interpret the result:

$$A_{x:\overline{n|}} = \frac{(M_x - M_{x+n}) + (M_x - M_{x+n-1}) + \cdots + (M_x - M_{x+1})}{n \cdot D_x} =$$
$$= \frac{1}{n} \cdot \frac{(M_x - M_{x+n})}{D_x} + \frac{1}{n} \cdot \frac{(M_x - M_{x+n-1})}{D_x} + \cdots + \frac{1}{n} \cdot \frac{(M_x - M_{x+1})}{D_x}$$

(10.30.)

We can see, that – according to (10.30.) – the net single premium of the “staged” Term insurance is the sum of the single premiums of n “normal” Term insurance with sum assured 1/n, but different terms (n, n-1, … and finally 1 years).

But if we rewrite (10.26.) by using, that

$$C_x = M_x - M_{x+1} \tag{10.31.}$$

then we get the following premium formula:

$$A_{x:\overline{n|}} = \frac{n}{n} \cdot \frac{M_x - M_{x+1}}{D_x} + \frac{n-1}{n} \cdot \frac{M_{x+1} - M_{x+2}}{D_x} + \cdots + \frac{n-1}{n} \cdot \frac{M_{x+n-1} - M_{x+\mathrm{n}}}{D_x} \tag{10.32.}$$

Here the interpretation of the parts of the equation is a little bit different. These are 1 year term special Term assurances with decreasing death sum assured. The speciality of them is, that somebody makes the contract at his/her x age, but they go into force after a 0, 1, …n-1 years "deferment period". In another words, if the insured dies during the "deferment period", then the insurer does not pay any benefit payment.

The single premium pure endowment with premium refund is a pure endowment with death benefit. In case of the death of the insured, the insurer pays back the (gross) premiums paid by the policy holder so far – so in case of the single premium version, the single gross premium. From this angle this can also be considered as a special endowment policy. But in fact it is only a "little bit" insurance, it is almost the same as a savings account. The difference of the two, that in case of death the beneficiary does not get interest. That is why the policy can be considered as the "simulation" of a non-insurance product.

In the premium calculation – compare to the logic of the classical premium calculation described above – there is a "twist", namely we need the gross premium already at the first step, when we calculate the net premium, but the calculation of the gross premium happens in the last step from the net premium.

If we note the gross single premium by AG, then we get the following equation for the single premium pure endowment with premium refund (which is a special endowment policy):

$$A_{x:\overline{n|}} = AG_{x:\overline{n|}} \cdot A^{1}_{x:\overline{n|}} + A_{x:\overset{1}{\overline{n|}}} \tag{10.33.}$$

The classical premium calculation technique becomes uneasy at this insurance, because we have to anticipate, the difference of the gross and net premium. For this kind of calculation, the modern profit testing technique is more appropriate, because with the classical method it is hard to calculate the gross premium of this policy.

Further problem, that the pure endowment with premium refund is in almost every occasion a regular premium policy. But the death benefit of the regular premium pure endowment with premium refund will be different than the single premium version of it, so we can not use (10.33.). Therefore, the regular premium pure endowment with premium refund causes further problems in the logic of classical premium calculation, so later we will revisit later this topic.

10.1.4. Joint Life Single Premium Insurance

Almost all types of insurances have a two or more person version, as we have seen in case of annuities. Now, and in the further discussion we will only deal with two person insurances, and within these only the term insurance and the pure endowment insurance. We are on the opinion that, on the one hand the relations of the other two person insurances – if necessary – can be derived in an analogous way from the relations of single life insurances, and on the other hand from the practical point of view the relations of single life insurances are much more important, since currently mostly these dominate the market.

In case of two lives we simply regard as death the death of either of the two insured persons (i.e. the first death), and as living until maturity if both insured persons are alive at the end of the term.

In case of the **two person term insurance** we try to think the following way: if all possible couples of ages *x* and *y* take out the policy of *n* years term and 1 Forint sum assured, then the expected value of benefits paid by the insurer yearly will be:

$$\left(l_x \cdot l_y - l_{x+1} \cdot l_{y+1}\right); \left(l_{x+1} \cdot l_{y+1} - l_{x+2} \cdot l_{y+2}\right); \dots; \left(l_{x+n-1} \cdot l_{y+n-1} - l_{x+n} \cdot l_{y+n}\right) \tag{10.34.}$$

If we denote the single premium in question by $A^1_{xy:\overline{n|}}$, then the equivalence equation is:

$$l_x \cdot l_y \cdot A^1_{xy:\overline{n|}} = v^1 \cdot \left(l_x \cdot l_y - l_{x+1} \cdot l_{y+1}\right) +$$
$$+v^2 \cdot \left(l_{x+1} \cdot l_{y+1} - l_{x+2} \cdot l_{y+2}\right) + \dots + v^n \cdot \left(l_{x+n-1} \cdot l_{y+n-1} - l_{x+n} \cdot l_{y+n}\right) \tag{10.35.}$$

Which gives:

$$A^1_{xy:\overline{n|}} =$$
$$= \frac{v^1 \cdot \left(l_x \cdot l_y - l_{x+1} \cdot l_{y+1}\right) + v^2 \cdot \left(l_{x+1} \cdot l_{y+1} - l_{x+2} \cdot l_{y+2}\right) + \dots + v^n \cdot \left(l_{x+n-1} \cdot l_{y+n-1} - l_{x+n} \cdot l_{y+n}\right)}{l_x \cdot l_y} \tag{10.36.}$$

Multiplying by v^x, and using the two person commutation numbers, the already (in single life version) introduced D_{xy} and (the not yet introduced) N_{xy} introduced earlier we get:

$$D_{xy} \cdot A^1_{xy:\overline{n|}} = \mathrm{v} \cdot \left(D_{xy} + D_{(x+1)(y+1)} + \dots + D_{(x+n-1)(y+n-1)}\right) - \left(D_{(x+1)(y+1)} + D_{(x+2)(y+2)} + \dots + D_{(x+n)(y+n)}\right) =$$
$$= \mathrm{v} \cdot \left(N_{xy} - N_{(x+n)(y+n)}\right) - \left(N_{(x+1)(y+1)} - N_{(x+n+1)(y+n+1)}\right) \tag{10.37.}$$

Where $D_{xy} = l_x \cdot l_y \cdot v^x$, and $N_{xy} = D_{xy} + D_{(x+1)(y+1)} + \dots + D_{(x+n-1)(y+n-1)} + \cdots$
From this (by anticipating the premium of annuities):

$$A^{1}_{xy:\overline{n|}} = \frac{\mathrm{v} \cdot \left(N_{xy} - N_{(x+n)(y+n)}\right) - \left(N_{(x+1)(y+1)} - N_{(x+n+1)(y+n+1)}\right)}{D_{xy}} = \mathrm{v} \cdot \ddot{a}_{xy:\overline{n|}} - a_{xy:\overline{n|}}$$

(10.38.)

The equivalence equation of the **two person pure endowment insurance** is:

$$l_x \cdot l_y \cdot A_{xy:\overline{n|}}^{\;\;\;\;1} = v^{\mathrm{n}} \cdot l_{x+n} \cdot l_{y+n}$$

(10.39.)

So:

$$A_{xy:\overline{n|}}^{\;\;\;\;1} = \frac{v^{\mathrm{n}} \cdot l_{x+n} \cdot l_{y+n}}{l_x \cdot l_y}$$

(10.40.)

Using commutation numbers:

$$A_{xy:\overline{n|}}^{\;\;\;\;1} = \frac{D_{(x+n)(y+n)}}{D_{xy}}$$

(10.41.)

10.2. Single net Premium of Annuities

Annuities can be paid in advance (annuity-due) or in arrears (annuity in arrears). This classification is purely technical. In case of annuity-due the insurer provides the annuity payment at the beginning of each insurance period, in case of the arrears payment it is paid at the end of the insurance period. In advance and in arrears annuity payments only differ when the annuity commences, since the advance payment provides the first payment at once, while the arrears only pays one insurance period later. So, if we take away the first payment from the advance version, we get the arrears annuity – at least in the case of lifetime annuities (in case of temporary annuities there is a further outpayment at the end of the term, if the insured is alive). Because of this, later on we will only deal with the annuity paid in advance, and if not stated explicitly otherwise, all discussion refers to this case.

In the title of the subchapter we are speaking about "single" premium, but later we do not discuss "regular premium" life annuities. It can be said, that annuities are "ab ovo" single premium products, because is seems pointless, that somebody is getting the annuity in the same time when he/she is paying the premium. (At least in the majority of cases, when the policy holder, the insured and the beneficiary are the same. If the policy holder and the insured/beneficiary are different persons, then this solution has point, but this kind of contracts are made only in special circumstances.) Namely – in the majority of cases – the insurer gets the whole premium before the inception of the annuity, so for them this product is a single premium policy. Naturally, it is possible to pay the single premium from the sum assured of a regular premium insurance at the same insurer, so the policy holder can feel the annuity as a regular premium one. But in this case the two stages (accumulation and decumulation) of the annuity is clearly separated to each other. And in the majority of cases the insurer handles the two stages with the help of different products. The reason is simple: it does not want to be engaged in point of annuity premiums already at the beginning of the accumulation period. Namely the premiums probably will increase because of the continuous increase of the life expectancy ("longevity").

10.2.1. The Premium of Immediate Lifetime Annuity

As mentioned earlier, this is the most important annuity type in a sense, since many other annuity types can be derived from this one.

Let $\ddot{a}_x$ be the single net premium of the immediate lifetime annuity with 1 Forint annuity payment paid in advance, if the entry age of the insured is *x* years.

When deducting the premium formula, we apply once more the suppositions used earlier in the chapter. According to this the income side of the equation is:

$$l_x \cdot \ddot{a}_x \tag{10.42.}$$

Since the first annuity payment is immediately due to the living, which means all insured, so the expected value of the first payment is:

$$l_x \cdot 1 = l_x \tag{10.43.}$$

One year later only l_{x+1}, and two years later only l_{x+2} insured will be alive. This way the expected value of the second payment is l_{x+1} Forints, the third is l_{x+2} Forints, etc… Discounting the payments the payout side of the equivalence equation will be:

$$l_x + l_{x+1} \cdot v^1 + l_{x+2} \cdot v^2 + \cdots + l_\omega \cdot v^{\omega-x} \tag{10.44.}$$

From this we get the following relation for the net premium:

$$\ddot{a}_x = \frac{l_x + l_{x+1} \cdot v^1 + l_{x+2} \cdot v^2 + \cdots + l_\omega \cdot v^{\omega-x}}{l_x} \tag{10.45.}$$

Using commutation numbers and multiplying by v^x we get:

$$ä_x = \frac{D_x + D_{x+1} + D_{x+2} + \cdots + D_\omega}{D_x} \tag{10.46.}$$

Here we have to face the same problem as before, namely that the formula is still too long. This way we introduce the commutation number N_x. The definition is:

$$N_x = D_x + D_{x+1} + D_{x+2} + \cdots + D_\omega \tag{10.47.}$$

Using N_x we can formulate the equation of $ä_x$ in a very simple form:

$$ä_x = \frac{N_x}{D_x} \tag{10.48.}$$

The formula of lifetime annuity in arrears (which is denoted simply by „a”) is simple on the basis of the above mentioneds:

$$a_x = \frac{D_{x+1} + D_{x+2} + \cdots + D_\omega}{D_x} = \frac{N_{x+1}}{D_x} \tag{10.49.}$$

The difference of the two

$$ä_x - a_x = \frac{D_x}{D_x} = 1 \tag{10.50.}$$

so these differ only in the first outpayment.

10.2.2. The Premium of Deferred Lifetime Annuity

The deferred annuity differs from the immediate type in that the first payment is due not at the commencement of the insurance, but after the deferred period of *m* years, if the insured is still alive at that time. (If the insured dies during the deferred phase, the insurance ends without any benefit payment.)

Let $_{m|}ä_x$ be the single net premium of a lifetime annuity paying 1 Forint yearly in advance, after a deferred phase of *m* years, in case of an insured of age *x*.

The payout side of the equivalence equation differs from the right side of the previous equation in that the first payment will be paid in *m* years, only to l_{x+m} number of insured, who are still alive at that time. So, the equation serving as a theoretical basis of the premium calculation can be written in the following form:

$$l_x \cdot {}_{m|}\ddot{a}_x = l_{x+\mathrm{m}} \cdot v^m + l_{x+\mathrm{m}+1} \cdot v^{m+1} + \cdots + l_\omega \cdot v^{\omega-x}$$

(10.51.)

Using commutation numbers:

$$_{m|}\ddot{a}_x = \frac{D_{x+\mathrm{m}} + D_{x+\mathrm{m}+1} + \cdots + D_\omega}{D_x} \quad (10.52.)$$

Or in a different form:

$$_{m|}\ddot{a}_x = \frac{N_{x+\mathrm{m}}}{D_x} \quad (10.53.)$$

We could also have derived this result through a different train of thought. According to this, the deferred lifetime annuity is the combination of a pure endowment insurance of n years term, and an immediate annuity (*m* years from now), where the sum assured of the pure endowment insurance serves as the single premium of the immediate lifetime annuity.

This way, the sum assured of the pure endowment insurance has to be $\ddot{a}_{x+m}$, since the insured who currently is *x* years old will be *x*+*m* years old *m* years from now, and the single net premium of the immediate lifetime annuity starting then will be $\ddot{a}_{x+m}$. The single net premium of the pure endowment insurance with 1 Forint sum assured and *m* years term – as we now – is:

$$A_{\substack{\quad 1 \\ x:\overline{m|}}} = \frac{D_{x+m}}{D_x} \quad (10.54.)$$

So we get:

$$_{m|}\ddot{a}_x = A_{\substack{\quad 1 \\ x:\overline{m|}}} \cdot \ddot{a}_{x+m}$$

(10.55.)

Since we know that

$$\ddot{a}_{x+m} = \frac{N_{x+m}}{D_{x+m}}$$

substituting into the above equation we get:

$$_{m|}\ddot{a}_x = \frac{D_{x+m}}{D_x} \cdot \frac{N_{x+m}}{D_{x+m}} = \frac{N_{x+\mathrm{m}}}{D_x} \quad (10.56.)$$

because, we can see that the two D_{x+m} factors cancel each other out.

With the aid of deferred annuities we can also form a relation between annuities with payments in advance and in arrears. If we think about it, we discover that the annuity with payment in arrears is an annuity paying in advance with a deferred phase of 1 year, so:

$$a_x = {}_{1|}\ddot{a}_x \tag{10.57.}$$

We have already got to know, that the difference between the annuity deferred for 1 year and the immediate annuity is that the payment of 1 Forint is missing at the beginning of the term. This way:

$$a_x = {}_{1|}\ddot{a}_x = \ddot{a}_x - 1 \tag{10.58.}$$

10.2.3. The Premium of the Temporary Annuity

The temporary annuity differs from the "simple" lifetime annuity in that payments stop after a certain period under all circumstances, so the insured receives the annuity until alive, but only until the end of the term of *n* years.

Let the single net premium of the temporary annuity with 1 Forint yearly payment and n years term in case of an insured of entry age x be: $\ddot{a}_{x:\overline{n|}}$

The temporary annuity can be derived from the "simple" lifetime annuity very easily. Think about it! The temporary annuity paying for *n* years is the difference of an immediate lifetime annuity and a lifetime annuity deferred by *n* years:

The immediate lifetime annuity with *n* years term

Year	0	1	2	...	n-1	n	n+1	n+2	...
Immediate annuity	1	1	1	...	1	1	1	1	...
Annuity deferred for *n* years	-	-	-	...	-	1	1	1	...
Difference: immediate, *n* years term temporary annuity	1	1	1	...	1	-	-	-	...

Table 10.1.: Benefits received during the term by different types of annuities

Based on this we can also formulate the net premium of the temporary annuity for *n* years:

$$\ddot{a}_{x:\overline{n|}} = \ddot{a}_x - {}_{n|}\ddot{a}_x \tag{10.59.}$$

The same formula with commutation numbers:

$$ä_{x:\overline{n|}} = \frac{N_x}{D_x} - \frac{N_{x+n}}{D_x} = \frac{N_x - N_{x+n}}{D_x} \tag{10.60.}$$

If we recede one step, and we use D instead of N, then we get, that:

$$ä_{x:\overline{n|}} = \frac{D_x + D_{x+1} + \cdots + D_{x+n-1}}{D_x} \tag{10.61.}$$

In case of annuity in arrears, the formula is consistent with reason

$$a_{x:\overline{n|}} = \frac{D_{x+1} + D_{x+1} + \cdots + D_{x+n}}{D_x} \tag{10.62.}$$

Out of this it is already clear, that the difference of annuity-due and annuity in arrears is:

$$ä_{x:\overline{n|}} - a_{x:\overline{n|}} = \frac{D_x - D_{x+n}}{D_x} = 1 - A_{x:\overline{n|}}^{\;\;1} \tag{10.63.}$$

of which the limit in case of n→∞, i.e. in case of lifetime annuity is the already discussed 1.

10.2.4. Certain annuities

Certain annuities cannot be considered insurance in a strict sense, since the fundamental feature of every insurance is that the benefits, or the degree of benefits provided by the insurer depend on the occurrence or non-occurrence of some random event. In case of this annuity, there is no such random event influencing the existence, the degree or the duration of the benefits paid by the insurer. Certain annuity means that for a specified period, an annuity with specified payment or a payment varying according to specified rules will certainly be paid to the insured or the inheritor of the insured (beneficiary). It is important to talk about them all the same, because it can be an important complementing element of annuities and other types of insurance.

Most of the above categories can be applied also to the certain annuity. Accordingly, we can talk about certain annuities paid in advance or in arrears, immediate or deferred, temporary (paid for a certain period of time) and … not "lifetime" annuity – since the payment does not depend on the fact that the insured is alive or not –, but instead of this "infinite" annuity, which means that theoretically it is to be paid by the insurer forever to somebody (the current owner of the annuity).

We denote the net premium of the certain annuity with a and ä, like the life annuities. The difference is clear, because in case annuity certains we do not note the age. The number in the right subscript means the term, emphasized by a "bend". So

$\ddot{a}_{\overline{n|}}$: is the single net premium of the certain annuity of 1 Forint yearly paid in advance, if the term is n years,

$\ddot{a}_{\overline{\infty|}}$: is the single net premium of the certain infinite annuity of 1 Forint yearly paid in advance.

According to the above discussions it is easy to see that:

$$\ddot{a}_{\overline{n|}} = v^0 + v^1 + v^2 + \cdots + v^{n-1} = \frac{1 - v^n}{1 - v} \tag{10.64.}$$

It can be proved that the infinite annuity is derived from the temporary annuity by increasing its term to infinity, choosing n to be infinite. If $\lim_{n \to \infty} v^n = 0$. So, formula 10.64. is transformed to the following form:

$$\ddot{a}_{\overline{\infty|}} = v^0 + v^1 + v^2 + \cdots + v^{n-1} + \cdots = \frac{1}{1 - v} \tag{10.65.}$$

The annuity in arrears versions are

$$a_{\overline{n|}} = v^1 + v^2 + \cdots + v^n = v \cdot \frac{1 - v^n}{1 - v} \tag{10.66.}$$

and

$$a_{\overline{\infty|}} = v^1 + v^2 + \cdots + v^n + \cdots = v \cdot \frac{1}{1 - v} = \frac{1}{i} \tag{10.67.}$$

because $v = \frac{1}{1+i}$, so $1 - v = \frac{i}{1+i}$.

10.2.5. Annuity with Guarantee Period

Annuity with guarantee period is the connecting key on the one hand between lifetime annuities and certain annuities, and on the other hand between single life annuities and joint life annuities, which will be discussed in the next chapter. The guarantee period means that for a certain time period the insurer pays the annuity to the inheritor of the insured or another person declared by the policyholder, even if the insured dies during this period. So, we can say that the annuity with guarantee period is a joint life annuity concealed in the form of a single life annuity, or that the annuity with guarantee period is a certain type of “widows’ annuity” insurance (that will be discussed later).

The guarantee period can theoretically have two forms:

- guarantee period at the beginning: where the insurer guarantees that starting from thecommencement of the annuity for a period of g years (the guarantee period) it will be paid certainly, even if the insured dies before the end of year g.
- guarantee period at the end: where After the insured’s death the insurer will pay the annuity for exactly g years.

The guarantee period can be used to loosen certain inhibitions that are in the way of signing the annuity policy. One of these inhibitions is that someone pays 1 Million Forints as the single premium of an annuity, and receives in exchange, say 10 thousand Forints while alive. But if death occurs in the first month, then – looking at it subjectively, from the insured's point of view – 990 thousand Forints remain at the insurer, so this money was needlessly "thrown out the window". But in a case like this a guarantee period of 5 years means that someone (the insured or the declared beneficiary) will certainly receive 60 months of annuity payment, that is, 600 thousand Forints (disregarding interest and discounting) from the insurer. This way the subjective uncertainty feeling of possibly throwing money out the window by purchasing the annuity insurance is considerably subdued.

We can also explain the situation without referring to subjective factors, but a kind of objective uncertainty. There are a lot of people, who applies a kind of preference order: first they spent their money for themselfes, second they would leave the rest to their children. The ratio, which is unknown in advance, is determined by the the actual lifetime – posteriorly. Therefore, they would like to postpone annuitization, partly because by it they surely have to give up bequeathing and partly because they do not want to run out of money if they would live for too long. The annuities with guarantee period make a kind of balance between these two motivations.

Let

$^{|g}\ddot{a}_x$:[97] be the single net premium of the immediate lifetime annuity of 1 Forint yearly payment in advance, with a guarantee period of *g* years, supposing an insured of *x* years,

The same way let

$^{g|}\ddot{a}_x$: be the net single premium of the life annuity with guarantee period at the end with similar parameters.

What are these single premiums?

If we think about it, then the annuity with guarantee period at the beginning is the sum of an immediate temporary certain annuity with a term of *g* years, and a deferred lifetime annuity with *g* years deferred period, so the premium in question is:

$^{|g}\ddot{a}_x$= the premium of an immediate certain temporary annuity with term *g* years + the premium of a deferred lifetime annuity with *g* years deferred period

$$^{|g}\ddot{a}_x = \ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x \tag{10.68.}$$

[97] The notations of the annuities with guarantee period is not standard, the problem not appeared in Paris in the late19th century.

Or using commutation numbers:

$$^{|g}\ddot{a}_x = \frac{1-v^g}{1-v} + \frac{N_{x+g}}{D_x} \tag{10.69.}$$

The same way we can discover that the annuity with guarantee period at the end is an immediate certain annuity with term g and an "immediate" annuity having the interesting feature that the insurer pays every payment only g years after its due date. (So the last payment, that the insured would receive immediately before his death is only paid to the beneficiary g years after that.) The role of the certain annuity here naturally is that the insured receives the annuity payments also during the g years before actually receiving the first real annuity payment. So, the premium calculation:

$$^{g|}\ddot{a}_x = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x \tag{10.70.}$$

Using commutation numbers:

$$^{g|}\ddot{a}_x = \frac{1-v^g}{1-v} + v^g \cdot \frac{N_x}{D_x} \tag{10.71.}$$

In the above formula the g year shift of paying the annuity payments is indicated by the v^g discount factor.

Naturally the annuity with guarantee period at the end can also be regarded in a way that the annuity at the end of the term is a kind of death benefit (the death benefit of a whole life insurance!), that the beneficiary receives in the form of an annuity. The present value of this annuity, when it begins is exactly $\ddot{a}_{\overline{g|}}$, so then the annuity with guarantee period is the sum of a "simple" annuity and a whole life insurance:

$$^{g|}\ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x = \frac{N_x}{D_x} + \frac{1-v^g}{1-v} \cdot \frac{M_x}{D_x} \tag{10.72.}$$

10.71. and 10.72. are equivalent with each other. To prove this we use the following relation between commutation numbers:

$$C_x = d_x \cdot v^{x+1} = (l_x - l_{x+1}) \cdot v^{x+1} = D_x \cdot v - D_{x+1} \tag{10.73.}$$

From this:

$$M_x = \sum_{k=x}^{\omega} C_k = v \cdot \sum_{k=x}^{\omega} D_k - \sum_{k=x}^{\omega} D_{k+1} = N_x \cdot v - N_{x+1} \qquad (10.74.)$$ [98]

10.74. can be transformed into a more convenient form, with the help of (10.75.):

$$N_{x+1} = N_x - D_x \qquad (10.75.)$$

Substituting 10.75. into 10.72. we get:

$$^{g|}\ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x = \frac{N_x}{D_x} + \ddot{a}_{\overline{g|}} \cdot \frac{M_x}{D_x} = \frac{N_x}{D_x} + \ddot{a}_{\overline{g|}} \cdot \frac{N_x \cdot v - (N_x - D_x)}{D_x} =$$

$$= \frac{N_x}{D_x} + \ddot{a}_{\overline{g|}} \cdot \frac{N_x \cdot (\mathrm{v} - 1)}{D_x} + \ddot{a}_{\overline{g|}} = \ddot{a}_{\overline{g|}} + \frac{N_x}{D_x} + \frac{1 - v^g}{1 - v} \cdot \frac{N_x \cdot (\mathrm{v} - 1)}{D_x} = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x \qquad (10.76.)$$

This is exactly 10.70..

10.2.6. The Premium of Joint Life Annuities

The same way as in case of other types of insurances, we can also talk about single life and multiple life annuities, that is, insurances, where there is not only one insured, but two or more. In the following we will only consider the premium calculation of one type, the single premium of the two person, immediate life annuity.

In case of the two person, immediate annuity, the same way as in the case of other two person insurances, death means the death of the insured to die first. (It may be strange that payments last only until the first death, although it is needed much more

[98] As a supplementary conclusion we can say, that the net single premium of the Term insurance is

$$A^1_{x:\overline{n|}} = \frac{M_x - M_{x+n}}{D_x} = \frac{N_x \cdot v - N_{x+1} - (N_{x+n} \cdot v - N_{x+n+1})}{D_x} =$$

$$= v \cdot \ddot{a}_{x:\overline{n|}} - a_{x:\overline{n|}}$$

similarly the situation discussed at the joint life insurances. The meaning of the equation: all the outpayments of the annuity-due are “shifted” by one year from the due time (the „v” shows this). If somebody reaches the end of the first year alive, then he/she would get the “shifted” outpayments of the annuity-due, but he/she also immediately has to pay it back, because of the negative annuity in arrears. This is happening every year, unless the insured dies. At the end of that year the outpayment of the annuity-due will be due without the obligation to pay it back since the annuity in arrears will cease.

after the death of the spouse than before that. But this annuity type only serves technical purposes, and the premium of most of the “real” two person, joint life annuities can be easily derived from this one.) According to this, the insurer pays the annuity payments in case of a two person immediate annuity until both insured persons are alive.

Let $\ddot{a}_{xy}$ denote the single net premium of the two person, immediate life annuity with 1 Forint yearly payment supposing that the insured persons are x and y years old.

When calculating $\ddot{a}_{xy}$, we start from the supposition that all possible couples of years x and y take out the above policy, which means $l_x l_y$ number of couples all together. Every couple receives the first annuity payment, so then the payout of the insurer is $l_x l_y$ Forints.[99] The second annuity payment goes only to those couples, where both are still alive a year later, which means $l_{x+1} l_{y+1}$ number of couples, and so on. So, the equivalence equation is:

$$l_x \cdot l_y \cdot \ddot{a}_{xy} = l_x \cdot l_y + l_{x+1} \cdot l_{y+1} \cdot v^1 + \cdots \tag{10.77.}$$

From this:

$$\ddot{a}_{xy} = \frac{l_x \cdot l_y + l_{x+1} \cdot l_{y+1} \cdot v^1 + \cdots}{l_x \cdot l_y} \tag{10.78.}$$

If we multiply both sides of the equation with v^x, then we can use the commutation numbers for two lives.

By noted $D_x \cdot l_y$ simply with D_{xy}, we can write the above equation in the form:

$$D_{xy} \cdot \ddot{a}_{xy} = D_{xy} + D_{(x+1)(y+1)} + \cdots \tag{10.79.}$$

The above formula can be significantly simplified if we introduce also the N commutation number for two lives. The definition of this is:

$$N_{xy} = D_{xy} + D_{(x+1)(y+1)} + \cdots \tag{10.80.}$$

Then the above equation takes the following form:

$$D_{xy} \cdot \ddot{a}_{xy} = N_{xy} \tag{10.81.}$$

so

$$\ddot{a}_{xy} = \frac{N_{xy}}{D_{xy}} \tag{10.82.}$$

We see that the joint life annuity is totally analogous to the single life annuity.

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[99] We can see that the implicit assumption behind this, that the mortality of the couple is independent of each other. Naturally, this is not necessarily true, but modelling the connection is difficult. This is the “classical” approach, which is simple, but sometimes not totally accurate.

10.2.7. The Premium of Annuities in p Payments Yearly

In case of all the above derived annuities we supposed that the payment is due once every (insurance) year, in one instalment, at the beginning of the year. But in real life people need annuities where payment is performed not once yearly, but more often, say 12 times (i.e. monthly). So, in the following section we show through a few examples how the above premium formulae change if we suppose not 1, but *p* number of instalments yearly. We are still supposing the payment of 1 Forint yearly, but this is performed in *p* number of instalments, so every time the insured receives *1/p* Forints.

Let's determine the single net premium of an annuity with 1 Forint yearly payment in *p* instalments, for an insured of *x* years! The notation of the single premium in this case is: $\ddot{a}_x^{(p)}$.

$\ddot{a}_x^{(p)}$ could be determined exactly if we knew a life table where the "distance" between neighbouring age categories is not 1 year, but *1/p* years. This can naturally be constructed from the existing mortality table by interpolation, but this already puts approximate values in the place of the exact ones. Below we review a method, or a formula of this kind, but first we'll derive based on simple (not totally correct) logic a simple approximate formula that can be easily and well applied in practice.

The method is based on the analogy of annuities paid in advance and in arrears. As we have seen in 10.58.:

$$\ddot{a}_x - a_x = 1 \tag{10.83.}$$

so the difference between the single net premium of the in advance and the in arrears case is 1, i.e. the annuity paid in arrears differs from the one paid in advance in that payment starts 1 year later than in the other case. From this we can deduct that

$$\ddot{a}_x - \frac{\mathrm{m}}{p} \tag{10.84.}$$

is the single premium of the annuity, where the yearly payment is due *m/p* year(-fraction) after the commencement of the insurance year[100]. It is clear that the possible values of *m* are: 0,1,2,..., *p*-1. If we substitute these *m* values into the above expression one after the other, and then add them up, we get the single premium of an annuity where the insured receives 1 Forint annuity payment at the beginning of every *1/p* year-fraction, so all together *p* Forints in the year. This will be the single premium of an annuity with *p* Forints yearly payment, but paid in *p* number of instalments yearly, so the formula is:

[100] Naturally this is not the exact supposition, but it is not too far from reality!

$$p \cdot \ddot{a}_x^{(p)} = \ddot{a}_x + \left(\ddot{a}_x - \frac{1}{p}\right) + \left(\ddot{a}_x - \frac{2}{p}\right) + \cdots + \left(\ddot{a}_x - \frac{\mathrm{p}-1}{p}\right) =$$

$$= p \cdot \ddot{a}_x - \frac{1 + 2 + \cdots + (p-1)}{p} = p \cdot \ddot{a}_x - \frac{p \cdot (p-1)}{2 \cdot p} = p \cdot \ddot{a}_x - \frac{p-1}{2}$$

(10.85.)

i.e.:

$$\ddot{a}_x^{(p)} = \ddot{a}_x - \frac{p-1}{2 \cdot p}$$

(10.86.)

So in case of monthly payment (when p=12, so the monthly annuity payment is 1/12 Forints) the net premium is:

$$\ddot{a}_x^{(12)} = \ddot{a}_x - \frac{11}{24}$$

(10.87.)

i.e. in case of monthly annuity the premium is somewhat lower (by 11/24 Forints) than the premium of the annuity paying the total 1 Forint yearly payment in one instalment at the beginning of the year.

Now let's look at the case of instalment payments within the year in case of annuities paid for a certain term (temporary annuities).

We know that the net premium of the temporary annuity of n years term is

$$\ddot{a}_{x:\overline{n|}} = \ddot{a}_x - {}_{n|}\ddot{a}_x \qquad (10.88.)$$

and, on the basis of (10.55.) we know, that the deferred annuity is the combination of a pure endowment insurance and an immediate annuity.

Using commutation numbers it is easy to see that:

$${}_{n|}\ddot{a}_x = \frac{N_{x+n}}{D_x} = \frac{N_{x+n}}{D_{x+n}} \cdot \frac{D_{x+n}}{D_x} = A_{x:\overset{1}{\overline{n|}}} \cdot \ddot{a}_{x+n}$$

So the temporary annuity can be written in the form:

$$\ddot{a}_{x:\overline{n|}} = \ddot{a}_x - A_{x:\overset{1}{\overline{n|}}} \cdot \ddot{a}_{x+n} \qquad (10.89.)$$

From this it follows that if we pay the temporary annuity in p number of yearly instalments, then we get the following relation on the net premium:

$$\ddot{a}^{(p)}_{x:\overline{n|}} = \ddot{a}^{(p)}_{x} - A_{x:\overline{n|}}^{\ \ 1} \cdot \ddot{a}^{(p)}_{x+n} =$$

$$= \ddot{a}_x - \frac{p-1}{2\cdot p} - A_{x:\overline{n|}}^{\ \ 1} \cdot \left(\ddot{a}_{x+n} - \frac{p-1}{2\cdot p}\right) =$$

$$= \ddot{a}_x - A_{x:\overline{n|}}^{\ \ 1} \cdot \ddot{a}_{x+n} - \frac{p-1}{2\cdot p} + A_{x:\overline{n|}}^{\ \ 1} \cdot \frac{p-1}{2\cdot p} =$$

$$= \ddot{a}_x - A_{x:\overline{n|}}^{\ \ 1} \cdot \ddot{a}_{x+n} - \frac{p-1}{2\cdot p} + A_{x:\overline{n|}}^{\ \ 1} \cdot \frac{p-1}{2\cdot p} =$$

$$= \ddot{a}_{x:\overline{n|}} - \frac{p-1}{2\cdot p} \cdot \left(1 - A_{x:\overline{n|}}^{\ \ 1}\right)$$

(10.90.)

so

$$\ddot{a}^{(p)}_{x:\overline{n|}} = \ddot{a}_{x:\overline{n|}} - \frac{p-1}{2\cdot p} \cdot \left(1 - A_{x:\overline{n|}}^{\ \ 1}\right)$$

(10.91.)

We would conclude to the same result, if we start instead of (10.83.) with (10.63.), and follow the logic of (10.84-86.).

Based on the above pattern, the version of instalment payments within the year can be derived in case of other types of annuities, too.

And now let's look at a more precise approach! As we have already stated, the l_x-s within the year are constructed by interpolation:

$$l_{x+\frac{k}{p}} = \left(1 - \frac{k}{p}\right) \cdot l_x + \frac{k}{p} \cdot l_{x+1} \qquad (10.92.)$$

For the sake of precision, we are using compound interest within the year.

Then, the expected present value of the p number of annuity payments in the year *t* for the starting l_x lives at the beginning of year t is:

$$p \cdot E^{(p)}_{x+t} = \sum_{k=0}^{p-1} l_{x+\frac{k}{p}} \cdot v^{\frac{k}{p}} = \sum_{k=0}^{p-1} \left(\left(1 - \frac{k}{p}\right) \cdot l_{x+t} + \frac{k}{p} \cdot l_{x+t+1}\right) \cdot v^{\frac{k}{p}} =$$
$$= l_{x+t} \cdot \sum_{k=0}^{p-1} \left(1 - \frac{k}{p}\right) \cdot v^{\frac{k}{p}} + l_{x+t+1} \cdot \sum_{k=0}^{p-1} \frac{k}{p} \cdot v^{\frac{k}{p}}$$

(10.93.)

The equivalence equation is:

$$l_x \cdot \ddot{a}^{(p)}_{x:\overline{n|}} = \sum_{t=0}^{n-1} E^{(p)}_{x+t} \cdot v^t = \frac{1}{p} \cdot \sum_{t=0}^{n-1} \left(l_{x+t} \cdot \sum_{k=0}^{p-1} \left(1 - \frac{k}{p}\right) \cdot v^{\frac{k}{p}} + l_{x+t+1} \cdot \sum_{k=0}^{p-1} \frac{k}{p} \cdot v^{\frac{k}{p}} \right) \cdot v^t =$$
$$= \frac{1}{p} \cdot \sum_{k=0}^{p-1} \left(1 - \frac{k}{p}\right) \cdot v^{\frac{k}{p}} \cdot \sum_{t=0}^{n-1} l_{x+t} \cdot v^t + \frac{1}{p} \cdot \sum_{k=0}^{p-1} \frac{k}{p} \cdot v^{\frac{k}{p}} \cdot \sum_{t=0}^{n-1} l_{x+t+1} \cdot v^t$$

(10.94.)

Since:

$$\frac{\sum_{t=0}^{n-1} l_{x+t} \cdot v^t}{l_x} = \ddot{a}_{x:\overline{n|}}$$

(10.95.)

and

$$\frac{\sum_{t=0}^{n-1} l_{x+t+1} \cdot v^t}{l_x} = \frac{\sum_{t=0}^{n-1} l_{x+t+1} \cdot v^t}{l_{x+1}} \cdot \frac{l_{x+1} \cdot v}{l_x} \cdot (1+i) = \ddot{a}_{x+1:\overline{n|}} \cdot A_{x:\overset{1}{\overline{1|}}} \cdot (1+i)$$

(10.96.)

so

$$\ddot{a}^{(p)}_{x:\overline{n|}} = \ddot{a}_{x:\overline{n|}} \cdot \frac{\sum_{k=0}^{p-1} \left(1 - \frac{k}{p}\right) \cdot v^{\frac{k}{p}}}{p} + \ddot{a}_{x+1:\overline{n|}} \cdot A_{x:\overset{1}{\overline{1|}}} \cdot (1+i) \cdot \frac{\sum_{k=0}^{p-1} \frac{k}{p} \cdot v^{\frac{k}{p}}}{p}$$

(10.97.)

The same for the lifetime annuity is:

$$\ddot{a}^{(p)}_x = \ddot{a}_x \cdot \frac{\sum_{k=0}^{p-1} \left(1 - \frac{k}{p}\right) \cdot v^{\frac{k}{p}}}{p} + \ddot{a}_{x+1} \cdot A_{x:\overset{1}{\overline{1|}}} \cdot (1+i) \cdot \frac{\sum_{k=0}^{p-1} \frac{k}{p} \cdot v^{\frac{k}{p}}}{p}$$

(10.98.)

10.2.8. Some Special Annuities

Earlier we have discussed "pension insurance", "widow's/widower's annuity" and "orphan's annuity". We have also given a few examples of these in the sub-section on annuities. Now let's look at what the premium formula of the annuities given as examples of "widow's/widower's annuity" would be. In these formulae we will use the premiums deducted in the earlier sub-section.

One of the examples of "widow's/widower's annuity" is a symmetric two person annuity. Here the annuity of 1 Forint yearly payment is paid only after the death of the first insured, to the other insured as beneficiary, until the death of this second insured. If this is a single premium product, then the premium can be determined the following way:

$$\ddot{a}_x + \ddot{a}_y - 2 \cdot \ddot{a}_{xy}$$

(10.99.)

As we know, $ä_{xy}$ denotes the two person annuity of 10.78., which pays 1 Forint to the two insured persons until the first death. The meaning of the above formula is that both insured persons receive 1 Forint from the insurer until they are alive, so during the period in which both are alive, together they receive 2 Forints. But during the time when they are both alive, they pay these 2 Forints back to the insurer, in other words, they do not receive anything. Immediately as one insured dies, his 1 Forint payment stops, and the second insured, who is still alive, doesn't have to pay back his 1 Forint to the insurer any more, so from this point on he receives net 1 Forint until his death.

If we want to generalise the above two person annuity and suppose that the insured receive *C* Forints together, and after the death of the other, the insured of age *x* receives *A* Forints, and the insured of age *y* receives *B* Forints, then we get the following formula:

$$A \cdot ä_x + B \cdot ä_y - (A + B - C) \cdot ä_{xy} \tag{10.100.}$$

The other example is an asymmetric two person annuity. Let the primary insured (whose death "widows" the other insured) be *x* years old, and the secondary insured (the possible widow/widower) be *y* years old. This is a single premium construction. After the death of the primary insured the secondary insured receives 1 Forint yearly annuity until alive, if the secondary insured is alive when first insured dies. If the secondary insured dies before the death of the first insured, the insurance is terminated without any benefit payment. The single premium of this insurance is:

$$ä_y - ä_{xy} \tag{10.101.}$$

The meaning of the formula is: the secondary insured receives a yearly 1 Forint annuity starting from the commencement of the insurance, but until the primary insured is also alive (i.e. both are alive), they pay this 1 Forint yearly annuity back to the insurer.

Naturally a number of other special annuity types can be imagined beside the above discussed ones.

10.3. The Net Premium of Regular Premium Payment Insurance

In case of the regular premium insurance the premium is paid not all at once, at the beginning of the term, but spread through the whole insurance term, in instalments. Most life insurances have both a single premium and a regular premium version (an exception is e.g. the immediate annuity, which doesn't have a regular premium version[101], and the term fix insurance, where the single premium version would be problematic[102]).

To keep the matter simple, we suppose that the single premium is paid at the beginning of each insurance year, in equal payments, except in the sub-chapter discussing the specialities of premium frequencies other than annual. This annual premium is derived from the single premium of the insurance with the same parameters, but the single premium version. Obviously, this cannot be done simply by dividing the single premium with the number of years in the term to get the annual premium. This has two causes:

Compared to the single premium insurance, the insurer suffers **interest loss** in case of the regular premium insurance, since the greater part of the premium is received only years later, and, until then, the insurer doesn't earn interest on these parts.

The insurer receives the total single premium. But in case of the regular premium, the insurer cannot be totally certain about receiving all of the premium payments, because if the **insured dies during the term**, then further premium payment ceases.

Due to these causes, the annual premium will be higher than the single premium divided by the number of years in the term.

As we have indicated, the period of premium payment can be equal to the insurance term, but it can also be shorter. If m denotes the number of years of premium payment, and the usual n denotes the insurance term, then it is always true that:

$$\boldsymbol{m \leq n}$$

The reason of this the practical insurance principle according to "the premium always goes to the insurer in advance". Namely, the insurer can enforce premium payment from the policy holder by refusing paying the insurance benefits in case of arrears of premiums. If the insurer gave the benefits in advance (e.g. if the premium payment period exceeds the term), then it would loose this simple but effective tool, and would be able to enforce the premium payment only by expensive methods which made the insurance premiums disproportionately high.

Hereunder we suppose that the premium payment period equal to the term. The permiums of cases different from this could be derived simply from this case.

In order to be able to derive the annual premium, we have to realise that the annual premium is technically the same as a temporary annuity paid in advance, where the

[101] Although even this can be imagined if the policyholder and the insured are not the same. Then the policyholder pays the premium in instalments while the insured already receives annuity payments. This construction was used e.g. in Hungary in the 1990s, when the disability pension liability of closed mines was transferred to insurers. On the other hand, these are usually individual annuity constructions, such products are not developed, because people typically take out annuities for themselves. Of course a greater market demand can be imagined, in which case these products would appear!

[102] We will discuss the term fix insurance under 10.1.3.

annuity payment is the annual premium, and the term of the annuity equals the premium payment term, only the annuity payment is not paid by the insurer to the insured, but vice versa. This last circumstance does not influence the value of the annuity.

Starting out from the above consideration, the equivalence equation can be written the following way in case of all regular premium insurances, if the annual premium is P:

$$\ddot{a}_{x:\overline{n|}} \cdot P = A$$

(10.102.)

since the expected income of the insurer is exactly the same as the value of the annuity paid to the insurer from the client, i.e. it is $\ddot{a}_{x:\overline{n|}} \cdot P$. The expected value of payout is the same as in case of the single premium insurance, since the single and regular payment versions do not differ in this respect. This way:

$$P = \frac{A}{\ddot{a}_{x:\overline{n|}}}$$

(10.103.)

Let's apply this general relation to concrete insurances! We are on the opinion that having derived the formulae of the single premiums now it is enough in most cases to just write the concrete formulae.

10.3.1. The Regular Net Premium of the Single Life Insurances

The regular premium of the **term insurance** is:

$$P^1_{x:\overline{n|}} = \frac{A^1_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}}$$

(10.104.)

The regular premium of the **pure endowment insurance** is:

$$P_{x:\overline{\overset{1}{n|}}} = \frac{A_{x:\overline{\overset{1}{n|}}}}{\ddot{a}_{x:\overline{n|}}}$$

(10.105.)

The regular net premium of the **endowment insurance** is:

$$P_{x:\overline{n|}} = P^1_{x:\overline{n|}} + P_{x:\overline{\overset{1}{n|}}}$$

(10.106.)

The regular net premium of the **whole life** insurance:

$$P_x = \frac{A_x}{\ddot{a}_x}$$

(10.107.)

- at least theoretically. In practice the whole life insurance is calculated quite often as an endowment until a very high age (e.g. 85), so its premium is:

$$P_{x:\overline{85-x|}} = P^{1}_{x:\overline{85-x|}} + P_{x:\overline{85-x|}}^{\ \ \ 1}$$

(10.108.)

Naturally, this is not a real whole life insurance, but it is called this way.

The net regular premum of the **"staged" term insurance**:

$$P_{x:\overline{n|}} = \frac{\dfrac{n \cdot M_x - (R_{x+1} - R_{x+n+1})}{n \cdot D_x}}{\ddot{a}_{x:\overline{n|}}}$$

It is worth to note, that this premium quite often leads to a negative reserve, which has to be avoided. That is why in the case of this insurance the premium payment period is quite often shortened – not because of marketing considerations, but because it is a kind of professional necessity. This topic is treated in detail at reserves.

The Net Regular Premium of the **Term Fix Insurance**:

In case of the term fix (or "à terme fix" after the French name) the insurer pays the sum assured at the end of the term under all circumstances, regardless of whether the insured is alive or dead. But premiums are paid until maturity, or until the death of the insured, if this happens earlier. From this we can see why the earlier remark was made concerning the term fix insurance, which stated that the single premium term fix insurance would be problematic, however at (10.24.) we have already showed, that what would be the single premium, what we can use here in the calculation of the premium of the regular premium version:

$$P_{x:\overline{n|}} = \frac{A_{\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{v^n}{\ddot{a}_{x:\overline{n|}}} \qquad (10.109.)$$

With the calculation of the regular net premium pure endowment with premium refund we are dealing with separately later, together with the calculation of its gross premium.

10.3.2. Regular net premiums of joint life insurances

It is obvious, that the net annual premium of the Term insurance for two lives is:

$$P^{1}_{xy:\overline{n|}} = \frac{A^{1}_{xy:\overline{n|}}}{\ddot{a}_{xy:\overline{n|}}} \qquad (10.110.)$$

And the net annual premium of the pure endowment for two lives is:

$$P_{xy:\overset{1}{\overline{n|}}} = \frac{A_{xy:\overset{1}{\overline{n|}}}}{\ddot{a}_{xy:\overline{n|}}} \qquad (10.111.)$$

The net annual premium of the other joint life insurances can be calculated similarly.

10.4. Calculation of Gross Premiums

The above calculated premiums were all net premiums. These cover only the liabilities undertaken by the insurer in the insurance policy. But the insurer also has expenses related to the insurance, and would like to achieve a certain profit through this activity. This way clients have to pay a gross premium for the insurance that is higher than the net premiums calculated so far.

The gross premium is traditionally calculated from the net premium by adding a loading factor. The loading factor itself is calculated from data concerning the expenses. Expenses related to the insurance can be divided into three groups:

α – expenses: expenses related to the issuance of the policy, above all, the commission and the expenses of underwriting (e.g. medical examination). This expense part is usually regarded as proportional to the gross premium of the insurance in case of (single premium) annuities, but as proportional to the sum assured in case of the other types of insurance. This factor is denoted by α.

β – expenses: expenses related to the collection of the premium belong to this group. It is usually considered as proportional to the gross premium, and the factor is denoted by β. These expenses arise during the premium term (usually *n*, but if it is different, then *m*) in case of regular premium insurances.

γ – expenses: other expenses of the insured related to the insurance (e.g. wages, maintenance expenses, data-processing expenses, etc…). This factor is considered proportional to the (sum assured, single net premium of the (temporary) annuity) multiple. The factor is denoted by γ. These expenses are due throughout the whole insurance term in case of regular premium insurances. (The whole insurance term lasts until maximum the death of the insured, except in case of term fix insurance, where always until the expiration.)

On a broader sense the calculated profit and the lost yield (because of the prescribed liquidity) of the solvency capital holded in suitably liquid form are also expenses. These can be calculated principally as β, or possibly as γ expenses, but – mainly in case of the profit – it has to be also take into account, that the classical premium calculation also contains a priori certain hidden profit sources, and a part of the yield of the reserve also a profit source. (See also the Chapter 18.!

In case of the α-expenses the basis of defining the ratio as percentage of the sum assured is that the commission is usually also defined as a percentage of the sum assured, and this is the largest of the α expenses. The purpose of defining the commission as a percentage of the sum assured principally is that it works for the agents as an incentive of longer term insurances (that are more favourable to the insurer), since with the same annual premium the client can purchase a policy of higher sum assured if the term is longer, so the agent receives higher commission. On the other hand, the commission is not always defined as a percentage of the sum assured, but also as a percentage of the premium, but then the commission rate depends on the insurance term. In these cases it is more complicated to calculate the α factor. Here we will deal with the traditional case.

Let's look at the single premium insurances first!

10.4.1. The Gross Premium of Single and Regular Premium Insurances

If the gross premium of single premium insurances is denoted by $AG_{x:\overline{n|}}$, then based on the above formula:

$$AG_{x:\overline{n|}} = A_{x:\overline{n|}} + \alpha + \beta \cdot AG_{x:\overline{n|}} + \gamma \cdot \ddot{a}_{x:\overline{n|}} \tag{10.112.}$$

So:

$$AG_{x:\overline{n|}} = \frac{A_{x:\overline{n|}} + \alpha + \gamma \cdot \ddot{a}_{x:\overline{n|}}}{1-\beta} \tag{10.113.}$$

If the annual gross premium is denoted by $PG_{x:\overline{n|}}$, then the corresponding equations are:

$$PG_{x:\overline{n|}} \cdot \ddot{a}_{x:\overline{n|}} = A_{x:\overline{n|}} + \alpha + \beta \cdot PG_{x:\overline{n|}} \cdot \ddot{a}_{x:\overline{n|}} + \gamma \cdot \ddot{a}_{x:\overline{n|}} \tag{10.114.}$$

So:

$$PG_{x:\overline{n|}} = \frac{A_{x:\overline{n|}} + \alpha + \gamma \cdot \ddot{a}_{x:\overline{n|}}}{(1-\beta) \cdot \ddot{a}_{x:\overline{n|}}} \tag{10.115.}$$

10.4.2. The Difference Between Premiums Calculated for Annual and Monthly Premium Payment

Insurers generally calculate premiums for annual payment, which means that they use the above introduced formulae. But in Hungary in most policies the two parties agree in a more frequent premium payment mode, the most common is the monthly payment. This means that the insurer gives a discount to the client, since the monthly frequency is less favourable to the insurer compared to the annual. This discount given to the client naturally has to be compensated in the premium paid by the client. Since the insurer calculates annual premium payment, i.e. supposes that if the insured is still alive, then the premium is received at the beginning of the insurance year all at once. If the insurer gives a discount, than it will have two deficiencies that have to be compensated:

1. Interest- and expense loss, since the greater part of the premium is not received at the beginning of the year, but continuously throughout the year, and premium collection has to be performed 12 times a year, not only once, so its expenses will be higher. Insurers usually try to compensate this loss by raising the 1/12th of the annual premium by a few percentages in case of monthly payment allowance. This increase depends on the current interest rates, since these indicate the degree of interest loss that the insurer suffers. (A practice also exists where the insurer increases the 1/12th part of the annual premium by a fix sum plus a few percentages of the premium, since the expense increase due to the more frequent premium collection doesn't depend on the interest rate.)
2. The premium loss due to mid-year deaths. This arises because in case of annual premium payment the insurer receives the total premium of the year of death, but in case of monthly payment allowance the monthly premiums following the date of death, that are due in that insurance year are not received. The usual solution to this problem is that the insurer stipulates in the policy terms and conditions that the policyholder is obliged to pay to the insurer the total annual premium due in the year of death, or if it is not paid, the insurer subtracts the remaining premium payments from the death benefit.[103]

It is also possible practice that the insurer calculates monthly premiums from the beginning. In these cases the formulae of regular premiums only differ from formulae determined above, that the single premium has to be divided by $\ddot{a}^{(12)}_{x:\overline{n|}}$ instead of $\ddot{a}_{x:\overline{n|}}$.

So instead of the above

$$P_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} \tag{10.116.}$$

[103] Nowadays insurers often refrain from this, but then excess loss of this kind has to be included in the so-called "frequency loading" as the losses under point 1.

annual premium we use

$$P^{(12)}_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}^{(12)}_{x:\overline{n|}}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}} - \frac{p-1}{2\cdot p}\cdot\left(1 - A_{x:\overline{n|}}^{\;\;1}\right)}$$

(10.117.)

the so-called “annualised” monthly premium, and 1/12th of this will be the actual monthly premium. (Of course here we can also use the more precise monthly annuity formula in the calculation!) As we see, the “annualised” monthly premium is higher than the annual premium, because the right hand sides of the above two equations only differ in that the denominator of 10.117. is somewhat lower than that of 10.116.

1. In case of monthly calculation some things change compared to the annual calculation. Here the annual premium payment (or the payment less frequent than monthly) will be the exception. If the client undertakes the annual premium payment, then he chooses conditions more favourable to the insurer, that the insurer awards with premium discount. The premium discount has the same two sources as previously the premium loading:
2. In case of annual premium payment the insurer receives the greater part of the annual premium earlier than calculated, so it earns interest on the account of the insurer until the premium payment is due according to the calculation. This interest can be returned to the policyholder in the form of premium discount. Naturally the level of this discount also depends on the current interest rates.

If the client chooses annual premium payment and dies during the year, then principally he “overpaid” in that year, since according to the calculation, premium payment is only due until the moth of death, and not for the months remaining from the insurance year. This part of the premium, that has arrived to the insurer, but that the insurer is principally not yet entitled to use, the so-called “unearned premium” is to be paid back to the client.

Due to the above findings we can make interesting statements about the movement of annual and monthly calculated premiums compared to each other as the interest rate changes. In case of annually calculated premiums the higher the interest rate, the higher the factor defined as a percentage will be that shows how much higher the monthly premium is than 1/12th of the annual premium. In case of monthly calculation the discount given to the client from 12 times the monthly premium for choosing annual payment frequency will be greater and greater. So, if we take the difference between the monthly premiums in the two calculation versions, then the higher the interest rate, the greater this difference will be, and vice versa.

What we have stated about the monthly premium – annual premium relation naturally holds – *mutatis mutandis* – for the semi-annual premium – annual premium, quarterly premium – annual premium, etc. relations, too.

An alternative of the above loading and discount based solutions could principally be that the insurer declares separate premium rates for annual, semi-annual, quarterly and monthly premium payment, this way neither the loading, neither the discount is necessary. Of course, this has the disadvantage that the tariff book will be much thicker, and the client won't be able to see through the advantages and disadvantages of the different premium frequencies, this way this method is rarely used by insurers in the practice. Nowadays this distinction is naturally theoretical, because tariff books are went out of use and intermediaries tend to set out premiums the clients have to pay by using online programs. But the logic of this calculation remained the same.

10.4.3. A special case: the gross regular premium of the pure endowment with premium refund

In the subchapter 10.1.3. we have already shortly presented the single premium pure endowment with premium refund and the problems attached to it. We know, that this policy exists almost exclusively in the regular premium version. This insurance does not fit into the logic of the classical premium calculation (and later we will see: into the logic of classical reserving) in more than one points. According to this classical logic, the calculation of the gross premium is the last step, but in this case we have to anticipate this already at the first step – it is obviously possible only as a later calculable parameter. It is also a usual assumption that the benefits of the single and regular premium versions of a policy are the same. It is true for all classical life assurances (presented in this book), except the pure endowment with premium refund, where we can not originate the regular premium version from the single premium version directly, because the death benefits are totally different. That is why we have to create the "single premium" for the regular premium pure endowment with premium refund, to be able to use the step 1 of the classical premium calculation.

The sum assured of the insurance are shown on the following graph:

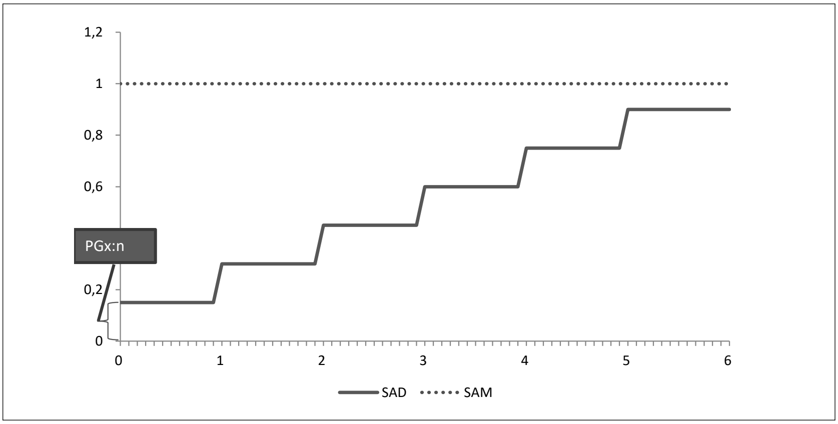

Figure 10.3.: The death (SAD) and maturity (SAM) sum assureds of the regular premium pure endowment with premium refund (for the case n=6)

We can see, that the benefit of the maturity part of the insurance is the usual, but the death benefit part is also "staged" as the "staged" term, with the difference, that there it is decreased year by year, but here it is increasing. We can use a similar trick, than there: we can divide the death benefit into "strips". The death benefit consists of the following "strips":

- Term insurance with sum assured $PG_{x:\overline{n}|}$ between the 0 and n anniversaries +
- Term insurance with sum assured $PG_{x:\overline{n}|}$ between the 1 and n anniversaries +
- …
- Term insurance with sum assured $PG_{x:\overline{n}|}$ between the n-1 and n anniversaries.

Accordingly, the net "single" premium of the regular premium pure endowment with premium refund is:

$$A_{x:\overline{n}|} = A_{x:\overline{n}|}^{\;\;\,1} + PG_{x:\overline{n}|} \cdot \frac{M_x - M_{x+n}}{D_x} + PG_{x:\overline{n}|} \cdot \frac{M_{x+1} - M_{x+n}}{D_x} + \cdots + PG_{x:\overline{n}|} \cdot \frac{M_{x+n-1} - M_{x+n}}{D_x} =$$

$$= A_{x:\overline{n}|}^{\;\;\,1} + PG_{x:\overline{n}|} \cdot \frac{(M_x + M_{x+1} + \cdots + M_{x+n-1}) - n \cdot M_{x+n}}{D_x} \tag{10.118.}$$

Using the commutation function R_x introduced earlier, we can write it into the following form:

$$A_{x:\overline{n|}} = A_{x:\overline{n|}}^{\;\;1} + PG_{x:\overline{n|}} \cdot \frac{R_x - R_{x+n} - n \cdot M_{x+n}}{D_x} =$$

$$= \frac{D_{x+n} + \mathrm{PG}_{x:\overline{n|}} \cdot (R_x - R_{x+n} - n \cdot M_{x+n})}{D_x}$$

(10.119.)

where

$$PG_{x:\overline{n|}} = P_{x:\overline{n|}} \cdot \left(1 + \lambda_{x:\overline{n|}}\right)$$

(10.120.)

and

$$P_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{D_{x+n} + P_{x:\overline{n|}} \cdot \left(1 + \lambda_{x:\overline{n|}}\right) \cdot (R_x - R_{x+n} - n \cdot M_{x+n})}{N_x - N_{x+n}}$$

(10.121.)

Out of (10.121.) we can get $P_{x:\overline{n|}}$:

$$P_{x:\overline{n|}} \cdot \left(1 - \frac{\left(1 + \lambda_{x:\overline{n|}}\right) \cdot (R_x - R_{x+n} - n \cdot M_{x+n})}{N_x - N_{x+n}}\right) = \frac{D_{x+n}}{N_x - N_{x+n}}$$

so

$$P_{x:\overline{n|}} = \frac{D_{x+n}}{N_x - N_{x+n} - \left(1 + \lambda_{x:\overline{n|}}\right) \cdot (R_x - R_{x+n} - n \cdot M_{x+n})}$$

(10.122.)

So finally we got a formula to which we have to be some idea about the size of the loading factor, because using the classical (10.115.) formula would be uneasy here.

The pure endowment with premium refund is a great test of the actuarial skills, but it is questionable, whether is there a point of this insurance, because it is almost non-insurance, being very close to the regular savings.

The following table shows, that what is the relative difference between the annual premium of a pure endowment with premium refund[104] and the annual instalment of a

[104] We considered as net premium not simply (10.122.), but a version of it, where λ=0.

simple savings account with a final 1 sum. In the majority of cases the difference almost undetechtable, only at higher entry ages and longer terms or higher technical interest rates will be discoverable this difference.

Interest	1,0%	2,0%	3,0%	1,0%	2,0%	3,0%	1,0%	2,0%	3,0%	1,0%	2,0%	3,0%
Age	20	20	20	30	30	30	40	40	40	50	50	50
Term												
1	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,01%	0,02%	0,02%
2	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,01%	0,01%	0,02%	0,03%	0,05%
3	0,00%	0,00%	0,01%	0,00%	0,01%	0,01%	0,01%	0,02%	0,02%	0,03%	0,06%	0,10%
4	0,00%	0,01%	0,01%	0,00%	0,01%	0,01%	0,01%	0,03%	0,04%	0,05%	0,11%	0,16%
5	0,00%	0,01%	0,01%	0,01%	0,01%	0,02%	0,02%	0,04%	0,06%	0,08%	0,17%	0,25%
6	0,01%	0,01%	0,02%	0,01%	0,02%	0,03%	0,03%	0,06%	0,09%	0,12%	0,24%	0,36%
7	0,01%	0,01%	0,02%	0,01%	0,03%	0,04%	0,04%	0,09%	0,13%	0,17%	0,33%	0,50%
8	0,01%	0,02%	0,03%	0,02%	0,04%	0,05%	0,06%	0,12%	0,18%	0,23%	0,45%	0,67%
9	0,01%	0,02%	0,03%	0,02%	0,05%	0,07%	0,09%	0,17%	0,25%	0,30%	0,59%	0,87%
10	0,01%	0,03%	0,04%	0,03%	0,06%	0,09%	0,12%	0,23%	0,34%	0,38%	0,75%	1,11%
11	0,02%	0,04%	0,05%	0,04%	0,08%	0,12%	0,15%	0,30%	0,45%	0,48%	0,95%	1,39%
12	0,02%	0,04%	0,07%	0,05%	0,10%	0,15%	0,20%	0,39%	0,58%	0,60%	1,17%	1,73%
13	0,03%	0,05%	0,08%	0,07%	0,13%	0,19%	0,26%	0,51%	0,75%	0,73%	1,44%	2,11%
14	0,03%	0,06%	0,09%	0,08%	0,16%	0,24%	0,33%	0,65%	0,95%	0,89%	1,74%	2,56%
15	0,04%	0,08%	0,11%	0,10%	0,21%	0,30%	0,42%	0,82%	1,20%	1,07%	2,10%	3,07%
16	0,05%	0,09%	0,13%	0,13%	0,26%	0,38%	0,52%	1,02%	1,49%	1,28%	2,50%	3,65%
17	0,05%	0,11%	0,16%	0,16%	0,32%	0,47%	0,64%	1,24%	1,82%	1,52%	2,96%	4,31%
18	0,06%	0,13%	0,19%	0,20%	0,40%	0,59%	0,77%	1,50%	2,20%	1,79%	3,47%	5,04%
19	0,08%	0,15%	0,22%	0,25%	0,50%	0,73%	0,93%	1,80%	2,62%	2,09%	4,04%	5,85%
20	0,09%	0,18%	0,26%	0,31%	0,61%	0,90%	1,10%	2,13%	3,10%	2,43%	4,68%	6,76%
21	0,11%	0,21%	0,31%	0,39%	0,75%	1,10%	1,30%	2,51%	3,64%	2,82%	5,41%	7,78%
22	0,13%	0,25%	0,37%	0,48%	0,92%	1,34%	1,52%	2,93%	4,24%	3,27%	6,24%	8,93%
23	0,16%	0,30%	0,44%	0,58%	1,13%	1,63%	1,77%	3,40%	4,91%	3,78%	7,17%	10,22%
24	0,19%	0,36%	0,53%	0,71%	1,37%	1,98%	2,05%	3,94%	5,66%	4,36%	8,23%	11,67%
25	0,22%	0,43%	0,63%	0,85%	1,65%	2,38%	2,37%	4,53%	6,50%	5,04%	9,44%	13,31%
26	0,26%	0,51%	0,75%	1,02%	1,97%	2,84%	2,72%	5,19%	7,41%	5,83%	10,83%	15,16%
27	0,32%	0,62%	0,89%	1,21%	2,33%	3,35%	3,12%	5,91%	8,42%	6,82%	12,56%	17,43%
28	0,38%	0,74%	1,07%	1,43%	2,74%	3,92%	3,55%	6,71%	9,51%	7,94%	14,47%	19,90%
29	0,46%	0,88%	1,27%	1,67%	3,19%	4,55%	4,03%	7,58%	10,70%	9,22%	16,60%	22,61%
30	0,55%	1,06%	1,52%	1,94%	3,68%	5,25%	4,56%	8,53%	11,99%	10,68%	18,99%	25,59%

Table 10.2.: The relative differences of the net annual premium of the pure endowment with premium refund and the annual installment of a savings account with different interest rates, terms and entry ages (calculating with 2014 Hungarian Male Population Mortality Table)

This is no wonder, because these two financial products differ from each other (above the method of the charging the costs – in the premium versus in the interest rate) only in that sense, that in case of death the beneficiary gets back the installments so far plus interests, but in case of pure endowment with premium refund only the instalments, without interests.

We can see it easily, if we examine how the pure endowment with premium refund insurance will change, we promise not only the premiums in case of death, but also the interests. Let us see the single premium version first!

$$A_{x:\overline{n|}} = A_{x:\overline{n|}}^{\ \ 1} + A_{x:\overline{n|}} \cdot (1+i) \cdot \frac{M_x - M_{x+1}}{D_x} + A_{x:\overline{n|}} \cdot (1+i)^2 \cdot \frac{M_{x+1} - M_{x+2}}{D_x} + \cdots + A_{x:\overline{n|}} \cdot (1+i)^n \cdot \frac{M_{x+n-1} - M_{x+n}}{D_x} =$$

$$= A_{x:\overline{n|}}^{\ \ 1} + A_{x:\overline{n|}} \cdot \left[(1+i) \cdot \frac{C_x}{D_x} + (1+i)^2 \cdot \frac{C_{x+1}}{D_x} + \cdots + (1+i)^n \cdot \frac{C_{x+n-1}}{D_x}\right] =$$

$$= A_{x:\overline{n|}}^{\ \ 1} + A_{x:\overline{n|}} \cdot \left[\frac{d_x \cdot v^x}{l_x \cdot v^x} + \frac{d_{x+1} \cdot v^x}{l_x \cdot v^x} + \cdots + \frac{d_{x+n-1} \cdot v^x}{l_x \cdot v^x}\right] =$$

$$= A_{x:\overline{n|}}^{\ \ 1} + A_{x:\overline{n|}} \cdot \frac{l_x - l_{x+n}}{l_x} = A_{x:\overline{n|}}^{\ \ 1} + A_{x:\overline{n|}} \cdot \left(1 - \frac{l_{x+n}}{l_x}\right)$$

from which

$$A_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}^{\ \ 1}}{\frac{l_{x+n}}{l_x}} = v^n \tag{10.123.}$$

We conclude, that we get the simple savings account.

The regular premium version will be a little bit more difficult:

$$A_{x:\overline{n|}} = A_{x:\overline{n|}}^{\ \ 1} + P_{x:\overline{n|}} \cdot (1+i) \cdot \frac{M_x - M_{x+1}}{D_x} + \left(P_{x:\overline{n|}} \cdot (1+i)^2 + P_{x:\overline{n|}} \cdot (1+i)\right) \cdot \frac{M_{x+1} - M_{x+2}}{D_x} + \cdots + \left(P_{x:\overline{n|}} \cdot (1+i)^n + \cdots + P_{x:\overline{n|}} \cdot (1+i)\right) \cdot \frac{M_{x+n-1} - M_{x+n}}{D_x} =$$

$$= A_{\overset{1}{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot (1+i) \cdot \left[\frac{(1+i)^1 - 1}{i} \cdot \frac{C_x}{D_x} + \frac{(1+i)^2 - 1}{i} \cdot \frac{C_{x+1}}{D_x} + \cdots + \frac{(1+i)^n - 1}{i} \cdot \frac{C_{x+n-1}}{D_x}\right] =$$

$$= A_{\overset{1}{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot \frac{1+i}{i \cdot D_x} \cdot [(1+i)^1 \cdot C_x + \cdots + (1+i)^n \cdot C_{x+n-1} - C_x - C_{x+1} - \cdots - C_{x+n-1}] =$$

$$= A_{\overset{1}{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot \frac{1+i}{i \cdot D_x} \cdot [(1+i)^1 \cdot C_x + (1+i)^2 \cdot C_{x+1} + \cdots + (1+i)^n \cdot C_{x+n-1} - (M_x - M_{x+n})]$$

(10.124.)

Using, that:

$$(1+i)^1 \cdot C_x + (1+i)^2 \cdot C_{x+1} + \cdots + (1+i)^n \cdot C_{x+n-1} =$$

$$= (1+i)^1 \cdot d_x \cdot v^{x+1} + (1+i)^2 \cdot d_{x+1} \cdot v^{x+2} + \cdots + (1+i)^n \cdot d_{x+n-1} \cdot v^{x+n} =$$

$$= (d_x + d_{x+1} + \cdots + d_{x+n-1}) \cdot v^x = (l_x - l_{x+n}) \cdot v^x = D_x - \frac{D_{x+n}}{v^n}$$

(10.125.)

and, that

$$M_x = v \cdot N_x - N_{x+1} = v \cdot N_x - (N_x - D_x) = D_x - (1-v) \cdot N_x = D_x - d \cdot N_x$$

(10.126.)

So

$$-(M_x - M_{x+n}) = -D_x + d \cdot N_x + D_{x+n} - d \cdot N_{x+n}$$

(10.127.)

We get, that:

$$A_{x:\overline{n|}} = A_{\overset{1}{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot \frac{1}{d \cdot D_x} \cdot \left[D_x - \frac{D_{x+n}}{v^n} - D_x + d \cdot N_x + D_{x+n} - d \cdot N_{x+n}\right] =$$

$$= A_{\overset{1}{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot \frac{1}{d \cdot D_x} \cdot \left[d \cdot (N_x - N_{x+n}) + D_{x+n} \cdot \left(1 - \frac{1}{v^n}\right)\right] =$$

$$= A_{\overset{1}{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot \left[\ddot{a}_{x:\overline{n|}} - A_{\overset{1}{x:\overline{n|}}} \cdot \frac{\ddot{a}_{\overline{n|}}}{v^n}\right]$$

(10.128.)

The regular premium is:

$$P_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{A_{x:\overline{n|}}^{\;\;1}}{\ddot{a}_{x:\overline{n|}}} + P_{x:\overline{n|}} \cdot \left[1 - \frac{A_{x:\overline{n|}}^{\;\;1}}{\ddot{a}_{x:\overline{n|}}} \cdot \frac{\ddot{a}_{\overline{n|}}}{v^n}\right]$$

(10.129.)

Rearranging this to $P_{x:\overline{n|}}$, we get, that:

$$P_{x:\overline{n|}} \cdot \frac{A_{x:\overline{n|}}^{\;\;1}}{\ddot{a}_{x:\overline{n|}}} \cdot \frac{\ddot{a}_{\overline{n|}}}{v^n} = \frac{A_{x:\overline{n|}}^{\;\;1}}{\ddot{a}_{x:\overline{n|}}}$$

So:

$$P_{x:\overline{n|}} = \frac{v^n}{\ddot{a}_{\overline{n|}}}$$

(10.130.)

So, the pure endowment with premium refund becomes really a simple savings account.

10.4.4. „Ideological" reasons and implications

The reason why insurers are selling this policy that can be calculated only in a very difficult way, instead of a very simple, almost identical financial product, is that insurers are not allowed to sell simple savings products, because they are "bank" products. And the regulation says that an insurer is only allowed to sell insurance, and a bank only bank products, and they are not allowed to sell (as own product, at the most as intermediary) bank or insurance products, respectively. Naturally, the question was raised: what is the difference between "bank" and "insurance" products? The usual answer to this is that an insurance product is what contains an insurance element. But firstly, this is not so obvious as it seems, secondly, it forces insurers to "ram down" unnecessary services to the client. An example: it follows from this that a pension insurance has to contain a death element, but it is not sure that the client really needs it. Another way: if the beneficiary will get from the insurer the payments made by the insured (who was saved for his/her pension purpose) plus interests in case of the earlier death of insured, it is not insurance, but if the beneficiary gets 101% of this, then it become insurance. Namely, a big savings element + small death element is already acceptable as insurance, because – after all – the endowment is a similar construction.

But it can be "proved", that in this sense the simple savings account itself is an "endowment" insurance. The proof is the following:

10.4.4.1. Single premium version

If we give to the insurer HUF $A_{\overline{n|}} = v^n$ for annual i interest, for n years, then at every anniversary the reserve (anticipating a little bit the material of the next chapter) will be the following:

$$V_t = A_{\overline{n-t|}} = v^{n-t}$$

This product is a kind of endowment policy, where the maturity sum assured is 1, but the death sum assured is varying, it is in every year exactly the reserve, namely

$$S_t = A_{\overline{n-t|}} = v^{n-t}$$

(10.131.)

So the net single premium of the death part of the endowment policy will be:

$$A_{x:\overline{n|}} = \sum_{t=1}^{n} \frac{d_{x+t-1}}{l_x} \cdot S_t \cdot v^t$$

(10.132.)

To transform this we get

$$A_{x:\overline{n|}} = \sum_{t=1}^{n} \frac{d_{x+t-1}}{l_x} \cdot v^{n-t} \cdot v^t = \frac{v^n}{l_x} \cdot \sum_{t=1}^{n} d_{x+t-1} = \frac{l_x - l_{x+n}}{l_x} \cdot v^n = v^n - {}_{n|}p_x \cdot v^n$$
$$= A_{\overline{n|}} - A_{x:\overline{n|}}{}^{1}$$

(10.133.)

namely it is the difference of the premiums of the simple savings and the pure endowment, so it can also be considered as a single premium of a special Term insurance, which complements pure endowment into a (special) endowment insurance.

10.4.4.2. Regular premium version

If we pay to the insurer $P_{\overline{n|}}$ at the beginning of each year during an n years period, which will be accumulated (together with the interests) into 1 until the end of the n years, then $P_{\overline{n|}}$ will be the following:[105]

$$P_{\overline{n|}} = \frac{v^n}{\ddot{a}_{\overline{n|}}} = \frac{1}{\ddot{a}_{\overline{n|}}} - d$$

(10.134.)

Its reserve will be:

$$V_t = v^{n-t} - \ddot{a}_{\overline{n-t|}} \cdot \frac{v^n}{\ddot{a}_{\overline{n|}}} = v^{n-t} \cdot \frac{1}{\ddot{a}_{\overline{n|}}} \cdot \left(\ddot{a}_{\overline{n|}} - \ddot{a}_{\overline{n-t|}} \cdot v^t\right) = v^{n-t} \cdot \frac{\ddot{a}_{\overline{t|}}}{\ddot{a}_{\overline{n|}}}$$

(10.135.)

[105] Some connections, e.g., $P_{\overline{n|}} = \frac{1}{\ddot{a}_{\overline{n|}}} - d$, or $P_{x:\overline{n|}} = \frac{1}{\ddot{a}_{x:\overline{n|}}} - d$ or are anticipated from the supchapter 10.6.!

The regular premium of the pure endowment:

$$P_{x:\overline{n|}}^{\ \ 1} = P_{x:\overline{n|}} - P^{1}_{x:\overline{n|}} = \frac{1}{\ddot{a}_{x:\overline{n|}}} - d - P^{1}_{x:\overline{n|}}$$

(10.136.)

The difference of the two:

$$P_{\overline{n|}} - P_{x:\overline{n|}}^{\ \ 1} = \frac{1}{\ddot{a}_{\overline{n|}}} - d - \frac{1}{\ddot{a}_{x:\overline{n|}}} + d + P^{1}_{x:\overline{n|}} = \frac{1}{\ddot{a}_{\overline{n|}}} - \frac{1}{\ddot{a}_{x:\overline{n|}}} + P^{1}_{x:\overline{n|}}$$

(10.137.)

The death sum assured here will be the reserve of the savings account:

$$S_t = v^{n-t} \cdot \frac{\ddot{a}_{\overline{t|}}}{\ddot{a}_{\overline{n|}}}$$

(10.138.)

The single premium of the Term insurance will be:

$$A_{x:\overline{n|}} = \sum_{t=0}^{n-1} \frac{d_{x+t}}{l_x} \cdot S_{t+1} \cdot v^{t+1}$$

$$A_{x:\overline{n|}} = \sum_{t=0}^{n-1} \frac{d_{x+t}}{l_x} \cdot v^{n-t-1} \cdot \frac{\ddot{a}_{\overline{t+1|}}}{\ddot{a}_{\overline{n|}}} \cdot v^{t+1} = \frac{v^n}{\ddot{a}_{\overline{n|}}} \cdot \sum_{t=0}^{n-1} \frac{d_{x+t}}{l_x} \cdot \frac{1 - v^{t+1}}{d} =$$

$$= \frac{v^n}{\ddot{a}_{\overline{n|}} \cdot d} \cdot \left(\sum_{t=0}^{n-1} \frac{d_{x+t}}{l_x} - \sum_{t=0}^{n-1} \frac{d_{x+t} \cdot v^{t+1}}{l_x} \right) = \frac{v^n}{\ddot{a}_{\overline{n|}} \cdot d} \cdot \left(\frac{l_x - l_{x+n}}{l_x} - A^{1}_{x:\overline{n|}} \right)$$

$$= \frac{v^n}{\ddot{a}_{\overline{n|}} \cdot d} \cdot \left(1 - {}_{n|}p_x - A^{1}_{x:\overline{n|}} \right) =$$

$$= \frac{v^n \cdot \left(1 - A^{1}_{x:\overline{n|}}\right) - A_{x:\overline{n|}}^{\ \ 1}}{\ddot{a}_{\overline{n|}} \cdot d} = \frac{\left(1 - \ddot{a}_{\overline{n|}} \cdot d\right)\left(1 - A^{1}_{x:\overline{n|}}\right) - A_{x:\overline{n|}}^{\ \ 1}}{\ddot{a}_{\overline{n|}} \cdot d} =$$

$$= \frac{1 - A^{1}_{x:\overline{n|}} - \ddot{a}_{\overline{n|}} \cdot d + \ddot{a}_{\overline{n|}} \cdot d \cdot A^{1}_{x:\overline{n|}} - A_{x:\overline{n|}}^{\ \ 1}}{\ddot{a}_{\overline{n|}} \cdot d} =$$

$$= \frac{1 - A_{x:\overline{n|}}^{\ \ 1} - A^{1}_{x:\overline{n|}}}{\ddot{a}_{\overline{n|}} \cdot d} - 1 + A^{1}_{x:\overline{n|}} = \frac{\ddot{a}_{x:\overline{n|}} \cdot d}{\ddot{a}_{\overline{n|}} \cdot d} - 1 + A^{1}_{x:\overline{n|}} =$$

$$= \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{\overline{n|}}} - 1 + A^{1}_{x:\overline{n|}}$$

(10.139.)

So, the regular premium

$$P_{x:\overline{n|}} = \frac{1}{\ddot{a}_{\overline{n|}}} - \frac{1}{\ddot{a}_{x:\overline{n|}}} + P^{1}_{x:\overline{n|}}$$

(10.140.)

Ant this is just the available premium!

10.5. The net premiums on another way – connections between single premiums

Below we will show what kind of important connections exis between the premium formulae,. To this we invert the method of presentation: the individual insurances are not built from simple elements, but we break up a general financial product into elements. This general financial product is the perpetuity, because as it can be said, that as the light of the Sun can broke into the all colours of the rainbow, and so the white light contains all the colours, the simplest annuity, the perpetuity contains all the possible annuity types and the other financial products. In the followings we will see how they are laid in the perpetuity. I restricht my analisys (for the sake of simplicity and perspicuity) to annuities-due, but the formulae are very similar to the annuities payable in arrears. The appropriate, standardised notations will be introduced according to the order of the explanation.

So, we can say, that „in the beginning was the perpetuity". The simple, standardised form of it, if somebody will get after his/her HUF 1 capital, in each year, at the end of the year *i* interest and (s)he depletes it. In this way the principal will remain (nominally)[106] unchanged for ethernity, and also the rent from it lasts for ethernity, which can be called as annuity because of its regularity. The payments of this annuity are due always at the end of the year, so this is an annuity payable in arrears. How can we make from this with HUF 1 principal and *i* interest rate an annuity-due perpetuity? The question can be reformulated as what amount can be deducted from the HUF 1 capital at the beginning of the year that with *i* interest rate will again reach HUF 1 until the end of the year? If we denote (following the tradition) this with *d*, than it can be deducted from the following equation:

$$(1 - \mathrm{d}) \cdot (1 + \mathrm{i}) = 1$$

(10.141.)

[106] Naturally, our analysis can be extended without many problems for principal unchanged in real terms. In this case, we have to split the nominal interest rate into two parts and the real interest rate will play the same role as the nominal interest rate in this analysis. In practice the land rent as a kind of annuity is the closest to the perpetuity and it represents a more or less unchanged principal in real terms.

from which:

$$d = 1 - \frac{1}{1+i}$$

(10.142.)

Earlier we have already introduced *v* the reciprocal of (1+i), the so called discount factor:

$$v = \frac{1}{1+i}$$

So:

$$d = 1 - v$$

(10.143.)

from which first

$$v = 1 - d$$

(10.144.)

second

$$d = 1 - \frac{1}{1+i} = \frac{i}{1+i} = i \cdot v$$

(10.145.)

So, *d* is the discounted value of *i*. It is logical, because the question could have been formulated also the following way: what is the interest rate instead of *i*, if we have to get it one year earlier? The answer naturally the discounted value of *i*, i.e. .

We get our first standardized annuity-due, where the annual payment (in advance) for HUF 1 principal and *i* interest rate is *d*. The notation of the single premium of the perpetuity (for annual HUF 1 payment in advance):

$$\ddot{a}_{\overline{\infty|}}$$

So, the equation of the perpetuity with annual *d* payment is:

$$1 = d \cdot \ddot{a}_{\overline{\infty|}}$$

(10.146.)

This express that the client for HUF 1 gets an ethernal cash-flow with annual *d* always in advance at the beginning of the year.

Followings, when I wrote an equation and one of the sides of it stands 1, it express that the client which kind of products can get for HUF 1, which are logically equal with the part of the perpetuity.

Before I do this it is edifying to calculate $ä_{\overline{\infty|}}$ from the equation above:

$$ä_{\overline{\infty|}} = \frac{1}{d} = \frac{1}{i \cdot v} = \frac{1+i}{i} = 1 + \frac{1}{i}$$

(10.147.)

where $\frac{1}{i}$ is the well known formula of the perpetuity payable in arrears with annual HUF 1 payments ($a_{\overline{\infty|}}$). The annuity-due perpetuity differ from this only with the HUF 1 payment right at the beginning.

The perpetuity is a cash-flow which lasts for ethernity. Naturally we can „cut" this cash-flow in different „places" in its lifetime. What will happen e.g. if we want to get the HUF *d* annual annuity-due payment after our HUF 1 capital only until our death? Then obviously our heir[107] right after our death (i.e. at the anniversary of the perpetuity after it) will get back the HUF 1 principal (what, if (s)he wants can put again into a perpetuity), because the point of the perpetuity, that the whole original capital will restore again and again after one year of the interest payment. But this means from another angle, that the uniform perpetuity is „dismembered" into two financial products:

1. An annuity-due lasting until our death with annual d payment, i.e. to a life annuity, plus
2. A life assurance with HUF 1 sum assured, the payment due to at our death (its technical name: whole life)

We can buy this two financial products exactly for HUF 1. The usual notation of the net single premium of the annuity-due life annuity with annual HUF 1 payment lasting until lifetime is $ä_x$, where x is the entry age of the insured (until whose death the annuity lasts) at the commencement of the contract. The notation of the net single premium of the whole life assurance with HUF 1 sum assured (benefit) is $\boldsymbol{A_x}$. In booth cases the implicit assumption is, that the financial institution (here: life insurer) invests our capital with annual *i* interest rate, and does not charge anything for its service.[108] Using these notations we can write this „dismemberment" of the perpetuity, as:[109]

$$1 = d \cdot ä_x + A_x$$

(10.148.)

[107] Naturally the insurer pays the sum assured to the beneficary independently from the inheritance, because – legally – the sum assured – if there is a formal beneficary – is not part of the bequest. But the real meaning of it is legacy.

[108] Or at least it makes to do with the interest's „marge" between what it really earned and what it payed to the client.

[109] Most of the actuarial books contains this equation. See e.g Bein–Bogyó–Havas p. 182, Bowers et al. p. 131, Gerber p36, Krekó p. 30, Neill p. 63. But the majority of further equations shown here are missing from them.

Naturally we can „cut" the cash-flow of the perpetuity at other places as well. There are two other important cases of the standardized annuities:

1. An annuity until a predetermined term (for the sake of simplicity, until whole number of years), then we ask the principal back
2. Annuity until a predetermined term, subject to a limit of the time of our death, then we ask back the pricipal (or to transfer it to our heir).

In the first case obviously have to be true, that

$$1 = d \cdot \ddot{a}_{\overline{n|}} + v^n$$

(10.149.)

where I note with *n* the (whole) number of years of the term, with $\ddot{a}_{\overline{n|}}$ the net present value of HUF 1 annual annuity-due and certain, and v^n the present value of HUF 1 due in *n* years.

In the second case obviously have to be true, that

$$1 = d \cdot \ddot{a}_{x:\overline{n|}} + A_{x:\overline{n|}}$$

(10.150.)

where $\ddot{a}_{x:\overline{n|}}$ is the net present value of the HUF 1 annual life annuity-due until death, but maximum for *n* years, while $A_{x:\overline{n|}}$ the net single premium of a life insurance for death and maturity („endowment") with HUF 1 sum assured.

We first have cut the perpetuity above when somebody dies. This event can be logically extended to more than one (in the simplest case: two) deaths. Then we get the following equation (with two insureds):

$$1 = d \cdot \ddot{a}_{xy} + A_{xy}$$

(10.151.)

Where $\ddot{a}_{xy}$ the net single premium of a HUF 1 annual annuity-due for two insureds, which lasts until both of them are alive. (One of them is x, the other is y years old at entry, and below – for the sake of simplicity – x and y insureds.) A_{xy} is the single net premium of a whole life insurance with two annuitants, which pays HUF 1 when any of them dies.

Naturally the joint life annuity also has a temporary variation. Then

$$1 = d \cdot \ddot{a}_{xy:\overline{n|}} + A_{xy:\overline{n|}}$$

(10.152.)

equation will be true, where $A_{xy:\overline{n|}}$ is the net single premium of a joint life endowment, which pays the HUF 1 sum assured when any of the insurded persons dies but after n years as latest, while the $\ddot{a}_{xy:\overline{n|}}$ single premium joint life annuity lasts until both annuitants is alive but maximum for n years.

A further extension is to add a guarantee period of g years to the annuity. If we seek the practical reasoning of it, then we can say the followings. In the cases of so far analysed (not conditional) annuities it can happen that the annuitant dies soon after the commencement of the contract, and the payments of the annuity will cease. This possibility scares off many potential annuity buyers from buying the annuity who vacillates whether (s)he depletes his/her accumulated wealth or leaves it to the children, and (s)he makes it dependent on his/her life expectancy, which is unkonwn for him/her. By guaranteeing some of the payments of the (lifelong) life annuity is intented to remove this psychological barrier of buying an annuity. The guarantee period can be put at the beginning of the term, or at the end. In case of annuity with guarantee period at the beginning (let note it with ${}^{g|}\ddot{a}_x$) the annuitant (or his/her heir) surely will get the payments of the first *g* years even if (s)he dies meanwhile. Naturally, this guarantee will not be an effective one, if the annuitant will die after g years.

An annuity with guarantee period will obviously provide more benefit than without a guarantee period, so its premium is also higher. From this follows that HUF 1 is not enough (above the annual d payment) for the previous whole life benefit, only its modified form. This modified form means, that in the first *g* years will in no way be paid the HUF 1 benefit, even if the insured would have died in this period. If the insured dies during the first *g* years, the beneficiary would get the HUF 1 at the end of the *g*-year period. This is as if the first *g* years would count a time unit, and later the time unit would be again one year. Let us note (say) this with *g* in the lower left corner (where *g* means the length of the first such a period). Then we can write the following equation:

$$1 = d \cdot {}^{|g}\ddot{a}_x + {}_{g}A_x$$

(10.153.)

The annuity with guarantee period at the end (let us note it, say, with ${}^{g|}\ddot{a}_x$) means, that the beneficiary will get the payments until g years after the death of the insured. So, it is obvious, that also the sum assured of the whole life insurance will due g years after the death, i.e. – can we say – we „shift" the death benefit with g years. This can be express also in the way, that in case of death not the HUF 1 is due, but only its discounted value, so our equation will vary to the following:

$$1 = d \cdot {}^{g|}\ddot{a}_x + v^g \cdot A_x$$

(10.154.)

The parts of the above equations we can further dismember. The following equations are obviously true:

$$ä_x = ä_{x:\overline{n|}} + {}_{n|}ä_x$$

(10.155.)

i.e. the lifetime annuity can be broken up into a n years (n ‹ ω) temporary annuity-certain and a deferred (life) annuity. Accordingly, the equation $1 = d \cdot ä_x + A_x$ can be also written into the following form:

$$1 = d \cdot \left(ä_{x:\overline{n|}} + {}_{n|}ä_x\right) + A_x$$

(10.156.)

(A deviation from the basic formula, that here it is not sure that all parts will have paid benefits, because the deferred annuity also a contingent one, and so it can cease without payment if the insured dies within the first n years period.)

Analogously to the annuities also A_x can be broken into the sum of a „temporary" and a „deferred" life insurance. The „temporary" life insurance is the classical term insurance, and the „deferred" one's classical notation is ${}_{m|}A_x$! (Remarks: 1. To emphasize that the length of this deferment not necessarily the same as at the annuities, I noted this one with m and not n; 2. ${}_{m|}A_x$ is different from the previously introduced ${}_{g}A_x$, the meanigs of the two are different. The vertical at the first note a contingent benefit, but at the second there is not a condition.)

Accordingly true, that

$$A_x = A^1_{x:\overline{m|}} + {}_{m|}A_x$$

(10.157.)

where $A^1_{x:\overline{m|}}$ the traditional notation of the single premium of a term (death) insurance with m years term and HUF 1 sum assured. So, we can further break up our basic equation $1 = d \cdot ä_x + A_x$ into the following form:

$$1 = d \cdot \left(ä_{x:\overline{n|}} + {}_{n|}ä_x\right) + A^1_{x:\overline{m|}} + {}_{m|}A_x$$

(10.158.)

(In this formula either the classical term insurance or the deferred whole life insurance surely will cease without paid benefit, but it is also possible that the deferred annuity will not have any paid benefit.)

I do not deal with the topic, but I remark, that

1. Also the temporary annuities can be broken up into the sum of a shorter temporary and a deferred temporary annuity.
2. The joint life annuities with two or more insureds can be broken up analogously to the one insured annuities discussed above.

Annuities with guarantee period are complex products so – naturally – they also can be broken up.

The annuity with guarantee period at the beginning is the sum of an annuity-certain and a deferred annuity, that is

$$^{|g}\ddot{a}_x = \ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x \tag{10.159.}$$

So the equation $1 = d \cdot {}^{|g}\ddot{a}_x + {}_gA_x$ is changing in the following:

$$1 = d \cdot \left(\ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_x\right) + {}_gA_x \tag{10.160.}$$

The annuity with guarantee period at the end obviously gives more guarnatee, than the annuity with guarantee pediod at the beinning, because:

1. It includes the guarantee of the other one, namely in the first g years the annuity is surely will be paid,
2. Contrary to the annuity with guarantee period at the beginning, the payments will surely last longer here, than without guarantee time. In case of annuity with guarantee period at the beginning, the guarantee is not effective, if the insured dies after the guarantee period, but in case of annuity with guarantee period at the end there is no such a caesura.

The annuity with guarantee period at the end can be interpreted in two ways:

1. It is a sum of a g years annuity-certain (because in the first g year the insured surely gets payments, even if he/she immediately dies after concluding the annuity contract) and such a „normal" life annuity, of which all the payments are „shifted" by g years, i.e.:

$$^{g|}\ddot{a}_x = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x \tag{10.161.}$$

This interpretaion is totally compatible with the „shifted" whole life insurance in the equation $1 = d \cdot {}^{g|}\ddot{a}_x + v^g \cdot A_x$. Then the equation got the following form:

$$1 = d \cdot \left(\ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x\right) + v^g \cdot A_x \tag{10.162.}$$

2. It is naturally a normal lifelong annuity plus such a whole life insurance, where the sum assured is paid as an immediate, g years annuity-certain, so:

$$^{g|}\ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x \tag{10.163.}$$

Then the equation $1 = d \cdot {}^{g|}\ddot{a}_x + v^g \cdot A_x$ got the following form:

$$1 = d \cdot \left(\ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x\right) + v^g \cdot A_x \tag{10.164.}$$

It is taken for granted, that here $d \cdot \ddot{a}_{\overline{g|}} \cdot A_x + v^g \cdot A_x$ is just A_x.

Naturally these two forms are equivalent to each-other, because if we replace A_x in the $\ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x$ with $1 - d \cdot \ddot{a}_x$, then we get, that

$$^{g|}\ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot A_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} \cdot (1 - d \cdot \ddot{a}_x) = \ddot{a}_x + \ddot{a}_{\overline{g|}} - \ddot{a}_{\overline{g|}} \cdot d \cdot \ddot{a}_x =$$

$$= \ddot{a}_x + \ddot{a}_{\overline{g|}} - \frac{1 - v^g}{1 - v} \cdot (1 - v) \cdot \ddot{a}_x = \ddot{a}_x + \ddot{a}_{\overline{g|}} - (1 - v^g) \cdot \ddot{a}_x = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_x \tag{10.165.}$$

Naturally, it is also possible to guarantee the temporary life annuity, however the justness of this kind of solution is not so clear than in case of lifetime annuity. However, the equations will also in this case the extension of the above ones', that is:

$$^{|g}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{\overline{g|}} + {}_{g|}\ddot{a}_{x:\overline{n|}} \tag{10.166.}$$

where, naturally g≤n. (In case of g=n $^{|g}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{\overline{g|}}$ and ${}_{n|}\ddot{a}_{x:\overline{n|}} = 0$.) In case of temporary life annuity with guarantee period at the end, there is no such a restriction:

$$^{g|}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{\overline{g|}} + v^g \cdot \ddot{a}_{x:\overline{n|}} \tag{10.167.}$$

Here there will be also true, that:

$$^{g|}\ddot{a}_{x:\overline{n|}} = \ddot{a}_{x:n} + \ddot{a}_{\overline{g|}} \cdot A_{x:\overline{n|}} \tag{10.168.}$$

Because the annuity-certain in fact a guaranteed annuity, so the guarantee period has specific meaning only in case life annuities. It is also possible to apply guarantee period for the joint life annuities, but the relevance of it here probably much less, considering that the aim of a guarantee time to pass a part of the annuity payment to another person, the same as the aim of joint life annuity. With other worlds: the annuity with guarantee period is a (nos so good) simulation of a joint life annuity. However, if it is necessary, on the basis of the above mentioneds, it is easy to add guarantee period into a temporary or a joint life annuity.

There is a clean-cut order of magnitude amongst the different annuities. It is unambiguous, that:

$$\ddot{a}_{\overline{n|}} > \ddot{a}_{x:\overline{n|}} > \ddot{a}_{xy:\overline{n|}} > \ddot{a}_{xyz:\overline{n|}}$$

or, that

$$\ddot{a}_x > \ddot{a}_{xy}$$

because in case of $\ddot{a}_{\overline{n|}}$ we surely get payments for n years, but in case of $\ddot{a}_{x:\overline{n|}}$ for maximum n years, considering, that the insured can die earlier. Furthermore, if not only one insured's death, but a further one's or two's can make cease the flow of annuity payments, it means, that the length of this flow would be even shorter. The same can be told in case of lifelong one person and two persons (etc.) life annuities.

This also means, that the differences of these single premiums will be higher than zero. Fortunately, their meanings is quite clear: $\ddot{a}_x - \ddot{a}_{xy}$: x annually gets HUF 1, but he pays back it until y is alive (negative annuity), consequently x only gets payments after the death of y. If x dies before y, than would not happen any payment. So, this is a contingent annuity: the condition is the death of y. We can note this by $\ddot{a}_{x|y}$ so

$$\ddot{a}_{x|y} = \ddot{a}_x - \ddot{a}_{xy} \tag{10.169.}$$

Then our equation will transform into the following:

$$1 = d \cdot \left(\ddot{a}_{xy} + \ddot{a}_{x|y}\right) + A_x \tag{10.170.}$$

$\ddot{a}_{x|y}$ can also be considered a kind of asymmetric widow annuity. It is rational if the financial situation of the two insured is different, y fends for x and wants to secure the income of her after his death. The death of x financially not affects y.

$\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}}$: annuity paid for those years of an n years period, when x not alive any more. If x is alive at age x+n, than the annuity payments would not start at all. So, this is also a contingent annuity, we can note it e.g. by $\ddot{a}_{\overline{n|}x}$.[110]

The meaning of this is clear: a beneficiary gets annuity payments (let us say: „orphan" annuity) until a certain age (for n years), but only if the insured dies until this age of the beneficiary. If the insured remains alive, the annuity payments would not start, because the condition is not realized.

In this case our basic equation will alter into the following form:

$$1 = d \cdot \left(\ddot{a}_{x:\overline{n|}} + \ddot{a}_{\overline{n|}x}\right) + v^n \tag{10.171.}$$

It is rational, if this annuity is a joint life one:

$$\ddot{a}_{\overline{n|}xy} = \ddot{a}_{\overline{n|}} - \ddot{a}_{xy:\overline{n|}} \tag{10.172.}$$

In this case the „orphan-annuity's" payments will start if any of the parents dies until a certain age of the child.

[110] In Chapter 4 in connection with the term insurance we have mentioned, that the clients in reality need instead of this a conditional annuity. In the majority of the cases it would be naturally a regular premium one, because this is the consumer need for such a cover. The statement, that this regular premium annuity is a special term insurance can be realised by the method below. The single premium of the conditional annuity can be converted into the same form as the single premium of that special term insurance which provide as benefit in case of the death of the insured an annuity certain for the remaining term. The two conversions are the following:

$$\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}} = \frac{d_x \cdot \ddot{a}_{\overline{n-1|}} \cdot v + d_{x+1} \cdot \ddot{a}_{\overline{n-2|}} \cdot v^2 + \cdots + d_{x+n-1} \cdot \ddot{a}_{\overline{0|}} \cdot v^{n-1}}{l_x}$$

or

$$\begin{aligned} l_x \cdot \left(\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}}\right) &= l_x \cdot \left((1 + v^1 + \cdots + v^{n-1}) - \left(\frac{l_x}{l_x} \cdot 1 + \frac{l_{x+1}}{l_x} \cdot v^1 + \cdots + \frac{l_{x+n-1}}{l_x} \cdot v^{n-1}\right)\right) \\ &= (l_x - l_x) \cdot 1 + (l_x - l_{x+1}) \cdot v^1 + \cdots + (l_x - l_{x+n-1}) \cdot v^{n-1} \end{aligned}$$

$$\begin{aligned} & d_x \cdot \ddot{a}_{\overline{n-1|}} \cdot v + d_{x+1} \cdot \ddot{a}_{\overline{n-2|}} \cdot v^2 + \cdots + d_{x+n-1} \cdot \ddot{a}_{\overline{0|}} \cdot v^n = \\ &= d_x \cdot (v^1 + v^2 + \cdots + v^{n-1}) + d_{x+1} \cdot (v^2 + v^3 + \cdots + v^{n-1}) + \cdots + d_{x+n-2} \cdot v^{n-1} = \\ &= d_x \cdot v^1 + (d_x + d_{x+1}) \cdot v^2 + \cdots + (d_x + d_{x+1} + \cdots + d_{x+n-2}) \cdot v^{n-1} = \\ &= (l_x - l_{x+1}) \cdot v^1 + (l_x - l_{x+2}) \cdot v^2 + \cdots + (l_x - l_{x+n-1}) \cdot v^{n-1} \end{aligned}$$

Generally it is not so important to take into account, that – with a slim chance – also the child can die, but if it also counted, then we can create a three persons joint life annuity:

$$\ddot{a}_{x:\overline{n|}yz} = \ddot{a}_{x:\overline{n|}} - \ddot{a}_{xyz:\overline{n|}}$$

(10.173.)

According to this, until all the three insureds are alive, there will no any annuity payment, and this situation remains if all insureds mature the end of the n years term. If any of the z and y dies, than the annuity payment will start for x until his/her x+n age, but maximum until his/her earlier death. If x dies before z and y, the annuity payments would never start. (In this example the parents were y and z, contrary to the previous one, where x and y!)

Our basic equation will alter into the following forms in these cases:

$$1 = d \cdot \left(\ddot{a}_{xy:\overline{n|}} + \ddot{a}_{\overline{n|}xy}\right) + v^n$$

(10.174.)

or

$$1 = d \cdot \left(\ddot{a}_{xyz:\overline{n|}} + \ddot{a}_{x:\overline{n|}yz}\right) + A_{x:\overline{n|}}$$

(10.175.)

10.6. The net premiums on another way – regular premiums

The general meaning of the classical equations, e.g. (10.138.): me, as client can get in exchange for HUF 1 from the provider, partly an annuity and partly a lump sum later. This is an exchange of cash-flows and the type of these cash-flows are:

1. the client pays now a lump sum – later he/she gets also a lump sum
2. the client pays now a lump sum – he/she gets an annuity continually

If we change the position of the client and the provider, then we get another financial products, namely:

Instead of 1: the client gets now a lump sum – he/she later pays a lump sum

Instead of 2: the client pays an annuity continually – he/she gets now a lump sum

By both exchanges we get totally different financial products, namely here to credits, which are beyond insurances.

We can change the type 2 exchange in another sense: we can put the benefit the client gets from the beginning of the term into the end of it. Here the client pays an annuity continually – but he/she gets a lump sum later. The expected present value of this later benefit naturally has to be the same what he/she would get now. (And naturally, we can change here the position of client and provider too, namely it is possible the client gets an annuity continually and he/she pays a lump sam later. But focus on the previous version!)

In case of 2nd exchange, the classical formulae:

$$d \cdot \ddot{a}_x = 1 - A_x \tag{10.176.}$$

$$d \cdot \ddot{a}_{\overline{n|}} = 1 - A_{\overline{n|}} \tag{10.177.}$$

$$d \cdot \ddot{a}_{x:\overline{n|}} = 1 - A_{x:\overline{n|}} \tag{10.178.}$$

The meaning of these: if somebody pays continually an annual d until the end of his/her life (n years or the combination of the two), then he/she would get now the sum on the right side of the equation. If we put the provider's outpayment onto the end of the annuity, then the sum on the right side is the expected present value of this outpayment.

We know, that the expected present value of HUF 1 outpayment in the future will be in the above mentioned cases one by one A_x, $A_{\overline{n|}}$, and $A_{x:\overline{n|}}$. Now let us see the interpretation of (10.177.)! This is:

the client pays an annual d for n years and will get a sum, of which present value now is $1 - A_{\overline{n|}}$, and its value then ("future value") $\frac{1-A_{\overline{n|}}}{A_{\overline{n|}}} = \frac{1}{A_{\overline{n|}}} - 1 = \frac{1}{v^n} - 1$.

The limit value of the present value is 1, if n→∞. The present value is less than 1 by a sum, of which future value is just 1.

If somebody would pay an annual $\frac{1}{\ddot{a}_{\overline{n|}}}$, than would get then a sum of which future value is $\frac{1-A_{\overline{n|}}}{d \cdot \ddot{a}_{\overline{n|}}} = 1$.

Namely it is not just approaching to 1 infinitely, but equel to 1. The present value of this is $\frac{1}{v^n}$.

The limit value of $\frac{1}{\ddot{a}_{\overline{n|}}}$:

$$\lim_{n\to\infty}\frac{1}{\ddot{a}_{\overline{n|}}}=\lim_{n\to\infty}\frac{d}{1-v^n}=d$$

(10.179.)

namely the two annual instalments are approaching to each other. It can be said, that the difference of $\frac{1}{\ddot{a}_{\overline{n|}}}$ and d, is the price to get the 1 (in present value) not in somevere the

infinity, but already in a finite time horizon. To this it is missing just a HUF 1 (on future value) outpayment in the finite time, that is why if we pays the difference of the two, than in limit value we get 0, but in finite time always the following (in present value):

$$\frac{1}{\ddot{a}_{\overline{n|}}}-d\to 1-\left(1-A_{\overline{n|}}\right)=A_{\overline{n|}}$$

(10.180.)

namely in future value always 1! Obviously the present value of this 1 (and also the premium of it) is approaching to 0, if we increase the time horizon!

This train of thought can be applied onto all of the equations above and so we get the formulae of the regular premiums of the whole life and endowment assurances and the regular savings account. ($\frac{1}{\ddot{a}_x}-d,\frac{1}{\ddot{a}_{x:\overline{n|}}}-d$ and $\frac{1}{\ddot{a}_{\overline{n|}}}-d$)

Another train of thought: with an annual $\frac{1}{\ddot{a}_{\overline{n|}}}$ payment (and with their interests) we can accumulate until the end of an n years period $\frac{1}{v^n}=(1+i)^n$.

If we pay annually only a smaller amount (smaller by d), then we accumulate only $\frac{1}{v^n}-1=(1+i)^n-1$ until the end of an n years period. The first is HUF 1 initial capital,

together with its (compound) interests, the second is the (compound) interests of the HUF 1 capital, without the capital itself. The difference of the two HUF 1, the accumulation of the capital itself.

This – after all – can be seen already in the formula itself. The gradual accumulation of $\frac{1}{\ddot{a}_{\overline{n|}}}-d$ can also be seen, that the interest due after the HUF 1 capital (which would

be I at the end of the year, but in the beginning of the year it is only d) is deducted ab ovo, so only the capital is accumulating. The weight of the capital become 0 in limit value.

Another way to formulate this: if we start to accumulate a sequence of d, then we get in present value, that:

$$d + d \cdot v + d \cdot v^2 + \cdots = d \cdot \frac{1}{1-v} = 1 \tag{10.181.}$$

Its future value on a finite time horizon is:

$$d \cdot (1+i)^n + d \cdot (1+i)^{n-1} + \cdots + d \cdot (1+i) = d \cdot (1+i)^1 \cdot \frac{(1+i)^n - 1}{(1+i) - 1} = (1+i)^n - 1 \tag{10.182.}$$

If we use $d \cdot (1+i) = i$ then this can also be written as

$$d \cdot (1+i)^n + d \cdot (1+i)^{n-1} + \cdots + d \cdot (1+i) = i \cdot (1+i)^{n-1} + i \cdot (1+i)^{n-2} + \cdots + i$$
$$= (1+i)^n - 1$$

Obviously

$$1 + i \cdot (1+i)^{n-1} + i \cdot (1+i)^{n-2} + \cdots + i = (1+i)^n \tag{10.183.}$$

It also can be fragmeneted as:

$$1 + i \cdot [(1+i)^{n-1} + (1+i)^{n-2} + \cdots + 1] = (1+i)^n \tag{10.184.}$$

and we can give to this the following interpretation: there is on the right side the HUF 1 capital and its compound interest together, and on the left side the same, but the capital and its (compound) interest are separated.

One of the results of this train of thought is that according to the (10.180.), the annual "premium" of a regular savings for an aimed HUF 1 is $\frac{1}{\ddot{a}_{\overline{n|}}} - d$.

This can be originated directly from (10.177.) by the following way

$$A_{\overline{n|}} = 1 - d \cdot \ddot{a}_{\overline{n|}} = \left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) \cdot \ddot{a}_{\overline{n|}} = P_{\overline{n|}} \cdot \ddot{a}_{\overline{n|}} \tag{10.185.}$$

Analogously, from the other two can be also originated the annual premium of the whole life and endowment on the following way:

$$A_x = 1 - d \cdot \ddot{a}_x = \left(\frac{1}{\ddot{a}_x} - d\right) \cdot \ddot{a}_x = P_x \cdot \ddot{a}_x \tag{10.186.}$$

or:

$$A_{x:\overline{n|}} = 1 - d \cdot \ddot{a}_{x:\overline{n|}} = \left(\frac{1}{\ddot{a}_{x:\overline{n|}}} - d\right) \cdot \ddot{a}_{x:\overline{n|}} = P_{x:\overline{n|}} \cdot \ddot{a}_{x:\overline{n|}}$$

(10.187.)

Finally, it is worth to see, how we can deduct – with the help of the equations above – the annual net premium of the term fix insurance. Namely, not in the "classical" sense as a special endowment policy, but as a combination of a savings and a conditional annuity. Our starting point is (10.185.), which shows the size of the annual instalments of a simple savings for HUF 1 in n years.

A term fix policy consist of this plus a risk component, which is practically a regular premium conditional annuity with an annual $\left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right)$ sum assured.

It means, that in case of the death of the insured, the insurer start to pay this annual savings instalment into the reserve of the term fix insurance, instead of the insured.

Also, on the basis of the equations above, the single premium of a single life conditional annuity with annual HUF 1 sum assured and with term n years is $\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}}$, namely the single premium of a conditional annuity with annual HUF $\left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right)$ sum assured is

$$\left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) \cdot \left(\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}}\right)$$

(10.188.)

what is – on the basis of the conversion below –

$$\left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) \cdot \left(\ddot{a}_{\overline{n|}} - \ddot{a}_{x:\overline{n|}}\right) = \left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) \cdot \left(\frac{\ddot{a}_{\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - 1\right) \cdot \ddot{a}_{x:\overline{n|}}$$

equal to an annual premium:

$$\left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) \cdot \left(\frac{\ddot{a}_{\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - 1\right)$$

(10.189.)

To this we have to add the annual $\left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right)$ "savings" instalment, so we will get, that the annual net premium of a term fix insurance is:

$$P_{x:n} = \left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) \cdot \left(\frac{\ddot{a}_{\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - 1\right) + \left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) = \left(\frac{\ddot{a}_{\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - 1 + 1\right) \cdot \left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right) = \frac{\ddot{a}_{\overline{n|}} \cdot \left(\frac{1}{\ddot{a}_{\overline{n|}}} - d\right)}{\ddot{a}_{x:\overline{n|}}} = \frac{v^n}{\ddot{a}_{x:\overline{n|}}}$$

(10.190.)

what is equal to (10.109.).

11. THE PREMIUM RESERVE

KEW WORDS

Policy anniversary
Paying up the policy
Premium reserve
Policy loan
Non-forfeiture options
Surrender
Zillmerization

Without any explanation, we have already used the term premium reserve several times. From these references, but also from different reports appearing in the press, it is obvious that this is a term of cardinal importance. The size of life insurance companies is characterized – besides premium income figures – by the size of the premium reserve. It can easily happen that the annual premium income of an insurance company and the premium reserve that it handles differ by several orders of magnitude to the advantage of the premium reserve. But what exactly is this premium reserve?

The premium reserve – referred to as mathematical reserve in the official terminology – is the sum of money accumulated by the insurer from the premiums paid by the members of the risk community to cover later benefits paid to the members of the risk community who have suffered loss. The premium reserve is primarily interpreted on the whole risk community, but because of practical purposes it is accounted on individual policies, and so the total premium reserve is the sum of the premium reserves of all individual policies. This is also due to the fact that the premium reserve is created from individual payments. To sum up, we can say that the premium reserve is the money of the risk community on which the insurer promised later benefits, but not yet provided any. The individual insured (policyholders) as members of the risk community can be regarded as owners of proportional parts of the premium reserve, but only to the extent that it doesn't interfere with the functioning of the risk community.

The premium reserve is the cause – as we have already referred to it – of dividing insurances into the branches of life and non-life, since in case of life insurance it is typical that the insurer collects (parts of) the premium paid by the client for decades to cover the later benefits. When we were discounting the above formulae, we have tacitly supposed that the decrease in premium payment due to premiums decreased by discounting is compensated by the investment yields of the insurer. But what does the insurer invest? Naturally the premium reserve. The premium reserve cannot be the summarized and compounded value of all premiums paid by the client, since the insurer also has to cover from the received premiums its expenses and the benefits paid during the term. So, the premium reserve – although somewhat similar – is more complex than a bank deposit. Let's examine the premium reserve in detail! Since the premium reserve behaves in a significantly different way in case of term insurance and pure endowment insurance, we will discuss these separately in the following.

It is important to note a dichotomy about reserves:

1. below we discuss the value of the reserves proportional to a single insurance contract and we present the method of the calculation of the reserve of a single contract, but
2. insurance supposes a priori, that there are many insureds and many contracts and the reserve belongs to these many contract together.

Calculating reserve of a single contract is a useful convention, because it helps to accomplish calculations, since the calculation procedure is the same for an insurance portfolio of any size: the final size of the reserve is proportional to the size of the portfolio. However, it is important to note that the reserve primarily belongs to the whole risk community, and only secondly to the individual contracts. The relevance of this statement will be clear eminently at non-forfeiture options. As a resulf of it in case of lapse the costumer will not simply get the individual reserve of his/her contract, but some part of it is deducted or – sometimes – the whole reserve is withheld.

11.1. The Premium Reserve of the Term Insurance

Let's take an example!

We suppose that 1,000 persons take out a term insurance of 1 Forint sum assured and *n* years term at the same time. All are men, *x* years old and pay the premium annually. How do we determine the necessary premium?

The most simple case would be if everyone would pay q_{x+t} every year (t = 0, 1, …, n-1), where q_{x+t} – as we know – is the probability of a man $x+t$ years old dying within one year. But since q_{x+t} – as we also know! – increases as x and t increase, i.e. as a person becomes older, the probability of death is higher, so in case of this type of premium construction the premium increases from year to year. This method can be imagined in case of group insurance, where the policy is renewed yearly. But it has been discovered several hundred years ago that in case of individual policies that are several years long, the premium increase following the increase of mortality rate has a very negative psychological effect, so other kinds of premium construction methods were used.[111]

Namely that the insurer requires the client to pay the same premium every year, which means that in the first years of the term the premium is somewhat higher than the

[111] On the other hand, the yearly renewable term insurance that can be regarded as an element of Unit Linked insurance works this way, so in case of modern insurances this is a possible, although complementary type of construction!

risk, at the middle of the term it is the same and finally at the end of the term it is lower, as shown by *figure 11.1*.

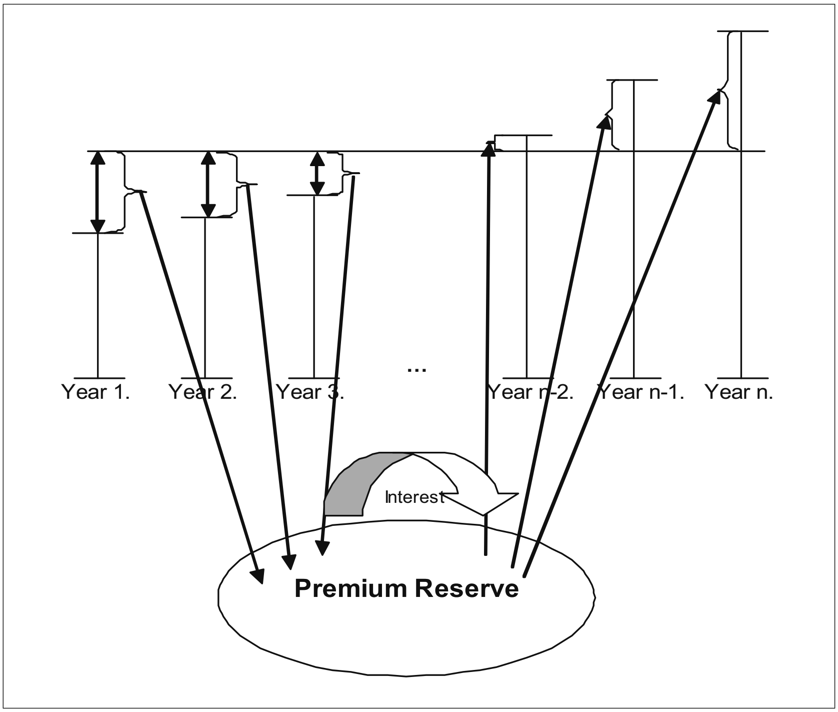

Figure11.1.: The relation of the annual premium needed and the actual premium in case of term insurance

The vertical columns show the value of premiums needed in years 1, 2, … n. The horizontal line shows the value of the level annual premium. The difference of the premium needed and the actual premium paid goes to the premium reserve in the first years of the term, and after that the premium deficiency arising in the later years is gradually supplemented from the reserve.

The insurer accumulates the *excess premium* paid at the beginning of the term, invests it, receives interest on it, and at the end of the term gradually uses it up to cover the premium deficiency calculated for this period. In case of the term insurance, this premium accumulated at the beginning of the term is the premium *reserve*. This increases roughly until the middle of the term, and gradually decreases after that, until it becomes zero at exactly the end of the term, as we can see in the following example. The example also shows that the premium reserve of the term insurance never becomes a significant value, and because of this, it generally (at most insurance companies, but not all, and not only because of this) it will not become the base of profit-sharing, neither of non-forfeiture option. (In other words, regular premium term insurance cannot be paid up or surrendered.) We will discuss non-forfeiture options later.

Example: If the client takes out the following term insurance, then the value of the premium reserve on the individual policy anniversaries can be as shown below.

Starting data	
Age:	40 years
Insurance term:	20 years
Sum assured:	HUF 10,000,000
Gender:	Male
Annual premium:	HUF 129,897

Year	Premium reserve	Year	Premium reserve
0	0	11	484,214
1	68,011	12	483,573
2	132,455	13	473,912
3	193,661	14	454,193
4	251,534	15	422,989
5	305,527	16	377,534
6	354,287	17	315,283
7	396,780	18	233,534
8	431,753	19	129,390
9	458,261	20	0
10	475,794		

Table 11.1.: The premium reserve of a term insurance – before premium payment

The path that the premium reserve of a term insurance runs is shown in figure 11.2. We have also indicated in the figure the effect of premium payment at the beginning of the year, this made the figure “crisscrossed”. Normally we only show the premium reserves at anniversaries, before premium payment, which makes the curve more “smooth”, as it can be seen on the other figures. We also indicated the level of the annual premium, so the order of the premium reserve can be imagined better (we see that after the first premium payment is the same as the level annual premium, and one year before the end of the term after premium payment it equals exactly the amount needed for the death benefits paid in that year!). The figure also shows what the level of death benefits paid form the premium reserve in each year is in this concrete case.

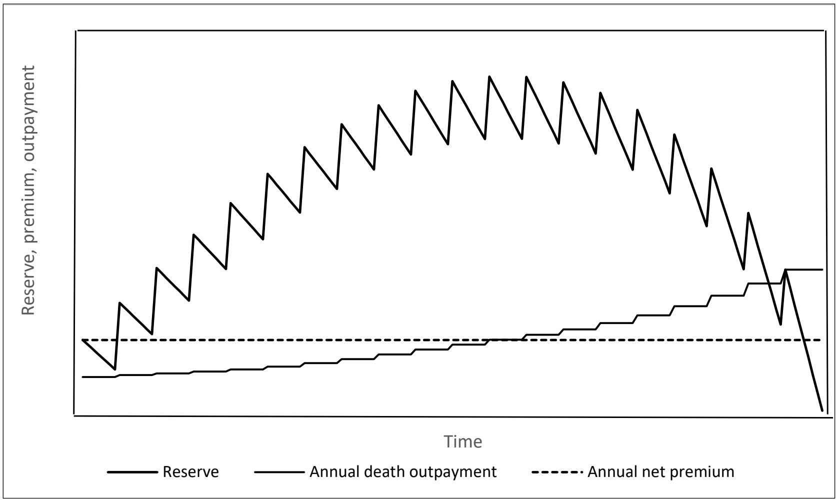

Figure 11.2.: The premium reserve of a term insurance

We should look at a few more things concerning the premium reserve of a term insurance. E.g. – we can declare the following rules concerning its level:

Supposing the same entry age, the longer the term, the higher the maximum of the premium reserve will be, and the closer it will relatively be to the end of the term.

Supposing the same term, the higher the entry age is, the higher the maximum of the premium reserve will be, and the closer it will get to the end of the term.

Both rules have the same cause: mortality rates – as we have seen – increase exponentially with age. This way the gap between the level annual premium and the death premium necessary in the year will increase, which has to be filled by the premium reserve. The difference is especially great at the end of the term, and increases with age exactly here, so a longer part of the term has to accumulate for the "premium deficiency" of the last years.

The above relations are shown on figures 11.3. and 11.4.:

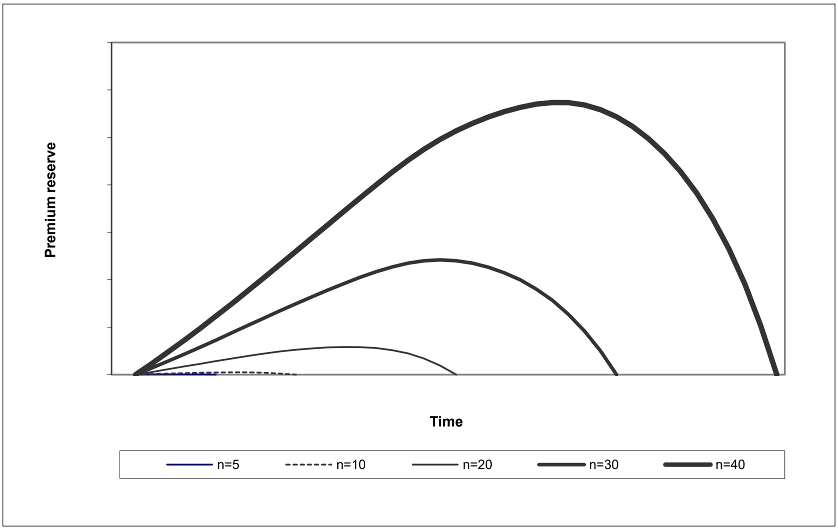

Figure 11.3.: The premium reserve of term insurance with the same entry age and different insurance terms

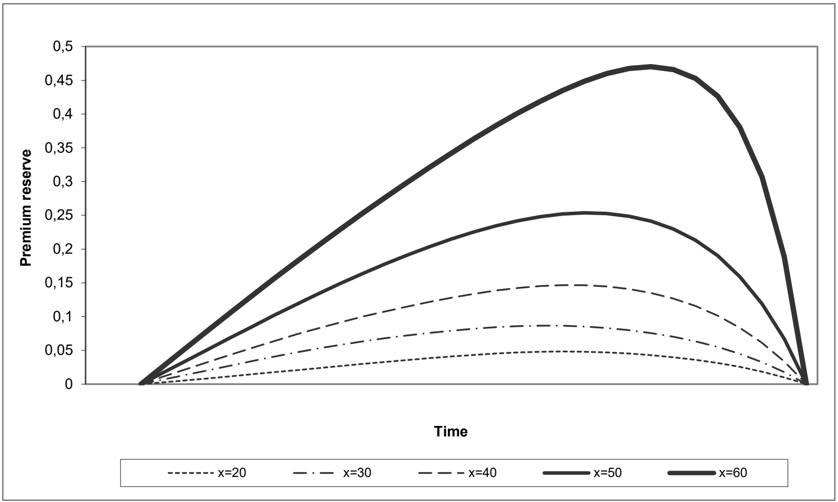

Figure 11.4.: The premium reserve of term insurance with the same insurance term and different entry ages

We also get a very interesting figure if we draw the curve of the term insurance with shortened premium payment term. In figure 11.5. the premium term is shortened to 10 years (the insurance term is 30 years!). After the end of the premium term insurances with shortened premium payment term works like a single premium insurance, so their premium reserve also equals that of the single premium insurance. In the figure the dashed line shows the reserve of the single premium term insurance, and the "filled line" shows that of the term insurance with shortened premium term. It is clear that after the end of the premium term the two curves are the same, and until then the single premium reserve at the end of the premium term has to be accumulated through premium payments.

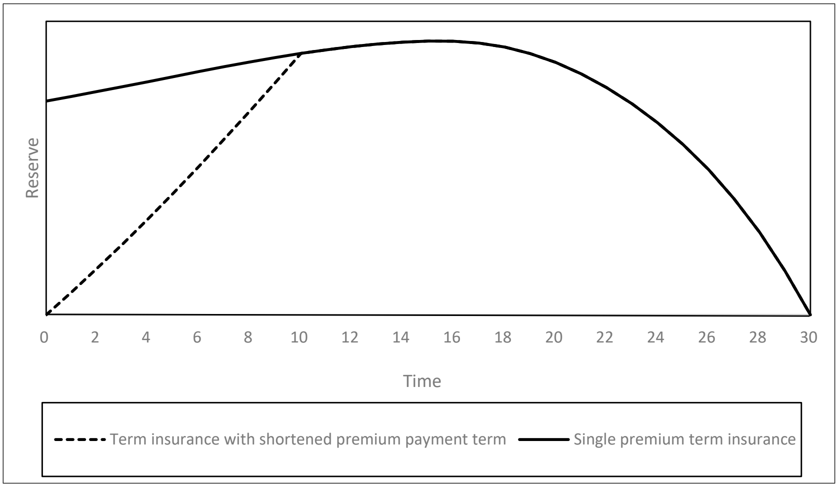

Figure 11.5.: Reserve of a Term insurance with shortened premium term

11.2. Premium Reserve of Pure Endowment and Endowment Insurance

The premium reserve of the *pure endowment* insurance is different. Here the sum paid to the client at maturity has to be gradually accumulated during the term. (This is why pure endowment, term fix and endowment insurances are called *saving type insurances*.) Here the premium reserve gradually increases, and at the end of the term it equals exactly the sum of benefit payment. The (risk part of) premium payments increase the reserve of the pure endowment insurance similarly as an interest earning deposit. The only difference is that if all other conditions are the same, the premium reserve of the pure endowment insurance increases faster than the deposit account, since the insurer subdivides the payments of those who die during the term among the “accounts” of those who are still alive.

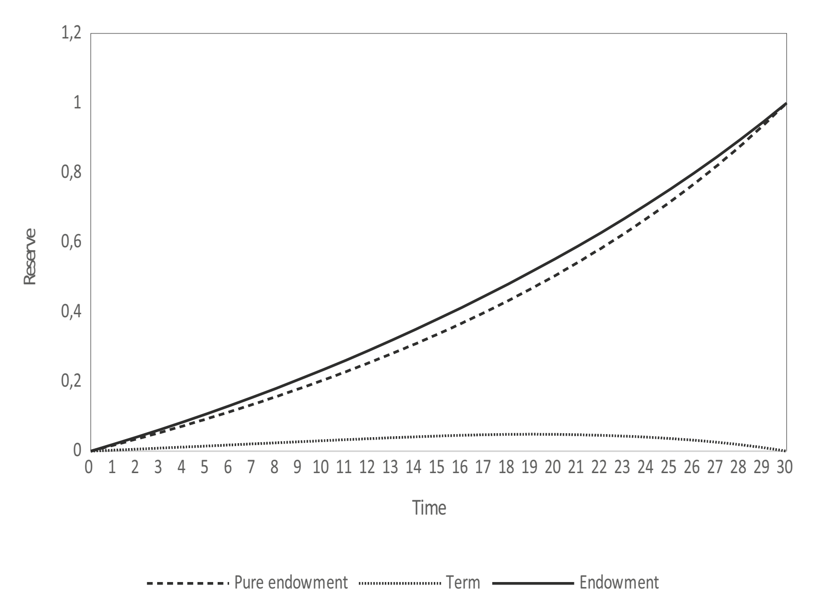

Figure 11.6.: The premium reserve of a regular premium term, pure endowment and endowment insurance

The premium reserve of the *endowment insurance* is the sum of the reserve of a *term insurance* and a *pure endowment insurance*, so it is zero at the beginning of the term, and equals the sum assured at the end of the term. The rate of increase of the premium reserve of an endowment insurance is lower than that of a similar deposit, since this price has to be paid for the security of death benefit payments.

Figure 11.6 shows the curve that a regular premium pure endowment and endowment insurance follows.

The premium reserve of a pure endowment insurance is described by a curve increasing by an accelerated measure. This accelerated increase has the following sources:

- the regularly arriving premium;
- the yield of the premium reserve (to the level of the technical interest rate);
- part of the premium reserve of those who die during the term.

The premium reserve of the single premium pure endowment and endowment insurance is somewhat different from the above. Here we have no premium arriving regularly, but the insurer receives the premium of the whole insurance term (that is, of course is not the same as the term multiplied by the annual premium) at the beginning of the term, so the premium reserve doesn't start from zero. Figure 11.7. illustrates this.

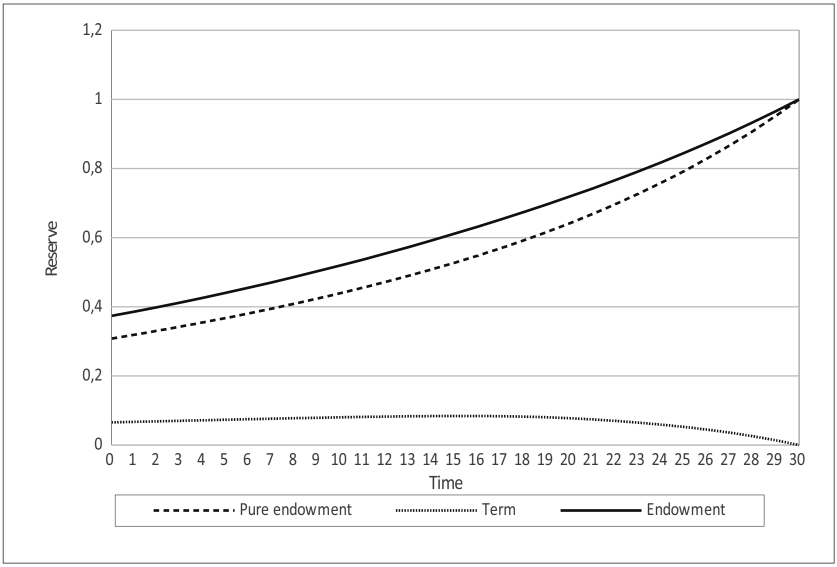

Figure 11.7.: Premium reserve of single premium pure endowment and endowment insurance

We can say the same about the pure endowment (and endowment) insurance with shortened premium term as we have said in case of the term insurance. Here is an example:

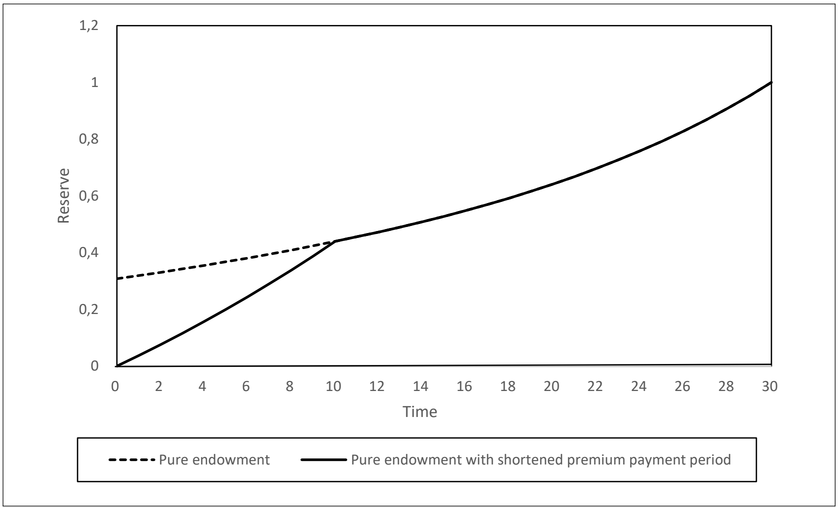

Figure 11.8.: Pure endowment insurance with shortened premium term

We can see by simply analysing the sources that the value of the premium reserve at a given moment cannot be determined by compound interest calculation. This requires special actuarial knowledge. This is even true for the premium reserve of the endowment insurance, that doesn't only depend on the earlier mentioned 3 (increasing) factors, but there is also a constant decreasing effect, namely that the dependents (beneficiaries) of the deceased are also satisfied from the premium reserve.

The calculation of the premium reserve is even more complicated. This is caused by zillmerization, a procedure named after Zillmer, a German actuary.

11.3. Zillmerization[112] and Other Problems

Zillmerization has been created to solve a problem of timing. The cover of the insurer's expenses is the expense loading, that arrives to the insurer (in case of regular premium insurances) gradually throughout the term, in equal payments. There wouldn't be any problem with this, if the expenses of the insurer would also arise this way. But the situation is different. There are expense parts (e.g. premium collection), that arise the same way. But these are the more negligible expense parts. The more significant

[112] Chapter 13. discusses zillmerization in detail.

expenses (contracting, underwriting, and policy issue) arise right at the inception of the policy. These are the expenses of commission, medical examination, administration, etc… that are due when the policy is signed. If the insurer wouldn't zillmerize, then this would have two major consequences:

The insurer would credit acquisition costs – that are to be debited to the policyholder, since they arise because of him – to the policyholder for a long period of time, that the policyholder would pay back gradually through the expense loadings.

If the policyholder surrenders the policy at the beginning of the term, then the insurer couldn't recover these expenses, since further premium payment ceases. This is one of the causes why insurers who do not apply zillmerization define a waiting period for non-forfeiture options.

Zillmerization solves this problem in the following way: that the insurer takes the premiums (in excess of the current death benefit payments) of the first 0.5-2.5 years – depending on the insurance term – and uses all of it to cover expenses (i.e. borrows the premium reserve of the first 0.5-2.5 years from the policyholder) and pays this loan back gradually later on from the premium loading. The effect of this is demonstrated by the following example:

Example: The premium reserve of a regular premium pure endowment insurance of 10 years term with and without zillmerization follows the curves of *figure 11.9.* (exaggerating the effect of zillmerization on purpose!):

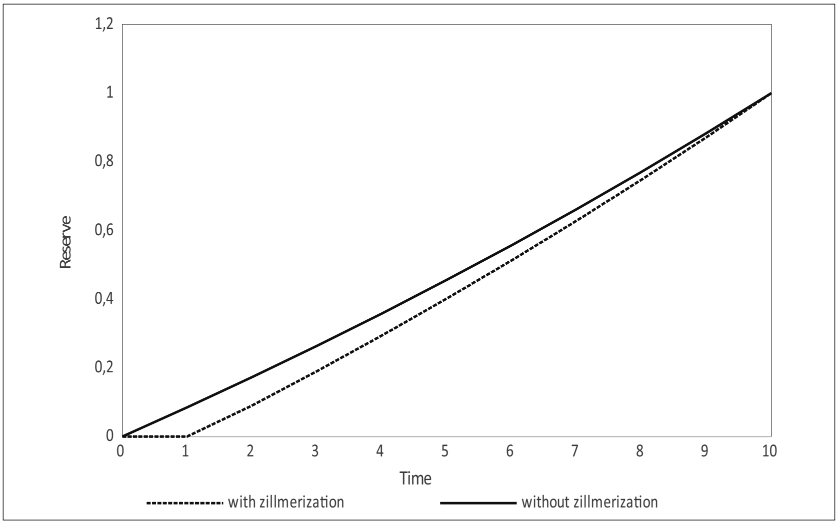

Figure 11.9.: The premium reserve of a regular premium endowment insurance with and without zillmerization

We see that the zillmerized reserve is lower than the non-zillmerized reserve in the whole term. Zillmerization has several consequences. One is that while the premium reserve is zero, there is no profit-sharing. The other is that the value of non-forfeiture options during this period is also zero.

Zillmerization won't result in negative premium reserve, but improper actuarial design can have the effect that the reserve of the product becomes negative somewhere during the term. Since this would mean that the insurer lends money to the client, and the client can surrender the policy any time (contrary to the insurer, who is not allowed to terminate the life insurance policy), these situations have to be avoided under all circumstances!

Non-forfeiture options have appeared several times before, so let's see what these are!

11.4. Non-forfeiture Options and Policy Loan

11.4.1. The Types of Non-forfeiture Options

The basis and the cause of non-forfeiture options is the premium reserve. The premium reserve could also be defined as the sum of money that the insurer has collected to provide some benefits – as stated in the policy – later on. But if the policy is terminated, then this promise of later benefits is gone, and the insurer has to account for the premium reserve. This accounting liability is called from the client's point of view *non-forfeiture options*.

There are several types of non-forfeiture options (and although it is not a non-forfeiture option, it is useful to discuss policy loan in the same section!). When categorising these, let's start from the fact that the insurer would like to keep the client under all circumstances. What does it do then?

First of all, it is possible that the client wants to terminate the policy because of *temporary financial difficulties*, needs a larger sum of money, and is on the opinion that the premium reserve could be used for this purpose. In this case, in order to avoid the termination of the policy, the insurer offers the possibility of *policy loan*. As we have already mentioned, this is not a true non-forfeiture option yet. The base of the loan (and this way also its limit) is the premium reserve. The insurer regards the loan as an investment possibility, and demands an interest on the loan that is the same as interest earned on its other investments. If the insurance company functions properly, then this yield, i.e. the interest of the loan is high, maybe higher than the interest of other loans.

There are a lot of misunderstandings concerning policy loans, that have to be cleared. For one, as we have seen, taking out the policy loan is not a particularly advantageous business for the policyholder. If someone takes out a life insurance policy to be able to take out a policy loan later on, then that person made significant miscalculations, since the sum of the loan remains for a number of years below the sum of premiums paid so-far. (A totally different case is when someone is required by the bank to buy a credit life insurance as a prerequisite of a loan.) Neither is it the best business for the insurer, since investing larger sums of money has less expenses than lending it in small portions. This way the policy loan is only used by insurers if everything else fails. Generally the regulators do not like it either, because it makes possible to abuse the tax benefits, so it is common to restrict it by losing the tax benefit as a sanction.

Secondly: If the client has longer lasting financial difficulties and cannot pay premiums any more (naturally this can only happen in case of regular premium insurances) but doesn't need the premium reserve, then the insurer offers the possibility of *paying up the policy*[113]. The essence of paying up the policy is that the insurer regards the premium reserve accumulated so far as the single premium of an insurance of the same kind, that has a term equal to the remaining years of the original policy's term, and a sum determined by the current age of the insured. This sum assured will naturally be lower than the original sum, since there is no further premium payment.

Thirdly: if the client gets into permanent financial difficulties, that hinder further premium payment, moreover the client needs the money accumulated in the premium reserve, then he *surrenders the policy*[114]. In this case the insurer terminates the policy and gives the premium reserve back to the policyholder. To be precise, not the whole premium reserve in most cases, only the greater part of it. The smaller part, that the insurer "nips off" or withholds partly serves to counterbalance the effect of anti-selection, that is created because the insurance is probably surrendered in a greater portion by those whose health has not deteriorated meanwhile, and have a greater chance of living until maturity. Those, whose health has deteriorated tend to "leave" the greater sum assured to the lower premium reserve in a smaller portion. This "nipping off" is closely related to our general statement, that the premium reserve primarily belongs to the risk community, and only secondly to the concrete client, where the insurer keeps record of it. Withholding further parts of the premium reserve partly gives a kind of compensation for the profit lost, and partly tries to make "painful" the surrender for the costumer, urging him/her to keep the policy. (Paradoxically, this kind of "penalty" can be valuable also for the costumer – as it already was mentioned in subchapter 7.3. -, because the main practical virtue of the life insurance for many its illiquid feature. Because it is hard (time-consuming and very expensive) to liquidate the capital in life

[113] This is already regarded as a non-forfeiture option by insurance regulation, but if we think about it, in reality this is only a standardized policy transform option!

[114] This is a true non-forfeiture option, and the only one in a sense, since in reality only this case requires "final settlement" between the insurer and the client!

insurance before the conctractual end of the policy, this feature also prevent the myopic depletion of it.)

11.4.2. Limiting Non-forfeiture Options

The basis of non-forfeiture options is the premium reserve, but not all insurances with premium reserve have non-forfeiture options, or the insurer doesn't usually offer all non-forfeiture options to all policies with premium reserve. This is caused by the *anti-selection* arising due to lapses.

Let's take an example! We start from the case when all non-forfeiture options are offered to all insurances with premium reserve. (Of course this is not true.) E.g. the *term insurance* also has the option of surrender. Let's suppose that two insured persons having a term insurance are thinking about surrendering the policy at the same time. If one is in great shape, and will probably live until maturity, then he will certainly surrender the policy. The other, who is almost dying, will probably not surrender the policy saying that his family will receive much more as sum assured than as surrender value. This way the anti-selection has materialized, which the insurer tries to avoid in case of the term insurance by not offering any non-forfeiture options. (There may be a difference in the health status of the insured persons despite the medical examination and same health status at the beginning of the term, because significant changes might happen compared to the health status of the insured persons at the commencement of the policy.)

The anti-selection realized through non-forfeiture options in case of term insurance is based on the fact that those, whose health status has not deteriorated significantly during the term tend to terminate the policy more. But when calculating the premium, we supposed that healthy people pay the premium through the whole term, and this covers the death benefits of those, whose health has deteriorated. So, if healthy people have a chance of leaving the insurance through a non-forfeiture option, this puts the security of the calculation at risk.

This works exactly the opposite way in case of the pure endowment insurance. Here those are tempted to leave the insurance, who know that death is near, and after their death it is not the inheritors who receive the accumulated premium reserve. In this case they surrender the policy if the chance is given. Meanwhile, the pure endowment insurance is naturally calculated in a way that a few insured have to die so that their premium reserve can be subdivided between the remaining insured. Because of this, surrendering or paying up the policy is not allowed in case of the pure endowment insurance. This causes the pure endowment insurance to be a very inflexible construction. If the client doesn't want to pay premiums any more, he loses all payments made so-far. This is why insurance companies don't like to offer this construction.

Because the *immediate annuity* is the series of pure endowment insurances, the same is true for annuities.

In certain cases insurers offer non-forfeiture options to term or pure endowment insurances despite the above. But these have special conditions.

If the business politics of the insurance company is such that the independent term and pure endowment insurance is sold together, quasi as an endowment insurance, then non-forfeiture options similar to that of the endowment insurance can be offered on both policies. But e.g. surrender in case of the pure endowment insurance can only be allowed if surrendering the term insurance at the same time, otherwise the insurer provides a good chance for anti-selection. Namely the client can surrender one policy and keep the other depending on how his health status changes, which he could not do with a united endowment insurance.

A reason for offering non-forfeiture options on a term insurance could possibly be that it is expected to represent a small fraction of the insurers total life portfolio, so the death premium reserve paid out as surrender value is a negligible sum compared to the whole reserve of the portfolio.

Some insurers provide non-forfeiture options in case of pure endowment and annuity insurance subject to medical examination, and only allow the insured in perfect health status to use these options. So, the medical examination is a way to ease the offer of non-forfeiture options, but since it is expensive, it is only worth using it in case of large premium reserve, i.e. large sum assured.

12. CALCULATION OF THE PREMIUM RESERVE

We only use the term premium reserve (life insurance mathematical reserve in the official terminology) for the traditional life insurances. We call the reserve of unit linked insurance – that practically has the same function, but requires technically somewhat different handling – the reserve of unit linked insurance, this way separating it from the general life insurance mathematical reserve. Here we will mostly deal with the reserve of traditional insurances, but at the end of the chapter, in a separate sub-section we'll discuss the reserve of unit linked insurance.

12.1. The Calculation of the Premium Reserve Generally

When we deducted the formulae of net premiums in the chapter on premium calculation, then our starting point was the equivalence equation, namely equivalence of the expected present value of all income and all payout. Clearly this equivalence has to hold not only at the commencement of the coverage, but at any point during the whole term (naturally discounting and compounding the appropriate values to this point in time).

Let's suppose that we consider the equivalence equation at the end of the t^{th} year of the insurance term. If at this point we denote the appropriate values the following way:

B_1^t: compounded sum of all income up to the t^{th} point of time (so the income due exactly at the t^{th} point is not included),

K_1^t: compounded sum of all benefit payments up to the t^{th} point of time,

B_2^t: discounted sum of all income expected until maturity,

K_2^t: discounted expected value of benefit payments until maturity,

then, according to the equivalence equation:

$$B_1^t + B_2^t = K_1^t + K_2^t \tag{12.1.}$$

From this we get by simple transformation:

$$B_1^t - K_1^t = K_2^t - B_2^t \tag{12.2.}$$

The left hand side of the above equation is the excess of all income received so far, remaining after the benefits paid so far, and the right hand side is the excess that indicates how much the expected future benefit payments exceed the expected future income. Both sides show the *premium reserve* at this point in time, and give calculation methods to the computation of this value. So, calculation can be performed in two ways:

1. The left hand side $(B_1^t - K_1^t)$ shows the so-called ***retrospective*** method (looking back into the past), according to which we subtract from the compounded value of income the compounded value of payout, and this way get the excess of accumulated income that can be used for future expected benefit payments.
2. The right hand side $(K_2^t - B_2^t)$ shows the so-called ***prospective*** method. According to this, the value of expected future income has to be subtracted from the value of expected future benefit payments, and this way we get the sum that has to be reserved, so that the insurer will be able to cover all future liabilities.

Since the reserves calculated by different methods are equal to each other, so we could say that we can choose between the two methods to our liking. But the regulation generally prefers, what is more, specifies the prospective method, and this could seem a little bit unusual. To understand the logic of the regulation it is worth to analyse more deeply the meaning of these formulae.

We can establish the reserves at anniversaries already at the beginning of the term using either of the formulae. This means that we assume that the informations we have taken into consideration in the calculation remain unchanged during the whole term which can be even some decades. So, behind the equality of the prospective and retrospective reserves stands the implicite assumption that the parameters underlying the calculation are unchanged. These parameters are the mortality and the technical interest rate. It is unsesessary to change these items if they were determined well. But it can happen the opposite.

If – for example – it turns out that the technical interest rate, which was promised and calculated into the premiums, is too high, namely the insurer probably would not able to reach it from its investment in the future, then it is not an adequate response to this (not prudent solution) to supplement the arisen deficit of the reserve from the capital in each year. The only prudent solution in this case if the insurer "eats the leek" in one piece, in other words foots the bill in advance for the whole remaining term. It means, that henceforth the insurer calculates the prospective reserve with a lower technical interest rate. In this case there will be naturally a disruption between the prospective and retrospective reserves. The value of the prospective reserve "takes a leap" (while the retrospective reserve remain unchanged) and the difference of the two have to be covered from the capital of the insurer. In this case the insurer is not allowed to argue that the value of the retrospective reserve is lower yet. The only important thing, which eventually determines the necessary level of reserves, is that the insurer has to be able to fulfil its service in the future promised to the clients at all events.

A similar problem can emerge if the mortality changes systematically (i.e. not one-time) on a non expected scale and because of it a mortality loss arises in a portfolio. In this case, the mortality table used for reserving needs a modification and this also causes a split between prospective and retrospective reserves.

Theoretically, for an insurer it is not a problem if mortality profit appears systematically in all years because of the applied mortality table and the insurer expects it. So, in the past the systematic mortality profit was not a motive to change the mortality table used for reserving. But at the beginning of the years 2000 a new point of view has emerged, the investors' of the insurers. They want to know exactly the profit or loss potential of an insurance portfolio, so they require to see the exact expected future results in the books of the insurer right now. Because of this, investors urged such a regulation that the reserves have to be calculated by the best available (projected) mortality table at all times. This type of regulation has already been introduced in many countries. Therefore, the need for change in mortality table can also appear in the case of mortality profit. In this case the reserve has to be reduced and the redundant reserve became profit. This also emphasize the importance of prospective reserving. (The details of the technical income statement we are dealing with in the Chapter 18.)

It is important to note that the abovementioned changes in the prospective reserve do not affect the premium paid by the client, because it is fixed in the contract, even if it has turned out meanwhile that this premium was not adequate, was too low.

Therefore, below we review first the prospective reserve formulae, then we examine the retrospective formulae compared to the prospective ones. Thus we first seek the values of and :

- K_2^t: the discounted value of future benefit payments, as the single premium of an insurance the same type and having the same sum assured as the original, that an insured of $x+t$ years could take out for a term of $n-t$ years (or a lifetime term). (This description in this simple form is not true for those life insurance products, where the sum assured depends on the elapsed time (t) – that is the "staged" term insurance and the pure endowment with premium refund insurance.) In other words the net single premium and actually express the same, since according to the equivalence principle the net present value of the claim payments equal to the single premium.
- B_2^t: the discounted value of future income in case of regular premium insurances is the capital value of the temporary annuity with payments in advance, which has an annual payment equal to the annual premium of the original insurance, where the insured is $x+t$ years old and the term equals the remaining years of the original term, $n-t$ years.

In case of single premium insurances our job is easier than in the case of regular premium payment, because here $B_2^t = 0$, since we do not expect premium income after the single premium payment at the beginning of the term.

The situation is a little more complicated than the above if the sum assured and/or the premium varies as a function of elapsed years, and in the case of premium refund insurances. These require further considerations, naturally keeping the validity of the basic equation $B_1^t - K_1^t = K_2^t - B_2^t$.

We denote the value of the premium reserve at the end of year t by V_t.

The premise of "end of year" means – beside other things – that the changes occurring exactly at the t point in time – e.g. insurance payment due in t – are not included in V_t. Further on we will give these V_ts for 1 Forint sum assured, as usual.

Earlier, when the insurers aspired to produce mortality profit each year (and which aspiration the investors do not like), there was a widespread practice the so called "conservative" method of reserve calculation. Under this term it was meant that in reserve calculation it was supposed the entry age of the insured to be 1 year higher than the actual, or the "technical" age adjusted by increase (due to poorer health status, occupation, etc…) This one year increase doesn't affect the form and logic of the following formulae.

12.2. The Calculation of the Annual Prospective Premium Reserve

Based on the above it is easy to give the formula of single premium insurances.

For the **term** insurance:

$$V_t = A^1_{x+t:\overline{n-t|}}$$

(12.3.)

so the value of the current reserve equals the single net premium of an insurance having a term of the remaining years *(n-t)* and an insured of age $x+t$.

The **pure endowment** insurance:

$$V_t = A_{x+t:\overline{n-t|}}^{\;\;\;\;\;\;\;\;1}$$

(12.4 .)

It is obvious that for the **endowment** insurance:

$$B_1^t + B_2^t = K_1^t + K_2^t$$

(12.5.)

Since the premium formulae of the different annuities are very similar, it will be enough if we look at the prospective reserve formula of the **single premium immediate, temporary, lifetime annuity**.

$$V_t = \ddot{a}_{x+t:\overline{n-t|}}$$

(12.6.)

For regular premium insurances it is enough to determine the formula generally, which is, based on the above, in the prospective case simply:

$$V_t = K_2^t - B_2^t = A_{x+t:\overline{n-t|}} - \ddot{a}_{x+t:\overline{n-t|}} \cdot P_{x:\overline{n|}}$$

(12.7.)

Since this formula is completely unambiguous and we have already derived all necessary sub-results in case of concrete insurances, here we won't introduce the reserve formulae in more detail except some non-trivial cases. néhány nem triviális eset kivételével.

In case of Term fix insurance $K_2^t = A_{\overline{n-t|}} = v^{n-t}$. It is important to note here, that the speciality of this insurance, that it does not cease after the death of the insured, but the insurer does not except any premium payment after that, so the reserve formula will not $K_2^t - B_2^t$ be any more but only K_2^t.

It was mentioned, that in case of "staged" term insurance and the pure endowment with premium refund, the K_2^t will be a little bit more complicated, than in case of the remaining traditional insurances.

In case of "staged" term insurance K_2^t would be the following on the basis of (10.29.):

$$A_{x+t:\overline{n-t|}} = \frac{(n-t) \cdot M_{x+t} - (R_{x+t+1} - R_{x+n+1})}{(n-t) \cdot D_{x+t}}$$

(12.8.)

but it is **incorrect**, because it is the single premium of an insurance where the sum assured at the beginning (when the insured's age is x+t years) is 1 and this is decreasing annually by $\frac{1}{n-t}$. This is obviously incorrect, because we need here such a single premium where the sum assured is already $\frac{n-t}{n}$ at the t. anniversary and furthermore the pace of the annual decrease is $\frac{1}{n}$. Because of this the right K_2^t similarly deduced as (10.25.):

$$K_2^t = \frac{\frac{n-t}{n} \cdot d_{x+t} \cdot v^1 + \frac{n-t-1}{n} \cdot d_{x+t+1} \cdot v^2 + \cdots + \frac{1}{n} \cdot d_{x+n-1} \cdot v^{n-t}}{n \cdot l_{x+t}}$$
$$= \frac{(\mathrm{n}-\mathrm{t}) \cdot C_{x+t} + (n-\mathrm{t}-1) \cdot C_{x+t+1} + \cdots + 1 \cdot C_{x+n-1}}{n \cdot D_{x+t}}$$
$$= \frac{(M_{x+t} - M_{x+n}) + (M_{x+t} - M_{x+n-1}) + \cdots + (M_{x+t} - M_{x+t+1})}{n \cdot D_{x+t}}$$
$$= \frac{(\mathrm{n}-\mathrm{t}) \cdot M_{x+t} - (M_{x+t+1} + \cdots + M_{x+n})}{n \cdot D_{x+t}} = \frac{(\mathrm{n}-\mathrm{t}) \cdot M_{x+t} - (R_{x+t+1} - R_{x+n+1})}{n \cdot D_{x+t}}$$

(12.9.)

Only in the denominator is some difference between the right (12.9.) and the incorrect (12.8.)!

As long as in case of "staged" term insurance the abovementioned K_2^t can be applied both single and regular premium insurances, the case of pure endowment with premium refund is even more difficult, since we know that here there is an essential difference between the single and regular premium variants, so here we have to construct the "single premium of the regular premium pure endowment insurance with premium refund" – which seems squaring the circle. Thus the K_2^t below can be applied only for regular premium pure endowment insurance with premium refund.

Aside from these restrictions, we can realize similarly as above, that K_2^t here is not simly (10.119.) by replacing x:n with x+t:n-t, but instead of this we have to modify (10.118.) consistent with reason:

$$K_2^t = A_{x+t:\overline{n-t|}}^{\;\;1} + (\mathrm{t}+1)\cdot PG_{x:\overline{n|}}\cdot\frac{M_{x+t}-M_{x+n}}{D_{x+t}} + PG_{x:\overline{n|}}\cdot\frac{M_{x+t+1}-M_{x+n}}{D_{x+t}} + \cdots + PG_{x:\overline{n|}}\cdot\frac{M_{x+n-1}-M_{x+n}}{D_{x+t}} =$$

$$= A_{x+t:\overline{n-t|}}^{\;\;1} + PG_{x:\overline{n|}}\cdot\frac{t\cdot M_{x+t} + (M_{x+t}+M_{x+t+1}+\cdots+M_{x+n-1}) - n\cdot M_{x+n}}{D_x}$$

$$= A_{x+t:\overline{n-t|}}^{\;\;1} + PG_{x:\overline{n|}}\cdot\frac{t\cdot M_{x+t} + R_{x+t} - R_{x+n} - n\cdot M_{x+n}}{D_{x+t}}$$

(12.10.)

The formula (10.119.) obviously is a variant of this in case of t=0.

12.3. The Retrospective Premium Reserve Formulae

Although regulation prefers prospective premium reserve calculation not on the basis of technical/calculation considerations, if we try to calculate the retrospective reserve on the basis of (12.2.) it is easily apparent that we not really get practicable formulae unlike above in the prospective case. Let us see the case of single premium term insurance. We could think, that here the following formulae are true:

$$B_1^t = A_{x:\overline{n|}}^1 \cdot (1+i)^t$$

$$K_1^t = A_{x:\overline{t|}}^1 \cdot (1+i)^t$$

(12.11.)

because at the beginning of the term the income was $A_{x:\overline{n|}}^1$, which bears interest, so at the end of t. Year it will be $A_{x:\overline{n|}}^1 \cdot (1+i)^t$, and because the present value of the outpayments of the first t years in the 0. Time is $A^1_{x:\overline{t|}}$, which became $A^1_{x:\overline{t|}} \cdot (1+i)^t$ to the t. Anniversary. Because the prospective and retrospective reserves have to be equal, the difference of the two should have to be $A^{\ 1}_{x+t:\overline{n-t|}}$, but will not be, since the difference is:

$$\left(A^1_{x:\overline{n|}} - A^1_{x:\overline{t|}}\right) \cdot (1+i)^t = \frac{(M_x - M_{x+n}) - (M_x - M_{x+t})}{D_x \cdot v^t} = \frac{M_{x+t} - M_{x+n}}{D_x \cdot v^t} \tag{12.12.}$$

instead of the righ
$\frac{M_{x+t}-M_{x+n}}{D_{x+t}}$, that is the denominator is $l_x \cdot v^{x+t}$ instead of the right $l_{x+t} \cdot v^{x+t}$. Therefore, the formulae **(12.11.)-(12.12.) are incorrect,** (12.12.) shows smaller value than the real (sith the denominator here is larger).

What could be the reason of this, sith the (12.11.) formulae are seemingly right? What is the root of the excess compare to (12.12.)? After a short speculation we could realize, that in the case of prospective reserving we do not need to care about the changing insured portfolio. However, in the case of retrospective reserving we have to take into account the reserves of the deceaseds and we have to distribute it among the livings (yet). Therefore, here B_1^t has a plus element. However, it means, that a retrospective premium reserve formula for an anniversary contains all the previous anniversary reserves, that is the retrospective formulae are recursive – at least in the first approach.

Naturally, the recursive formulae inhere the advantage – beside some disadvantages – that they show how we have reached one phase from an other. And we often need to know how the reserve changes and what factors cause the change. In the usual, prospective reserve formulae this remain hidden, so it is useful to convert them with a recursive (and so with retrospective) approach. Since it is very instructive, so we are dealing with these recursive, retrospective premium reserve formulae – and their interpretation – more detailed.

It is worth to note, that calculation of the prospective reserve – although it is the default – not always possible, especially the promised fix benefit is missig. In the case of the reserves of the modern UL insurances we can calculete them quasi only with retrospective method. Maybe this is the reason, the regulation handles their and the traditional life insurances' reserve separately. In the case of group life insurances also the retrospective reserve calculation is plausible.

12.3.1. The Change of the Premium Reserve of Single Premium Insurances

The prospective formula for the recursive, retrospective reserve one is a good starting point. In case of the pure endowment insurance these are for the t. and t+1. policy anniversaries:

$$V_t = A_{x+t:\overline{n-t|}}^{\;\;1} = \frac{D_{x+n}}{D_{x+t}} = \frac{l_{x+n} \cdot v^{n-t}}{l_{x+t}}$$

$$V_{t+1} = A_{x+t+1:\overline{n-t-1|}}^{\;\;1} = \frac{D_{x+n}}{D_{x+t+1}} = \frac{l_{x+n} \cdot v^{n-t-1}}{l_{x+t+1}}$$

(12.13.)

We can easily write the reserve for the t+1. anniversary as the function of the reserve for the t. anniversary as follows:

$$V_{t+1} = \frac{l_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i)$$

(12.14.)

In another form this is:

$$V_{t+1} = \frac{l_{x+t+1} + d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) = V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i)$$

(12.15.)

It is well shown here, that from the one anniversary to the other, the previous anniversary reserve not only bears interest ($V_t \cdot (1+i)$, but the reserve of the deceased d_{x+t} is distributed among the yet livings (l_{x+t+1}).

The propective reserves of the single premium term insurance are:

$$V_t = A^{\;1}_{x+t:\overline{n-t|}} = \frac{d_{x+t} \cdot v^1 + d_{x+t+1} \cdot v^2 + \cdots + d_{x+n-1} \cdot v^{n-t}}{l_{x+t}}$$

$$V_{t+1} = A^{\;1}_{x+t+1:\overline{n-t-1|}} = \frac{d_{x+t+1} \cdot v^1 + d_{x+t+2} \cdot v^2 + \cdots + d_{x+n-1} \cdot v^{n-t-1}}{l_{x+t+1}}$$

(12.16.)

It is differ from the recursive formula of the pure endowment insurance, that here from V_t we have to remove $d_{x+t} \cdot v^1$ which refers to the deceaseds in the age of x+t, when we turn to the reserve of the t+1. anniversary. So, the recursive, retrospective reserve fomula will be the following:

$$V_{t+1} = \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(V_t - \frac{d_{x+t} \cdot v^1}{l_{x+t}} \right) \cdot (1+i)$$

(12.17.)

We can rearrange it into some rational forms:

$$V_{t+1} = \frac{l_{x+t}}{l_{x+t+1}} \cdot (V_t \cdot (1+i) - q_{x+t}) = \frac{l_{x+t+1} + d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} =$$
$$= V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} =$$
$$= V_t \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot \big(1 - V_t \cdot (1+i)\big)$$

(12.18.)

The next to last formula includes (12.15.)[115], and, in addition, it contains the deduction because of the deaths, i.e. the HUF 1 sum assured which was paid each deceased (d_{x+t}) has to distribute among the yet livings at the end of the year (l_{x+t+1}). However, we can also interpret (see the last formula), that the death benefit which has to distribute among the yet livings in fact only $\big(1 - V_t \cdot (1+i)\big)$, sith a part of it can be financed from the reserve of the deceased. We can also say that for the insurer, the "sum at risk" is not the whole HUF 1 sum assured, but only a part of it above the reserve $\big(1 - V_t \cdot (1+i)\big)$. The life reinsurance contracts for example are generally concluded for this sum, not for the original sum assureds.

The prospective reserve formulae of the temporary, in advance annuity are:

$$V_t = \ddot{a}_{x+t:\overline{n-t|}} = \frac{l_{x+t} \cdot v^0 + l_{x+t+1} \cdot v^1 + \cdots + l_{x+n-1} \cdot v^{n-t-1}}{l_{x+t}}$$

$$V_{t+1} = \ddot{a}_{x+t+1:\overline{n-t-1|}} = \frac{l_{x+t+1} \cdot v^0 + l_{x+t+1} \cdot v^1 + \cdots + l_{x+n-1} \cdot v^{n-t-2}}{l_{x+t+1}}$$

(12.19.)

We can see, that we can reach t+1 from t with the following formula:

$$V_{t+1} = \frac{l_{x+t}}{l_{x+t+1}} \cdot (V_t - 1) \cdot (1+i)$$

(12.20.)

We can rewrite it into another form:

$$V_{t+1} = \frac{l_{x+t+1} + d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{l_{x+t}}{l_{x+t+1}} \cdot (1+i) =$$
$$= V_t \cdot (1+i) + \frac{d_{x+t}}{l_{x+t+1}} \cdot V_t \cdot (1+i) - \frac{l_{x+t}}{l_{x+t+1}} \cdot (1+i)$$

(12.21.)

[115] Which – naturally – does not mean that the reserve of the term insurance is the reserve of the pure endowment insurance lowered by something!

According to this formula, the reserve at the beginning of the policy year bears interest and the reserve of the deceaseds (plus interest) are distributed among the livings. The benefits paid for the livings (plus its interest) are deducted and distributed among the reserve of the yet livings at the end of the policy year.

If we compare (12.15.), (12.18.) (nex to last) and the formula above, then we can say, that the first two elements (the previous reserve plus interest and the distribution of the deceased's reserve among the yet livings) are common elements, but the deductions during the term because of the benefit payments are different ones.

Henceforth, we do not deal with the retrospective reserves of the other insurances, but they can be constructed similarly.

12.3.2. The Retrospective Formula of the Premium Reserve of Regular Premium Insurances – Without Zillmerization

On the basis of the abovementioneds we can easily deduct the recursive, retrospective reserve formulae of the regular premium insurances – temporarily without zillmerization, but it will also be examined later. First we analyse the term insurance, and generalise it without examining the others in detail.

The prospective reserve formula of the regular premium term insurance is:

$$V_t = A^1_{x+t:\overline{n-t|}} - \ddot{a}_{x+t:\overline{n-t|}} \cdot P^1_{x:\overline{n|}}$$

and

$$V_{t+1} = A^1_{x+t+1:\overline{n-t|-1}} - \ddot{a}_{x+t+1:\overline{n-t-1|}} \cdot P^1_{x:\overline{n|}} \tag{12.22.}$$

If we put (12.19.) into (12.20.), we get

$$\ddot{a}_{x+t+1:\overline{n-t-1|}} = \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(\ddot{a}_{x+t:\overline{n-t|}} - 1\right) \cdot (1+i) \tag{12.23.}$$

Analogously, after (12.18.) we know, that

$$A^1_{x+t+1:\overline{n-t-1|}} = \frac{l_{x+t}}{l_{x+t+1}} \cdot A^1_{x+t:\overline{n-t|}} \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \tag{12.24.}$$

Because of it, we can transform (12.22.) into the following form:

$$V_{t+1} = A^1_{x+t+1:\overline{n-t|-1}} - \ddot{a}_{x+t+1:\overline{n-t-1|}} \cdot P^1_{x:\overline{n|}} =$$

$$= \frac{l_{x+t}}{l_{x+t+1}} \cdot A^1_{x+t:\overline{n-t|}} \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} - \left(\frac{l_{x+t}}{l_{x+t+1}} \cdot \left(ä_{x+t:\overline{n-t|}} - 1\right) \cdot (1+i)\right) \cdot P^1_{x:\overline{n|}} =$$

$$= \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(\left(A^1_{x+t:\overline{n-t|}} - ä_{x+t:\overline{n-t|}} \cdot P^1_{x:\overline{n|}}\right) + P^1_{x:\overline{n|}}\right) \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}}$$

(12.25.)

That is

$$V_{t+1} = \frac{l_{x+t}}{l_{x+t+1}} \cdot \left(V_t + P^1_{x:\overline{n|}}\right) \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} =$$

$$= \left(V_t + P^1_{x:\overline{n|}}\right) \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot \left[1 - \left(V_t + P^1_{x:\overline{n|}}\right) \cdot (1+i)\right]$$

(12.26.)

This differ from form (12.18.) only because here we take into account the annual net premium just arrived at t. anniversary.

On the basis of this we can construct all the other formulae of the recursive, retrospective regurlar premium reserves, so we do not treat them in detail.

12.4. The premium reserve formulae in another way

Similarly to the subchapter 10.5. where we rewrote some classical premium formulae into another form, we show here that some classical reserve formulae also have another form. The starting point there was three classical formulae in connection with three financial products: the whole life insurance (10.148.), the simple savings (or single premium term fix insurance – 10.149.) and the endowment insurance (10.150.).

By using these formulae and the (10.181.) formulae for net premiums which were deducted from these, we can write three new reserve formulae for these 3 products.

In case of **Endowment insurance** on the basis of (10.150.) the new single premium reserve formula:

$$V_t = A_{x+t:\overline{n-t|}} = 1 - d \cdot ä_{x+t:\overline{n-t|}}$$

(12.27.)

This formula can be interpreted in the following way: the sum assured of the endowment insurance is 1, and it has to be reached by the end of the term in such a way that the reserve not only increases by interest accretion but also decreases because of the payments to the deaths.

The regular premium (prospective) reserve of the endowment insurance – by using the previous (12.27.) formula – is:

$$V_t = A_{x+t:\overline{n-t|}} - ä_{x+t:\overline{n-t|}} \cdot P_{x:\overline{n|}} = \left(1 - d \cdot ä_{x+t:\overline{n-t|}}\right) - ä_{x+t:\overline{n-t|}} \cdot \left(\frac{1}{ä_{x:\overline{n|}}} - d\right) =$$

$$= 1 - \frac{ä_{x+t:\overline{n-t|}}}{ä_{x:\overline{n|}}}$$

(12.28.)

In the case of the **Whole life** insurance the single and regular premium reserves (and its interpretations) are – analogously – the followings:

$$V_t = A_{x+t} = 1 - d \cdot ä_{x+t}$$

(12.29.)

and

$$V_t = A_{\overline{n-t|}} - ä_{\overline{n-t|}} \cdot P_{\overline{n|}} = 1 - d \cdot ä_{\overline{n-t|}} - ä_{\overline{n-t|}} \cdot \left(\frac{1}{ä_{\overline{n|}}} - d\right) =$$

$$= 1 - \frac{ä_{\overline{n-t|}}}{ä_{\overline{n|}}}$$

(12.30.)

In case of the simple savings

$$V_t = A_{\overline{n-t|}} = 1 - d \cdot ä_{\overline{n-t|}}$$

(12.31.)

and

$$V_t = A_{\overline{n-t|}} - ä_{\overline{n-t|}} \cdot P_{\overline{n|}} = 1 - d \cdot ä_{\overline{n-t|}} - ä_{\overline{n-t|}} \cdot \left(\frac{1}{ä_{\overline{n|}}} - d\right) =$$

$$= 1 - \frac{ä_{\overline{n-t|}}}{ä_{\overline{n|}}}$$

(12.32.)

In case of the certain annuity, using the applicable formula

$$ä_{\overline{n|}} = ä_{\overline{n-t|}} + v^{n-t} \cdot ä_{\overline{t|}}$$

(12.33.)

we can transform (12.32.) into another form:

$$V_t = 1 - \frac{ä_{\overline{n-t|}}}{ä_{\overline{n|}}} = v^{n-t} \cdot \frac{ä_{\overline{t|}}}{ä_{\overline{n|}}}$$

(12.34.)

In case of pure endowment we can also write an applicable formula:

$$A_{x:\overline{n|}}^{\ \ 1} = A_{x:\overline{t|}}^{\ \ 1} \cdot A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1}$$

(12.35.)

Using this and the analogous formula with (12.33.)

$$\ddot{a}_{x:\overline{n|}} = \ddot{a}_{x:\overline{t|}} + A_{x:\overline{t|}}^{\ \ 1} \cdot \ddot{a}_{x+t:\overline{n-t|}}$$

(12.36.)

we get that:

$$V_t = A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1} - \ddot{a}_{x+t:\overline{n-t|}} \cdot P_{x:\overline{t|}}^{\ \ 1} = A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1} - \ddot{a}_{x+t:\overline{n-t|}} \cdot \frac{A_{x:\overline{n|}}^{\ \ 1}}{\ddot{a}_{x:\overline{n|}}} =$$

$$= A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1} - \ddot{a}_{x+t:\overline{n-t|}} \cdot \frac{A_{x:\overline{t|}}^{\ \ 1} \cdot A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1}}{\ddot{a}_{x:\overline{n|}}} = A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1} \cdot \left(1 - \frac{A_{x:\overline{t|}}^{\ \ 1} \cdot \ddot{a}_{x+t:\overline{n-t|}}}{\ddot{a}_{x:\overline{n|}}}\right) =$$

$$= A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1} \cdot \frac{\ddot{a}_{x:\overline{n|}} - A_{x:\overline{t|}}^{\ \ 1} \cdot \ddot{a}_{x+t:\overline{n-t|}}}{\ddot{a}_{x:\overline{n|}}} = A_{x+t:\overline{n-t|}}^{\ \ \ \ \ \ 1} \cdot \frac{\ddot{a}_{x:\overline{t|}}}{\ddot{a}_{x:\overline{n|}}}$$

(12.37.)

12.5. The Calculation of Mid-year Premium Reserve

In the previous sections we have learned how the premium reserve can be calculated at the end of the tth insurance year. But we know that the premium reserve changes gradually (the interest is added to the reserve gradually, and deaths also occur spread through the year), so the value of the reserve within the year will be different than at the anniversary. We often need this mid-year reserve for practical calculations. Also, for practical purposes, the mid-year reserve is usually calculated simply as the interpolated value (weighted average) of the annual reserves.

Let's suppose that we want to know the level of the reserve of single premium insurances within the *(t+1)* insurance year, at the *(t+τ)* point, where $0 < \tau < 1$.

Then the two neighbouring premium reserves will be V_t and V_{t+1}. The weighted average of these is:

$$V_{t+\tau} = (1-\tau) \cdot V_t + \tau \cdot V_{t+1}$$

(12.38.)

In case of regular premium insurances the above picture changes, because we also have to take into account that the next annual premium payment ($P_{x:\overline{n|}}$, or simply *P*) arriving to the insurer at the beginning of the year, that is immediately added to the premium reserve. So, at the end of year *t* the premium reserve is V_t but one moment later, at the beginning of year *t+1* it will be $V_t + P$. According to this, the mid-year premium reserve will be:

$$V_{t+\tau} = (1-\tau)\cdot(V_t + P) + \tau \cdot V_{t+1} \tag{12.39.}$$

In case of regular premium insurances we also have to deal with another problem. In the above reserve formula we supposed annual premium payment. But it is possible that the insurer calculated the premium by monthly payment. In this case – if we want to be precise – we have to calculate also the reserve with the monthly value. But instead of this there is a good approximation if we also use (12.38.) in case of regular (montly) premium cases.

12.6. A negative Premium Reserve

In case of the above discussed products the premium reserve was evidently a positive value. The phrasing („the part of earlier premium income reserved for later claims payment „) also suggests the non-negative sign. The positivity is natural in case of single premium insurances. But there might be regular premium insurances, where the positivity of the premium reserve doesn't hold. This means – if we think about it – that the insurer has provided more benefits looking at it time-proportionally than the amount of premium that has already been paid, so the client owes the insurer.

Since the client can terminate the policy at any time, situations like this have to be avoided, since in case of surrender the insurer might not be able to recover its money, the sum of which is indicated by the negative reserve. This is why we say that it is a **professional mistake** to construct insurances with negative premium reserve, although it happens all the same (or at least it used to happen). A good example of this (and maybe the only one?!) the "staged" term insurance (which was under the label "credit life insurance"[116] popular in the Hungarian market in the '70s and '80s, that was purchased as a cover of loans for OTP apartments. [117] As it was already mentioned,

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[116] The term „credit life" naturally refers to generally all life insurances that are used as a coverage of a loan. In case of the product discussed here the term "Credit Life" was the name of this product, at a time when the number of life insurance product offered was quite limited.

[117] An other example could be the modified term insurance we proposed in the Chapter 4, which pays annuity for the remaining term after the death of the insured. About this we have proved in Chapter 10 that it is a conditional annuity. In case of regular premium, the problem of negative reserve also appears here.

in this product the sum assured decreased from year to year, simulating the sum of the remaining outstanding debt. If the insured dies before the loan is repaid, then the remaining debt is paid by the insurer.

The cause of the negative premium reserve of the credit life insurance was that while the annual premium was fixed, the risk undertaken by the insurer – due to the decreasing sum assured – decreased from year to year, – or at least there was some decreasing sections of it. The risk of the insurer is demonstrated by the following figure (calculated by 2014, Hungarian, male population mortality table and by 0% technical interest rate, assuming a 20 years old entry age insured). The risk of the insured is the product of the decreasing death sum assured and the increasing death probability:

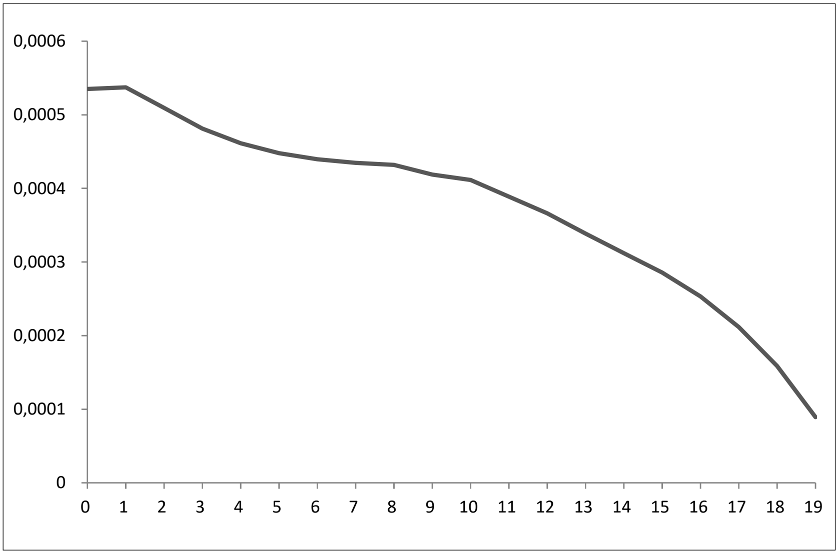


Figure 12.1.: The risk of the insurer in case of “staged” term insurance with 20 years entry age and 20 years term

The decrease of the insurer’s risk is caused by the moderate increase of the death probability in such a low entry age. Because of this, the decreasing sum assured is dominant during the calculation of the risk of the insurer. But the picture becomes a little bit different if the entry age is increasing:

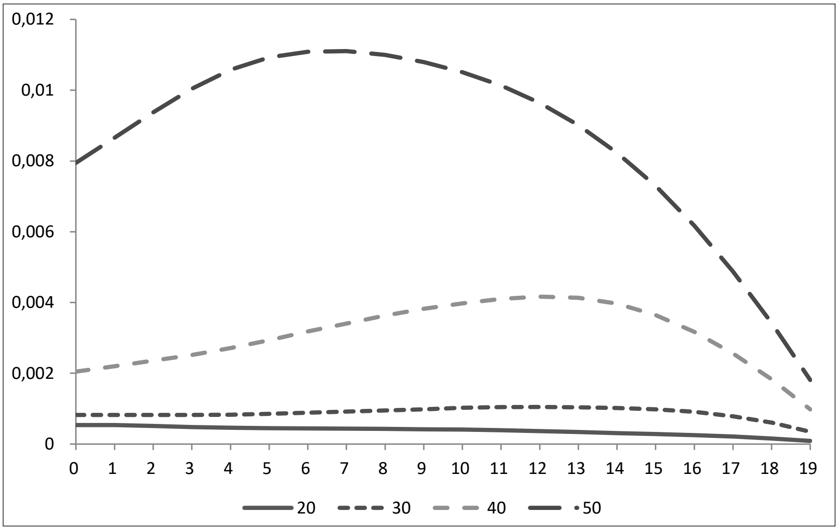
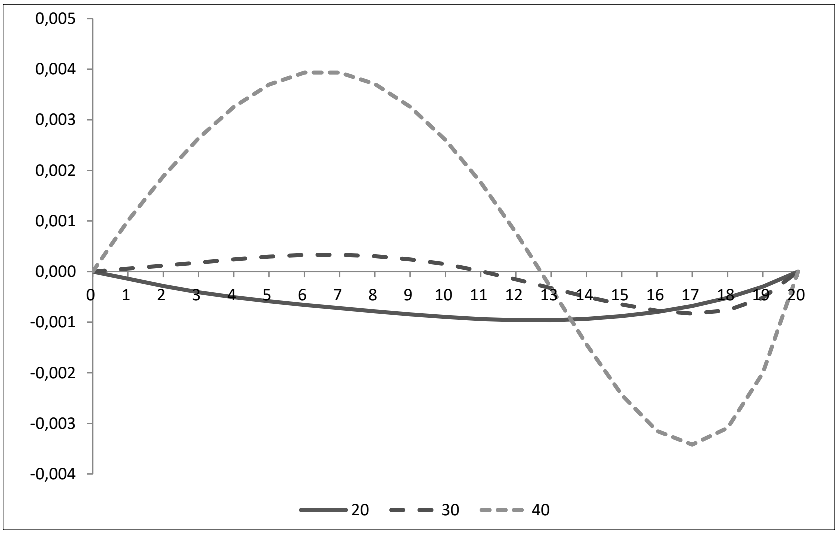


Figure 12.2.: The risk of the insurer in case of “staged” term insurance with various entry age and 20 years term

We can conclude, that the decreasing risk during the whole term is generally not true in case of higher entry ages, but even in these cases there are decreasing sections towards the end of the term. And this is already enough for the negative reserve, because such a way there will be some intervals when we use the premium before it flows into the reserve – at least if the premium payment period is equal to the term of the insurance.

We do not repeate the premium and reserve formulae here, because we already presented them earlier. An the same time we present the reserve curve in case of various entry ages:



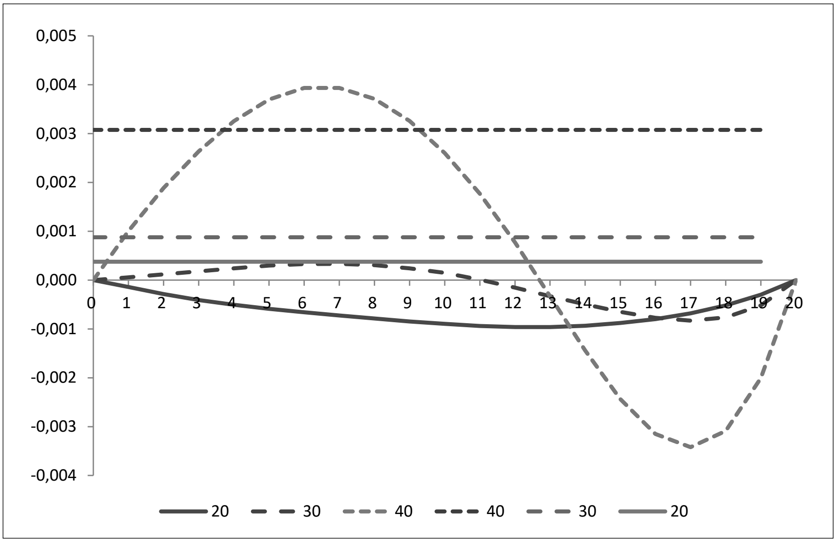


Figure 12.3.: The reserve and annual premiums of the "staged" term insurance with various entry age and 20 years term

(From the figure we have left out the reserve in case of 50 years entry age, because the values of the lower entry ages are much lower, so we can better see the differences.)

It is shown, that at 20 years entry age, when the annual risk of the insurer is decreasing during the whole term, the reserve will be negative during the whole term. However, in case of higher entry ages, where the risk is increasing at the beginning of the term (as the situation is at the majority of the insurances) the reserve also increasing for a while, but this is switching into decrease such a way, that after some time the reserve become negative.

It is shown in the Figure 12.3., that the negative reserve can reach the double of the annual premium, which can cause significant loss for the insurer if the policy holder ceases the insurance.

We can make the reserve positive everywhere if we push the curve ("line") of the already paid premium above the curve of the insurer's risk. If we do not want a varying (decreasing) annual premium, then we can reach this by shortening the premium payment period. The following figure (to which we have drawn the curves of the other relevant informations) can show this:

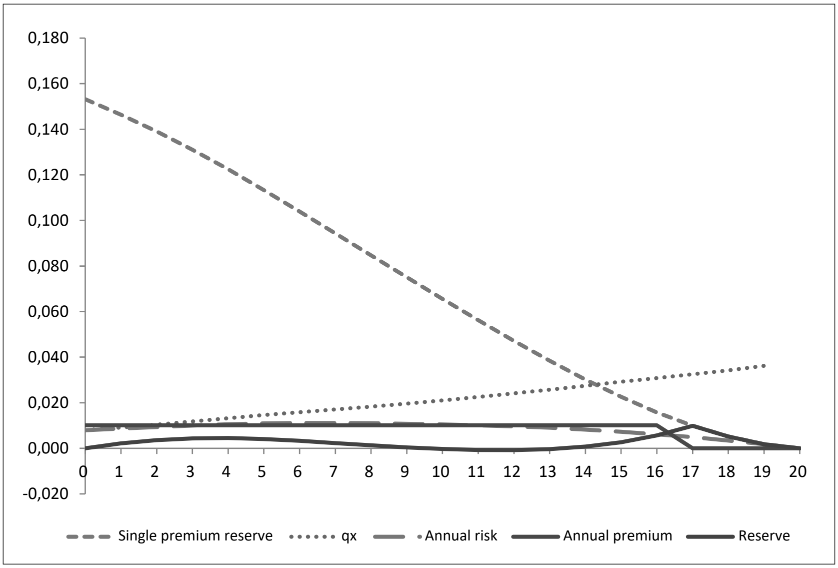


Figure 12.4.: The premium, reserve and the risk of the insurer of a "staged" term insurance. The entry age is 50 years, the technical interes rate is 0%, the premium payment period is shortened to 17 years

We can see on the Figure 12.4, that in case of these parameters there will be a small section where the reserve is negative, but in case of 16 years premium payment period (Figure 12.5.) the reserve will be already positive everywhere.

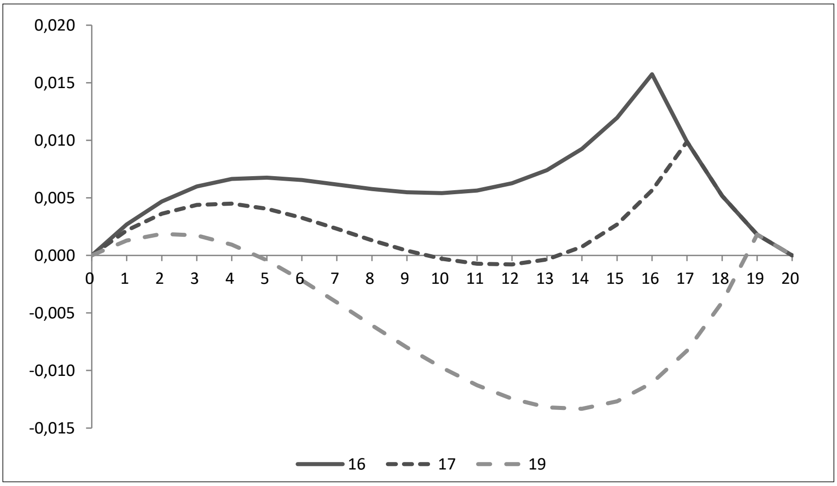


Figure 12.5.: The reserve of a "staged" term insurance with 50 years entry age, 0% technical interest rate and various shortened premium payment periods

It could be logical to construct such a regular premium orphan's annuity product, where the parent pays the premium regularly until the end of the term, or his (or the later beneficiary's) death. If the beneficiary was alive at the death of the insured, and the insurance term hasn't ended, then until the end of the term, but at most until the death of the (second) insured (beneficiary) the insurer pays him an annuity.

The benefit of this insurance decreases over time, since the longer the parent lives, the shorter the time of annuity paid to the child will be. This way the premium reserve will be negative also here. This can be avoided – for example – by building in a simple term insurance, so in the event of death of the parent, not only the annuity, but a lump sum is also paid to the child (widow). The positive premium reserve of the term insurance with a correctly chosen sum assured can compensate the negative premium reserve of the orphan's annuity. Naturally the net premium of this insurance is also higher than in the former case of the product resulting a negative premium reserve.

It is a widespread opinion that zillmerization makes the premium reserve negative at the beginning of the term. As we will see in the next chapter discussing zillmerization in detail, this is wrong, the premium reserve will never be negative due to zillmerization. Formulae are sometimes not handled precisely enough in relation to zillmerization, and this might cause the image of negative premium reserve.

12.7. Cash Flows in Unit Linked Insurance

As we have already mentioned, we do not use the term premium reserve for the reserve of unit linked Insurance, but its function is completely analogous to the premium reserve of traditional insurance. This way we will briefly examine this reserve based on the traditional, retrospective reserve formula of endowment insurance.

UL insurances – at least in their widespread, non-guaranteed version – essentially do not promise a fix, predetermined benefit, so in their case we can not use the prospective reserve calculation.[118] The cash-flow of the UL insurances resemble most of all to endowment insurance, so its recursive, retrospective reserve can be a good starting point. We can imagine it on the basis of (12.26.) on the following way (since the recursive, retrospective reserve formula of the endowment insurance is – formally – equal to the term insurance's):

$$V_{t+1} = \left(V_t + P_{x:\overline{n|}}\right) \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot \left[1 - \left(V_t + P_{x:\overline{n|}}\right) \cdot (1+i)\right] \tag{12.40.}$$

It is worth to apply some modification on this formula (i.e. on the formula of the premium reserve of the regular premium endowment insurance at anniversary before premium payment without zillmerization), and to turn it to the following form:

$$V_{t+1} = (V_t + P) \cdot (1+i) - \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+i)^{\frac{1}{2}} \cdot \left[SA - (V_t + P) \cdot (1+i)^{\frac{1}{2}}\right] \tag{12.41.}$$

The three modifications:

- instead of sum assured HUF 1 we apply the more general SA („sum assured"),
- we assume, that death benefit payments happen right after the death, not at the end of the policy year – i.e. on average at the half of the policy year,
- we left the index of P, because in determining the premium of the UL insurances the age and the term play not as an important role as in case of the traditional life insurances.

The formula above shows an endowment insurance, if we assume, that the fixed sum assured is equal to the reserve at the end of the term, i.e.:

$$SA = V_n \tag{12.42.}$$

But of course in case of UL insurances the reserve at the end of the term is not

[118] However, internationally its guaranteed version is spreading, which is called – quite misleadingly – as „variable annuity". But these guarantees are different from the guarantees applied in case of traditional life insurances, these are practically built-in options, the pricing of which is not presented in this book.

known, so we can not use this sum assured definition. Then again we can look at reserve formulae as if they were describing the essential part of the insurance cash flow (but they do not contain the premium part covering expenses, and we also disregard the – by the way relatively simple – modifications/generalizations due to annual premium payment), i.e. as if they were describing the insurance in general. According to this, the most general unit linked insurance construction can also be described by the equation defining its reserve. This reserve-equation can be derived from the equation of the endowment insurance with the following modifications:

- We split the expense part that was until now implicitly handled and united ("premium loadings") into parts and make it partly explicit. The dynamics of the different expense parts from now on will not be the same. The different expense parts are[119]:
  - » Administration fee: we express it separately and explicitly in the new cash flow formula. Its increase can depend on how P changes, moreover it can also be defined as an absolute value, independent from P. This means a more even spreading of expenses between insurances, since the administration fee proportional to P (that is customary in traditional insurances) is not justified, since it costs the same to handle policies of larger and smaller sum assured. The same way it changes independent of whether the policyholder has raised the premium meanwhile or not.
  - » Fund management fee: it depends on the value of the reserve handled and not the premium, and is subtracted from the gross yield.
  - » Bid-offer spread: it is an expense part proportional to the net premium, similar to the premium loading of traditional insurances, that is not included in the cash flow formula, but appears explicitly to the client – contrary to traditional insurances.
- Technical interest rate: instead of the unique, unchanged indicator of investment returns that is known beforehand, we introduce two different kinds: 1. an indicator of investment earnings that can only be known subsequently, 2. a technical type, projected indicator (we will still call this technical interest rate and denote it by i!)
- Sum assured: formula (12.42.) concerning the sum assured is changed to a conditional formula: $B_t = max\left(0; SA - (V_t + P - a_t) \cdot (1+i)^{\frac{1}{2}}\right)$ where B_t is the sum assured projected at the beginning of the year, that has to be paid in addition to the value of money in the funds in the event of death of the insured. The formula explicitly includes the administration fee, since in the traditional case P already does not include this, but in case of Unit Linked insurance it does. There is no strict determination of SA here, but insurers generally give a maximum

[119] Not mentioning conditional expense parts such as the fee of transfer between funds, since this is only due if the client initiates the transfer.

value (usually greater than zero) depending on the minimum premium, the term and the insured's age, and the policyholder can freely choose within this.

- Instead of the $\frac{d_{x+t}}{l_{x+t+1}}$ factor insurers generally use its loaded factor, that can be regarded as another hidden expense element. But mortality probability is often decreased due to positive selection.
- Naturally the risk premium is most often subtracted not annually, but e.g. monthly, and due to this, the formula has to be – slightly – altered, but we will disregard this (as we have done in case of traditional insurances!).
- The premium of supplementary risks (insurance riders) is also subtracted from the reserve regularly, but we will disregard this as well.
- Hungarian insurers often use the technique of initial units, that is forbidden in several places, that we also do not discuss in the above formula.[120]

According to these, the new formula describing the cash flow of unit linked insurance is (keeping the notation V of the reserve!):

$$V_{t+1} = (V_t + P - a_t) \cdot \big(1 + (h_t - w)\big) - \frac{d_{x+t}}{l_{x+t+1}} \cdot (1+\lambda)^{\frac{1}{2}} \cdot B_t \tag{12.43.}$$

where we used the following special notation:

$$B_t = max\left(0; SA - (V_t + P - a_t) \cdot (1+i)^{\frac{1}{2}}\right) \tag{12.44.}$$

and:

h_t: gross yield of year t (earned between the tth and t+1th anniversary)
w: fund management fee, defined as a percentage of the value of the managed fund
λ: loading factor used to calculate the death risk premium
a_t: administration fee applied in year *t* and subtracted at the beginning of the year
SA: minimum benefit paid upon death.[121] Its value is: $0 \leq SA \leq H(x,t,n,P)$
H(x,t,n,P): limit on the minimum death benefit defined as a function of the insured's entry age, the elapsed time, the term and the annual premium.
B_t: the insurer's projected death benefit above the reserve.

The precise value of the above formula can only be determined after t+1. But we already have to know the value of B_t at *t*, because this is when the projected death risk premium is subtracted from the reserve. Due to this, the yield has to be estimated, and *i*, the technical interest rate serves this purpose.

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[120] The creation of initial units serves a similar purpose as zillmerization in case of traditional insurances, and we are consistent with ourselves in that neither was zillmerization included in the cash flow formula (12.40) of traditional endowment insurance.

[121] In reality it could have an index to express that SA can change. The index was left out to indicate that this change isn't necessarily tied to anniversaries.

On the basis of (12.43.) the risk premium to be deduced from the reserve is $\frac{d_{x+t}}{l_{x+t+1}} \cdot (1+\lambda)^{\frac{1}{2}} \cdot B_t$, which can be odd a the benning, because at first we perhaps would like to multiply the sum assured (plus costs) not by $\frac{d_{x+t}}{l_{x+t+1}}$ but by $q_{x+t} = \frac{d_{x+t}}{l_{x+t}}$, by the death probability.

But it is not right and we demonstrate it in simple terms. We set aside the precise formula of the capital above and we simple note its size in t by C_t. We also set aside the cost component of the risk premium, and we introduce the notation P_{x+t} for the net risk premium for year t. Then we can formulate the abovementioned problem in the first instance, that it seems, that the annual risk premium is the following:

$$P_{x+t} = (SA - C_t) \cdot q_{x+t} = (SA - C_t) \cdot \frac{l_{x+t} - l_{x+t+1}}{l_{x+t}} = (SA - C_t) \cdot \frac{d_{x+t}}{l_{x+t}} \tag{12.45.}$$

if either $SA > C_t$, or 0, where $SA - C_t$ is the sum at risk.

The problem, that if we deduct this risk premium from C_t, then the sum at risk immediately increases with P_{x+t}, i.e. we have to determine a risk premium also after this, and so on. I.e. the right risk premium is an infinite series:

$$P_{x+t} = (SA - C_t) \cdot q_{x+t} + (SA - C_t) \cdot q_{x+t} \cdot q_{x+t} + (SA - C_t) \cdot q_{x+t} \cdot q_{x+t} \cdot q_{x+t} + \cdots \tag{12.46.}$$

where

$$q_{x+t} + q_{x+t}^2 + q_{x+t}^3 + \cdots = q_{x+t} \cdot \frac{1}{1 - q_{x+t}} = \frac{l_{x+t} - l_{x+t+1}}{l_{x+t}} \cdot \frac{1}{1 - \frac{l_{x+t} - l_{x+t+1}}{l_{x+t}}} = \frac{l_{x+t} - l_{x+t+1}}{l_{x+t+1}} \tag{12.47.}$$

I.e. the exact risk premium is not (12.45.), but

$$P_{x+t} = (SA - C_t) \cdot \frac{l_{x+t} - l_{x+t+1}}{l_{x+t+1}} = (SA - C_t) \cdot \frac{d_{x+t}}{l_{x+t+1}} \tag{12.48.}$$

Therefore we have to calculate not with q_{x+t}, but with its modified version (with a little bit higher number). At the same time it is important to note, that:

- the sum at risk can fluctuate after its deduction, because of the fluctuation of C, which is a natural phenomenon, and this can modify the necessary risk premium by much higher degree, than this correction (instead of $\frac{d_{x+t}}{l_{x+t}}$ we use $\frac{d_{x+t}}{l_{x+t+1}}$).[122] The defence against this anyway: not annual, but much frequent risk premium deduction – naturally calculating by the proportional part of the annual premium, and building in some safety substitute above the risk premium.

- naturally, the risk premium a priori have to be supplemented by some substitute, because of the costs of the deduction as process.

13. ZILLMERIZATION

Due to its outstanding importance we discuss the mathematical relations of zillmerization in detail, in a separate chapter. Zillmerization, in point of the traditional life insurances can be regarded as general practice in Hungary, so when calculating the gross premium reserve of regular premium insurances we cannot leave it out.

Initially zillmerization has had a strict logic, which has supposed synchronous fulfilment of some elements and it has complexly solved more problems. This strict logic has already loosened and zillmerisation has simplified today into a cash-flow (some degree) improving method. Considering this, first I describe the original logic of zillmerization in detail, what I will call as "conservative approach", then briefly the modern approach. (I do not deal with here the detailed mathematics of this latter. The interested can find it in Banyár [2001], and [2003]).)

The zillmerization in this form can be used for traditional life insurances. A similar problem also emeges in case of modern life insurances, for which the initial unit technique a possible (wrong) solution. The aim of this technique is legitimate, but its form problematic because it is based on the delusion of the client. The equivalent (with zillmerization), non-delusive technique in case of UL insurances would be an explicit distraction from the first collected premiums at the beginning of the term.

[122] This is the reason why the following joke was born about actuaries. A sheep-farmer shows off his sheeps in his farm to an actuary. The sheeps are in two sheep-pens. The farmer asks the actuary to estimate the number of the sheeps. The actuary response is quick: 1006. The farmer is surprised, and asks the reason of this precise estimation. The actuary says: in the first sheep-pen there is 6 sheeps, in the other about 1000.

13.1. Zillmerization – in the Conservative View

Zillmerization, with conservative approach, means borrowing part of the premium (or the premium reserve) of the first year at the beginning of the term, that will be repaid later on from the premium loadings in equal annual instalments.

The three important, interrelated elements of the "conservative approach" are:

1. The insurer borrows only from the first year's premium, and so
2. There is an upper limit of the borrowing,
3. The frequency of the premium payment is annual (and not – e.g. – monthly).

According to this, the maximum sum borrowed can be a part of the first year's gross premium so that the remaining part of the premium still covers the continuous expenses[123] and the first year's death benefits. Consequently before premium payment the premium reserve is zero, and after premium payment even at the commencement of the insurance and at the first anniversary it is a positive value, since a part of the first net premium, and the second (greater) net premium totally goes to the reserve, and perhaps even the first premium has a part above the benefit payments that can fill the premium reserve. The limitation is necessary, because in the "modern" practice this limit is not followed, which very often causes the reserve of the first anniversary to be zero (i.e. neither the first, nor the second premium fills the reserve above the part covering the death benefit payments of the given year).

We will use the following notation:

- z: portion of the sum assured ("zillmer percentage" or "zillmer quota") that shows how much more we use of the first year's premium than the cover of continuous expenses (primarily used to cover acquisition commission)
- P_1: net premium of the first year, that remains from the gross premium after the first year's expenses
- PZ: net premium of the consequent years, the so called "zillmer"-premium that already contains the repayment of zillmerization
- P: original annual net premium (for the sake of simplicity we left the index of it, however in this aspect we will not be always consistent – sometimes we use it)
- PG: gross annual premium
- C: original premium loading (cost compontent)
- CC: continuous premium loading
- p: net premium necessary to cover the risks of the first year

The following equations can easily be written based on the above notation:

[123] Under continuous expenses here we mean that the original premium loading is divided into two parts: the repayment of zillmerization and the part covering continuous expenses.

$$PG = P + C = P_1 + z + CC = PZ + CC$$

(13.1.)

That is the gross premium can be decomposed partly in traditional manner (net premium + costs), partly the following way:

- in the first year: net premium necessary to cover the risks of the first year + the “borrowed” part because of zillmerization + “the rest” cost component
- in the other years: the new, higher net premium (PZ) + the new, lower cost compontent (CC).

The Figure 13.1. shows these (and other) relationships:

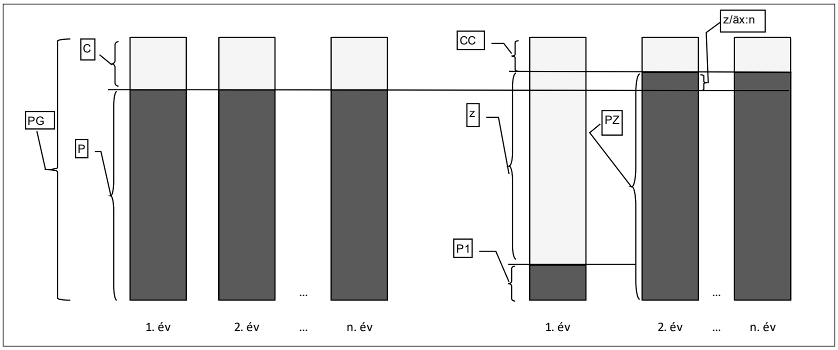

Figure 13.1.: Zillmerization – the conservative approach

One of the important further equalities is the size of the “instalment” i.e. what the insurer put back into the reserve. In other words: the difference between the “zillmer” premium and the normal net premium, which will be $\frac{z}{ä_{x:\overline{n|}}}$, that is there will be true, that

$$PZ + CC = P + \frac{z}{ä_{x:\overline{n|}}} + CC$$

(13.2.)

This is partly “forseen”, because we have to reimburse z during n years in advance instalments such a way, that we take into account the reserve of the deceased in the meantime, partly we will deduct it from the other equilities, sith:

Instead of the original equivalence equation concerning the net premiums,

$$P \cdot ä_{x:\overline{n|}} = A_{x:\overline{n|}}$$

(13.3.)

it is more correct to write the following in case of zillmerization (while that equation also holds, since we get the value of P from there):

$$P_1 + PZ \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right) = A_{x:\overline{n|}} \tag{13.4.}$$

Because in the two equations above on the right-hand sides are the same values, so the left-hand sides have to be also the same values, i.e. the following has to be true

$$P_1 + PZ \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right) = P \cdot \ddot{a}_{x:\overline{n|}} \tag{13.5.}$$

On the basis of (13.1.) it is clear, that

$$P_1 \ + \ z \ = \ PZ \tag{13.6.}$$

To replace PZ in (13.5.) by this we get, that:

$$P_1 + (P_1 \ + \ z) \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right) = P \cdot \ddot{a}_{x:\overline{n|}} \tag{13.7.}$$

i.e.:

$$P_1 \cdot \ddot{a}_{x:\overline{n|}} = P \cdot \ddot{a}_{x:\overline{n|}} - z \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right) \tag{13.8.}$$

so

$$P_1 = P - z \cdot \frac{\ddot{a}_{x:\overline{n|}} - 1}{\ddot{a}_{x:\overline{n|}}} \tag{13.9.}$$

If we put back into (13.6.) we will get, that

$$PZ = P_1 \ + \ z \ = \ P - z \cdot \frac{\ddot{a}_{x:\overline{n|}} - 1}{\ddot{a}_{x:\overline{n|}}} + z = P + \frac{z}{\ddot{a}_{x:\overline{n|}}} \tag{13.10.}$$

Which is (13.2.) if we leave CC from both sides.

The premium reserve can be written more precisely according to the above:

$V_0 = A_{x:\overline{n|}} - P_1 - PZ \cdot \left(\ddot{a}_{x:\overline{n|}} - 1\right)$ and (13.11.)

$V_t = A_{x+t:\overline{n-t|}} - PZ \cdot \ddot{a}_{x+t:\overline{n-t|}}$, if $1 \leq t$ (13.12.)

Using (13.10.), we can also write the reserve formula as:

$$V_t = A_{x+t:\overline{n-t|}} - \left(P_{x:\overline{n|}} + \frac{z}{\ddot{a}_{x:\overline{n|}}}\right) \cdot \ddot{a}_{x+t:\overline{n-t|}} \tag{13.13.}$$

And in the state after premium payment this changes to the following form:

$$A_{x+t:\overline{n-t|}} - \left(P_{x:\overline{n|}} + \frac{z}{\ddot{a}_{x:\overline{n|}}}\right) \cdot \left(\ddot{a}_{x+t:\overline{n-t|}} - 1\right)$$

(13.14.)

According to (13.4) it is straightforward to see that (13.11), i.e. the starting premium reserve before premium payment is $V_0 = 0$.

For this reason, the starting premium reserve right after the first premium payment is P_1.

z can be freely chosen within certain limits. This limit is, that the first year's gross premium has to be enough for z. This can also be put into shape, that z should not crowd out the remaining risk premium (P_1) to such an extent, that the rest will not be enough to cover that year's risks. This way, if we denote the net premium necessary to cover the first year's risks, then the following criterion has to be satisfied:

$$P_1 \geq p$$

(13.15.)

From this we can deduct a maximum criterion for z, about which we can say, that it is also a criterion of whether the zillmerization is „conservative" or „modern" (if it comes true, then „conservative", if not, then „modern").

Using (13.9.), and the definition of the net premium we can write (13.15.) in the following way:

$$P_1 = P - z \cdot \frac{\ddot{a}_{x:\overline{n|}} - 1}{\ddot{a}_{x:\overline{n|}}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - z \cdot \frac{\ddot{a}_{x:\overline{n|}} - 1}{\ddot{a}_{x:\overline{n|}}} \geq p$$

(13.16.)

Rearranging this we can get for z, that:

$$\frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - p \geq z \cdot \frac{\ddot{a}_{x:\overline{n|}} - 1}{\ddot{a}_{x:\overline{n|}}}$$

(13.17.)

On the right-hand side we can get z (for which we are seeking a maximum criterion), if we divide with $\frac{\ddot{a}_{x:\overline{n|}}-1}{\ddot{a}_{x:\overline{n|}}}$, which will be positive by all means (at least we dismiss such an extremity as 1 year term, annual premium insurance – in which case the zillmerization is a priori impossible):

$$\left(\frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} - p\right) \cdot \frac{\ddot{a}_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}} - 1} \geq z$$

(13.18.)

The left-hand side of this we can transform into a more handleable form ont he following way:

$$\left(\frac{A_{x:\overline{n|}}}{ä_{x:\overline{n|}}}-p\right)\cdot\frac{ä_{x:\overline{n|}}}{ä_{x:\overline{n|}}-1}=\frac{A_{x:\overline{n|}}}{ä_{x:\overline{n|}}-1}-p\cdot\frac{ä_{x:\overline{n|}}}{ä_{x:\overline{n|}}-1}=\frac{A_{x:\overline{n|}}}{ä_{x:\overline{n|}}-1}-p\cdot\frac{ä_{x:\overline{n|}}-1+1}{ä_{x:\overline{n|}}-1}=\frac{A_{x:\overline{n|}}-p}{ä_{x:\overline{n|}}-1}-p$$

(13.19.)

To the continuation it would be expedient to find a formula for p, i.e. for the net premium necessary to cover the risks of the first year. It is logical to write, that

$$A_{x:\overline{n|}}=p+A_{x:\overline{1|}}^{\ \ 1}\cdot A_{x+1:\overline{n-1|}}$$

(13.20.)

This formula is true, because the single premium necessary to pay now of an insurance with term of n years (not necessarily „Term") can be divided into two parts:

1. the (net) premium necessary for the first year, i.e. p
2. the premium necessary for the other years.

This latter is not identical to $A_{x+1:\overline{n-1|}}$, because the insured would have to pay it in a year's time in his/her age x+1 years. But only if he/she will be alive then, which is not sure. If somebody has to pay the premium for the last n-1 years already now, then we can interpret this, that he/she has to buy a one-year-term pure endowment policy with such a sum assured from which he/she will be able to by a cover then for the remaining n-1 years. If he/she would die meantime this would be unnecessary. Altogether he/she has to pay less for this now as if he/she would buy it one year later provided he/she remain alive.

By rearranging (13.20.) we can get a useful formula for the numerator of (13.19.). But also for the denominator of it, because if we assume, that $A_{x:\overline{n|}}=ä_{x:\overline{n|}}$, then we know, that here p=1, sith in the first year of an immediate, in advance annuity it has to be paid exactly 1. So, we can write (13.18.), using (13.19.) and (13.20.), in the following way:

$$\frac{A_{x:\overline{n|}}-p}{ä_{x:\overline{n|}}-1}-p=\frac{A_{x:\overline{1|}}^{\ \ 1}\cdot A_{x+1:\overline{n-1|}}}{A_{x:\overline{1|}}^{\ \ 1}\cdot ä_{x+1:\overline{n-1|}}}-p=\frac{A_{x+1:\overline{n-1|}}}{ä_{x+1:\overline{n-1|}}}-p=P_{x+1:\overline{n-1|}}-p\geq z$$

(13.21.)

With (13.21.) we got an elagant inequality for maximal possible value of z.

We have not give a value for z which is depending on age and term, because the practice of the insurers, that they apply an only one zillmer-precentage for reserving. But it is obvious from (13.21.), that the maximum possible value of the z depends on x and n, so the final possible maximum value of z is the minimum of z-s for the all possible (x;n) combinations used by the insurer.

It is useful to calculate the values of the net premium part necessary to cover the first year's risks – p – in case of some traditional insurances. We can do this based on formula (13.02.).

$$p = A_{x:\overline{n|}} - A_{x:\overline{1|}}^{\;\;\;1} \cdot A_{x+1:\overline{n-1|}} \tag{13.22.}$$

In the **pure endowment** case:

$$p = A_{x:\overline{n|}}^{\;\;\;1} - A_{x:\overline{1|}}^{\;\;\;1} \cdot A_{x+1:\overline{n-1|}}^{\;\;\;\;\;\;\;1} = \frac{D_{x+n}}{D_x} - \frac{D_{x+1}}{D_x} \cdot \frac{D_{x+n}}{D_{x+1}} = 0 \tag{13.23.}$$

since in this case there is no benefit payment during the term.

In case of **term** insurance:

$$p = A_{x:\overline{n|}}^{1} - A_{x:\overline{1|}}^{\;\;\;1} \cdot A_{x+1:\overline{n-1|}}^{1} = \frac{M_x - M_{x+n}}{D_x} - \frac{D_{x+1}}{D_x} \cdot \frac{M_{x+1} - M_{x+n}}{D_{x+1}} =$$
$$= \frac{M_x - M_{x+1} - M_{x+n} + M_{x+n}}{D_x} = \frac{C_x}{D_x} = q_x \cdot v \tag{13.24.}$$

Since the **endowment** insurance is the sum of a pure endowment insurance and a term insurance, this way the value of p in case of the endowment insurance is equal to (13.24.). The situation is the same in the case of whole life insurance.

In case of the **term fix** insurance:

$$p = A_{\overline{n|}} - A_{x:\overline{1|}}^{\;\;\;1} \cdot A_{\overline{n-1|}} = v^n - \frac{D_{x+1}}{D_x} \cdot v^{n-1} = v^n \cdot \left(1 - \frac{l_{x+1}}{l_x}\right) = v^n \cdot q_x \tag{13.25.}$$

The result is not surprising if we remember that the term fix insurance can be regarded as an endowment insurance with varying sum assured, which has a death sum assured equal to the value of the maturity benefit discounted to the time of death (and were the death benefit is – according to the policy – immediately changed to a single premium term fix insurance).

We can get the value of p similarly In the case of the other insurances.

Returning to the reserve formulae, instead of (13.13.) it is a conventionality to use the equivalent, but a little bit more easily usable formula:

$$V_t = A_{x+t:\overline{n-t|}} - PZ \cdot \ddot{a}_{x+t:\overline{n-t|}}$$

if

$$A_{x+t:\overline{n-t|}} - PZ \cdot \ddot{a}_{x+t:\overline{n-t|}} > 0 \tag{13.26.}$$

and

$$V_t = 0$$

if

$$A_{x+t:\overline{n-t|}} - PZ \cdot \ddot{a}_{x+t:\overline{n-t|}} \leq 0 \tag{13.27.}$$

Here the differentiation of the first year's premium is missing, so the formula can even give a negative result, but then we make the reserve equal to 0.

Beside simplicity, that is an advantage of this formula, it also has its disadvantages:

- It is not so obvious to define the reserve after premium payment from the reserve before premium payment, as in case of the "precise" version.
- Some are incited to investigate the mystic meaning of the negative reserve.

As it can be seen from the above train of thought, in reality **negative reserve isn't created by zillmerization**[124], its minimum value is 0, as it can be seen from the precise form. Only the simplified form makes it seem like a negative reserve is created by zillmerization.

Although the "negative reserve" is a misunderstanding, but its value can be given a meaning according to the following. Let's compute the value of the following formula!

$$V_0 = A_{x:\overline{n|}} - PZ \cdot \ddot{a}_{x:\overline{n|}} = A_{x:\overline{n|}} - \left(P_{x:\overline{n|}} + \frac{z}{\ddot{a}_{x:\overline{n|}}}\right) \cdot \ddot{a}_{x:\overline{n|}} = A_{x:\overline{n|}} - P_{x:\overline{n|}} \cdot \ddot{a}_{x:\overline{n|}} - z = -z \tag{13.28.}$$

This is nothing else but the sum (or its negative) that the insurer borrows from the first year's premium, and that the insurer starts to repay already at the beginning of the term, from the first year's premium loading.

This is understandable, since formulated in a different way this means that the initial premium reserve after premium payment is the following:

$$V_0 + PZ = -z + PZ = P_1 \tag{13.29.}$$

[124] At the same time the negative reserve – as we have seen – is an existing phenomenon, only not because of zillmerization! We have also seen that the negative reserve generally is a professional mistake.

which we have already deducted. And we know that the value of the initial reserve after premium payment is lower than the zillmerized net premium, because a part of the first premium is taken out of the reserve (i.e. from the first premium).

We also can generalize the above relation. If for some t>1-re a (13.27.) negatív lesz, vagyis teljesül, hogy

$$A_{x+t:\overline{n-t|}} - PZ \cdot \ddot{a}_{x+t:\overline{n-t|}} \leq 0$$

(13.30.)

then this means that the value of z has been set so high, that the first year's zillmer premium is not enough to cover it, and parts of the further premiums also have to be subtracted to cover it.

13.2. Zillmerization Today, illetve a zillmerezés értelmezése

If we examine the recent, modern practice of the zillmerization, we can find that any of the important elements of the conservative approach is not kept, namely:

1. the insurer borrows not only from the first year's premium, but even form the 3-4, that is
2. the upper limit for the zillmer quota in (13.21.) is not kept, and
3. the premium payment frequency is (at least in Hungary and typically) monthly.

Because of these, though the insurer borrows from the reserve of the client, but this borrowed money will not cover (unlike in case of classical zillmerization) the immediate costs of the insurer relating to the contract, mainly the (initial) acquisition commission. If this is higher, than the first year's premium, then the insurer have to complement it from its capital. The situation is similar, if the cover of the (already paid) initial commission is missing because of the montly premium payment frequency.

What happens – for example – if during the determination of the value of z, the insurer hasn't taken into account the limit, that z has to "fit into" the annual gross premium, i.e. z and the loading were set independently? Then naturally the part of z due is subtracted from the gross premium, although the insurer has to fill the missing part from some other source.

Naturally we are not certain that any problem would arise because of exceeding the limit, since it is not certain that the true expenses of the insurer and the determination of z are synchronised. It is possible that the insurer sets z higher than the expense coverage he needs. But now it will not be examined here!

The question arises what happens if the policy is surrendered in the phase with 0 reserve? This question is especially interesting in the case of premium payment more frequent than annual. The possibility of lapses throws light on the significance of the classical approach and why annual used to be the dominantly supposed premium frequency. Since in these cases the expenses behind z are mostly covered from premium income, so zillmerization actually gave the insurer more sources.

Nowadays, on the other hand, when premium payment is usually not annual, and exceeding the "natural limit" of z can be considered general, zillmerization almost causes more problems than it solves. In case of non-annual premium payment, namely, the insurer is not relieved from having to finance acquisition expenses – at least for a while – from its own capital. This problem is only intensified if its level exceeds the value that the first year's zillmer premium can cover.

Moreover, in case of lapse the insurer not only has to claw back commission paid to the agent for disciplinary purposes, but also because it was fundamentally the insurer's money advanced for acquisition. So, in these cases – contrary to the classical approach – the insurer has used up a z that hasn't even arrived yet.

So in cases when the premium payment is not annual, zillmerization cannot fulfil its classical function. Then from the original Zillmerien thoughts the only one remaining is that zillmerization somewhat refines the insurer's cash flow, since a greater part of premium can be used to cover expenses than in the non-zillmerized case. But in these cases other methods have to be applied beside zillmerization to solve the financing of the insurer. This is even more true if z exceeds the classical limit.

Finally let's discuss a little bit how the zillmerized "negative reserve" can be handled from the accounting point of view! Naturally emphasizing that "negative reserve" is not created through zillmerization, we can give an accounting meaning to the negative value calculated by the simplified formula of zillmerization – even more so, since we know the meaning of this value.

The reason of zillmerization: the initial expenses related to the insurance (mainly acquisition expenses) arise already at the beginning of the term, and the cover of these expenses is the "z" part nipped off the first (or first k+1) premium(s).

So the insurer signs the policy, the first year's premium arises and is divided into three parts:

1. The part necessary to cover the first year's risks (or a little more than this) fills the premium reserve
2. $1/z^{th}$ part of the sum assured goes to cover initial expenses
3. The continuous premium loading covers continuous expenses.

If the client terminates the policy during the first year, then the insurer can return the client the current sum of the reserve and still be "at the money", supposing that:

1. Premium payment was annual in reality
2. Z has been determined the conservative way
3. Initial expenses have not exceeded $1/z^{th}$ of the sum insured.

If initial expenses have exceeded $1/z^{th}$ of the sum insured, then the insurer has to bear the consequences, and if they have not exceeded it, then the insurer can be happy. But anyway the insurer can suppose that $1/z^{th}$ of the sum insured describes the initial expenses, even if he knows this is only approximately true.

If, on the other hand, the premium payment was not annual and/or zillmerization exceeds the conservative limit, then surrendering the insurance within the first k years could result that the insurance premiums paid do not cover the acquisition costs – regardless of zillmerization! This phenomenon is – in a certain sense – contrary to the principle of zillmerization, which also shows that it has been developed for annual premium payment (and k=1).

It this case, the following tactic could be applied (that in the end the insurer uses even if there is no zillmerization):

- we suppose that the initial expenses equal the "negative premium reserve" that can be calculated at the beginning of the term through zillmerization.
- the insurer has paid out this sum in reality, but it has not arrived yet from premium payment (because of the non-annual premium frequency)
- the sum paid out for initial expenses can be considered as recovered if the zillmerized reserve becomes positive "on its own"
- until this premium reserve is negative, and to the extent of its negativity, the negative part is accounted as a profit neutralizing factor (deferred charges), since it is an expense that will soon be covered
- if it should not be recovered (the policy is terminated in the "negative" phase), then the deferred charges on the premium reserve of the terminated policy are resolved (i.e. accounted as a loss)
- the loss can be decreased by efficient commission claw back from the agent.

In the above case the key was that we identified initial expenses (for the sake of simplicity) with the negative premium reserve, and their recovery with the decreasing negativity of the reserve.

If we had not zillmerized, then the initial expenses would have been recovered from the premium loadings in much smaller portions and during a longer time. In this case the deferred charges have to be maintained for a much longer period of time (if this is allowed by accounting regulation).

It is important to mention that the above procedure is justified if our applied reserve formula is sensitive to net premiums actually arriving (and the very general linear approximation is not used during the year, e.g. even in case of annual premium payment).

14. POSSIBLE METHODS OF HANDLING INFLATION

KEY WORDS

Current sum assured	Value reserving techniques
Current premium reserve	Yield
Premium increase options	Investment profit sharing

From time to time appears periods in a developed country, when the inflation is high, however sometimes – like in Japan after 1990 or on the whole World after 2008 – just the opposite of it, the deflation causes problems. Since we can not exclude it, it is expedient if the insurance products can handle this problem, because inflation causes many problems, both for the client and the insurance company. The main problem for the client is that inflation deteriorates the insurance benefit undertaken by the insurance company. For the company, the most important problem is that the profit loading calculated gradually becomes inadequate. Problems of fairness also arise. Namely, above we counted with the technical interest rates. As it was pointed out, this is always a relatively low interest rate (generally it is between 2-4%[125]), guaranteed by the insurance company as the yield of the premium reserve.

In times of inflation, however, nominal yields are significantly higher than the yield that would correspond to the technical interest rate. Who should have this surplus? Considering fairness, it belongs to the client, since it is his money that accreted that interest. Thus, in times of inflation, the question of ''profit-sharing'' arises emphatically. These are the reasons that make it inevitable to apply techniques of handling (not ceasing, eliminating) inflation.

At first sight we may say that in case of life insurances there are two (parallel) techniques to be applied:

- Premium increase and/or
- Profit sharing.

The description below is applying to the traditional life insurances. Their tight construction can not be easily undone and originally it does not contain the problematic of inflation handling, and profit sharing (the yield due to the costumer was fixed, not varying). In the case of modern life insurance products, however, the hangling of the

[125] Although in the 1990s in Hungary there were examples of technical interest rates of 7%, and the general rate was 5,5%, it decreased to 4% and later to 2,9% in the '2000s years and to 2,4% in the '2010s years. Later this decrease – worldwide – has continued and has appeared the technical interest rates below 1%, while previously the most common rates were above 3%.

varying yield is part of the basic construction and the indexation of the premium or only the deducted expenses independently from the premium and the indexation of the death (or other risk) sum assured is easily solvable, so it does not cause a technical problem we have to discuss it seperately.

14.1. Premium Increase

In times of inflation, usually every insurance company offers premium increase to its clients. This is not an obligation for the client but an opportunity offered by the insurance company, which has nothing to do with any rises in prices. (We have discussed the question of the price of the insurances in the Chapter 7.) Premium increase means that the insurance company makes it possible to recalculate the client's insurance – which is deteriorating because of inflation – without exposing him to a new process of underwriting. Premium increase could only be called a rise in prices, if the insurance company increased the premium without increasing the insurance benefit it provides, or, if it calculated the price of the increased insurance benefit on the basis of rates less favourable for the client. Thus, in case of life insurances, a rise in the prices means that the insurance company changes its rates, so it provides the same unit of insurance benefit for higher prices. Premium increase is a different case. This is important to note, because many people who have a life insurance are confused about this difference. It is widely believed, that if an insurance company increases premium by a larger extent than the rate of inflation of the previous year (due to technical reasons that will be discussed later), it deceives its client. In fact, the insurance company does not make the insurance more expensive by increasing premium (if the increase is based on the same rates), even if the extent of this increase is not related at all to the extent of inflation.

The lack of underwriting means a higher level of anti-selection risk for insurance companies, so, according to their best practices, they offer this opportunity only to those clients who regularly accepted premium increase previously, while others, who did not, lose their right to do so. This protects the companies from those who first think of following up inflation rates on their death-beds.

The principle of premium increase is simple: the insurance company considers that for the extra premium (the difference between the increased premium and the premium paid in the previous year), the policyholder buys the same kind of insurance with *regular* premium payment for the rest of the term, calculating with the current age of the insured. The premium of this new insurance is simply added to the previous insurance premium.

It depends on the insurance company to determine the basis for the premium increase. Since the reason of the option is inflation, one of the most frequent solutions is to declare the inflation rate as the basis. For instance, the extent of increase is the same as the inflation rate of the previous year, or 80% of the inflation rate of the previous year, etc. Since inflation only becomes dangerous above a certain level, it is general to set a lower limit to premium increase, e.g. it is not offered if the inflation is below 5, or 10%!

Another method of premium increase is the technique of revalorization, where the possible extent of premium increase equals the investment profit of the insuraner in question. The technique of revalorization integrates premium increase and profit sharing systems into one complex technique, which will be discussed later on in more detail.

By what extent will the sum assured rise as a result of the premium increase? There is no general answer to this question, because it depends on the age and sex (of the insured), the insurance term, and the type of insurance. But it is certain that in case of endowment insurances, the sum assured increases less than the premium. This has two reasons:

1. The insured advances in age, and the older he becomes, the higher his probability of death will be, i.e. the more expensive his insurance will become.
2. As time goes by, the remaining term is shorter, which means that the time for saving gets shorter too, and this also results in the increase of the premium.

Thus, as time goes by, premium increase has gradually less impact. This is the reason why a number of insurance companies deny premium increase some years before maturity.

Compared to premium increase, investment profit sharing has a totally different effect.

14.2. Investment Profit Sharing

Investment profit sharing is in relation with the investment nature of savings-type insurances, thus this technique is applied not only to handle inflation. The reason why it is discussed here is that it happens to be an effective method for handling inflation, too.

As it was mentioned before, investment profit is the profit gained on the investment of the premium reserve above the technical interest rate.

There are several modes of investment profit sharing (for instance to draw it among the clients), although only two modes appear in practise. Either it is expended for *raising the insurance benefit*, or it credited to the client as an *interest earning deposit*. (As we will see later on, the technique of revalorization is in general a special case of the first mode.)

When the first mode is applied, the profit corresponding to the technical interest rate is subtracted from the total profit gained (since it was originally calculated into the premium), and then the surplus is shared, in a given ratio, between the insurance company and the client. The client's share then is considered to be the *single premium* of such a life insurance, that has a term equal to the remaining term of the original insurance (so its maturity is the same as that of the original insurance), and the client's entry age is his current age. On the basis of this, the sum assured is calculated, and the original insurance benefit is raised by this amount, the client's share of the investment profit is added to the premium reserve.

The result of the inflation handling technique applied by the insurance company in practice will basically depend on the outcome of the investments made by the insurance company. However, it can never be sure that the economic situation will enable the insurance company to offer elimination of the effect of inflation, in the sense that – resulting from the joint effect of (not compulsory) premium increase and profit share – the sum assured will increase by the same extent as the inflation. Because of this, it is better to talk about *handling inflation* than *following up inflation*.

14.3. The Technique of Revalorization

As it was already mentioned, techniques of premium increase and investment profit sharing are usually applied together, though their technical transactions are separated. This causes several problems that can be solved by the integrated system of premium increase and investment profit sharing, which is called the technique of revalorization (indexation technique). These problems are the following:

1. It was already mentioned above, that the extent of increase in the sum assured, which results from the premium increase, is gradually seceding from the extent of the premium increase. Although technically this is absolutely reasonable, in the course of time it makes the client less and less interested in increasing the premium.
2. The increase in the sum assured, which is less than that of the premium, is partly compensated by the raising of insurance benefit provided by investment profit sharing, but it is impossible to calculate precisely how the rise in the insurance benefit (resulting from the two techniques together applied together) is related to the extent of premium increase. Natural expectation of the client would be that if the premium increases by e.g. 20%, then the sum assured (in total, together with the profit share) should be increased by 20%, since this seems – at least on the face of it – to be fair, and what is more important, this seems to be practical, since in this way the expenses of the client and the level of the insurance benefit are increased by the same extent.

The increase in the total sum assured resulted in the parallel application of the two techniques is not equal to the premium increase, because they depend on different factors; the rate of premium increase depends on the inflation rate, while investment profit sharing depends on the yields the insurance company could achieve. In order to make the increase of the sum assured resulting from the premium increase precisely supplement the increase caused by investment profit sharing, premium increase has to depend on the extent of profit share.

But how much should the premium increase if the total increase in the sum assured is to be the same as the premium increase?

Before answering this question, let us examine the reason of the fact that if the premium is increased at the policy anniversary by 100% (e.g.), then the sum assured will be increased less? (Although the answer to this question was already given previously, now another approach is shown.)

If somebody wants to take out an endowment insurance for a term of 10 years, and the annual premium is 10,000 Forints for a sum assured of 100,000 Forints, then he will get an insurance of 200,000 Ft-s for an annual premium of 20,000 Forints. Namely, if we double the amount of the premium at the beginning of the term, the sum assured will be increased by the same extent: it is also doubled. But after a year we will have a different situation. By the time of the first policy anniversary, the insurance company accumulates a certain amount of reserve by using the premium paid by the client for the first year. This will be needed for providing the insurance benefit undertaken by the insurance company. The amount of this reserve depends on the sum assured. If the sum assured is 100,000 Forints, then this will need half as much reserve as a sum of 200,000 Forints would need by the time of the first anniversary. Thus, if the client doubles the premium for the rest of the term at the first policy anniversary, then the insurance company cannot double the sum assured, because in the first year it has accumulated a premium reserve which is enough only for one half of the sum assured. So, if the premium reserve and the premium are increased by the same extent, then the sum assured could be increased by that extent, too.

In every year the premium reserve increases – in addition to the pre-calculated amount – in the proportion of the investment profit of the premium reserve, since it is the investment profit of the premium reserve, and its extent is determined in the ratio of that. So, if premium increase follows the extent of investment profit of the premium reserve, then the sum assured will increase precisely by the extent of the investment profit of the premium reserve.

This is the technique of revalorization.

This technique has the advantage of being transparent, thus it needs less registration and computer technology than other techniques of premium increase and investment profit sharing that are used separately. However, the result is the same. The increase in the sum assured can be divided into two parts: the impact of premium increase and the impact of investment profit sharing. If we did so, we would find the same as above, namely that the extent of increase in the sum assured caused by the premium increase is gradually decreasing, while the extent of increase caused by the investment profit sharing is constantly growing.

Revalorization is thus an elegant technique with only one significant disadvantage compared to the separately applied techniques of premium increase and investment profit sharing. Namely, that the extent of premium increase depends on the activity of the insurance company, instead of other objective indicators such as the rate of inflation. However, this is not a problem but an advantage, if clients have greater confidence in the insurance company than in the state (since the rate of inflation is determined by national institutions). Thus it is highly important for insurance companies using this technique to acquire their clients' confidence.

15. THE CALCULATION OF INFLATION PREMIUM INCREASE AND INVESTMENT PROFIT SHARING

15.1. Premium Increase Independent of Profit Sharing

Most insurer offer their clients at every policy anniversary the opportunity of increasing the premium – and based on this also the sum assured – by the extent of the inflation of the previous year (or by a given percentage of it). (Previous year hereby means the last calendar year of which there is an official rate of inflation available, thus in January of year x ”previous year“ can also be year x-2.) However, this is usually offered to clients only if the rate of inflation is above 10%. Clearly enough, possibility of premium increase is offered only for clients holding insurances of regular payment.

The extent of premium increase is given. But by how much is the sum assured increased? There are basically two ways for this to be calculated. In both ways, the extra premium is considered to be the annual premium of a new insurance. The difference lies in the tariffs they use to calculate this new insurance. The two possible ways:

- To use a normal tariff, as if the policyholder effected a new insurance;
- To use a preferential tariff. Naturally, the absolute upper limit of the allowance given is that the insurance company provides the increased insurance for a net premium, what naturally never happens, because there would not be a cover for the increased cost of the insurer because of the inflation.

Let PG_t denote the annual premium that has to be paid at the beginning of insurance year t.

If k denotes the inflation rate in the ‘’previous” year, and if it is assumed that this is the rate by which the insurance company lets premium increase, the new premium will be:

$$PG_{t+1} = PG_t \cdot (1 + k) \tag{15.1.}$$

Thus the extra premium will be:

$$dPG_{t+1} = PG_{t+1} - PG_t = PG_t \cdot k \tag{15.2.}$$

Assuming that the insurance company uses the first method (does not give any allowance), and stands for the gross annual premium of 1 Ft sum assured, then the increase of the sum assured is:

$$dSA_{t+1} = \frac{dPG_{t+1}}{PG_{x+t:\overline{n-t|}}}$$

(15.3.)

where , and is the sum assured valid throughout the insurance year *t*.

If we do not have to use a different reserve formula in a certain part of the term due to zillmerization, then the premium reserve of the end of year *t* (before premium payment) can simply be calculated with the following formula:

$$V_t = SA_t \cdot \left(A_{x+t:\overline{n-t|}} - \ddot{a}_{x+t:\overline{n-t|}} \cdot \mathrm{PZ}\right)$$

(15.4.)

where *PZ* is the ''reserve-'' or „zillmer"-premium, i.e. the net premium that fills the reserve from year to year.

Guaranteed Insurability Option (GIO) does not belong here due to its subject matter, but because of its similar technical implementation it should be mentioned here. The policyholder possessing such an option has the opportunity from time to time (e.g. every three years) to increase his premium above the inflation premium increase. This does not serve for compensating inflation but for covering his increased insurance needs arising from his permanently improved financial situation. Calculation of premium increase due to GIO is technically the same as it is described above.

15.2. Profit Sharing Independent of Premium Increase

Insurance companies usually give their clients the greater part of the premium reserve's yield over the technical interest rate in the form of profit sharing. The extent of this share varies – as well as the technical interest rate – as it depends on the insurance company in question. If *i* stands for the technical interest rate and *h*[126] denotes the annual yield of the premium reserve, then the investment profit (*h'*) is:

$$h' = h - i$$

(15.5.)

126 Below – implicitly – we assume, that the "yield" is the "gross yield" in investment sense. If the investor of the reserve is the insurer itself, then this is generally true. However, if it employs a trustee, this has became a kind of net yield from which the fee of the trustee already has been deducted. If the trustee is a firm inside the insurance group, then the whole gross yield is realized by the group.

For example:

Technical interest rate:	5%
Profit share of the client:	90%
Yield:	10%

Here the extent of the investment profit percentage going to the client (h'') is the following percentage of the average premium reserve of the previous year:

$$h'' = 0{,}9 \cdot (10\% - 5\%) = 4{,}5\% \tag{15.6.}$$

Total from the yield for the insurance company:	$0{,}1 \cdot (10\% - 5\%) = 0{,}5\%$
Total from the yield for the client:	$5\% + 4{,}5\% = 9{,}5\%$

The insurance company can give this investment profit back to the client in two different ways:

- opens an „account" for the client, where the profit share will be credited from year to year, and where it will carry interest together with the premium reserve,
- considers the profit to be the single premium of a single premium insurance, so it will be added to the premium reserve, thus increasing the sum assured.

The first possibility does not need any mathematical explanation, so here we only discuss the second one, where we promptly face a problem. "Time" for insurances differs from "time" for investments. Namely, if an insurance was effected on the 24th of March, then the "insured year" will last from the 24th of March till the 23rd of March. But results of investments are usually reviewed according to calendar years, and profits are shared between contracts similarly. From this it follows that it is problematic to decide what the basis for investment share should be for a contract? It is usually the average premium reserve of the previous year.

Let's suppose that the insurance had its t^{th} policy anniversary *(*that was the end of "insured year" number *t*) in the previous calendar year (the investment share of which we are interested in). From the beginning of the previous calendar year until the t^{th} policy anniversary τ (fragment) year elapsed. Then, the average annual premium reserve (without investment share) will be the simple arithmetic average of the premium reserve at the beginning and at the end of the calendar year.

The premium reserve at the beginning of the calendar year is shown by (12.39.) which is the following:

$$V_{t+\tau} = (1-\tau) \cdot (V_t + P) + \tau \cdot V_{t+1} \tag{15.7.}$$

The premium reserve at the end of the calendar year is:

$$V_{t+1+\tau} = (1-\tau) \cdot (V_{t+1} + P) + \tau \cdot V_{t+2} \tag{15.8.}$$

Thus the average premium reserve of the calendar year is:

$$\frac{V_{t+\tau} + V_{t+1+\tau}}{2} \tag{15.9.}$$

Naturally, above we latently supposed that the contract did not commence, and is not terminated in the calendar year in question, and that its premium is paid regularly, annually (this is why the correction factor *P* is present in the premium reserve at the beginning of the insurance year). Usually in case of the first and the last fragment year a proportional part of the premium reserve is taken, while in case of single premium insurances, *P* factors are simply missing from the above formula. The premium reserve of the investment profit shares that have been distributed before is interpolated in the above described way and added to the basis of profit sharing.

On the basis of the average premium reserve calculated this way the insurance company provides the profit share, the extent of which is h''. This means that the investment profit (coming to the client) is:

$$h'' \cdot \frac{V_{t+\tau} + V_{t+1+\tau}}{2} \tag{15.10.}$$

But this investment profit is directly added to the client's premium reserve from the beginning of the following calendar year. However, we do not know by how much the sum assured will be increased.

Insurance companies usually change investment profit to sum assured by using a net tariff, i.e. without charging expenses. But, because the number of the years remaining from the term is not a whole number at the beginning of the calendar year, single premiums also have to be interpolated to the beginning of the calendar year. Since the beginning of the given calendar year falls between policy anniversaries *t* and *t+1,* the single premiums $A_{x+t:\overline{n-t|}}$ and $A_{x+t+1:\overline{n-t-1|}}$ have to be interpolated. So, the interpolated single premium is:

$$A_{x+t+1+\tau:\overline{n-t-1-\tau|}} = (1-\tau) \cdot A_{x+t+1:\overline{n-t-1|}} + \tau \cdot A_{x+t+2:\overline{n-t-2|}} \tag{15.11.}$$

Accordingly, the increase of the bonus sum assured (dBSA) from the new calendar year is:

$$dBSA = \frac{h'' \cdot (V_{t+\tau} + V_{t+1+\tau})}{2 \cdot A_{x+t+1+\tau:\overline{n-t-1-\tau|}}} \tag{15.12.}$$

15.3. Integrated Premium Increase and Investment Profit Sharing System – the Technique of Revalorization

The two methods of managing inflation described above are usually applied together, though technically separately. But another technique – the "revalorization" – was also mentioned, which integrates the two and has the advantage that all important values – premium, premium reserve, sum assured – are increased by the same extent, which extent is the investment profit share (%) returned to the client by the insurance company.

The technique of revalorization is based on the finding that if the premium is raised by $x\%$ on a policy anniversary, then the sum assured cannot be raised by the same extent, (it can only be increased by a lower degree – e.g. in the case of endowment insurances, where this technique is most often used), because that part of premium reserve that should have been accumulated starting from the beginning of the insurance to cover the $x\%$ extra sum assured is missing. The shortfall is precisely $x\%$ of the premium reserve up to that date.

Here comes the technique of revalorization, saying that the percentage of investment profit share shows exactly the extent of increase in the premium reserve. And, if this percentage is also the percentage of the premium increase, then the increase of the sum assured resulting from the premium increase and that resulting from profit sharing will exactly complete each other to the extent of the percentage of the investment profit share.

However, this all seems so simple only at first sight, but when going into details, we find several problems. Thus, not all problems will be discussed in the following, only the basics. We use some simplifying assumptions.

1. In case of every age-term combination the same loadings (determined as a percentage of the net premium) are added to the net premiums, thus the extent of the net premium increase is the same as that of the gross premium increase.
2. The premium reserve is not zillmerized.
3. Investment profit share is allocated according to insurance year instead of calendar year. This brings along technical problems for the insurance company, namely that it needs to know the actual yield at any moment. But a more serious problem, which prevents this technique from being applied without further considerations is the demand of clients that their investment profit shares should be calculated on the basis of authorized investment profit data. Such data are, however, available only per calendar year, some months after closing the calendar year in question.
4. The basis of the investment profit is not the annual average premium reserve, but the premium reserve at the end of the calendar year.

First of all, it has to be proved that, under these conditions, the extent of the increase of the sum assured resulting from the investment profit sharing and that resulting from the premium increase really equal to the extent of the investment profit going to the client (*h”*).

We prove this for the first increase and profit sharing. If P_1 denotes the net premium of the first year, where – as we know –

$$P_1 = SA \cdot \frac{A_{x:\overline{n|}}}{ä_{x:\overline{n|}}} = SA \cdot P_{x:\overline{n|}} \tag{15.13.}$$

then the premium of the second year is:

$$P_2 = P_1 \cdot (1 + h'') \tag{15.14.}$$

Then the increase of the sum assured resulting from the premium increase is (here we can calculate with net premiums, using the assumption that the loadings are equal):

$$\frac{P_2 - P_1}{\frac{A_{x+1:\overline{n-1|}}}{ä_{x+1:\overline{n-1|}}}} = \frac{h'' \cdot SA \cdot P_{x:\overline{n|}}}{\frac{A_{x+1:\overline{n-1|}}}{ä_{x+1:\overline{n-1|}}}} \tag{15.15.}$$

The increase of the sum assured resulting from the profit share is (here we make use of the assumption that the basis of the profit share is the premium reserve at the end of the year):

$$\frac{V_1 \cdot h''}{A_{x+1:\overline{n-1|}}} = \frac{SA \cdot \left(A_{x+1:\overline{n-1|}} - ä_{x+1:\overline{n-1|}} \cdot P_{x:\overline{n|}}\right) \cdot h''}{A_{x+1:\overline{n-1|}}} \tag{15.16.}$$

What needs to be verified is:

$$\frac{h'' \cdot SA \cdot P_{x:\overline{n|}}}{\frac{A_{x+1:\overline{n-1|}}}{ä_{x+1:\overline{n-1|}}}} + \frac{SA \cdot \left(A_{x+1:\overline{n-1|}} - ä_{x+1:\overline{n-1|}} \cdot P_{x:\overline{n|}}\right) \cdot h''}{A_{x+1:\overline{n-1|}}} = h'' \cdot SA \tag{15.17.}$$

Dividing this by *SA*, breaking up the parenthesis and making simplifications we get the following equation:

$$\frac{h'' \cdot P_{x:\overline{n|}} \cdot ä_{x+1:\overline{n-1|}}}{A_{x+1:\overline{n-1|}}} + \frac{A_{x+1:\overline{n-1|}} \cdot h'' - ä_{x+1:\overline{n-1|}} \cdot P_{x:\overline{n|}} \cdot h''}{A_{x+1:\overline{n-1|}}} = h'' \tag{15.18.}$$

Since the positive and negative elements drop out each other, we get the

$$h'' = h''$$

identity, which means that the statement is verified.

Without going into any more details, we only mention what happens if the client refuses the premium increase at a policy anniversary, or if – due to its slight extent – the insurance company does not offer this option. In this case the best solution is to choose the "account method" for handling the generated profit share, because this will not ‘’ruin’’ the future possibility of applying the technique of revalorisation.

16. MODERN PREMIUM AND RESERVE CALCULATION

KEY WORDS

Field of actuarial control	Unbundled products
Internal rate of return (IRR)	Net present value
Yearly renewable term insurance	Profit test
Level premium	

In the previous chapter the traditional method of premium calculation was presented. It is characterised by the strict differentiation between net and gross premium, and by its dealing with the impact of relatively few factors. Nowadays, modern methods of calculating premium are spreading, existing side by side with the traditional modes. The main feature of these methods is that through computer program packages they can explicitly consider much more factors effecting the premium. Due to the nature of the subject they do not work with closed formulae, but with the method of trial and error: different assumptions are being varied until an ''acceptable'' and stable premium is calculated. In these models profit is the main outcome-variable, so the method itself is called profit test.

The modern premium calculation method is equal to the company valuation method of the finance, the technique of the net present value. Thus, this method is more flexible than the traditional premium calculation technique, which has accommodated to the speciality of life insurance.

In the followings, we introduce the main points of profit tests, and we show through a case study that using simple assumptions we can make such models for ourselves.

16.1. The Profit Test[127]

The first profit testing softwares were made in actuarial consulting companies[128] in order to check the calculations of clients, but it was soon realised that these are marketable products on their own.

Profit testing programs appeared at the time when unbundled products were spreading, which do not contain fixed elements (endowment, term fix, etc. – i.e.

[127] This sub-chapter is based on Tibor Edvi's lecture on the topic.
[128] E.g. the Prophet, the product of Bacon & Woodrow that became part of Deloitte & Touche is the most popular such program in Hungary today.

traditional life insurances as this book calls them), but they are constructed by visible elements (universal life, unit linked – modern life insurances as this book calls them). In these programs all parts of the premium are visible and are handled like an account. (As we have seen, this is the combination of an account and a yearly renewable term = YRT insurance.) This development gave momentum to new pricing methods, because applying classical methods to these products is rather difficult (since classical methods of premium calculation were developed to '’suit’' traditional products).

The new methods made it necessary, while the development of the environment made it possible for new pricing methods to come into life. Main elements of the development of the environment are:

1. Opportunities provided by information technology.
2. Given conditions (inflation, investment environment: shift from investment in bonds towards investments in shares).
3. A state of competition in which insurance companies ’’found’’ themselves.

As we have already seen, the change of the environment in itself contributed to and reflected on the appearance of modern insurances.

Modern principles used in the pricing process do not contradict the classic ones, but they are slightly rephrased. In case of modern techniques of premium calculation we do not talk about the principle of equivalence, but about the net present value of the premium being equal to zero, i.e. about the requirement of $NPV(PG)=0$.

One of the most important new concepts is the cash flow, which is the difference of and the time series of incomes and expenditures, and the internal rate of return (IRR), that – we can say – is the same as the technical interest rate in the classical case.

From the point of view of modern products, classic methods have the disadvantage that they do not deal with the following factors, problems:

- profit sharing (investments)
- inflation, i.e. the increase of premiums and expenses
- cancellations, policy pay-ups, money withdrawal, partial surrender, other options
- realistic expectations of the owner (return on capital)
- complicated expense structures (e.g. occasional expenses)

In comparison to this, the technique of net present value can be more easily adjusted to reality. The conditions of this are:

1. determining the realistic cash flow (of course, this often means only an expected value both in the area of mortality and other options. However, correlations between certain factors can be taken into consideration.)
2. applying a realistic discount rate

The elements of cash flow are (+ and – represent the direction of the flow of money from the insurer’s point of view):

+ premium income
+ investment earnings
- benefit payments (insurance benefits, surrender)
- expenses/commissions (acquisition, administrative/maintenance). After a kind of preliminary calculation these are put into a more easily handable form (so called „expense allowance") to make them an appropriate input of the profit test model.
- reserving expenditure (change of reserves)
- solvency (extra immobilisation of capital)

Two methods exist for handling the expected value (which partly means certain numbers by the piece and partly yields):

- deterministic and
- stochastic.

Within the deterministic method (classic premium calculation belongs here) mortality tables and multiple exit tables (Markov chains) are used. The problem is that this contains only the most probable cases (along the most probable picture of future). It is completed with, and made more subtle, by a sensitivity test, or maybe several future scenarios are created, and their weighted average used as a result. But this easily leads to inconsistency. Such problems are to be solved by the stochastic methods, which sometimes analyse mortality, but more often yields. Complicated stochastic methods are used only if deterministic methods fail (e.g. their predictions are not verified).

The most widespread technique is the sensitivity test, which is the profit test itself.

In a sensitivity test it is examined how ''robust'' the pricing of the product is, i.e. how much its profitability tolerates significant changes of certain parameters. It might be said, that sooner the product's cash flow returns early expenses, the more robust the pricing is. From this point of view we can say that the goal of zillmerization is to make the product more robust, since it changes the cash flow in the following way (see F*igure 16.1.*):

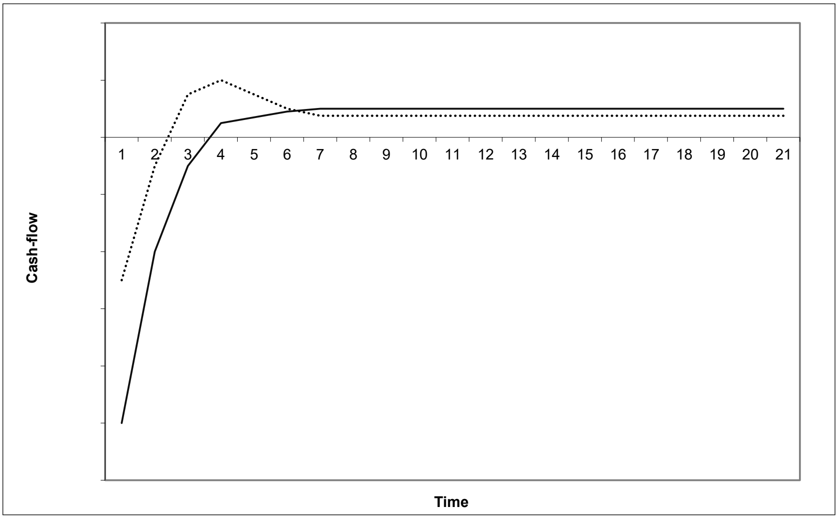

Figure 16.1: Cash flow with (dashed line) and without zillmerization

Negative cash flow during the term is not allowed – at least if this can be anticipated. If it is noticed, it must be eliminated, or reserves must be built beforehand to counterweight the negative cash flow.

Profit test is not simply a technique of premium calculation. It can be, and has to be performed not only during the development of the product, but also afterwards, when it is being sold (post-calculation).

The place of profit test within the life of the product is best illustrated by Goford's "Actuarial Control Cycle" (*Figure 16.2.*):

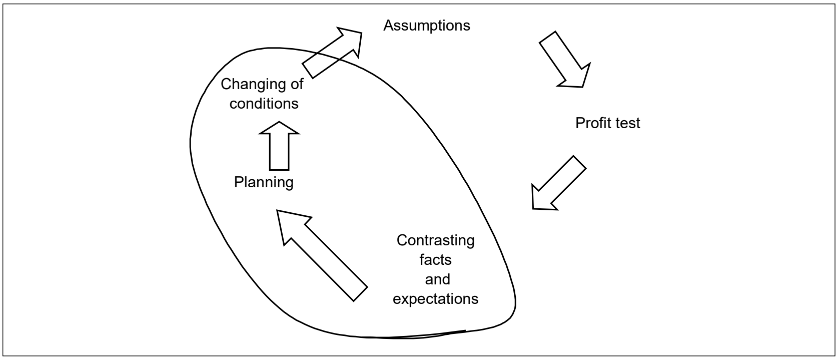

Figure 16.2: The Actuarial Control Cycle

Finally, the question arises what discount rate we should use during the calculation? Generally speaking, the answer is that this is usually determined by the owner himself, since this is the (minimally) expected yield of the owner. (He himself can determine this e.g. on the basis of CAPM)

16.2. Case Study: Calculation of the Expense Part of a Rider to Life Insurance Policies

As it is well known, European Union directives allow life insurance companies to offer accident- and sickness insurances as ''complementary risks'' along with life insurance policies. Thus, it is possible to sell life-, accident- and health insurances together, within one package. Usually this takes the form of constructing accident or sickness (sometimes life) insurance riders to the main policy.

Insurance riders still do not have so subtle methodology as the methods of main policies, and their models are often missing from the popular profit tests. This field is characterised on the one hand by the level premium independent of age and term[129] – mostly in case of accident insurance riders – and on the other hand by the lack of mathematical reserving. Also, the calculation of expenses is often made on the basis of ''feelings''.

In the followings I try to present a model more subtle than the usual[130], which naturally also has its limitations. I take it for a given condition that premiums were calculated with the consideration of age, so I will deal with the "fair" and more or less sensitive calculation of the expenses. (The question of "fairness" arises basically because of the premiums depending on age – in case of level premiums this is not a problem.) Here I am neither concerned with the long term equalisation of premiums, nor with reserving. This model is suitable to carry out a sensitivity test (profit test) on the effect of certain expense factors in an Excel model. But it is not my intention to discuss the Excel realisation of the model.

16.2.1. Should we use a level or an age-dependent premium?

Before introducing the model, let us summarize when level premium can be used, and in what cases it is not practical.

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[129] Differences between the risks according to sex are not examined here, it is considered to be given that these risks are different, so different premiums should be calculated in the case of the two sexes – even if they are equal.

[130] Of course, this does not mean that it is impossible that a more refined approach than the one presented here may exist in the Hungarian market.

Premium calculation of an insurance company basically depends on three factors:

1. nature of risk
2. statistics available
3. habits of other competitors

Let us go through these factors!

Level premium can, or rather has to be applied if the risk in question is independent of age. In such a case, the (regular) premium will be independent of the term, too. However, most probably we will not find a risk that satisfies this prerequisite. Accidental death is usually considered to belong to this category, but it is well known that accidental mortality of men in their late teenage years and early twenties is significantly higher than the average.

Contrary to this, in certain situations level premium may be adequate. Basically, in such cases when the risk contains no drastic trend depending on age, i.e. it has no tendency to rise, or even it significantly decreases with age.

In the following figures ("Basin", "Random fluctuation" "Slight trend") you can see cases when level premium is not only possible, but it is advisable.

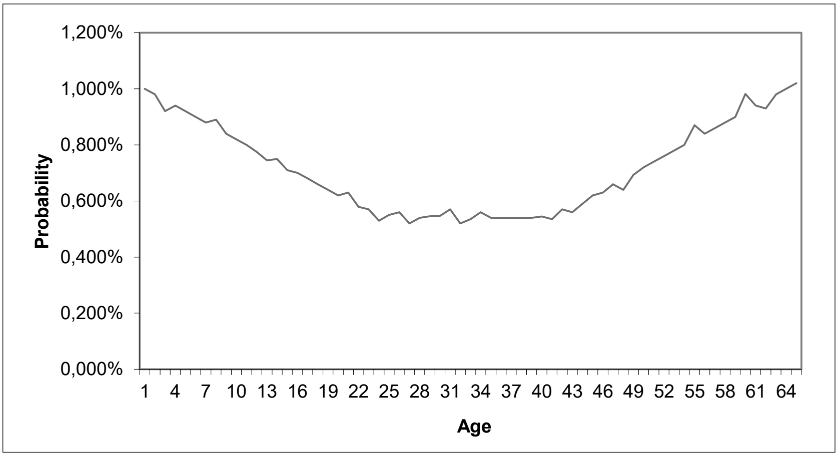

Figure 16.3: "Basin"

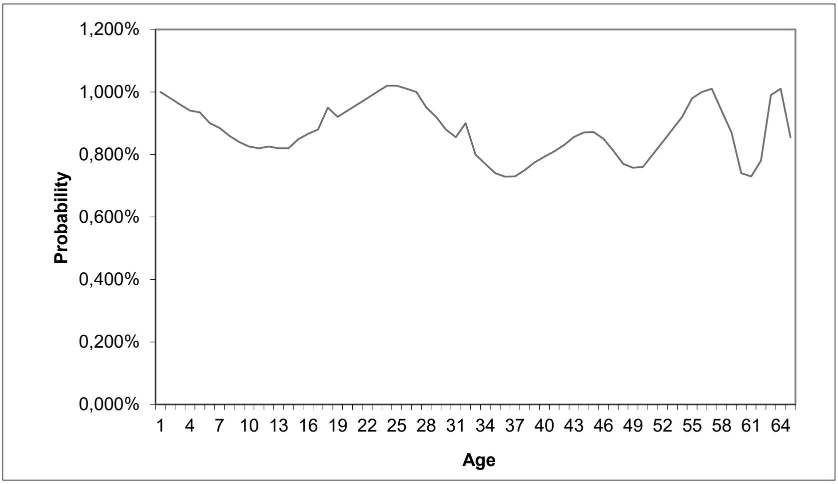

Figure 16.4. “Random fluctuation”

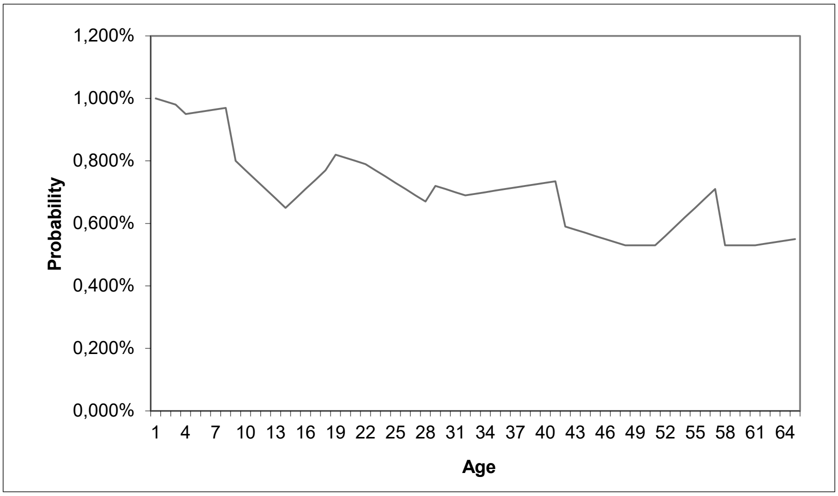

Figure 16.5: “Slight trend”

In the case illustrated by figure 16.6., level premium is still possible, but it already causes several problems.

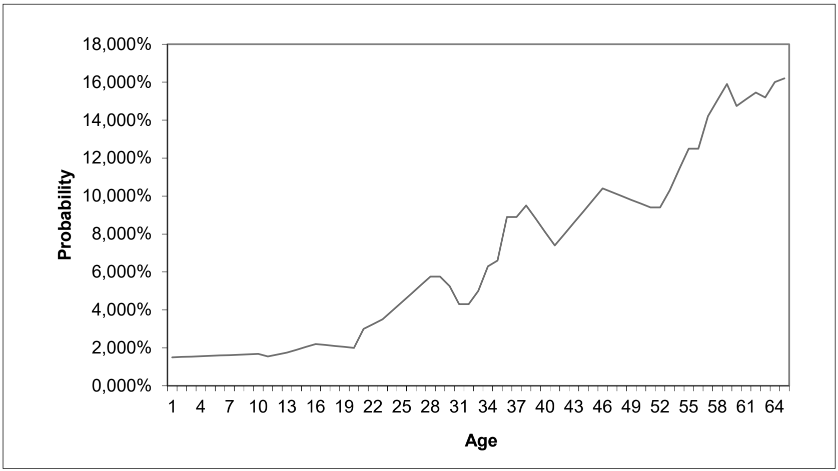

Figure 16.6.: "Drastic trend"

It is problematic, because if premiums are equalised, the portfolio will quite probably become auto-selected, which would mean that those who represent higher-than-average risk will consider the insurance to be exceptionally favourable, while those who represent lower-than-average risk will wait some time before taking out this type of policy. This will have the result that, all in all, expectations of the insurance company regarding the composition of risks within the portfolio will not be fulfilled. What is more, in such case it is worth for rival companies to enter the market with premiums differentiated by age.

If the insurance company uses age-dependent premiums, it is useful to consider the question of mathematical reserving.[131] But this also requires that the insurer keeps premiums of the individual policies fixed for a longer period of time (at least 3 years), i.e. there should be differentiation by age when entering the insurance, but aging during the term should not bring along changes in the premium paid.

There is currently a mixed practise regarding the equalisation of risk during the term and reserving. From a business aspect, a non-changing premium is more favourable, but accident and sickness insurance risks are not stable enough to enable the insurance company to engage itself for a longer period of time. Thus, from this perspective, this situation differs from the usual life insurance situations. As a consequence, insurance companies apply premiums that change yearly, or equalise premiums only for a

[131] In case of single premium and long term, reserving is necessary even if premiums are independent of age!

shorter period of time (e.g. for five years), or they keep the possibility of changing the premiums, or they use a combination of all these possibilities.

The available statistics also influence the premium calculation of the insurance company. If there are no available statistics for the risks according to age for example, (up to now, accidental risk has fallen in this category), the insurance company is prone to calculate a fix (level) premium based on the average of the risk community.

It is worth calculating premiums depending on age if there is a drastic trend according to age, even if there are no relevant statistics available. In such a case, the insurance company may use assumptions and analogies, or it may adapt existing data from abroad.

It often happens, that market ''practise'' has also influence on whether the insurance company uses level premiums or premiums depending on age. In Hungary for instance, in the case of accident insurances, premiums independent of age (and naturally sex[132]) are most characteristic; there are examples of both ways in the case of disability waiver of premium, while in the case of hospitalisation daily allowance insurance tariffs usually depend on age.

16.2.2. The Problem

Now the goal of the case study is more apparent. Some questions have been answered, some are still open. It is decided that we plan some kind of life-, accident or health insurance rider. As for the risk, we find a strong trend increasing by age in the data. Net premiums have been calculated. These are:

- premiums depending on age and sex[133], changing yearly (generally[134] rising)
- there is no mathematical reserve and the technical term[135] of the insurance is one year.

The task is: to calculate an expense part on the net premiums.

Beside others, the following questions arise:

- How can expenses be spread in a "fair" way?
- How can loadings be determined? (The goal is to be able to carry out sensitivity tests on loadings with a simple Excel model.)
- How can the effects of quota share reinsurance on the premium be planned?

The following sub-chapter deals with these questions.

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[132] This was the situation even before the Gender directive in this field.

[133] Though it is not possible anymore a premium differentiated according to the sex (however when this case study was prepared it was typical), I have not changed the logic of the case study, because here the logic of differentiation is important. It would have been substituted by other factors as well.

[134] Despite the rising trend, temporary falls may occur.

[135] It is up to the insurer to define a product with formally a term of one year automatically renewed, or fix term. The automatically renewable policy enables the insurance company to correct the premium easily, or to leave the market if risks increase, while a fix term is more appealing to clients, because it makes planning easier.

16.2.3. Spreading of expenses

In the case study I use the generally known notations with the following differences, and I introduce the following special marks:

nem: its possible values are: male, female

NP_x^{nem}: net premium of the insurance rider if the insured is of x years of age and of *nem* sex, in case of a sum assured of 1 Ft,

SA : the average sum assured of the rider at the commencement of the insurance (independent of age and sex)

P_x^{nem} : the size of the *sex* population of age x in Hungary

i_k : interest rate (discount factor) in the year number k. in the life of the product

fm: payback period of development expenses (year)

em: payback period of acquisition expenses (year)

tf_k : probability of surrender and termination (including termination due to death) in year number k. of the term

DB_k : number of new policies in year number k.

id_k : increase rate of the gross premium (and the average sum assured) in year number k.

In the model – which primarily serves for examining the effects of different expense factors – I assume an expense structure which is more subtle than the traditional expenses α, β, γ known from life insurances, but which is simpler than the structure used in big profit test models. The main idea is that I explicitly take into consideration the effect of the size of the portfolio and that of the average premium, versus the α, β, γ model; (although the effect of changes in the inner age structure of the portfolio and changes in time do not appear explicitly). I can do so, because I differentiate not only between expenses depending on the premium (and thus on the sum assured), but also between several other types of per policy expenses (that are independent of the level of premium). Of course there must be some hidden assumptions on the size of the portfolio and the level of premiums also in the α, β, γ models, but these are not explicit from the model's perspective.

The most important factor among those that I disregard here is the portfolio's inner structure (which is only represented by averages) and its development, and the ”transitionary” types of expenses that are theoretically possible between the single and regular expenses.

The expenses I take into consideration have the following dimensions (**not** independent of each another):

- Single or regular expense
- Expense proportional to the gross premium or per policy expenses
- Expense of the whole portfolio, the portfolio of the given year, or the given policy

Since these dimensions are not independent of one-another, the following six combinations can be imagined (with the names that I gave them):

fk: development expense – arises only once – spread over the whole portfolio
ekfd: single expense occurring at the inception of every policy, in Forints/policy
eksz: single expense that occurs at every policy inception as percentage of the gross premium
foksz: regular expenses as percentage of the gross premium
foka: regular expenses in yearly absolute value (for the whole first year premium of all insurance riders)
fokfd: regular expenses in Forints/policy – for the first year premium of the given insurance rider

A profit test can be carried out easily if all possible expenses are listed on an Excel worksheet, put into one of the above mentioned categories, then the values are summarized by expense types. These sums will be the input data of the formulae. After that the value of different expense factors can be varied in the table, so the sensitivity test of the premium can be carried out. Profit, allowances, provisions for adverse deviation, etc. should also be considered as expenses (e.g. as *foksz* expense), too.

Now, after the preparations, let's take a look at the possible problems that may arise in spreading the expenses. An evident solution is – so it is usually applied – that the *loading* is the same for all age groups, even if net premiums are different. But the solution developed for level premiums is not the best solution here.

Spreading expense factors proportionally on every Ft of the net premium would be a wrong method due to several of reasons:

1. Since the differences of net premiums by age are very high, certain age groups would bear a disproportionately great part of certain fix expenses, which is not fair.
2. Due to this unfairness, there would be an unnecessarily big difference between the gross premiums of different age groups.
3. But the most important thing that affects the insurance company is: the premium will not be ''self-financing'', i.e. if the age composition of the actual portfolio differs from the previous expectations, then the actual cover of expenses coming into the insurance company will also differ from the necessary amount. The reason for this is that expenses were not spread in the way as they arise.

In order to avoid these problems we create separate loadings for the different age groups and sexes, where our basic concept is to divide expenses between policies as precisely as possible where this division is unambiguous (disregarding differences occurring in the sums assured within the groups of the same sex, age and insurance type – that cannot be handled by a proportionate loading[136]).

[136] The step further is the solution already applied in the case of Unit Linked insurances, namely that certain expenses are completely separated from the premium and the sum assured, and are deducted monthly, in absolute value. Later this may be a possible way for insurance riders, too.

This concept can be easily represented in relation to most of the expense factors, since they have originally been formulated this way (ekfd, eksz, foksz, fokfd). However, it is a bit problematic in the case of the ”general” expenses like fk and foka. These first have to be spread over policies somehow. Now we choose the method of converting these two factors into one of the other four[137].

Evidently, these can be converted into the following expense factors:

- development expenses (fk): into single Forints/policy (ekfd) expenses
- yearly stock expenses (foka): into regular Forints/policy expenses.

The consideration that lies behind converting into per policy expenses is that general expenses should be shared equally per insured.

The conversion is done according to the following:

Converting development expenses: we assume that the payback period of development expenses is *fm* years, so it is loaded to the portfolio of the first *fm* years. Being a single expense, it is loaded to all riders taken out in this period of time, and for the sake of proportionality the loading is the same for all policies. We correct with a discount factor in the denominator of the formula, because the expense part with a longer payback period is carrying interest until it is refunded.

$$\frac{fk}{\sum_{j=1}^{fm} DB_j \cdot \prod_{k=1}^{j} \frac{1}{(1+i_k)}} \tag{16.1.}$$

Thus we raise the ekfd expenses per product according to formula 16.1.

Converting foka expenses: this is less complicated, since only one year is concerned. Similarly as above Fokfd expenses increase according to the following formula:

$$\frac{foka}{DB_1} \tag{16.2.}$$

Considering different factors:

fk and foka[138]: sharing them proportionally among all pieces.

ekfd (early Ft/piece expenses) – per product: here again we determine a payback period (*em* – it is practical to choose the same as *fm*, though their content is different), which means that these expenses have to be refunded on the portfolio alive during the payback period. These are spread in the ratio of the average gross premium. The extent of this factor is not influenced by the dynamics of portfolio development.

eksz (expenses expressed as percentage of the initial annual gross premium) – per

[137] In the case of Excel, this is only an intermediate conversion, it does not affect our assumption that the changes of these expense factors will also directly appear in the change of the premiums.
[138] This factor has to be recalculated every year.

product: nearly the same as the previous one, but here we do not have to care about the absolute value of the gross premium, but the payback period and probabilities of surrender. Dynamics of portfolio development is not a factor to count.

foksz (expenses expressed as percentage of the regular gross annual premium) – per product: simplest to deal with.

fokfd (regular Forints/policy/year expenses) – per product: divided by the average gross annual premium.

Considering the expense factors mentioned above and keeping our concept in mind, we get the following loading-formulae.

The loading expressed as percentage of the net premium can be calculated with the help of two kinds of expense factors:

1. expenses given in Forints/policy – these can be easily projected to the net premium. (ekfd, fokfd)
2. expenses given in the percentage of the gross premium (eksz, foksz), which have to be projected to the net premium.

If *b* represents expenses given in the ratio of the gross premium, and *n* represents the expenses given in the ratio of the net premium, then the loading can be calculated by the following formula:

$$\lambda = (1+\lambda)\cdot b + n \tag{16.3.}$$

From which:

$$\lambda = \frac{n+b}{1-b} \tag{16.4.}$$

$$\lambda_x^{nem} = \frac{ekfd}{SA \cdot NP_x^{nem} \cdot \sum_{l=1}^{em} \prod_{k=1}^{l} \frac{(1+id_k)\cdot(1-tf_k)}{1+i_k}} + \frac{fokfd}{SA \cdot NP_x^{nem}} + $$
$$+(1+\lambda_x^{nem}) \cdot \left[\frac{eksz}{\sum_{l=1}^{em} \prod_{k=1}^{l} \frac{(1+id_k)\cdot(1-tf_k)}{1+i_k}} + foksz \right] \tag{16.5.}$$

Expressing this explicitly (by using the relation deduced above):

In order to simplify the formula 16.6., let us introduce the notation *kdf* (combined discount factor) as follows:

$$\lambda_x^{nem} = \frac{\dfrac{ekfd}{SA \cdot NP_x^{nem} \cdot \sum_{l=1}^{em} \prod_{k=1}^{l} \dfrac{(1+id_k)\cdot(1-tf_k)}{1+i_k}} + \dfrac{fokfd}{SA \cdot NP_x^{nem}} + \dfrac{eksz}{\sum_{l=1}^{em} \prod_{k=1}^{l} \dfrac{(1+id_k)\cdot(1-tf_k)}{1+i_k}} + foksz}{1 - \left[\dfrac{eksz}{\sum_{l=1}^{em} \prod_{k=1}^{l} \dfrac{(1+id_k)\cdot(1-tf_k)}{1+i_k}} + foksz\right]}$$

(16.6.)

$$kdf = \sum_{l=1}^{em} \prod_{k=1}^{l} \frac{(1+id_k)\cdot(1-tf_k)}{1+i_k}$$

(16.7.)

Thus the formula above is simplified like this:

$$\boldsymbol{\lambda_x^{nem}} = \frac{\dfrac{\boldsymbol{ekfd}}{\boldsymbol{SA \cdot NP_x^{nem} \cdot kdf}} + \dfrac{\boldsymbol{fokfd}}{\boldsymbol{SA \cdot NP_x^{nem}}} + \left[\dfrac{\boldsymbol{eksz}}{\boldsymbol{kdf}} + \boldsymbol{foksz}\right]}{\boldsymbol{1} - \left[\dfrac{\boldsymbol{eksz}}{\boldsymbol{kdf}} + \boldsymbol{foksz}\right]} =$$

$$= \frac{\dfrac{1}{SA \cdot NP_x^{nem}} \cdot \left[\dfrac{ekfd}{kdf} + fokfd\right] + \left[\dfrac{eksz}{kdf} + foksz\right]}{1 - \left[\dfrac{eksz}{kdf} + foksz\right]}$$

(16.8.)

16.2.4. The Effect of Reinsurance on the Premium

The loadings above were calculated without the assumption of reinsurance. But it is not sure at all that the reinsurer will undertake the reinsurance on net premium, so this effect has to be taken into consideration (of course, only if the insurance company reinsures the portfolio at all). Only the simplest type of reinsurance, the quota share reinsurance is discussed here. However, it is also the most widespread, and in relation to new businesses, it is given preference by reinsurance companies.

Let us suppose that the reinsurance contract is a quota share with r% retention, and the insurance company pays back a ceding commission c% for the part handed over (for the gross premium). Since it is probable that c will not be equal to the expense part, the loading calculated without taking reinsurance into consideration will have to be corrected in a way, that the expense part, which is calculated on the basis of the new loading and which remains at the insurance company, should be equal to the expense part calculated with the former loading:

$$\lambda \cdot NP = r \cdot \lambda^{vb} \cdot NP + c \cdot (1-r) \cdot \left(1+\lambda^{vb}\right) \cdot NP$$

(16.9.)

Simplified:

$$\lambda = r \cdot \lambda^{vb} + c \cdot (1-r) \cdot \left(1+\lambda^{vb}\right)$$

(16.10.)

Expressing :

$$\lambda^{vb} = \frac{\lambda - c \cdot (1-r)}{r + c \cdot (1-r)}$$

(16.11.)

where

λ: is the loading calculated without taking reinsurance into consideration

λ^{vb}: is the loading calculated with regards to reinsurance

NP: is the net premium.

In order to check the calculation, if the commission is completely equal to the loading, i.e. $c=\lambda/(1+\lambda)$, then:

$$\lambda^{vb} = \frac{\lambda - \dfrac{\lambda}{1+\lambda} \cdot (1-r)}{r + \dfrac{\lambda}{1+\lambda} \cdot (1-r)} = \frac{\lambda \cdot (1+\lambda) - \lambda \cdot (1-r)}{r \cdot (1+\lambda) + \lambda \cdot (1-r)} = \frac{\lambda + \lambda^2 - \lambda + \lambda \cdot r}{r + r \cdot \lambda + \lambda - \lambda \cdot r} = \lambda$$

(16.12.)

If there is no reinsurance, i.e. if r=1, then the result is also:

$$\lambda^{vb} = \frac{\lambda - c \cdot (1-1)}{1 + c \cdot (1-1)} = \lambda$$

(16.13.)

IV. QUESTIONS REGARDING THE LIFE INSURANCE INDUSTRY

17. SOME PROBLEMS OF THE LIFE INSURANCE INDUSTRY

KEY WORDS

Insurance application	Administrator
Broker	Mortality profit
Investment profit	Sources of profit
Medical statement	Medical examination
Sales channels	Hidden profit sources
Network	Temporary decline
Commission system	Renewal commission
Commission regulation	Acquisition commission
Calculated profit	Solvency
Claims handling	Area director
Classical branch offices	Product development
Underwriting	Recruiting
Levelizing	Sum at risk
Expense profit	Surrender/lapse profit
Policy	Agent
Policy issue	Director of sales
Policy administration	Waiting period

In the following we try to deal with some of the problems regarding life insurance, though our list might be incomplete, we decided to create a kind of logical order. The reasons are mainly practical, as during his own business the author was mostly faced with these problems and has good reason to suppose that others are or will be confronted with them as well.

17.1. Some Problems of Founding a Life Insurance Company

It was mentioned previously that a typical life insurance is different from the other insurance types in two aspects:

1. Claims can be forecast with high accuracy,
2. The insurance contracts are usually signed for decades, and the premium paid by the policyholders creates the funds for claims and expenses gradually.

These special features are reflected in the profitability of the newly launched insurance companies. It is natural that every company, so as every insurance company, whether its main products are life or other kinds of insurance, show a deficit in the first years of its existence. The main reason for that is that the expenses of founding such a company (buildings, rental fares, salaries, devices, such as personal computers and software) has not been compensated yet by sufficient premium income. However, compared to property insurance companies, the life insurance companies have a specific initial source of loss. This specific source of loss can be connected to the problem mentioned at the topic of zillmerization, namely that (in the case of insurance with typical, i.e. regular premium payment) the expenses of the insurance companies resulting from the life insurance (commission, medical examination, policy administration) incur at the beginning of the term, while the cover of these expenses from premium loading are arriving in a relatively slow rate. One possible solution to this problem is zillmerization, which means that the insurance company borrows money from the premium reserve of the client. This sum can be that part of the risk premium in the first year or years, which is not essential for paying up the possible death in that year. If the insurance company determines the commission level to such a degree, that the sum borrowed by zillmerization meets the expenses of signing the insurance policy, this problem ceases to exist, which means that the newly founded life insurance company cannot be differentiated from the newly founded property insurance companies in the terms of initial losses. However, if the fights for the best agents in the business force the companies to decide on a higher commission rate, then this results in the above mentioned additional loss factor.

In these cases the insurance companies appropriate larger sums for the signing of an insurance policy than the premium income of the given policy in the first year. This means that the better the launching of the insurance company is and the faster it gets new insurance policies, the higher losses it has in the first few years, or until the new policies outnumber the old ones that managed to recover their initial losses. This period producing losses can last up to 5-10 years. This is usually longer than the loss-producing period of the newly founded property insurance companies, as in the case of (usually one year long) property insurance policies the initial loss factor caused by the commissions is missing. That is why life insurance companies are usually founded by firms with high capital investments that have the time to wait out this period of 5-10 years.

The life insurers' course of business is safer than the property and casualty insurers', due to the predictable feature of the claims. On the other hand, by means of the premium reserves bound by the policies for several years, the life insurance companies obtain sources that can be invested in long terms. In the case of property insurance, these incomes are missing or insignificant.

It is natural that even the experienced and calm owners would like to shorten the initial loss-producing period. That is why the pressure set by owners is particularly high to increase the premium loading in the case of newly founded life insurance companies. The effective barrier of this can be the competition, if its impact is not neutralized.

On the Hungarian market there was a big insurers' greenfield operation boom in the early 1990s. That time insurers struggled with similar problems, because the majority of them were newly established. One of the consequences of this, that the cost component of the life insurances were characteristically high and this was not really mitigated by the competitive pressure from older insurers.

17.2. Some Problems Arising in the Course of Company Operation

17.2.1. Product Development, New Policies

Product development is a strategically important area of every life insurance company, as it is the process of product development that determines the range of products defining the face of the company.

Most of their products can be traced back to the basic types of elemental life insurances mentioned earlier, however some details of the particular conditions (the possible entry age of the insured person, the possible duration of the policy, the age and sum limits of the medical examination, exclusions etc.) are usually different, characterizing a given company. The task of the product development process is to "monitor well" the needs of potential customers and to create a the "mix" that satisfy them perfectly from the available raw material.

During the process of development it is not only the marketing, but also the insurance technical aspect that must be considered. Let's see an example: if the insurance regulations provide special tax reductions in case of term insurance policies but not in the case of endowment ones[139], then the insurance companies might be incited to sell their endowment insurance policies not as an endowment one, but divided into independent term and pure endowment insurance. However, with this solution the range of possible premium increasing and profit sharing methods will be technically narrowed down. It is only the technique of revalorization that goes well with this solution, as this is the only technique where the insurance premium of both the term and the pure endowment insurance increases in the same degree and the initial proportion between the two sum assureds – that can be e.g. one to one – can last for the whole insurance term. If the company chooses a system for increasing the insurance premium that – let us say – depends on the inflation rate, then the increase of the sum insured in case of term insurance and pure endowment insurance will be different already in the first year, as in this system it is only the amount of premium increase that is given, but the increase of the sum assured depends on the proportion of premiums in accordance with the tariffs, and in the case of the above mentioned two different policies it can be the same only by chance.

There are certain cases when some information is missing during the process of product development and this has to be surmounted by technical solutions. For the most of Hungarian insurance companies such missing information can be the product type selection (mortality) table that shows the rate of mortality among those who choose different policies (such as term life insurance or life annuity insurance), since the auto-selection works in the way that among people choosing term life insurance the rate of mortality is higher from the average, while among people choosing annuity insurance it is lower. In the case of term life insurance the insurance company has the right to apply

[139] In Hungary – for example – tax benefits concerning to life insurances have been always changing after 1990. Sometimes all kind of life insurances were preferential, sometimes only a few. However, a fix element of the tax regime, that the insurance benefit (all kind of, not only life insurance) is tax free. Internationally about pension funds it is widespread the classification of EET etc, where E = exempt (from tax), and T=taxed. The first position marks the inpayment (premium), the second the yield, and the third the benefit. This can also be used for life insurances. Justifiable, because equitable regimes can be considered only EET and TTE. The EEE obviously rearranges the taxpayers money to the savers, however the TTT means undue taxing. The moderate versions of TTT are ETT and TTE. In Hungary for life insurances mainly the TEE is in effect, but for some products the EEE. In some periods EEE was the common. Mention must be made life insurance in Hungary is traditionally exempt from VAT, which is questionable and sometimes the need for change arises. However, it is problem, that not really clear what should be the basis of this tax, since the whole premium obviously not a good one. Internationally this problem is generally solved by separate insurance tax which was introduced in Hungary at the beginning of 2010s for some non-life branches. The theoretically right solution – without known international example – would be the definition and publication of the price of insurance (see Banyár-Vékás [2016]), and this would be the basis of VAT. The price is practically the the cost component in the insurance premium.

premium correction factors during the process of underwriting, which means that the selection table is not an urgent need. However, in the case of annuity policies generally there is no underwriting, (although this practice internationally is just changing), so there is no chance for correction, i.e. the effect of auto-selection must be calculated into the insurance premium beforehand. Without the selection table insurers usually use the method of age decrease, which means that during the calculation they consider every insured to be five years younger than their real age, which results in a higher premium.

The method of age decrease makes the calculations more simple by means of saving special mortality tables and it is even good from the aspect of marketing. For example, when price differentiation was possible between men and women, some insurers did not use a special mortality table compiled for women to calculate their insurance premium, but used the men's mortality table but such a way, that it considered the female customers to be five years younger than their real age.[140]

17.2.2. The Safety of the Insurance Company

If we want to formulate generally what the product sold by an insurance company is, we could say it is safety itself. That is why it is extremely important for the company to be stable, creating safety for itself as well. (Naturally need to know, that the 100% extent is not possible, and neither the regulator seeks to reach it. The aim of the EU's regulation to reach the 99,5% safety level.) This effort for safety can be traced back in all momentums of the functioning of an insurance company. Through the examination of the "life-cycle" of a concrete life insurance policy, let's have a look at the safety solutions that the insurer uses.

Everything starts with product development and the selection of the proper product. The product sold later must fit into the risk managing system of the insurance company (and vice versa: this system must be worked out in accordance with the product). That is why it is important to know the specialities of the risks embedded in the product itself. With a short bypass let's have a look at the traditional life insurance types that are important from this point of view. Where the risk of the company is the death of the insured person, the most considerable ones are the following: the term, the endowment and the term fix life insurance. If we look at the risks of the company, the order increasing by risk is term fix, endowment and term insurance. It is obvious that the term insurance carries larger risk than the endowment insurance, since in case of endowment insurance the pure endowment part decreases the effect of the risk part. The reason for the term fix to carry smaller risk than the term one is that the insurance company pays in both cases, but in case of the term fix benefit payment is always at the end of the insurance term, while

140 Since they usually live longer.

the endowment insurance pays in case of the earlier death of the insured before the end of the term. The lower risk is indicated by the lower fluctuation of claims. If the insurance company does not want to deal with underwriting too much, then the best choice is to sell term fix insurance, as the Austrian companies entering the Hungarian insurance market and selling life insurances illegally have done in the early 1990s.

After the insurance application, but before signing the policy the insurance company carries out a very precise underwriting procedure (discussed in more detail under the chapter 17.5.3) in order to filter out the adverse-selection.

If the risk is death type (and not pure endowment type as in the case of annuity insurance) and the sum assured is more than the risk bearing capacity of the insurer, then the insurer reinsures the policy.

The risk bearing capacity is connected to the variance of claims, which is connected to the level and distribution of the sum assured. The maximum level of variance can be such that the insurance company can pay incurred claims with high probability – using its capital resources and reserves.[141]

Reinsurers usually sign short term (mainly one year) contracts only for the death sum assured (to be more precise, for that part of the sum assured that is above the premium reserve – the so called "sum at risk") with the life insurance companies, which is why they do not create premium reserves of the undertaken risks. The net premium of the reinsurance is usually calculated with the help of a mutually accepted mortality probability sequence. After the issuance and the possible reinsurance of the policy, the insurance company starts to collect the insurance premiums and build up the premium reserve. To make sure that the premium reserve can cover the costs of benefit payments, some insurance companies use the method of conservative premium reserving, which means that in the premium reserve calculation – leaving everything else the same – the company considers the insured person to be one year older than his real age, thus resulting in a higher premium reserve all together (because this method doesn't have the same effect on every single policy) than without the age increase. As we have mentioned before, this method is called "conservative" premium reserving.[142]

The created premium reserve must be invested by the insurance company and in the case of traditional life insurance policies it has to reach the guaranteed yield in accordance with the technical interest rate. In this investment, safety is essential for

[141] Which are controlled by the minimum requirements, the so-called solvency regulations of the European Union.

[142] According to the new insurance regulation in the EU effective from 1 January 2016, the so called Solvency II, such hidden safety margines are theoretically forbidden, the reserve has to testify exactly the risk at that moment when it is valued on the basis of our actual whole knowledge. All the other (unexpected) risks above this is covered by the solvency capital.

insurer. This emerges differently at the traditional life insurances, where the yield is (partly) guaranteed and the modern ones, where not. In case of the reserve of the traditional life insurances the basic principle is that if the company has to choose between the secure but low yield and the insecure but high yield investment form, then they prefere the first option. That is why the clients of an insurance company do not expect especially high, but rather safe yields from their insurers. The safety of the investment is secured by the method of diversification, i.e. the insurance companies themselves use the strategy of self-insurance. Above this it is expedient to match the expiries of investments and the insurance contracts to each other (asset-liability matching = ALM). The large part of the modern insurances are provided any yield guarantee by the insurer, here the investment risk is the client's risk. It can also be interpreted, that the insurer solved the problem of safety it sought to achieve in the field of investment of the reserves by introducing the modern insurances.

And finally before benefit payment is due (in case of death, maturity, paying the annuity) the insurance companies investigate the justness of the claim carefully to avoid insurance frauds.

17.3. The Sale of Life Insurance, Sales Channels

The most important segment of a life insurance company from the point of profit (if there is a "most" important segment at all) is the sales network. As the life insurance policies are typically long term contracts, it is extremely important to make sure that the clients choose this type of insurance not because of a sudden decision but after a thorough consideration, which means that they consider the premium payment to be advantageous after several years as well. Since ordinary people in Hungary do not possess even a minimal level knowledge of life insurance and insurance at all, it can be presumed that they will receive the greater part of their knowledge on the subject from the insurance intermediary[143]. That is why it is especially important to have well educated intermediaries, whose interest is to offer real and not false information to the clients. This is the main reason why "passing" agents (who are not professionals, only try to sell a few occasional policies) are not suitable for the sales of life insurance. The best thing for an insurance company is to work with the same, stable educated team of agents, who would not sacrifice their "professional good name" for the sake of some short term advantages (insurance policies based on manipulated data, that are this way

[143] From the agen who represent the insurer and from the broker, who – theoretically – represents the client. In case of economically strong corporations, brokers probably really represent the client, but this quite questionable in the case of majority of the life insurances.

unstable). Unfortunately, in Hungary it happens quite often that the agents are bunglers not doing a satisfying job. (The scale of bunglers is very extensive. It starts with some basically honest agents whose only reason for selling a life insurance policy is "why not pay life insurance premiums instead of paying for 20 packets of cigarettes each month" and it ends with the type – acting in bad faith – who offer a very high and long term yield (exceeding the interest of the best bank deposit) after the premium reserve (acting as if the likvidity of the two would be the same).

Bunglers can cause great damage not only to the clients, but also to the insurance company whose policies they sell. It is not just about the dissatisfied clients who can deter everyone in their environment from the given company and even from signing a life insurance policy, but about the possibility that the level of undertaken risk is not clear to the insurer due to false or unsatisfactory data provided by the bungler agents, and so the insurance becomes effective with a premium lower than necessary.

There are several sales channels for selling life insurance policies. The traditional way is the sales through insurance intermediaries (agents or brokers[144]).

Apart from some exceptions the sales through **brokers** is not so widespread in Hungary, however, the multiple agent enterprises are quite widespread in life insurance. The main reason for this is that the brokers are usually specialised for the sales of the valuable property insurance policies and not for the sales of life insurance, that can be considered as a mass insurance type from this point of view. However, there are certain signs that in building their own network, insurance companies are using elements characteristic of broker firms (without transforming their own network into a broker firm). Some of the insurance companies start to build up networks with units owned by the manager or managers, that are in contractual relation with the insurance company and are selling exclusively their policies (so they are not brokers), and these managers employ the agents, who are usually enterpreneurs. Regarding the settlement of accounts, the process is the same as in the case of broker companies. Multiple agent firms, which organizationally independent from insurers, but which represent insurers towards clients are quite widespread, however their aggregate market share is fluctuant.

In Hungary life insurance companies sell their insurance policies primarily the traditional way, usually through their own or multiply agent network. Some specificity of the functioning of the insurer's own agent network we will discuss it later under subchapter 17.4.

The modern life insurance sales channels can be alternatives of the insurance company's own agent network:

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[144] In Hungary around the term „broker" there is some uncertainty. By definition this is the independent insurance intermediator who represents the client. But sometimes it is used as an insurance intermediator who works as enterprise, even when this enterprise is the angent of an insurer. This is a lax, but widespread wording.

- branch offices of banks and the Post (“bank insurance”)
- direct mail,
- call-centres,
- the Internet
- others.

Sales through the **branch offices of banks and the Post** is quite widespread in Hungary, and some insurers are owned by banks[145], and for some of them the only sales channel is the bank (or Post) branch office. The success of bank insurance depends strongly on the traditions of the given country. In some of the Western countries (e.g. Portugal, Spain and Belgium) this is the most important sales channel, but in other countries it has only marginal role – Hungary is in between the two. However, it is important to mention that it is only the very simple life insurance products that is suitable for sale in bank offices, which means that the structure of products offered by banks is very different from that offered by agent networks.

The method of **Direct Mail** (when the companies send letters to clients offering their life insurance products) is not present on the Hungarian market today. Although there were several attempts to introduce this method, it is not probable that it will be successful in Hungary.

The **Call-Centres** are in the same position as the direct mail method, but there were more attempts made in this area.

The Call-Centre as a sales channel means selling through phone, and it has two different types: the active and passive type.

The **Active Call-Centre** is when the insurance company operates with outgoing calls, i.e. it is the call-centre that turns to the potential clients offering them the services of the insurance company. In some countries (e.g. in Great Britain) these active call-centres are really successful in the sales of certain community insurance types (mainly in the case of house- and third-party insurance policies), but (until now) failed almost everywhere in the case of life insurance[146].

The **Passive Call-Centres** are much more common, and almost universal among insurers. The main point of this call-centre is that the insurance company advertises in the media the number of the call-centre, which is usually free and can be reached night or day, and the products that are sold by calling this number. There are many passive call-centres in Hungary, but the records show that the clients usually dial these numbers

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[145] However sometimes the owners are changing, also the banks are buying and selling insurance companies. The opposite owner structure (insurance company owns bank) is uncharasteristic, however there were some attempts (which proved unsuccessful).

[146] There were two unsuccessful and terminated attempts to introduce the active call-centres for the sales of life insurance in Hungary at the end of the 1990s.

not with the intention of purchasing but to make complaints and report claims. That is why their importance is chiefly not in sales, but in follow-up of the contracts.

It is possible to buy life insurances through **Internet,** but in practice it serves much more the information of the clients, and the clients with buying intention are directed to an agent of the insurer. It is done partly because of the so-called „chanel-conflict”. Namely the agent network resents, if the insurer try to by-pass them during the sales process.

From time to time insurance companies launch **other** sales channels as well. Some examples: they have tried to sell life insurance policies in post offices and through trade union members.

Lets have a detailed look at the historically most important (and also at present one of the most strong) sales channel: the insurer's own agent network.

17.4. Sales Through the Insurer's Own Agent Network

17.4.1. Network Organisation and Management

The agent network can be independent or be the part of a "general" network (i.e. the regional representative system of an insurer). In accordance with this there are two major types of networks in Hungary:

- classical branch offices, and
- networks dealing with sales only.

It is usually the non-life or composite insurance companies that have classical branch offices, (small regional offices to where the client can simply go into) because it is the property insurance where it is important to hold policy administration and claims handling in a branch close to the insured property. The main point of the classical branch office network is that the branch not only deals with sales, but also with risk underwriting, policy administration and claims handling. The sales itself is only one function of the network unit, and the leader of the sales team is not the same person as the manager of the branch but one of the substitutes.

In the case of specialised life insurance companies it is not reasonable to have a classical branch office network, as the customer service and claims handling can be carried out centrally as well. Here it is enough if the network deals with sales only. That is why some of the composite insurance companies have tried to establish an independent network which is different from the networks dealing with property insurance and deals only with life insurance sales.

All of the networks (apart from the small insurance companies) have different levels, i.e. hierarchically divided. The leaders and the staff of the different levels of sales networks have different names. Some of the companies split the country to regions and the leader of these branches are called regional directors. This is common in the case of larger insurance companies. In most insurance companies the biggest unit of the network is usually an “area” (e.g. a county) defined in different ways at different companies, with a leader called “area director” at the top. In the regional system there are several ’territorial “area directors” under the leadership of a regional director. The “area director” directs more than one team of agents. At the top of each team of agents (one team usually consist of 5-20 agents) is the so-called “director of sales” (in some places the “director of the branch” is also used, but this name can be misleading as it refers to the classical branch network). The official name of the sales person under the leadership of the director of sales is the agent, though some people use it unwillingly. They are usually called mediators, representatives or advisor.

The agents get commission for their work done. This is the most important factor for them, so the insurer can encourage agents for better achievement by means of the commission system. The possible types and scale of commission is laid down by the commission regulation. It is basically the commission regulation that transfers the will of the centre to the network. It can be said with a slight exaggeration that the only communication channel between the centre and the network is the commission regulation. Because of its importance it is discussed in a separate subchapter.

17.4.2. Commission System, Commission Regulation

Regarding to the commission rules – from the point of view of calculability i.e. actuary’s – there are more than one requirements. (However, fulfillment of these made satisfied not only actuaries.) The features of a satisfactory commission rules are the following:

It is **comprehensive** that is, it covers all aspects. Comprehensiveness means completeness from more than one point of view.

On the one hand the commission regulation must include all of the expenses incurred in connection with sales. These expenses (as long as such expenses exist) can be the money given to the directors and the agents to cover their costs of clothing, the costs of company cars, the fix salary of the directors of sales. These expenses only incur in connection with sales though they are not considered to be part of the commission system. From the point of view of expenses comprehensiveness is essential so that the commission regulation can provide a satisfactory basis for profitability calculations. In other words it means that the commission regulation must consider the sales network to be an independent profit centre, so it must include all of the expenses and there should be no costs in connection with sales that are not part of the commission regulation.

On the other hand the commission regulation must be complete from the point of view of agent hierarchy as well, since it has to include all of the different posts that are in any relation to sales. E.g. instructors supervising the network, brokers, possibly the officers of the bank who sell bank insurance, etc. must have their place in this hierarchy.

Thirdly the commission regulation must include the detailed description of all instruments used during the work of the agents such as the method of planning, supervising and examining the work-plan of agents etc.

Fourthly it must include every encouraging instruments that is different from the commission such as the system of the prizes of different competitions, the system of "encouraging" travels, career opportunities for the leaders (and leaders-to-be), that means not only advances in the hierarchy, but gaining some of the status symbols connected to the increase in the number of policies sold (e.g. company mobile and car, etc. regulating exactly the connection between the benefits and the acquired portfolio).

Fifthly it also must include the benefits of that team leader who establishes a new team of agents on his own that can become independent in the future. These benefits are essential in making the team leaders interested in encouraging their best agents to be independent and widen the network of sales.

The sixth feature of the commission regulation is that the regulation must exactly regulate all commission types that are due according to the retaining level of the portfolio of an agent or their leaders, so that keeping the portfolio becomes an automatic task instead of a question of occasional actions.

The good commission regulation is **stable** in two ways: on the one hand the system itself has to be unchanged for a long time, on the other hand the commission rates have to be constant as well. Every effort should be made to have less and less people possess the power to change the rates, as rarely as possible. (The best solution would be if only one person, the director in charge of the network could change the commission rates). The reasons for the change must be controlled precisely.

The commission regulation must be **multi-level,** which means that the salary of all the agent leaders of all levels must derive directly from the performance of the agents in their team exclusively (or at least primarily). The fix salary of the director of sales expedient to set to minimum and should be paid for a temporary period, which should be shorter and shorter with the development of the company!

It must be kept in sight that for an insurance company it is better to have no director of sales at all in one region than to have one with a very high fix salary and doing nothing for it.

Maximally proportional to performance: that partly means the multi-level feature in the above sense, and partly that the system must not contain any commission title (e.g. interventional and procedural commission) that doesn't increase strictly proportional to the volume of business, but exactly contrary to it. (The interventional commission[147] is highest when the paying morality of the clients is worst, i.e. when

[147] The point of it, that the agent is sent to the non-paying client to collect the insurance premium. This act is called as "intervention".

the insurance company has financial difficulties. Moreover these cannot be calculated beforehand, and their use is not in line with the requirement of predictability, without which profitability cannot be measured well.)

Of course, the interventional and procedural commission gave answers to real problems, so if they are eliminated, new solutions must be found taking over their functions. The new commission used is the internationally well known **renewal commission** continuous (possibly „maintenance", „portfolio management"), or – with an old Italien origin Hungarian insurance term – **sperative commission**. The commission proportional with the performance is variable cost , i.e. it can be paid from the current premiums, so it is easy to plan it.

Making the renewal commission general means at the same time the spreading of **service agents**. This means that all policies have a service agent at every moment whose task is to do every "agent type" job in connection with the policy (e.g. intervention, handing out letters of premium increase, providing information, etc.). (The system of service agents is a question of company philosophy. Its application means that company and the agent do not only deal with the client until the policy is signed, but the agent serves as a kind of personal financial advisor.)

It is the service agent who receives the renewal commission. At first the service agent is the agent who managed to sign the policy with the client, but if he leaves the company or the region then another agent receives the policy. This means that the system of the renewal commission is general, because in case of a policy the title to renewal commission never ceases to exist.

A performance proportional commission system means that the commission is always proportional to the premium income. If the acquisition commission is determined as a percentage of the first year's premium then the agent will receive it immediately when the client pays the annual premium, but will receive it in 12 instalments if the client chooses to pay the insurance premium monthly. This is called "earned as paid" commission in insurance terminology. It means that the insurance company can avoid the problem of commission chargeback.

The agents are encouraged to convince the clients to choose the one year payment which is the most secure method of payment from the company's point of view. It can happen that the acquisition commission does not depend on the frequency of payment but in these cases the rules of commission chargeback must be laid down precisely. The renewal commission is always due from the **beginning** of the second year in the same frequency as the client pays the insurance premium so it is similar to the acquisition commission from this aspect. The renewal commission encourages the agent to intervention automatically as without the premiums paid by the client the agent won't receive the commission. Of course the intervention must be backed up with an interventional list by the informatics and the leader of the agents has to supervise its execution. The authority of the leader of the agents has to be defined properly for taking away the service agent function from those agents who do not intervene satisfactorily.

The commissions are expedient to be "**levellized**" as much as possible. The **levellizing** are similar to "**droping**", but their meaning is different: the former one refers to the ratio of the initial (immediately payable) acquisition and the continuous sperative in the total commission. The more higher the proportion of the latter, the more levellized the commissions. For the insurer the good commission system is as levellized as possible at least from financial and incentive point of view, but for the agents, the situation is the opposite. The final ratio is a matter of the balance of the power. The two extreme cases are the only acquisition commission or only sperative commission. The first is the „heaped", the second is the „levellized" commission system.

Controlled in detail: nothing can be left to the invention of the person using the regulation. It would carry the risk of creating several different "mutations", or uses of the regulation in opposite to the original intention due to "legal gaps".

The computer record of agents, insurance policies and commissions must be controlled as well. It is a basic requirement to keep a record of the total commission history per policy and per agent. The system must allow service agents to move between policies and teams.

Flexible: it is important for the company to adapt itself to the changing circumstances without changing the structure and principles. Flexibility is a kind of principle that must be considered all the time during the development of the concrete commission system. Flexibility can be reached by some special solutions such as leaving blank positions in the hierarchy in case of introducing new levels (as yet of unknown purpose) later on.

For the sake of flexibility the insurance company may split the different sales channels without integrating them into **one** hierarchy, and plans more than one channels strictly divided and operating parallel to each other. One example of that is when the elite policies are sold separately from the common insurance policies by an independent team, having their own leaders and with a different commission system, etc.

Well-designed: The good commission regulation is well-designed, making it understandable for its users. This can be achieved by commentaries connected to the regulations. These commentaries are not part of regulation, but give useful help and hints to the user.

17.4.3. Recruitment

Organising the agent network is a continuous task due to the natural fluctuation and the development of the company. The official term for winning new agents over to the company is **recruitment**.

Recruiting is usually the duty of the unit leaders as they are the ones responsible for the satisfactory sales performance that cannot be reached without a team of adequate size and quality. During recruitment it is important for unit leaders to know what kind of agents they are looking for.

From all of the insurance types selling life insurance is the most difficult task. That is why it is fortunate – though not a basic requirement – if the agents selling life insurance policies have a college/university degree.

As the life insurance is a long-term financial commitment towards the clients, it is essential to have clients who trust the company. It is the agent who is the representative of the insurance company, so the agent has to be able to win the clients' confidence.

Life insurance is an instrument for making our lives predictable, therefore those people become clients who have a need (or this need has been aroused in them) for a consolidated life. The agent has to radiate this consolidation as well, this is why elderly or middle aged agents are more suitable for selling life insurance policies than younger ones. This is also related to the fact that the typical client is usually middle-aged. The agent who represents the insurer naturally has to be such to whom not only the client but also the insurer can let the management of a part of its finances, e.g. collecting the first premium without fearing that the agent will spend it to his/her own use (which happens time to time). Nowadays, the abstraction of the client's premium by the agent is tried to be avoided by the insurer not allowing the agent to collect money from the client.

The training of new agents is a very important part of the recruitment process. The agents must be educated and trained well enough to be able to give thorough information. Although training is most emphasized at recruitment and directly afterwards, it really is a continuous requirement as the knowledge of agents must always be adapted to the changing circumstances.

It is important that recruiting should aim at new agents instead of winning over well trained agents from other rivals. On the one hand it is a question of ethics, on the other hand those agents who are won over easily can also leave the new company any time.

17.5. Technical Duties Regarding the Signing and the Administration of a Life Insurance Policy

It is useful to go through the history of a life insurance from its inception to its termination to know the different duties that might have to be done in relation to it.

17.5.1. The Insurance Application

The insurance intermediary fills out the insurance application (or proposal) form of the policyholder and the insured person and collects the insurance premium of the first insurance period[148]. The money is transferred in 1-4 days – according to the business regulations – to the unit handling the contract (the centre or a given branch office). Simultaneously with the signature of the application, the medical statement form is also filled out about the health conditions of the insured person and/or (depending on the age of the insured, the type of insurance and the sum insured) makes an appointment with the M.D. of the insurance company who examines the insured person thoroughly.

The word **insurance application** has two meanings here. In spite of the fact that the active party is usually the insurance company (the agent representing the company), when a life insurance policy is signed it is the client (the policyholder) who takes the initiative formally by **proposing** the company to sign a life insurance contract. The contract comes into existence when the insurance company accepts the application of the insured person. It is a practice to use the term **insurance application** to the form which is filled out by the policyholder when proposing to the insurance company.

The **medical statement**, is an official form as well, where there are questions asked by the insurance company regarding the medical condition of the insured person. The medical statement must be filled by the insured person as part of the **reporting liability** of the insured and consciously incorrect statements might involve punitive sanctions and finally the refusal of the payment of the sum insured.

The (first) insurance period that the premium collected by the agent from the policyholder when signing the application depends on the frequency of the premium payment. In the case of a single premium insurance the insurance period is the whole insurance term, in case of monthly premium payment the insurance period is one month, in case of quarterly payment the period is 3 months, which means that the insurance period is the term covered by the premium.

For the sake of informing the client, the application is usually filled outin two copies, and the agent leaves one of copy by the policyholder. In our days more and morewidespread, that the forms are filled out a priori electronically and the intermediary prints a copy for the client to sign. Because of the same reasons the agent also hands over the conditions of the given insurance. These conditions have usually two levels: the **General Life Insurance Regulation** (or the General Conditions of Life Insurance) contains the general rules of life insurance policies of the given company including the most important life insurance regulations of the civil code. The **Policy Conditions** contain the conditions of the given life insurance product which might be different from the regulations laid down by the General Life Insurance Regulation. If the insurance

[148] Although today it is forbidden by many insurers. Instead this the clien pays this premium by postal cheque or electronic way, so the intermediary not handles cash from the client. The reason of this prohibition were the misuses in this field, when the intermediary used for his/her own the (often very high sum of) premium from the clients.

contract is accompanied by different insurance riders then these might each be regulated by different special conditions.

17.5.2. Policy Administration

The first task in connection with the insurance application is **policy administration,** that is a formal and technical check of the data supplied by the client and the completeness of the application. The person doing the policy administration is called the **administrator**. Policy administration – at least a huge part of it – nowadays is more and more automated, in case of electronic data collection it happens practically immediately

The objects of the supervision are:

- First of all whether all of the required columns are filled out by the client. If something important is missing then the administrator gives the insurance application back to the agent who has to visit the client (or clients) again.
- The correctness of calculations, i.e. whether the agent calculated correctly the following:
    - » the **age of the insured person** at the time of the signature of the contract. The Hungarian practice (which is not exclusive internationally) is that the age of the insured person is calculated by subtracting the client's birth year from the effective year of the policy (i.e. not the whole available information is used – otherwise: it is supposed as if everybody had been born on 1 of January).
    - » the insurance premium paid after the **initial sum insured**. The initial sum insured is the sum laid down in the application and which serves as the base to the insurer for calculating the initial premium. The initial sum insured can be different from the current sum insured – due to inflation handling techniques, premium increase and investment profit sharing.
    - » **the authenticity of the signatures**. (This is important for avoiding the possibility of agents signing the application instead of the clients to avoid the difficulties of handling incomplete information.)

If the application and the medical statement is complete and correct officially, than the policy is ready for underwriting.

17.5.3. Underwriting, policy issuance

To avoid adverse selection, the insurer must use the method of **underwriting** before almost all policy issues. What are the risks of the insurer in the case of the different types of products?

In the case of **Term Insurance** the risk is that the client will pass away "early" (before the maturity of the insurance). From the point of the insurance company, those clients whose expected life span is shorter than average signify bad risk. Underwriting is to avoid and manage the bad risks of a company.

In the case of pure endowment and annuity insurance policies the risk of the company is that the client will be alive at the time of maturity or that the client will live longer than average. It can be said that at this form of insurance it is the client with a bad health status who signifies a good risk. It would be logical to adopt an underwriting method similar to those used in the case of term insurance (medical examination for example) and if it turns out that the client is in perfect health condition the company would take the risk with increased insurance premium. However, there is no underwriting in the case of pure endowment insurance at all. One of the reasons for that is that this kind of insurance is usually not sold separately (because it is difficult to have it accepted by the client that in the event of death all the results of saving will be granted to the insurance company, or the risk community). The other reason is that from the client's part – as the probability of surviving is more likely than the probability of death – survival carries less chance of manipulation in itself, so the possible profit for the client is much smaller, which does not cause adverse selection on a big scale.

In case of annuity policies the insurance companies compensate the auto-selection occurring necessarily by the method of age decrease built in the policy, or by using some kind of projected mortality table. The insurer supposes that only those will sign an annuity policy whose chances of survival are the best, this way insurance premiums are higher, but there is no underwriting. This is why it is not worth signing an annuity insurance for those clients whose chances of survival are worse, which means that the insurance companies leave a significant potential segment of the market uncovered. This is why there is spreading a kind of reversed underwriting in case of annuity insurance policies, which means that if someone is ill then he/she can get reduction from the insurance premium.[149] In those countries where this is already used the underwriting is quite light, a simple self-asssessment, because the good faith of the clients is assumed. Naturally it is a question, whether in the longer term, if the life annuity penetration is increased and the clients are learnt the logic of this, they remain in good faith or not. It is questionable whether in countries such Hungary, where the good faint is less widespread is it possible to introduce this kind of simple practice or not.

Underwriting can be avoided by certain methods in certain cases. These methods are usually used in the case of cheaper mass types of insurance where the expenses do not contain the cover of underwriting or where the nature of the sales channel is not suitable for it (e.g. in banks or in case of commercial credit life insurance). The most common of these methods are:

[149] The name of these are impaired or enhanced annuity. More and more widespread the “bulk annuities” have purchased not individually, but in “group” by pension funds. These also have „medically underwritten” versions.

- group selling – not for selected groups
- applying exclusions of wide range
- gradual increase of the of term sum assured (i.e. the total sum will be valid in 3-5 years, by the time the effect of underwriting would be over. Until then the death benefit increases gradually starting from a low amount.)
- declaring a waiting period (this can be currently – since already decades, declared in the Civil Code, it is – maximum 6 months in Hungary)
- guaranteed issued product. In this case the insurer guarantees that in certain limits it accepts in every account the proposal for death risk, but in the first – let us say – two years the deatht benefit is paid only in case of accidental death.

Sometimes it is compulsory to have a **waiting period** in case of underwriting as well. When there is a waiting period and the insured event happens during this period, the insurer (apart from cases such as accidents) pays not the sum insured, but only the premium reserve to the beneficiary. If there is a medical examination before signing the insurance contract the insurance companies usually disregard the waiting period.

Do not mingle waiting period with cooling off period. This latter is provided in many countries for many products and services, including insurance. The point of this, that after concluding the contract, in some period (generally in 30 days) the client can quit from the contranct without any penalty. In this case the precontract situation is restored.

The underwriting is the process of examining whether the application carries a normal scale risk or it should apply higher premiums because of the higher health or occupational risk factors (sport, hobby). There are usually three categories of the risks:

1. Health (its size or the change of it slightly depends on the client's decision although it is influancable by the client actively)
2. Occupation, sport, hobby (its length strongly depends on the client's decision)
3. Financial
4. In accordance with this there are three types of underwriting:
5. Medical
6. Non-medical
7. Financial

The documents of the medical underwriting could be the medical statement and, in certain cases, the statement from the M.D. (that can have different degrees).

The inputs of the non-medical underwriting are usually the appropriate data of the application form and filling out a special questionnaire in some cases.

The financial underwriting is not so common, it is usual only in cases with very high sum insured. The task of financial underwriting is to examine whether the client's purpose to sign the policy is well-established enough on a financial basis. The insurance company must be sure that the client can pay the insurance premium (so that the energy invested in the acquisition of the policy was not wasted) and that there is no purpose of insurance fraud.

In the course of underwriting it is essential to clarify whether the documents supplied (application, medical statement, the records of the medical examination) contain enough information to accept the application.

If some additional information is needed, then the underwriters get it from the agents, by mail or sending the client to a new medical examination.

The results of underwriting can be different:

- in most cases the insurance application is accepted,
- in some cases, when the insured carries a higher risk (has a bad health or because of other reasons, e.g. has a dangerous job or pursue dangerous sports) the insurer can increase the premium or declare exclusions. Of course the premium increase is an alteration of the insurance application, and it has to be accepted by the client.
- in some cases the application can be refused.

One of the possible methods of increasing the premium is to raise the age of the insured (that finally results in premium increase). Nowadays it is more common to increase the premium directly with the starting point of the so called extra-mortality. The rate of premium increase is usually calculated from tables (generally compared by reinsurance companies). The increase can be expressed in the percentage of the premium, in per thousand of the sum insured or as a fix amount.

In addition or instead of premium increase it can happen that some events are excluded from the insurance.

It can happen that the insurance company does not accept the risk and refuses the application. The refusal can be permanent or temporary, when it is called temporary decline. The permanent refusal happens only in the case of extremely high risks such as a very serious fatal illness that was unknown by the client as well. Temporary decline is usual in case of pregnant women.

If the insurance company accepts the application, or the exclusion or premium increase is accepted by the client, the next step is the policy issue.

The **policy** is a document issued by the insurance company containing the fundamental conditions of the contract. By issuing the policy the insurance company accepts the application of the policyholder so the contract comes into force at this time.

The life insurance contract has two parts, the insurance application signed by the client or clients and the policy signed by the insurer.

17.5.4. Indexation, Indexation Letter

There are certain repeated tasks with every insurance during its term. These are usually connected to the **policy anniversary**. The policy anniversary is usually that day of every year when the policy came into force. Some time ago the practice of the Hungarian insurance companies was, that they settles the date of the anniversary to be the first day of the month following the inception of the policy, but this practise is died out. The commencement of the policy adjusted this way was called the "**technical commencement**".

In the case of traditional insurances it is usually the policy anniversary (or a given day of the year) when the investment profit originated from the investment of premium reserve is credited to the client who has the option of **premium increase** at this time (this is true for modern life insurance policies as well). Making use of premium increase options technically means the purchase of new policies (even in case of the revalorisation technique, though it is not so obvious) that have a term equal to the remaining term of the original policy, and the entry age of the insured equal to the current age of the insured person. In case of making use of premium increase options the premium reserve is the sum of the premium reserves of the initial sum insured and the increased sum insured.

Due to investment profit sharing and premium increase the **current sum insured** becomes higher. The current sum insured is the sum insured determined in the policy (the initial sum insured) raised with the sum insured of new policies chosen by the client as an increase option and/or with the sum insured in those single premium insurances that are obtained by the client through investment profit sharing. If the client did not make use of premium increase option or the policy doesn't receive investment profit sharing (or it is administered on a separate account), the current sum insured can be the same as the initial sum insured.

With the increase of the current sum insured the current premium reserve of the traditional insurances will be higher as well.

The premium reserve always consists of the following parts:

- the premium reserve of the main policy,
- the reserves of the investment profit sharing
- and the premium reserves of the new insurance policies purchased by the client through premium increase options.

About these changes (or about that there is no any change) the insurer annually informs the client generally via mail. Indexation is generally automatic, but the client can refuse it. In the mail the insurer also attracts attention of the client for this possibility. The annual mail in case of UL insurances contains a complex account about the bought and sold units and their exchange rate, its changes, the fund switches initiated by the client, etc.

17.5.5. Claims Handling – Making Use of Non-forfeiture Options, Insured Event, Maturity, Benefit Payment

Making use of non-forfeiture options and the occurrence of the insured event results in the insurer's liability of benefit payment. In case of benefit payment the contract usually ceases to exist, but not in all cases. In case of paying up the policy, partial surrender or if in unit linked life insurances money is withdrawn from the funds, the policy is continued. In the case of term fix insurance death is an insured event but the policy is not terminated. In case of some insurance types the maturity (or "expiry – depending on the contract) of the policy is an insured event as well (term fix, endowment insurance) but in other cases it is not (term insurance).

The sum paid is corrected with the sum originated from the differences of the frequency of premium payment at the time of calculating the benefit. If the insurance company supposes annual premium payment in its calculations and conditions but gives the allowance of monthly, quarterly or semi-annual payment, then at the time of making use of non-forfeiture options and when the insured event occurs, the part of the annual premium not paid in yet can be subtracted from the amount of benefit.

In case of supposing monthly payment as a basic principle the case is just the opposite. If the clients chooses not monthly but less frequent payment, a so called **unearned premium** here is formed, that the insurer pays back to the policyholder together when paying the benefits. The unearned premium is the premium for the whole months remaining from the given insurance period, as it has not been "earned" by the insurer yet. [150]

17.6. The Profit of the Life Insurance Company

The business insurance companies work for profit. (Formally it is not true for mutual insurers, but in practice the bigger a mutual insurer the more important also for them the profit.) At the end of discussing the subject of life insurance let's have a look at the different parts of the final goal of the operation of life insurance companies. Basically there are five different types of profit:

- calculated profit,
- expense profit,
- premium reserve (and capital) investment profit,

[150] In the insurance (mainly in non-life) practice this also has an other, although not an opposite meaning. This is deferred charges. When an insurer makes a balance-sheet it consider not he whole premium which came in to the insurer as the income of the actual year, only a proportional part of it. The part of the premium for the next year(s) risks is deferred to the next year. This is the "unearned premium".

- mortality profit, given from the difference between the calculated and real mortality rates
- hidden profit sources.

Of course all of these profit types can be negative as well, which means that the insurance company has loss. Let's have a look at these profit types one after the other:

1. The ***calculated profit*** is planned to be included in the premium loading and in the expenses of modern insurance types as one of the expense elements.

2. The ***expense profit*** is the result of the fact that the insurance company during its operation does not use the total premium loading collected to pay for its expenses.

3. As we have discussed before, the ***investment profit*** is the interest earned by the insurer on the premium reserve and the yield of the asset funds of unit linked life insurances. It is important to know that the profit of the insurance company and the yield of the premium reserve and asset funds are different, which means that the insurer can show a loss and still pay profit share to its clients after the yield of the premium reserves and also the yield of the asset funds can be favourable. Naturally the insurer doesn't have to share the yield of investing its own capital, it increases or decreases its profit and not the clients'.

4. The ***mortality profit*** or loss results from the difference between the calculated and the real mortality rate. If the real mortality is higher than the calculated one that means mortality loss in the case of term insurances but in the case of pure endowment (and annuity) insurances it means mortality profit and vice versa. If an insurance company is not specialised in annuities, then its portfolio is dominated by term insurance types (where the real risk of the insurer is death, as in case of term, endowment, term fix and unit linked life insurances), which means that the real risk is the higher-than-average mortality. If the insurer calculated correctly, there is no general increase in the rate of mortality among the insured population due to economic recession or similar reasons and the underwriting was correct, then the insurer has to realise mortality profit. The reason for this is that from the point of mortality the clients of an insurance company are in a better position than the national average which is the base of the calculation. Since people who sign a life insurance policy and can regularly pay its premium presumably live in good financial conditions and have long-term plans, which means that they care for themselves etc. In the process of underwriting the insurance company makes a selection among the population and this creates profit as well.

The mortality profit is not absolutely due to the insurance company under all conditions. In this regard the practice is different in Hungary. Most of the insurance companies keep the whole mortality profit for themselves, but some of them share it with the client according to the rate of the investment profit sharing. Sharing the mortality (or maybe also the cost-) profit in some countries can also be a legal requirement – mainly at mutual insurers, where this practice is logical, because their owners are the clients.

In the developed countries life expectancy has significantly and continuously increased in the past decades, and this tendency is expected to continue (comp. "longevity), this way in case of annuities the danger of the mortality loss is high, one can say that it is the mortality loss and not the profit that can be regarded as "default". However, there are some mortality tables taking the annuitant auto-selection into account and projecting the increase of life expectancy that can be a useful in fighting against mortality loss.

5. The ***hidden profit sources*** are not so much "in view" as the "open" profit sources discussed so-far, but they are just as important for insurers. We try to introduce – without discussing all types – three different hidden profit sources. These hidden profit sources are mostly typical in case of the classically calculated traditional insurances. The transparent structure of unit linked life insurances and the profit-test techniques used for their calculation make these profit types more open and realised in the above profit source categories.

The first hidden source is the **surrender profit**. On the one hand it comes from the fact that in case of surrender or if premium payment fails than the insurer doesn't pay all the premium reserve back to the client. On the other hand the source of this profit is that the insurance company takes back from the agent the acquisition commission paid in case of surrender before the end of a given period (for example 2 years). It is important to know that this kind of profit is not favourable for the insurance companies, because surrendering the policy means the reduction of the portfolio. On the other hand we have already mentioned that one purpose of holding back part of the premium reserve in case of surrender is to balance the expense-increasing effect of anti-selection. Also there are some cases when commission chargeback from the agent is not possible, if at the time of surrender the agent does not work for the insurance company any more.

The second hidden source is that the "**new" insurance created by premium increase** is usually given to the client according to normal tariffs and the commission is embedded into the premium loading part of the normal tariff. However, after premium increase the agent does not receive acquisition commission, which means that this remains at the insurer as a source of profit. In reality this hidden source is a type of expense profit.

The third hidden source comes from **zillmerization**. As we have already discussed, zillmerization means that the insurance company borrows part of the premium reserve of the client which is paid back gradually with all the interests during the insurance term. The interest paid to the client by the insurance company is the same as the technical interest rate, which – mainly at inflationary times – falls behind the market loan interest rates. This means that can be some interest profit deriving from zillmerization.

It is important to note that the existence of the hidden profit sources does not mean that the insurance company cheats on the clients as it is possible, that these hidden sources are the part of the normal profit similar to that of other companies in different sectors. If it would be possible to gain with this hidden profit sources systematically higher profit than the normal one, than this would attract new investors into the insurance business which would push down the level of profit into the normal level.

18. TECHNICAL INCOME STATEMENT

KEY WORDS

Premium due date	Commission chargeback
Direct debit	Mathematical reserve
Written premium	Unearned premium
Embedded value	Technical income statement
Profit dependent premium refund reserve	Individual outstanding claims reserve
Reserve for claims incurred but not reported	Surrender reserve

18.1. The Technical Income Statement in General

Every economical organisations has to know the processes and factors that influence its results. For Some companies, such as insurance companies the demonstration of these is officially regulated. A document of this kind is called technical income statement. In the case of life insurance companies – because of the special features of the business – calculating the profit is a very complicated process requiring many special calculations.

In the following we'll examine the technical income statement of the traditional life insurances, because in case of Unit Linked insurances – due to their transparent structure – it is far more simple to create such a statement. This means that by examining the traditional type, we deal with the more difficult task[151].

The technical investment statement is the appendix of the yearly balance sheet. The balance sheet tells us the value of reserves and profit on the turning day, but the processes resulting in these figures are not visible. The purpose of the technical investment statement (as usually the purpose of income- and result statements connected to the balance sheet as an appendix) is to show: from the values of the previous turning day what processes resulted during the elapsed time in the reserve- and profit figures stated in the balance sheet. The question is that during the accounting period what happened to the initial assets of money and to the new ones arriving in the meantime (mainly premium income) until they reach the final figures of the balance sheet?

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151 However the Unit Linked insurance stands on a totally different "product platform" than the traditional insurance, so the technical income statement of these insurance types must be examined in a different way.

The technical income statement can be closely connected to the **profit test,** but is very different from in some aspects. The main difference is that the profit test is a preliminary test for an imaginary portfolio, but the technical income statement explains the real processes on the real portfolio afterwards. This means that when making an income statement we have to use the same model as in the case of making a profit test. The difference is that we take the real figures as a starting point and considers the real happenings of the period in question.

Planning and **controlling** mainly differs from the technical income statement in that these give an acting line into the hands of the management and a report of the momentary status of its realization. Planning and controlling doesn't need to be comprehensive, precise, dealing with all the details. In the case of the technical income statement the most important things are comprehensiveness and the precise[152] equality of the appropriate data.

The technical income statement shows the processes resulting in the figures at the end of the year and thus it is able to split these processes into different parts.

In case of the technical income statement we have to give account of two things:

1. What happens to the money collected during the year? (Mainly to premium income, but there are other income types as well, e.g. increase of share capital.)
2. What happens to the money already reserved at the beginning of the year during the next year? (insurance technical reserves, share capital)

The profit consists of five different parts:

1. Mortality profit
2. Surrender profit
3. Investment profit
4. Profit correction factor
5. Expense profit

18.2. The Path of the Money Collected During the Year

First we look at figure 18.1. that shows the schematic path of the collected premium within the insurance company. This figure can be of our help later on as well, though not all of its branches (e.g. the change of the external sources and the share capital) will be dealt with and some of the branches (e.g. the risk profit and claims reserve) will be discussed in more detail.[153]

[152] It is essential to know that in the practice we have to make compromises and sometimes apply approximation solutions instead of calculating the precise value.

[153] It was my intention to show a figure that is general and not applied for a concrete life insurance company dealing with traditional life insurance policies.

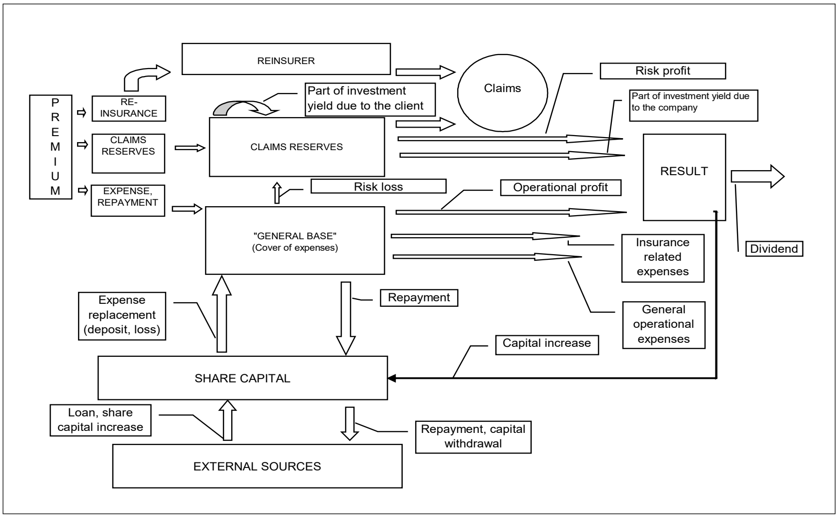

Figure 18.1.: The path of the premium within the insurance company, or the main cash-flows of the insurer

At first sight the premium income (at this point it is the **written** premium and not the actually collected **premium income**) is divided into two parts when arriving to the insurer:

- the net premium filling the reserve
- the premium loading covering expenses.

At second sight we have to make a correction, namely that the premium (in case of regular payment policies) is typically not divided between the net premium and the premium loading. Because of the gross premium reserving technique, i.e. zillmerization the division is changing in time. From the first premiums the sum covering the expenses is higher than the premium loading and much less or nothing fills the reserve, but in case of the later premiums one part of the premium loading also goes to the reserve.

This is problematic, because all premiums must be individually divided into reserve filling and expense covering parts. The parameters of the division are:

- the type of insurance[154]
- the nature of premium payment (single premium, regular premium or top-up payment) and the frequency of the payment (annual, semi-annual, quarterly, monthly)[155]

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[154] And – naturally – the structure of the concrete policy as well (main policy – insurance riders) but that is not discussed at this point.

[155] For the sake of simplicity we deal with regular, annual premium payment policies.

- the insurance term
- the entry age of the insured person
- the sex of the insured person
- the age increase applied by underwriting
- the sum of the premium[156]
- the number of the premium payment (this is important because the effect of zillmerization[157] is changing in time)

The current premium depends on whether there was a premium increase before or the client has accepted it or not. Fortunately this does not affect the internal ratios of the premium.

So, in the end, the premium income is used in two ways: it either goes to cover expenses or to build up reserves. The difference between the real expense and the sum covering the expenses is the expense profit or loss. The loss can be covered from the share capital. The reserves are used to cover claims and surrender benefit payments (and the transforming of policies, mainly and almost exclusively paying up the policy). The difference of the expected and real values of these are the sum of two factors: the mortality and surrender profit. The purpose of differentiating between the two is to monitor the difference between the "proper" termination of policies (which is the occurrence of the insured event) and the "non-proper" termination (surrender or transforming the policy).

In the above we have calculated based on written premiums because this is the logic of the traditional insurance and generally the computer system serves this logic as well. On the other hand the written premium can be paid earlier or later than the premium due date. The main reasons of the deviation are:

1. In case of Direct Debit, when the insurer withdraws the premium from the account of the client:
   a) The insurer withdraws the money not at the proper due-date but on a given date of every month, that can be in certain cases earlier or later than the due-date. If it is withdrawn earlier than the due-date, the money is put on a so-called parking account and entered into the books as premium income only on the due-date.
   b) There is no enough money on the account of the client so the insurer can only withdraw the money later than the due-day or never at all.

[156] It is important because of volume-depending reductions, but this is not discussed here.
[157] The situation is not that bad: zillmerisation not decreases but increases the net premium, so the ratio of the net premium and premium loading remains constant.

2. In case of paying by postal cheque or individual money transfer:
   a) The client forgets about paying the insurance premium or doesn't have enough money, the premium will be paid later or never at all.
   b) It happens rarely that the client pays the postal cheque earlier than the due-date.

In case of paying later the insurance company can ask for interest on overdue payments, but this is generally not practiced by Hungarian companies so there is no need to count with this correction factor (it would be part of the profit correction factor discussed later).

The insurance company gets a kind of latent interest profit after the premiums paid earlier but the premiums paid later mean latent interest loss. (Latent, because the interest of reserves and investment profit sharing is paid according to the written premium – and in the end this should be compensated by the interest on overdue payments). These latent profits and losses are not discussed later because:

- most of the premiums are paid by *direct debit,* which is quite trustworthy.
- in the case of *direct debit* the interests of premiums paid earlier and later equalize each other.
- the policies of considerable overdue payment usually become surrendered and the insurance company assert the interest loss at that time.
- Benefit payments could also be late, and the latent profit of overdue benefit payment compensates the latent loss due to overdue premium payment, and this is also not calculated.[158]

(In the technical sense the exact timing of premium payments is important only in the case of traditional insurances, where this is already built-in the reserve. In the case of modern UL insurances the exact timing is important only in business sense and the liquidity management point of view, because UL insurances are automatically adapting to the premium payment in any time.)

Altogether the results must be corrected at the end of the year by the following:

(premiums paid in, but not due in the given year) – (premiums not paid in yet but accounted).

I call the value calculated this way profit correction factor.

All **other income** must be examined properly according to the accounts. The most important of these is the commission chargeback (that was paid in effectively), the others are not significant factors.

If there are other expenses (not connected to the insurance activity), they have to be compared to other income, and the balance of the two has to be written to the expense profit.

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[158] Of course the correct thing would be to determine these latent factors as well, but it would be an enormous work to do so.

18.3. The Path of Money Already at the Insurer at the Beginning of the Year

There are two different types of these:

1. The reserves
2. The share capital of the insurance company

The **reserves** are divided into two main groups:

1. mathematical reserves
2. other reserves such as:
   a) unearned premium reserve
   b) surrender reserve
   c) outstanding claims reserve (individual+ IBNR159)
   d) profit dependent premium refund reserve (not every insurance company has to use it)

Since they behave differently (although their behaviour is connected to each other), they are discussed separately.

The **mathematical reserve** changes due to the following :

1. premiums filling the reserve
2. benefit payments (death, maturity[160])
3. distributing of the premium reserves of the deceased among those still alive.
4. the share of the clients from investment yields (temporarily it can be part of the profit dependent premium refund reserve – but it is not necessary)

The insurance companies generally use computer systems that calculate automatically the changing of the above mentioned first three factors with the help of the reserve functions built in the system, and it is not possible to show these factors separately.

The value of the 4th factor can be clearly determined when crediting the profit share.

The components of the other income elements can change because of different reasons.

In case of annual premium payment the **unearned premium reserve** – as the premium immediately becomes part of the reserve – is not a meaningful category, so it is not discussed here. The difference between the decrease and the increase would be an income modifying factor (expense profit or mortality profit). The value of the unearned premium reserve is usually unambiguous and can be easily taken from the computer system.

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[159] Incurred but not reported, international abbreviation.
[160] Of which a special case is the annuity.

The value of the **surrender reserve** is unambiguous as well. The difference between the value at the beginning and the end of the year can be accounted for in the expense profit (or maybe in the correction factor).

The mortality profit must be decreased by the difference between the value of the **outstanding claims reserve** at the beginning and the end of the year, because logically it belongs there.

The insurance company doesn't necessarily have **profit dependent premium refund reserve**. It is formed when the investment profit share is not credited continuously to the premium reserves. As we are supposing continuous crediting here, we don't use it. If it existed, then its value at the beginning of the year would be added to the premium reserves formally during the year.

The main factors effecting the **share capital**:

- using up the share capital, if the result is negative and not deferred, and a loan hasn't been taken out
- the yield of investing the share capital
- decision on increasing or decreasing the share capital (but this is of course not an income factor)

18.4. Calculating the Factors of Profit

18.4.1. Expense Profit, Income Correction

This profit factor is calculated as a remainder, so it is defined following way:

Expense profit = total profit – mortality, surrender and investment profit

On the other hand its name is justified because the main factor of this income is the expense saving (or exceed the expenses if the value is negative).

The profit correction is an important part of the income (according to a decision it can be accounted independently or can be considered part of the expense profit). This is formed because we calculate mortality- and investment profit to "theoretical" values, based on the written and not the actual premium income. With the help of the profit correction we adjust it to the actual premium income.

According to the above the expense profit is roughly: all actual expenses must be subtracted from that part of the premiums, written for the given year that is saved for covering expenses (this figure is corrected by the difference between the written and actual premium income). It is important to note that those expenses that were accounted somewhere else are not part of the total expenses incurred (primarily the fund management fee can be mentioned here).

The expense profit contains also the difference of other income and other expenses.

One part of the expenses is the interest paid for the capital loan. (The loan itself appears between the sources as a kind of external source, which equalizes the increase of assets. The repayment of the capital loan decreases the assets and sources on the same level, that is why we have to deal with the interest only regarding the profit.)

A separate part of the acquisition expenses should be isolated within the expense income. Namely: the sum paid for acquisition and that part of the zillmer premium that is saved for z must be matched. They should be corrected continuously with the decrease of deferred acquisition costs due to surrender, with the successful commission chargeback and with the unsuccessful commission chargeback accounted as loss.

18.4.2. Mortality (risk)[161] profit

The calculation of the mortality profit is discussed in detail in the following. For an accurate calculation we have to have the components with premium reserve in detailed figures.

The insurance portfolio is divided into the following groups by policies[162]:

1. From those components that were valid at the beginning of the year those that:
   a) are still valid at the end of the year or came to an end during the year when the insured event has occurred;
   b) came to an end with lapse or surrender during the year;
   c) were the policy was transformed or paid up.[163]
2. From the contracts signed during the given year:
   a) are still valid at the end of the year or came to an end during the year when the insured event has occurred.
   b) came to an end with surrender during the year.
   c) were the policy was transformed or paid up.[164]

Lets see how the mortality risk is calculated in each group:

In group 1.a. the calculation does not depend on the type of the insurance, it can be pure endowment, term or other kind as well and it also doesn't depend on whether

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[161] The term "risk" profit is more adequate if we also have risks such as accident, that doesn't originate from mortality differences.

[162] If the client has the option of adding and leaving components (insurance riders) from the main policy during the year, then here we have to use the term "components" instead of policies.

[163] Paying up the policy is in reality a kind of policy transformation, but – due to the usual Hungarian company policy – this at the same time the only type of policy transformation.

[164] This will be an empty set with great probability, group because it is a common policy not allow the policy being paid up in the first year, and clients are usually don't want to transform single premium policies.

the policy is single- or regular premium. Calculating the net premiums paid is not a simple task.

In groups 1.b. and 2.b. the mortality profit is not calculated (even if it could be), but accounted as part of the surrender profit.

In the case of 2.a. the formula of the mortality profit becomes simpler, since there is no reserve at the beginning of the year (i.e. it is 0) and there is no, profit sharing for clients or any other plus (shortly “bonus”) bonus paid during the year (because this is due only at the end of the first year) except the extra bonus in case of death.

In group 1.c. the policy transformation is considered as a surrender and a new policy at the same time. The difference between the premium reserve released and the single premium of the new policy (initial premium reserve) is accounted as surrender profit. From that point on the transformed policy is regarded as a new policy.

In group 2.c the procedure is almost the same as in group 1.c. There is only one difference, that here we don’t have to register two new policies, it is enough to register only the second one after the policy transformation. (Though it is almost certain that this group is will be empty.)

As we do not deal with components without premium reserve (e.g. accident riders), we only have to mention that the process is simpler there, because there is no premium reserve, no bonus and no interests.

The mortality profit for the whole portfolio is the sum of the above terms decreased with the balance of the outstanding claims reserve at the beginning and the end of the year.

18.4.3. Surrender Profit

In groups 1.b. and 2.b. of the partition of the previous section we simply take the premium reserves of the surrendered policies, (including earned but not paid bonuses) and subtract all surrender benefits.

The premium reserve released at the time of policy transformation must be added to this figure (i.e. we add the premium reserve before policy transformation – including the earned bonus – and subtract the premium reserve after the policy transformation).

Control possibility: calculating the mortality and surrender profit at the same time: if we do not differentiate between the individual components of the portfolio in the general formula of calculating the mortality profit then we get the mortality and surrender profit as one figure.

18.4.4. Investment Profit

There are two parts of the investment profit:

- the yield of the invested share capital (the share capital reduced by deferred loss).
- the yield of the premium reserve in the year (the bonus or technical interest paid to the client during the year – that can also be negative – must be subtracted from this figure).

From the above figures and from the results of long calculations we get a detailed picture of what happens to the money of the clients and the insurance company during the year.

18.5. Calculating the Mortality Profit

Calculating the mortality profit is very important, so let's look at it in detail.

18.5.1. Mortality Profit of Insurances With Single Premium

The recursive premium reserve formulae suggests that there is a mortality profit if the premium reserve of the deceased person not only covers the death benefits and the premium reserve credited to those still living, but reserve is released even above these. There is mortality loss when things happen in the opposite way (e.g. when none of the insured of pure endowment insurance die in the given year.)

According to the above mentioned, if we suppose that:

- the insurance year and the calendar year coincide,
- there is no profit sharing,
- there is no surrender and no new policies are signed,

then the mortality profit can be calculated the following way:

(all premium reserves at the beginning of the year compounded to the end of the year) – (the year end premium reserve of those still living) – (the death benefits compounded).[165]

(In the case of pure endowment naturally the value of death benefits is 0.)

In the end the coincidence of the insured- and the calendar year is not important, because the result will be the same if we left this supposition.

It is relatively easy to correct the figures with the help of profit sharing: the value paid during the year is added to the formula (compounded) and the effect of profit sharing is included in the year end premium reserve (as it usually is).

[165] Thus not the so called mortality services reduced with the premium reserves.

The arriving premium of the new policy must be accounted as premium income (and compounded to the year end). All the new policies of the year must naturally be included in the year-end portfolio (if the insured is still living).

But it has to be corrected by the surrenders, because the result of surrender is not entirely mortality profit – it is mainly surrender profit. The correction is the following: the initial reserve of those who surrender their insurance during the year is subtracted from the premium reserve at the beginning of the year and w perform a separate calculation on this portfolio. Naturally they are not included in the year end portfolio, which practically means group of "living" policies from that time.

According to the above mentioned, the definition of the corrected mortality profit for the not surrendered portfolio is the following:

(all premium reserves at the beginning of the year of those not surrendered during the year compounded to the year end) + (the initial premium reserve of those new policies signed during the year compounded to the year end) + (the profit share of policies terminated but not due to surrender compounded to the year end) – (the premium reserve of those still living at the end of the year and the compounded death benefits)[166]

In the case of surrendered/lapsed policies the procedure is the following: the premium reserves at the beginning of the year compounded to the time of surrender can be divided into three parts:

1. the part given back to the client (in case of surrender)
2. the mortality profit/loss for the last year fragment
3. profit/loss of surrender.

We can also say that the profit of surrender is what remains from the premium reserve compounded to the time of surrender after subtracting the first two parts (and this way we have defined its method of calculation).

Calculating the mortality profit is very simple:

(the premium reserves of the surrendered policies at the beginning of the year compounded to the time of surrender) + (the profit share given at policy anniversary compounded to the time of surrender (if there was a policy anniversary until surrender)) + (the value of the profit share not yet distributed at the time of surrender) – (the estimated profit share at the beginning of the year compounded[167]**) – (premium reserve at the time of surrender (including the effect**

[166] If we want to make this formula valid for annuities as well, we have to be more general – the sum insured already paid or the claims already compounded to the year end.

[167] Since the estimated value of the profit share not yet distributed is considered to be the part of premium reserve at the time of policy anniversary, that would be counted twice after distribution. Moreover the distributed value is a precise value, while at policy anniversary we only have an estimate.

of the possible indexation)) – (all benefit payments in the given year compounded to the time of surrender)

Naturally if the surrendered (lapsed) contract was signed during the year we have to calculate with the initial- and not the beginning-of-year premium reserve.

It is relatively hard to make a clear situation regarding the bonuses not yet distributed that should be paid after the surrendered policies in the given year. One part of these will be accounted as investment profit, another part as surrender profit. I think the following solution is an appropriate compromise (the above relation already reflects this):

- we should estimate the bonus due to the client at the time of surrender (even if we don't want to pay it out in this case),
- if there was no policy anniversary in the meantime, we have to subtract the estimated bonus credited at the beginning of the year (if there has been a policy anniversary we have already done this correction),
- this figure should mean the premium reserve at the time of surrender.

The mortality profit is the sum of mortality profits calculated for the surrendered and not surrendered contracts.

As a control we should examine whether the above mentioned formula is supported by the recursive formulae in case of term insurances or not.

The recursive reserve formula of the single premium term insurance is képletét a (12.18.) mutatja, amit átrendezhetünk a következő formára:

$$\frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)\right) = V_t \cdot (1+i) - V_{t+1} \tag{18.1.}$$

This can be interpreted easily. If there is no death, then the left side of equation (18.1) or the sum of this for all policies will be the mortality profit, that is:

$$\sum \frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)\right) \tag{18.2.}$$

since at this time we have the premium reserve compounded to the end of the year for all policies, but these subtracted terms are not needed for the further benefits.

Common misunderstanding: A general "everyday" definition of the mortality profit is the sum of the following for all policies:

= sums insured × death rate compounded to the end of the year – death benefit payments compounded to the end of the year

This formula is a more or less correct – though not precise – estimation of term insurance, that we see from the following formula:

$$\frac{d_{x+t}}{l_{x+t+1}} \cdot \left(1 - V_t \cdot (1+i)\right) = \frac{q_{x+t}}{p_{x+t}} \cdot \left(1 - V_t \cdot (1+i)\right) \tag{18.3.}$$

The $\frac{(1-V_t \cdot (1+i))}{p_{x+t}} \cdot$ is not the sum assured, but an estimation of it. This must be exactly one here.

$\frac{1}{p_{x+t}}$ is a number greater than one, but $\left(1 - V_t \cdot (1+i)\right)$ less than one, their product is somewhere around one, but not exactly.

18.5.2. Mortality Profit of Insurances With Regular Premium Payment

According to the relations concerning single premium insurances the reserve of insurances with regular premium payment (**if we do not use the zillmerization**) is different from the single premium insurances in only one aspect, the net premiums paid at the beginning of the year.[168] That is why in the definition of mortality profit this is the only factor that has to be modified. The mortality profit of insurances with regular premium consists of the following:

(All premium reserve at the beginning of the year of those who did not surrender or pay up[169] their policy during the year, compounded to the end of the year) + (The initial premium reserve (before premium payment) of those who took out new policies during the year, compounded to the end of the year) + (The profit share of those contracts that terminated but not because of surrender or paying up, compounded to the end of the year) + (The net premiums paid, compounded to the end of the year) – (The premium reserve of those still living at the end of the year) – (the compounded value of benefits paid[170])

It is a very important supposition in the above that premiums are paid in regularly, which means that the written premium and the actual premium payment is the same.

For the paid up and surrendered policies the following formula is used:

(The premium reserve at the beginning of the year of the paid up and surrendered policies compounded until surrender/pay-up) + (All net premiums paid, compounded to the same date) + (The profit share paid at policy anniversary compounded until surrender (if there was a policy anniversary until surrender)) + (The value of profit share not yet distributed at the time of surrender) – (The compounded profit share estimated at the beginning of the

[168] In case of more frequent premium payment (not annual) the sum of the reserve can change during the year.

[169] I.e. we want to account the effect of paid up policies in the surrender profit! In this case – above this – we also have to suppose a new, different type, single premium insurance.

[170] If we want to use this formula for annuity as well, we have to be more general – all paid sum insured compounded to the end of the year.

year) – (the premium reserve at the time of surrender (including the effect of the possible indexation)) – (Benefits paid in the given year, compounded until surrender)

We can regard paid up policies (by product type) as a separate group of single premium policies, that commence at the time of paying up, and for which we can calculate the mortality (or possibly surrender) profit according to the general rules. On the other hand the mortality profit relevant to the year fragment until paying up and surrender (both simply referred to as "surrender" from now on) increases the mortality profit of the contracts existing in the given year.

The **zillmerized** case differs from the above only in the aspect that the interpretation of "net premiums paid" must be modified to "that part of the zillmer premium which fills the reserve", as it has been discussed earlier. In the classical case this is P_1, in the non-classical case this is the sequence of P_i-s in the first k years, and $A_{x+k:n-k} - PZ \cdot \ddot{a}_{x+k:n-k} + PZ > p_{k+1}$ is a mid-value in year (k+1), after that it is PZ.

18.5.3. Deviation from the Standard Mortality Table

When calculating premiums, many of the companies suppose that due to the effect of underwriting the expected mortality rate will be less than the population-mortality in the first years, and after that gradually coming closer the expected mortality of the insured portfolio (used in the calculations) will reach population mortality in some years. The premiums calculated with this method are naturally lower than the premiums calculated all the way with the population mortality table.

In this case the premium calculation is typically (but not in every case) done with the method of profit testing, therefore there is no individual net premium (if there would be, the net premium would be lower than the net premium calculated the usual way). The net premium is also used in premium reserve calculations in the case of the profit test method. However, this net premium is calculated the usual way – i.e. it is calculated exclusively according to the population mortality table – and not with the mortality rates decreased at the beginning of the term.

This method of calculating the premium reserve supposes that a larger part of the premium must be set aside for benefits than what was taken into consideration when calculating the premium, which means that the practice doesn't use the same principles in every area. As the practice is not derived from strictly uniform principles, we have to choose between the two net premium (and also mortality benefit) calculation methods in case of calculating the mortality profit. The result won't be totally correct in either cases, so it is suggested to choose the simpler solution.

18.5.4. A Detour: What is the Benefit of the Term Fix Insurance?

When calculating the mortality profit (and generally: when accounting benefits) we have to face the problem: what exactly is the benefit of the (regular payment) term fix insurance? In the section below examine this question in detail.

The answer is not totally unambiguous, because the too obvious answers usually contain contradictions. Such obvious, but not satisfying answers are:

- The benefit is specified in policy conditions, that is the maturity sum paid at the time of maturity. The basic problem with this solution is that it supposes that it doesn't matter whether the insured dies during the insurance term or not, while we know that in this case the premium reserve is filled up. From the point of the risk community everything is right, since the premium paid by the clients still living is enough on the level of the risk community to cover all benefit payments at the end of the insurance term. However, on the individual level the accounting does not follow the real happenings.
- In case of death it would be logical to account the filling up of the reserve as a benefit. The problem with this is that in this case (one part of) the benefit paid at the end of the insurance term would be accounted twice.
- If the filling up of the reserve is considered to be a benefit, but the maturity sum paid after the policies of deceased is not (to avoid duplicated accounting), then we find ourselves in an obvious contradiction with the actual cash flow of the insured, as these benefits are not used for the filling up of reserves.

The solution to the problem is to **define** the **benefit of the policy** in a **different**, but equivalent way to make the complexity of the background more visible.

The traditional term fixinsurance is nothing else but a special endowment insurance (as we have already explained it in the subchapter 4.3.6.). Special, because its sum paid in case of death is the value of the maturity sum insured discounted to the time of death. According to this:

- If the insured person lives at the time of maturity then the insurer pays the maturity sum.
- If the insured person dies during the insurance term then the pure endowment insurance ends immediately, but the insurer pays the discounted value of the pure endowment part to the beneficiaries. However, the insurance contract implicitly contains that the beneficiary does not receive this money but purchases a special policy without any death risk (in fact a single premium term fix insurance), so he pays that money to the insurer as a premium and receives the benefit of the new policy at the time of the original maturity. By that time this sum is reaches exactly the original maturity sum due to the interests.

So what should be accounted as benefit?

If the insured person lives until maturity: only the payment of the maturity sum.

If the insured person dies during the insurance term, then the benefit can be divided into two parts:

- the discounted value of the maturity at the time of the death of the insured (the policy is terminated at this time but a new one is formed implicitly). But, at the same time, it must be accounted as a premium income of the single premium term fix. In fact, this implicit premium income will be the base of the further benefits, and this helps to avoid the duplicate accounting of the same benefit, but at the same time we follow the insurer's outgoing cash flow.
- at the end of the term the single premium term fix insurance is terminated and its benefit accounted.

The question might arise why is this process of implicit cash flows is necessary in the case of death. First of all, because the insurer has to react to the important events, such as the significantly different reserve after death and the lack of further premium payment. On the other hand, the beneficiary might think that the insurance company does not provide any benefit at that time, but later or continuously, that is why this payment has to be corrected by considering it to be premium income (which is the opposite of benefit payment).

The term fix insurance can placed in the above mentioned relation regarding mortality profit the following way:

- If the insured person lives there is no change.
- If the insured person dies, then the maturity benefit appears like a new insurance policy at the time of the death, which means that the compounded single premium must be considered "the value of the initial premium reserve (before premium payment) of those who entered the insurance during the year compounded to the end of the year", which is a plus figure. Since its source is a death benefit of exactly the same value, it must be subtracted as "the compounded value of benefit payment" – so altogether we can disregard the separate consideration of filling up of reserves at this time.
- In this case at the end of the year the "year-end premium reserve of the still living" will be the filled up reserve of course.
- The annuity benefit and the payment of the maturity sum are considered normal benefits.

Regarding the mortality profit of the term fix insurance, its benefit is the same as for the everyday people. In case of death the increase of the reserve presents itself only in the change of the "year-end premium reserves of the still living", so it is the simple process that is justified here as well.

18.6. Embedded Value[171]

The calculation of the *embedded value* (EV) does not belong strictly to the technical income statement, but its function is in one sense the same: to get a clearer view of the financial status of the life insurance company. The calculation of the embedded value is more and more common in Hungary and everywhere in the world as well. The EV itself is a method of policy valuation. It tells us the value of an already existing life insurance portfolio. It is useful to be familiar with its value all the time (e.g. manager remuneration can be connected to the increase of the EV), but it is extremely important when the insurance company or one part of the portfolio is for sale.

The generalisation of EV is the *appraisal value,* that can be defined as the sum of the EV and the goodwill, where the goodwill is the future development potential (the value of the portfolio that can be obtained in the future, that the insurance company, the knowledge of the management etc. "has inside itself").

There are many subjective values in the EV, that is why it is the change of the EV and not its actual value that counts.

In case of life insurances the EV gives a more realistic picture of the value of the insurance company than traditional accounting. (The traditional accounting might show a bad company as profitable compared to a good company if surrender rates are high in the bad company, although it is obvious that this is not a good sign for the future.)

The EV can be divided into the following parts:

$$\mathbf{EV = VIF + NAV}$$

where:
VIF = value of in force
NAV = net asset value

The VIF can be calculated with a profit testing software. The profit test must be performed on a portfolio starting from a given moment. The portfolio is usually not the whole portfolio of the insurance company, but a kind of simpler representation of it (e.g. the individual contracts are grouped according to age or gender).

The following formula represents the change of the EV:

$$\mathbf{\Delta EV = VIF{*}RDR + NAV{*}RFR + NBC}$$

Where:
RDR = risk discount rate
RFR = risk free rate
NBC = *new Business contribution* = NBAV = new business added value

[171] Based on a lecture held by Tibor Edvi in 2002!

It can be pointed out from this formula that the EV (the value of the company) can be increased mainly with new policies.

The ΔEV itself is not a closed formula, because we supposed that nothing changes. But in reality there can be several differences:

difference between facts and plans (variation) (+/-)

change of suppositions (that must be introduced in the documents attached to the calculation of the EV) (+/-)

The success of the management is usually measured with the changes of the EV purified from all other changes. Such other changes are e.g. tax, mortality, the change of the benchmark yield, so everything that is an environmental factor and does not indicate the performance of the management.

19. THE ACTORS AND RIVALS OF THE LIFE INSURANCE MARKET AND ITS SIGNIFICANCE IN THE NATIONAL ECONOMY

KEY WORDS

Bank insurance
Defined benefit system
Defined contribution system
Funded system

19.1. The Actors of the Life Insurance Market

The actors of the life insurance market (excluding the clients) can be divided into the following groups:

- suppliers
- competitors
- mediators
- associations
- regulator

The **suppliers** are the insurance companies themselves. The most important insurance companies of the Hungarian market are composite companies (according to their operational permit, though in reality not in every case).

The most important **competitors** of life insurance companies in Hungary (but the pattern is similar in many countries also) are **pension funds**, **banks** and **investment funds**. However, it is mainly co-operation and specialisation and not the rivalry that is characteristic of these institutions. It is very common that insurance companies direct financial groups or are members of such a group, which includes the above mentioned different competitor companies as well. The bank insurance is a good example of co-operation between insurance companies and banks and pension funds will be discussed in a more detailed way in chapter 19.2. Of course, in a wider sense, these companies remain competitors of the life insurance companies in spite of the co-operation, and the same way social security is a kind of competitor for life insurance as well. If we restrict the group of life insurance companies to the group of joint stock companies dealing with life insurances, then we can say that the life insurance mutuals are also rivals of these companies but the mutual insurance sector is characteristically important only in those countries, where the big insurance mutuals were created at least one century ago and preserved continouosly since then. Today already it is very hard to build up a new mutual insurer from the zero to reach a significant market share.

We have already dealt with two types of **intermediaries,** the independent **brokers** and the **agents** working for one (or sometimes several) insurance companies. Independent financial advisors or life-cycle planners are similar to intermediaries but they do not make mediation, they give only advice. These are already appeared in many developed countries, but in the Hungarian market their significance is marginal.

Associations are important members of the insurance market, of which can be more than one in each insurance subsectors – because they are organized on a voluntary basis. However, in a specific country the most important from these are the associations of the insurers and the independent intermediaries. In smaller countries – as in Hungary e.g[172] – insurers generally have an only one association, in bigger countries can be more, for example by activity (life insurance, non-life insurance, car insurance, etc.). Association for independent (or maybe dependent) intermediaries can be more than one even on such a relatively small market than the Hungarian one. They play an active part in of giving opinion on Bills regarding the insurance profession, in lobbying for more favourable regulations, operate ethical codex and a committees, examine and spread international experiences, etc.

The **regulator** of the insurance market is naturally the Hungarian state, which performs this task on the one hand by producing regulations, and on the other hand by operating institutions that monitor the compliance of these regulations. The rules are prepared (laws, government regulations) and published (ministry regulations) primarily by the Ministry of Finance. The compliance of regulations and laws are monitored by the organization responsible for the supervision of the financial organization. Until 2000 in Hungary this was the State Insurance Supervisory Authority, then the integrated "super supervisor", the Hungarian Financial Supervisory Authority (HFSA), which supervised all kind of financial institutions until 2013, when it was integrated into the Central Bank of Hungary. The Hungarian developments anyway on this field are following the international developmens – at least more or less – especially the practice of the UK (although not precisely). First in UK was created an integrated supervisor, the Financial Supervisory Authority-t (FSA), curtailing the supervisory licenses of the Bank of England. Later this Authority was splitted into two parts, a prudential and a consumer protection authority, and the prudential part was integrated into the central bank.

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[172] Although this is the matter of point of view. For example in Hungary also the agricultural insurance associations with almost immensely small market share have their own alliance. We could even consider it as a rival alliance, but it is not expedient.

19.2. The Connection Between Life Insurance and the Social Security System and Benefits

Being so significant, and being the functional supplement of the social security pension scheme (and also the health insurance scheme, to a lower degree) and – in a certain sense – its rival as well, the social security pension system – including the institution system of pension funds – is discussed in a separate chapter.

The Hungarian social security system (as it was already discussed in the Chapter 3) has two subsystems: the pension- and the health insurance subsystem. Life insurance has connections with both subsystems but the importance of these connection is not the same. The connection between life insurance and the social security health insurance subsystem mostly comes from (accident and sickness) insurances taken out as riders to life insurance policies – that would be non-life, but are accounted in the life branch. As these policies are important, but all the same only supplemental elements of the life insurance policy, we'll not discuss the relation of life insurance and social security health insurance in more detail.

The connection with the pension scheme is closer and the two spheres can be compared more easily. The aspects of comparison are:

- goal or function,
- operating system,
- relation between payments in and out,
- the reason for joining,
- profit.

The **goal** of the social security pension scheme and life insurance is almost the same: to provide for the financial safety of old age. However, this goal can be achieved in different ways.

There already are certain differences in the main goals. The social security pension scheme tries to be comprehensive and to provide some kind of benefit for every member of society (this goal can never be reached entirely). Life insurance, at least in case of a single company never has this kind of goal. The maximum is to provide this benefit for those members of society who represent a sound demand.

Apart from the common goal and function, life insurance has further functions as well. As we have already discussed them when discussing the different life insurance types, here is a brief list of the functions missing from the social security system (or performed only on a basic level):

- expenses of funeral and covering inheritance tax (in the case of whole life),
- savings for the benefit of a child (e.g. in the case of term fix),
- providing for relatives left behind (term insurance, widow and orphan annuity).

Life insurance operates only as a funded system, whilstthe social security pension scheme operated only in the so called pay-as-you-go system.

The main differrence between the two, that in the **funded system** all payments made by the insured in his active period are accumulated on a separate savings account and at retirement the client receives from the accumulated payments an annuity with a capital value equal to the accumulated sum. In other words we can say that everyone receives his own saved money as a pension, [173] and if the risk community would split up, every member would get his own money back (increased with interests).

In the case of the **pay-as-you-go system** the risk community does not accumulate anything, the total of all contribution payments is enough only for covering the pension benefits of those who are retired. If the risk community would split up, no-one would get their contribution payments back, and the persons retired wouldn't receive any pension benefits in the future, as there isn't any kind of reserve accumulated in the system. As the Hungarian social security system operates mainly in a pay-as-you-go system, those who are presently contributors must hope that the system will still operate at the time of their retirement. If it wouldn't operate at that time, they couldn't expect to receive any benefits from anyone.

This means that in contrast to the funded system, the pay-as-you-go system operates basically without reserves[174].

It is common to introduce another pair of categories used for describing pension systems: the concept of **Defined Contribution** and **Defined Benefit** systems. In the first case the contributions paid during the active years and their interests are strictly taken into account, and at retirement the valid pension annuity is calculated based on these. In the second case there are some regulations used for defining the valid benefits at the time of retirement, and contribution payments are adjusted to this. In the case of defined contribution there is a strict individual equivalence, while in the case of defined benefit the contribution payments and the received benefits equal only on the level of the whole risk community. [175] Life insurances are defined by contribution, but the present pay-as-

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[173] Or maybe during the accumulation is also created a risk community – esetleg a felhalmozás során is képeznek egy veszélyközösséget – cp pure endowment!

[174] Of course this is only one opinion. The other one is that the pay-as-you-go system has implicit reserves, namely the guarantee of the state. From the point of view of the state this sum is national debt, and since it is not accounted, the debt is implicit as well. That is why if the risk community would split up, the state would repay the implicit national debt – from other income – which means that it pays the pensions "earned" by contribution payment. Everything depends on what the state does. It have to mention, that – on the basis of what we explained in the Chapter 3 – it would be possible to transfer this system, by an appropriate reform, into a special funded system. The special capital (funding) here would be the human capital, rearing of a new contribution payer generation.

[175] While it is theoretically questionable whether in the social security system the in- and outpayments have to equal to each other or the human capital investment (child rearing) and the pension given for it, and the contribution payment is none other than the personal repayment of the human capital investment.

you-go system of the Hungarian social security pension scheme is defined by benefit. The individual pension funds that were part of the social security system for a while (between1998 and 2010) also operate in a defined contribution system.

It is worth to mention, that in 1997 a reform was implemented in the social security's pension system, which was practically terminated in 2010. The main goal of this reform to transform the exclusively as pay-as-you-go social security pension system– on the long term, in several decades – into partly (about 25%) a funded one. This transformation is gradual and long-term[176]. The organizational units of the funded system are the private pension funds that are similar to the voluntary funds.[177]

If we compare the two systems (the pay-as-you-go and the funded) we can discover further differences as well.

Life insurance can principally not operate in a system other than funded. This is closely connected to its voluntary nature. It would not be worth joining a voluntary life insurance scheme as a contribution payer in a pay-as-you-go system, therefore this system can operate only on a compulsory basis, regulated by the state.

There are huge reserves in the funded system that must be invested on the capital market. The investment of this enormous sum of money stimulates the stock exchange continuously and causes capital abundance.[178] In the funded system the financial knowledge of people dealing with these investments becomes more and more important, just as the role of the investing institutions and the capital markets in general.

In the pay-as-you-go system significant reserves are not created, that is why this system does not affect the stock exchange and doesn't stimulate the economy by investing the reserves. Due to the absence of reserves the role of the investment and financial knowledge and the role of the institutions is not so important. However, the bureaucracy and the corporations re-distributing the income have a very important role.

The question can be asked: Why is the Hungarian social security pension system operated exclusively on a pay-as-you-go basis?

The reason is quite simple and explains why the social security pension system was initially pay-as-you-go type in most countries. The reason is that the pay-as-you-go system can be introduced from one day to the other in the sense that it can immediately provide benefits to those who need it. Of course at the time of introducing this system

[176] In other words it can be said that this transformation makes the implicit reserves into explicit ones.

[177] Since after 2013 in Hungary the private pension funds were degraded into doomed rudiments with small membership, in this book in previous chapters it was superfluous to between „voluntary" and „private" pension funds, so we have mentioned only the "pension funds".

[178] Or it is simply encourages the state to spend towards its incomes, sith the insurers are ready to invest their reserves into state bonds, so to finance the budget deficit. In this case on the stock exchange only in the special, state bond section of it, will be abundant capital, and the private investment are crowded-out by the state spending.

none of its beneficiaries have any former savings[179] in exchange of the received benefits, although they have "earned the right" for a pension. Actually it is this saved amount – that should back up the payment of the first benefits – that will be missing from the pay-as-you-go system until the end[180], and this is the reason why after several decades of operation it cannot be guaranteed to people who have paid contributions all their lives that they will get their rightful pension if the system should fall down.

If the funded system had been introduced, then the first generation that can receive a complete pension from the system would be the generation which entered the labour market when the system was introduced, as they are the ones who have saved money for a long enough time. So, the funded system does not solve the problem of those who are currently in retirement age. That is why at that time the decision was made to introduce the pay-as-you-go system.

The pay-as-you-go system got into a serious depression by the end of the 1990s in Hungary (but not only here). The reason for this is that the system is very sensitive to demographic changes. This system works quite well until there are more people entering the labour market than retired age, as it is the relatively low payment of the working can cover the pension of the retired. It still works properly when the average age of the population does not increase. The increase of the average age nowadays increases primarily the time spent in retirement, when the person receives benefits from, and not pays contributions into the social security funds.

Altogether it can be said that the pay-as-you-go system works properly when the average age stagnates or decreases, but when the population is ageing, in the system has to face more and more problems. This is currently the case in Hungary, the population gets older and older and the birth rate is falling.

We have already discussed the **principle of equivalence**, which means – in the case of life insurances – that premium payment is proportional to the risk, and benefits paid are proportional to premium payment. As the outgoing payments depend on chance, we cannot say that the outgoing payments are equal to the incoming payments, but we can say that this is true for the discounted expected values.[181]

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[179] At least in the Samuelson's version of the PAYG system – which is presently the only one existing version of it. In the new interpretation of it – mentioned in the Chapter 3 – who rears contribution payers, deserves the pension, without contribution payment. In this book I also have interpreted the PAYG system on the basis of its own self-understanding, which is often not fit for a possible, reformed system. (see Banyár [2014])

[180] In many countries operating a pay-as-you-go system this amount saved had existed earlier, but has been destroyed in World War II. The pay-as-you-go system was introduced in most countries as the replacement of the former funded system, but the systems haven't been made funded later on. This does not change the validity of the above train of thought.

[181] This use of the principle of equivalence is sometimes called fair calculation, fairness or actuarial fairness. It means that there is no systematic redistribution of income between insured persons. The logic of competition definitely leads to this direction.

There is no equivalence in the social security pension scheme, only a so-called quasi-equivalence. The reason of this is the **principle of solidarity,** namely that some people in need receive higher benefits than their contribution payments, which can be done because those who are not in need receive less than their own payment. Because of the competition and the voluntary basis the principle of solidarity is necessarily missing from the operation of life insurance.

The principle of solidarity can be seen at many points of the social security system – however this (as we can see below) is changing dynamically:

- There is a so-called minimal pension that the pensioner receives whether this sum is "earned" by former contribution payments or not. This is a redistribution of income in favour of those people who had low wages or short time spent in work, or both.
- Contribution payments was counted in the pension in a degressive way, which means that the more someone paid in the less he received proportionally. In Hungary this was terminated by law as unfair in 2013. However, it is not entirely clear whether the pensionability exempt from any degressivity or with an appropriate degressivity is fair? Namely the people with higher pension will probably live a longer life so the non-degressive scale means the redistribution of pension rights towards them.
- The principle of solidarity works also in favour of women. In spite of the fact that the average life expectancy of women is much higher, so they will spend more time in retirement and consequently will receive more pension than the men, this does not appear in receiving a lower level of pension for the same salary. However, in pension schemes operating as funded systems and in case of annuities this was natural until 2012, when in the whole territory of EU the differentiation of the premium of insurances (include annuities) according to sex was prohibited.

We have already mentioned and pointed out the differences between the two systems, namely that joining the risk community of the life insurance companies is **voluntary,** but the social security pension scheme is **compulsory** by law. It is because of this compulsory feature that it can operate on a pay-as-you-go basis and that the principle of solidarity is taken into consideration.

Currently this difference is one of the main reasons that in Hungary the social security pension scheme disposes over sums significantly higher than all life insurance companies together. However, the formers are flow items, but the latters are stock ones (more or less) long term reserves.

The main business goal of companies selling life insurance products is **making profit**. (Naturally it does not pertain to the not for profit mutual insurance associations – which have in Hungary a very small market share – and the insurance cooperatives, which are possible according to the law, but do not exist in reality, only for the joint stock life insurance companies.) This can be closely connected to the fact that all of these companies are in private property.

The social security pension scheme is a typical **non-profit** scheme, the motivation of its operation is that it is the state's duty to provide pension for everyone and to satisfy the needs of the insured. The social security system is not a private property but a state bureaux.

The voluntary pension funds in Hungary were formed in 1994 as a supplement of the social security scheme. The private pension funds, that were formed in 1998, and mainly terminated after 2010 are following the model of the voluntary ones could be considered to be part of the social security pension system.

Without discussing the differences between pension funds and life insurance companies in detail, let's see some of their similarities and differences. (We use the term pension fund when we are referring to both of funds, but mention the different names when we are speak of the their differences.)

One of the similarities is that both funds and the life insurance companies operate in a funded and defined contribution system.

One common feature of voluntary pension funds and life insurance companies is the voluntariness, which makes them different from the social security pension scheme. The same can be said about the principle of solidarity, which is missing from not only the voluntary funds but from the private funds as well.

The owner structure of pension funds and the life insurance companies is different. The pension fund theoretically, and according to the law, is owned by its members and works on a self-governing basis. (However de facto they are ownerless or – from another angle – owned by (at least the majority of them) joint stock companies and managed by a management appointed by these companies – although formally elected by the general meeting of the members.)

The benefit of the pension funds is mainly the collection and investment of contributions paid by members, and not the risk elements that are characteristic of life insurance companies, e.g. the sum paid out in case of death and as an annuity is significantly higher than the sums paid in.

The pension funds are the rivals of life insurance companies. But apart from the competition there a co-operation between the two spheres can also be imagined, which became "part of the system" in Hungary:

1. Many insurance companies operate so-called "fund service companies" that provide services to the pension funds. These services can be various from the founding of pension funds to administration and fund management. These fund service companies are profit oriented, they do their job in return of fee received from the pension funds. In fact the biggest funds were founded by insurance

companies (and by banks in some cases) – though principally it was not allowed by law.

2. The insurance companies have larger risk communities than the voluntary pension funds, that is why for the voluntary funds it is worth to buy some insurance products from insurance companies instead of organizing it for themselves.

19.3. The Effect of Life Insurance on the National Economy

Life insurance has direct and indirect positive effects on the national economy. Lets list these effects.

One of the **indirect effects** is that life insurance creates the **feeling of safety** and in some very important cases safety itself. When we emphasize the role of the feeling of safety, we are not think of the insurance providing less than people expect, but already of the fact that the insurance itself makes a lot of people more relaxed and happier (which is not a negligible aspect for politics).

The economical effect of safety provided by life insurance – as by all insurance types – is the fact that in case of an unexpected negative event (death or becoming unable to work) the disadvantageous economical effects will **not spread,** which means that the negative effects will be located to the small circle of the people involved. And there the disadvantageous economical effects will be prevented by the benefit payment of the insurance company. For example if an enterpreneur who has a loan on the enterprise dies, it is certain that this person can be replaced in the enterprise. It can happen that the successors of the enterprise cannot pay the instalments and the enterprise will be bankrupt, that can have an effect on other enterprises as well, so the negative effect can spread. This can be stopped by a life insurance that can cover the instalments from the sum insured, which means that the enterprise won't be bankrupt if the enterpreneur dies but the successors can terminate it the normal way. (We haven't mentioned other, more distant positive economical effects of life insurances here.)

The **direct effects** are the following:

1. The companies selling savings type life insurances can be considered from the point of national economy to be companies that are collecting the small capitals of the economy and transferring it into a **big investable capital**. Without this accumulation these small capitals might not get into the production, or with a much lower efficiency than this way, grouped into a large capital together

withthe other small capitals. That is why life insurance companies are among the largest investment groups in the Western world and their role in stimulating the economy and producing new workplaces is essential.

2. Those business insurance companies that sell savings type life insurance take over certain benefits from the social security system. This is very important in those countries where the social security system has difficulties (as in most European countries, and in Hungary also).
3. The savings type life insurance is nothing but postponed consumption. In the first period until premium payments are more than benefit payments it has an anti-inflationary effect, since it drives purchasing power away from current consumption, this way decreasing the pressure of inflation.

As a summary we can say that:

1. Life insurance creates workplaces not only in the life insurance business but in businesses connected to the insurance such as banks, financial institutions, health service.
2. The premium reserve collected from the premiums paid by policyholders serves as a basis that is suitable for investing in the national- or private sphere. This aggregation has a positive effect on the state of the national economy.
3. By means of annuity insurance and creating pension funds [182]it provides pension annuity benefits to those who would not get this kind of safety anywhere else.
4. Insurance companies (through paying taxes) provide a significant income to the government. The domestic benefits provided to the insured avoid this money getting abroad.
5. Life insurance companies pay attention to making health services stronger and to encourage the population to better health care. So, these kind of social expenses could be decreased.
6. Life insurance has an important role in creating the financial stability of the population. It can be reached by giving the members of society the possibility of minimizing their unexpected financial losses.
7. The society can save a significant sum by organising given social functions in the scope of life insurance.

[182] Internationally the privately held pension funds, not their Hungarian version is widespread for pension savings. The Hungarian pension funds are much more a mutual insurance type institutions. From this point of view, the Hungarian regulation is quite unique and its expediency is quite questionable.

APPENDIX

ABBRIVIATION OF THE EUROPEAN COUNTRIES

Abbriviation	Country
AT	Austria
BE	Belgium
BG	Bulgaria
CY	Cyprus
CZ	Czech Republic
DE	Germany
DK	Denmark
EE	Estonia
ES	Spain
FI	Finland
FR	France
GR	Greece
HU	Hungary
IS	Iceland
IE	Ireland
IT	Italy
LI	Liechtenstein
LT	Lithuania
LU	Luxembourg
LV	Latvia
MT	Malta
NL	Netherlands
NO	Norway
PL	Poland
PT	Portugal
RO	Romania
SE	Sweden
SI	Slovenia
SK	Slovakia
UK	United Kingdom
HR	Croatia

Explanation of Terms

Active phase of life

The phase of life approximately from age 20-25 years to 60-70 years, when individuals make a living, provide for others and accumulate for later phases of their life from the income of their own work.

Adverse selection

Selection of the insured persons that is harmful for the insurer. The Hungarian terminology distinguishes between antiselection and autoselection according to whether bad faith or explicit intention of fraud from the part of the insured is present or not.

Age of insured

The insurer specifies the *entry age* (or the current age) of the insured by subtracting the insured's birth year from the commencing year (or the current calendar year) of the policy.

Age Pyramid

A diagram illustrating the composition of the population of a country by age and gender.

Life annuity

The (generally monthly) sum paid by the insurer regularly to the beneficiary declared in the insurance policy. The annuity can be "life"-annuity, when it is to be paid at most until the death of the insured, or certain annuity, when the payment is guaranteed within a given period (or it can also be the combination of these two).

Application, Proposal

Formally it is always the client (the policyholder) who initiates the signature of the insurance contract by making an application to the insurer to sign an insurance policy (even if the client wouldn't have thought of it on his own, but it was the insurer's agent who persuaded him into making the application). The policy becomes effective when the insurer accepts the application. The application form is the printed document, the client makes the application to the insurance company by filling out this form.

Beneficiary

The person who receives the benefit defined in the insurance policy from the insurer in case of the insured event. (If the insured event is living until maturity, then the beneficiary is usually the insured.)

Captive insurer

Insurer established by a big organization (e.g. a big corporation or the state), which insures exclusively the asset or employees of this organization.

Loss division

The early operating principle of the first insurance-type organizations. The members of the risk community divide among themselves the already occurred loss to finance it together and porteriorly. The opposite operating principle is the risk division.

Risk reduction

A risk management strategy. Using such solutions that lower the probability and/or the magnitude of the loss.

Prospect theory

Utility theory reformed by Kahneman and Tversky. According to it, the behavior of people towards losses and gains are asymmetric. People are risk-avoiding towards uncertain gains and risk-seekers towards uncertain loss. At the same time this bevior strongly depend on whether this uncertainty is related to small or high probabilities.

Risk sharing

The modern operating principle of insurers, whereby – in contrast to the loss division – it is not the already incurred claims, but the risks that are shared. So, under this principle the insurer collect premiums in advance, and pays the losses from the reserves made ut from these premiums.

Risk avoidance

It means as a risk management strategy, that somebody a priori does not start an activity whereby a certain type of risk can appear. As a prospect theory concept means to prefer smaller certain gains to those which expected value is higher, but uncertain.

Risk management

The possible strategies of the management of risk as risk avoidance, risk reduction, risk equalization, self insurance and the insurance.

Risk equalization

Risk management strategy within the framework of which we diversify our activity such a way, that the effects of a certain risk are opposite to the different activities so the outcome will be neutral.

Risk spreading
Strategies applied by insurers to divide risks. Main metods of it the the reinsurance and the pool.

Composite insurer
Insurer which is active in both life an non-life insurance.

Mutual insurer
Insurer owned by the insureds. The legal form of it is association or cooperative.

Cash flow
The Income and payout of a certain period from the point of view of an individual (institution, person) regarding a certain aspect (e.g. loan, maintaining an enterprise, etc.) all together, represented in chronological order.

Commencement date
The insurance policy becomes effective usually the day following the payment (to the account, the cash-desk, or to the agent of the insurer) of the premium advance (the premium of the first insurance period), supposing that the insurer hasn't rejected the application within the period available for underwriting (if there is no agreement stating otherwise).

Computability
The property of a stable system of relations, when its state can be predicted in the long-term relatively precisely and with high certainty.

Current premium reserve
The premium reserve of the insurance has the following parts:

- the premium reserve of the main policy,
- the premium reserve of the investment profit share, and
- the premium reserve of the insurances purchased by the chosen premium increases.

Financial planning of the life cycle
The projection of the cash flows of the remaining part of the life cycle, and the creation of a consumption and savings strategy that matches the long-term goals.

Frequency of payment
Premium payment can be *regular*:

- monthly (12 times a year),
- quarterly (4 times a year),
- semi-annual (twice a year),
- annual (once a year),

or *single* (when the total premium of the insurance is due at the inception of the policy).

In case of regular premium payment the policyholder can usually change the frequency of premium payment on policy anniversaries.

General Conditions

Insurers generally summarize in separate regulations the policy conditions that concern every single life insurance policy, regardless of insurance type. The General Conditions of different insurance companies are somewhat different – although very similar.

Cohort mortality table

A mortality table that follows the actual mortality of a group of people (e.g. the population of a country) born in the same period (e.g. in the same year), contrary to the usual mortality table, which is an artificial table constructed from the mortality of different generations living at the same time.

Indexation techniques

The initial sum insured gradually looses its real value as time passes due to inflation. Indexation techniques serve to subdue the effect of inflation. They are truly effective only if applied together. The indexation techniques are:

- premium increase options and
- investment profit sharing.

Premium increase is obviously only possible in case of regular premium insurances, while the investment profit sharing refers to both regular- and single premium insurances (generally with the exception of term insurance). The client decides whether to make use of the premium increase option or not, while profit sharing is automatically received.

Initial sum assured, initial sum insured

The sum declared in the insurance policy at the time of its inception, that the insurer used for calculating the initial premium of the policy. The initial sum insured can differ from the current sum insured (see there!) at a given point of the insurance term.

Insurance

The creation of business or mutual based risk community, a form of risk transfer.

Insurance benefit

The benefit that the insurer provides in case of the insured event, and/or the expiration of the policy. In case of life insurance this usually means the payment of the sum insured to the beneficiaries immediately after the insured event, or taking over further premium payment until the end of the policy, when the sum insured is paid out.

Insurance Event, Insured Event

The event, which if incurs, the insurer provides the insurance benefits based on the life insurance policy.

In case of life insurance the *insurance even* can be (theoretically):

- the death of the insured during the insurance term,
- the insured living at the maturity of the insurance policy.

However in practice sometimes also some non-life (mainly accidental and sickness insurance) events are acting as life insurance events, Besides marriage and child birth can also appear as insurance events in life insurance contracts.

Insured

The person whose life is bound to the insured event. (It is often the same person as the policyholder.)

Insured Period

The month/quarter/half-year/year counting from the day the policy becomes effective, that an individual premium payment covers. The length of the period consequently depends on the frequency of premium payment.

Life-, accident- and sickness insurance [183]

Grouping the different branches of insurance according to their subject. The subject of this branch is the life, health and well-being of the individual.

Life cycle

The financial aspect of the individual's life from birth until death.

Life expectancy

In any age the number of years that persons of that given age will live in average. A special case of it is life expectancy at birth, that is the remaining life expectancy at age 0.

[183] There is no single term to define these types of insurance, since the term "personal insurance" refers to insurances connected to persons, i.e. apartment-, car- etc. insurances beside the life-, accident- and sickness insurances!

Life expectancy at birth

The expected (average) age at the time of death of a group of people born at the same time.

Life insurance, life assurance

One of the insurance branches, that gives protection against insured events related to the individual's life (death, living until maturity).

Life insurance contract, life insurance policy

An agreement between the insurer and the policyholder, in which the insurer undertakes the liability against the premium payment of the policyholder of providing the insurance benefits if the insured event occurs. The general conditions of this contracting are regulated by the Civil Code.

Long Term Care

Special conditional life annuity which pays benefint (or pays larger sum) only if the insured (generally because of his/her high age) is not able to self-care and need care.

Maturity

The date when – if it hasn't been terminated earlier due to death, surrender, or lapse – the insurance policy expires.

Medical examination

Medical examination is necessary so that the insurer can precisely assess the mortality risk of the insured. Since the (mortality) risk of the insured increases on the one had with the (death) sum assured, and on the other hand with the age of the insured, insurers tie the automatic medical examination partly to a certain level of the sum assured and partly to a certain entry age. E.g. it is possible that an insurance company does medical examination every time, if the following age and sum assured limits are exceeded:

Entry age	Initial death sum assured
below 45 years	Above 2 million Forints,
46 to 55 years	Above 1 million Forints,
56 years	In all cases

Naturally there might be cases when the insured doesn't exceed the given limits, but the M.D. of the insurer still finds the medical examination necessary. This might be the case if the medical statement throws light on health problems, the severity of which cannot be assessed solely based on the medical statement.

The expenses of the medical examination are usually covered by the insurer.

Medical Statement

A form that has to be filled out together with the insurance application, containing questions about the insured's health status. Its purpose is that the insurer should get a satisfying idea of the death risk that the insured person represents.

Moral hazard

A phenomenon when the existence of the insurance raises the probability of the loss, because the sense of security encourages a more irresponsible behaviour.

Mortality or Life table

A table containing at least the number of lives by age starting from a given population, but generally also contains the mortality and survival rates.

Non-forfeiture options

The premium reserve of the policy is fundamentally the client's (the policyholder's) money. This way the insurance company has to be able to account for the reserve to the client if the policyholder cannot continue the insurance with the same conditions. Possible non-forfeiture options of life insurance policies are the following: surrender, paying up the policy and policy loan. (The policy conditions specify the concrete non-forfeiture options that a given insurance policy has.)

Paid-up

Paying up the policy is possible in case of *regular premium insurances* (if policy conditions allow). In this case, the client doesn't pay the premium any more, and the insurer leaves the policy in force without changing its term, and using the available premium reserve as a single premium to purchase an insurance of a lower sum insured.

Social contract among generations

A concept originated from Samuelson, according to which the pay-as-you-go pension system has to be organized in such a way that the current actives support the current elderly for the promise that in their old age, they will also be supported by the future actives.

Pay-as-you-go system

A functioning principle of social security systems popular mainly in developed countries, according to which the currently necessary benefit payments of beneficiaries are covered by the current contribution payments of contributors.

Phases of the life cycle

The most important continuous periods of the life cycle. From the economical point of view, the life cycle can usually be divided to active-inactive-active phases.

Policy

The printed document issued by the insurance company, that contains the most important elements of the insurance contract. The insurer accepts the application of the policyholder by issuing the policy, i.e. the life insurance contract becomes effective when the policy is issued.

Policy anniversary

Generally the date of the policy becoming effective every year. Earlier it was common in the Hungarian market that theHungarian insurance companies don't follow this practice, and adjust the policy anniversary to the first day of the month following the date of the policy becoming effective. The commencement of the policy tailored this way is also called "technical commencement date".

Policy conditions

The policy conditions contain those conditions of the insurance contract that only refer to the given type of main policy or insurance rider.

Policyholder

The person who takes out the insurance and generally performs premium payment.

Pool

Portfolio unification. A method of risk spreading when some insurers unify their individually too small similar insurance portfolios in the case of which the law of large numbers are not functioning well. The unified portfolio is managed by together from the risk management point of view.

Premium

A fee (premium) depending on the sum assured, the type and term of insurance, the insured's age, gender and medical status, occupation and hobbies, that the policyholder pays for the insurance benefits.

Premiums due

The insurance premium of an insurance period is due in advance on the first day of the given insurance period.

Premium increase options

Most insurers provide the option to clients of increasing the insurance premium (with certain limitations), and consequently also the sum insured every year depending on the inflation rate. Making use of these premium increase options technically means the purchase of new insurance, that have an insurance term equal to the remaining term of the original insurance, and the entry age will be the current age of the insured. In case of premium increase, the premium reserve will be the sum of the premium reserves belonging to the initial sum and to the increased sum(s).

Reserve

Every premium that the policyholder pays to the insurer serves two purposes. The greater part is to cover the risks undertaken by the insurer, the insurer pays its liabilities undertaken in the policy from this part (we could say that this is the risk community's "membership fee"). A smaller part is necessary to cover the insurer's expenses related to the insurance (actually this is the price of insurance or rather the larger part of it). The greater part is again split into two parts. One part (its value depends on the type of insurance) is paid out to cover the current death benefits the same year as it was collected. But the benefit of the insurer is only expected to be paid in several years (depending on the date of maturity or death). As time passes, the death risk increases and the time of maturity benefit is closer. This way the insurer has to form reserves from the earlier premium payments (from the other part) to cover these insured events occurring later. This reserve is called premium reserve. The value of the premium reserve depends on:

- the entry age of the insured,
- the gender of the insured,
- the insurance term,
- the type of insurance,
- the sum insured,
- the time passed since the commencement of the insurance,
- this way the premium reserve can have an infinite number of values.

Private insurance

Insurance not organised by the state, but mainly business based.

Probability of death = mortality rate

The probability that a person of age x years will not live to be x+1 years old.

Probability of survival

The probability of an individual of age x years living to be x+1 years old.

Profit sharing

If the insurer achieves a yield higher than the technical interest rate when investing the premium reserve, then the policy receives a share from this profit. The insurer divides a part of this as specified in the policy conditions (e.g. 90% of the yield above the technical interest rate) among the individual policies proportional to their premium reserve. The profit share increases directly by a lump sum either the premium reserve, or the balance of an account set up separately for the client. The insurer either transforms the sum of this increase to a sum assured, considering the current age of the insured and the remaining term of the policy (i.e. the former sum insured increases by this value), or simply handles it on the separate account. If the insurer uses the client's share of the investment profit to increase the sum assured, then the investment profit share is regarded as the premium of a single premium insurance.

The value of the premium reserve is important also in this case, since the achieved investment profit is distributed between the individual policies according to the ratio of premium reserves. This also means that it can happen that (since the premium reserve might be zero in the initial period) a policy doesn't receive an investment profit share on the first few occasions. Naturally this is only true for regular premium policies, since in case of single premium policies the premium reserve is significant from the first moment.

Reinsurance

The most common form of risk spreading. Insurance, used by insurers. The direct insurer transfers a part of the risk undertaken by it, which exceed its risk capacity, to an other insurer, the reinsurer.

Return on investment

The result of investing the premium reserve, that the insurer partly calculates in the premium right from the start (technical interest rate), and partly (in case of a higher return) shares with the policyholder (profit sharing).

Rider

A life, accident or sickness insurance, that cannot be taken out independently, only as a complementary to a life insurance as main policy. Its benefits serve to complement the benefits of the main policy.

Risk

A event occurring randomly and having a negative effect.

Risk commencement date

After the waiting period, or if there is no waiting period, or in cases that are not affected by the waiting period, 0 o'clock of the day following the payment of the premium advance, if the insurance policy later became effective, or would have become effective. I.e. if the insured event occurs after 0 o'clock of the day following the payment of the premium advance, and the policy would have become in force independent of the insured event, then the insurer pays the benefits specified in the policy to the beneficiary.

Risk community

A group organised to ward off or decrease a concrete risk, that operates from payments made by its members, and has the goal of (partly) compensating the negative financial effects in case of those members of the risk community, who have suffered a loss due to this risk.

Those who take out an insurance policy become members of a risk community. In case of life insurance the risk is death and/or living until maturity.

The basic principle of every risk community is that anyone can suffer a loss, but we do not know beforehand who and when. Those who are affected by the risk (and suffer a loss) cannot cope with it on their own. This is why the members of the risk community cover the loss of the effected persons together.

The above statements also mean that in case of life insurance those who live until maturity pay more – disregarding interests – than they receive from the risk community (moreover in some cases they only pay, but do not receive anything). But it is all the same worth joining the risk community, because in return the insured can feel secure.

The insurance company is an institution organising risk communities.

Risk transfer

Handing over the risk that we consider too high, against payment, to someone for whom this risk isn't high.

Selection Table

A mortality table that contains the mortality rates of a special selected population (primarily a group having a given type of insurance, e.g. term insurance, annuity).

Self-insurance

A risk-handling strategy, when the individuals lean primarily on their own reserves.

Social Security

A compulsory insurance system organised by the state, maintained from contribution payments, to provide primarily pension- and medical benefits. In a wider sense all kinds of state maintained systems (i.e. maintained from taxes) can be called (moreover, here we should call them!) social security systems, although strictly speaking this is not correct.

Solidarity

Solidarity is in the economic sense a kind of redistribution, it is a one-way income transfer from social groups in better financial position to those who are in a worse financial position, who are financially in need.

Sum assured, Sum insured, Face amount

The sum insured shows the level of insurance benefits if the insured even occurs. Since in case of a life insurance (e.g. an endowment insurance) there might be two insured events, we distinguish the death sum assured and the maturity sum assured (that are not necessarily, but generally the same). We also distinguish the initial and the current sum assured. The current sum assured can be higher, than the initial, because of the premium increases and profit sharings.

Surrender

Surrender means that the policyholder terminates the policy and demands the premium reserve from the insurer. The insurer doesn't pay out a certain percent of the premium reserve upon surrender. The purpose of holding back a part of the premium reserve is to compensate the deterioration of the portfolio due to the fact that surrender is more often chosen by the insured representing good risk, than the insured of bad risk. This premium reserve part held back is usually between 1-20%, and decreases as time passes. (The precise regulation varies from company to company.)

The other reason of decreasing the surrender value is that the premium reserve has to be mobilised at surrender, and the insurance company is forced to break up the investment portfolio. This can has additional costs. But this additional cost not necessarily appears, sith an insurer with not a running off portfolio has continuous premium income, which is not yet invested. For an insurer with a running of portfolio in turn its investments are continuously terminating.

Technical interest rate

Insurers calculate the premium of insurances by supposing that when investing the premium reserve, it will earn an interest of at least 2-4% every year. This supposed fixed (2-4%) interest rate, that varies from insurer to insurer is called technical interest rate. By signing the insurance policy, the insurer guarantees that the policyholder will receive this interest rate.

Insurers choose the technical interest rate between 2 and 4%, because in the long term (and life insurance generally is long term) the yield of investments cannot be predicted. In a consolidated economy with low inflation rates, a 4% interest rate is considered very good. This way no financial institution can take long term commitments above this level.

Naturally the investment yield of the insurer will exceed 2-4% in several years. In these cases the investment profit sharing will come into effect.

The possible maximum value of the technical interest rate for newly signed policies is regulated by directly in many countries, or indirectly in other countries by solvency capital which is increasing by the technical interest rate.

Term

The period of time starting with the inception of the policy and ending with the maturity date declared in the policy, that can usually be only in whole years.

Transparency

The requirement that the functioning of an institution or a financial product, and the financial affairs of an institution should be visible and understandable without significant effort for those who are interested. It is particularly important that the paths and levels of individual contributions and utilizations of participants can be followed at every moment by the participants.

Underwriting

The insurer examines whether the application covers a normal risk, or due to the possibly significantly worse heath status or occupational (sports, hobby) risk, an increased premium tariff should be used. It is possible that the insurer doesn't take the risk and rejects the application. The base of underwriting is the application, the medical statement, and – in some cases, e.g. higher sum insured – the data provided by a medical examination. In some cases – e.g. very rare occupation or dangerous hobby – the insurer might also request the filling out of another questionnaire.

In case of extremely high sum insured – to prevent insurance fraud – the insurer might ask separate questions, that try to determine if there is some hidden purpose of signing the application. Such questions might refer to the relation of the insured and the beneficiary, the financial status of the policyholder, etc.

Unearned premium

If in case of regular premium insurances the insurer calculates the premium tariff based on monthly premium payment, but the policyholder chooses a premium payment less frequent (quarterly, semi-annual, annual), then the premium that applies to the whole months remaining from the insurance period is called "unearned premium", since the insurer has not yet "earned" this part of the premium. If the client has unearned premium when the insured event occurs, then the insurer pays this back together with the benefit payment.

Waiting period

It is very common that insurers define a waiting period, within which there is no benefit payment. This period is usually half a year. If the insured event happens within this period, the insurer doesn't pay the sum insured to the beneficiary (apart from certain exceptions, e.g. accidents), only the premium reserve.

If the policy became effective following a medical examination, then insurers usually do not apply a waiting period.

Zillmer adjustment

In case of regular premium payment a lot of insurers make the premium reserve of the policy zero in the commencing period of the policy (the first 1-4 years, depending on the entry age of the insured and the insurance term), so it only becomes positive, a significant value compared to the paid premiums later during the term. The reason of this is that the greater part of the insurer's expenses connected to the policy arise immediately when it is signed. To cover these, the insurer uses the total premium of the first 1-4 years, so this does not fill the reserve. The excess premium used in the first years is given back from the premium loading of the later years to fill the reserve (and during that period the increase of the premium reserve will be faster than if the insurer didn't use this technique).

This type of reserving method is called zillmerization. It has an effect – among others – on the profit sharing and on non-forfeiture options (see there!). This way the investment profit share and – in case of surrender – the surrender value of the policy is zero or a low sum in the first years.

The Hungarian male and female Population mortality tables and commutation numbers of year 2015, with 2% technical interest rate

Male populatin mortality table

Age (x)	l_x	d_x	q_x	v^x	C_x	D_x	M_x	N_x	R_x	S_x
0	100 000	505	0,51%	1,00000	495,10	100 000,00	24 955	3 827 303	1 676 270	109 702 678
1	99 495	43	0,04%	0,98039	41,33	97 544,12	24 460	3 727 303	1 651 315	105 875 375
2	99 452	22	0,02%	0,96117	20,73	95 590,16	24 418	3 629 759	1 626 856	102 148 072
3	99 430	24	0,02%	0,94232	22,17	93 695,11	24 398	3 534 169	1 602 437	98 518 313
4	99 406	14	0,01%	0,92385	12,68	91 835,78	24 376	3 440 474	1 578 040	94 984 144
5	99 392	13	0,01%	0,90573	11,54	90 022,40	24 363	3 348 638	1 553 664	91 543 671
6	99 379	12	0,01%	0,88797	10,45	88 245,71	24 351	3 258 616	1 529 301	88 195 033
7	99 367	11	0,01%	0,87056	9,39	86 504,95	24 341	3 170 370	1 504 950	84 936 417
8	99 356	10	0,01%	0,85349	8,37	84 799,39	24 331	3 083 865	1 480 609	81 766 047
9	99 346	11	0,01%	0,83676	9,02	83 128,29	24 323	2 999 065	1 456 278	78 682 182
10	99 335	12	0,01%	0,82035	9,65	81 489,30	24 314	2 915 937	1 431 955	75 683 117
11	99 323	14	0,01%	0,80426	11,04	79 881,82	24 304	2 834 448	1 407 640	72 767 180
12	99 309	16	0,02%	0,78849	12,37	78 304,47	24 293	2 754 566	1 383 336	69 932 732
13	99 293	19	0,02%	0,77303	14,40	76 756,72	24 281	2 676 262	1 359 043	67 178 166

Age (x)	l_x	d_x	q_x	v^x	C_x	D_x	M_x	N_x	R_x	S_x
14	99 274	21	0,02%	0,75788	15,60	75 237,29	24 267	2 599 505	1 334 762	64 501 904
15	99 253	27	0,03%	0,74301	19,67	73 746,44	24 251	2 524 268	1 310 495	61 902 399
16	99 226	31	0,03%	0,72845	22,14	72 280,76	24 231	2 450 521	1 286 244	59 378 132
17	99 195	37	0,04%	0,71416	25,91	70 841,36	24 209	2 378 240	1 262 013	56 927 610
18	99 158	43	0,04%	0,70016	29,52	69 426,40	24 183	2 307 399	1 237 804	54 549 370
19	99 115	49	0,05%	0,68643	32,98	68 035,58	24 154	2 237 973	1 213 620	52 241 971
20	99 066	53	0,05%	0,67297	34,97	66 668,58	24 121	2 169 937	1 189 467	50 003 998
21	99 013	56	0,06%	0,65978	36,22	65 326,38	24 086	2 103 268	1 165 346	47 834 061
22	98 957	56	0,06%	0,64684	35,51	64 009,25	24 050	2 037 942	1 141 260	45 730 793
23	98 901	56	0,06%	0,63416	34,82	62 718,65	24 014	1 973 933	1 117 210	43 692 851
24	98 845	57	0,06%	0,62172	34,74	61 454,06	23 979	1 911 214	1 093 196	41 718 918
25	98 788	59	0,06%	0,60953	35,26	60 214,34	23 945	1 849 760	1 069 217	39 807 704
26	98 729	62	0,06%	0,59758	36,32	58 998,41	23 909	1 789 546	1 045 272	37 957 943
27	98 667	66	0,07%	0,58586	37,91	57 805,25	23 873	1 730 547	1 021 363	36 168 398
28	98 601	71	0,07%	0,57437	39,98	56 633,91	23 835	1 672 742	997 490	34 437 850
29	98 530	75	0,08%	0,56311	41,41	55 483,46	23 795	1 616 108	973 655	32 765 108
30	98 455	81	0,08%	0,55207	43,84	54 354,14	23 754	1 560 625	949 860	31 149 000
31	98 374	85	0,09%	0,54125	45,10	53 244,53	23 710	1 506 271	926 106	29 588 375
32	98 289	90	0,09%	0,53063	46,82	52 155,42	23 665	1 453 026	902 397	28 082 105
33	98 199	95	0,10%	0,52023	48,45	51 085,94	23 618	1 400 871	878 732	26 629 078
34	98 104	102	0,10%	0,51003	51,00	50 035,80	23 569	1 349 785	855 114	25 228 208
35	98 002	112	0,11%	0,50003	54,90	49 003,71	23 518	1 299 749	831 545	23 878 423
36	97 890	124	0,13%	0,49022	59,60	47 987,94	23 464	1 250 745	808 026	22 578 674
37	97 766	138	0,14%	0,48061	65,02	46 987,41	23 404	1 202 757	784 563	21 327 929
38	97 628	155	0,16%	0,47119	71,60	46 001,06	23 339	1 155 770	761 159	20 125 171
39	97 473	174	0,18%	0,46195	78,80	45 027,48	23 267	1 109 769	737 820	18 969 402
40	97 299	199	0,20%	0,45289	88,36	44 065,78	23 189	1 064 741	714 552	17 859 633
41	97 100	225	0,23%	0,44401	97,94	43 113,39	23 100	1 020 676	691 364	16 794 891
42	96 875	254	0,26%	0,43530	108,40	42 170,09	23 002	977 562	668 264	15 774 216
43	96 621	286	0,30%	0,42677	119,66	41 234,82	22 894	935 392	645 262	14 796 654
44	96 335	327	0,34%	0,41840	134,13	40 306,64	22 774	894 157	622 368	13 861 262
45	96 008	376	0,39%	0,41020	151,21	39 382,17	22 640	853 851	599 594	12 967 104
46	95 632	435	0,45%	0,40215	171,51	38 458,77	22 489	814 468	576 954	12 113 254
47	95 197	499	0,52%	0,39427	192,88	37 533,17	22 317	776 010	554 465	11 298 785
48	94 698	573	0,61%	0,38654	217,14	36 604,34	22 124	738 477	532 148	10 522 776
49	94 125	654	0,69%	0,37896	242,98	35 669,46	21 907	701 872	510 023	9 784 299
50	93 471	743	0,79%	0,37153	270,63	34 727,08	21 664	666 203	488 116	9 082 427
51	92 728	845	0,91%	0,36424	301,75	33 775,53	21 394	631 476	466 452	8 416 224
52	91 883	957	1,04%	0,35710	335,04	32 811,51	21 092	597 700	445 058	7 784 748
53	90 926	1 074	1,18%	0,35010	368,63	31 833,10	20 757	564 889	423 966	7 187 048
54	89 852	1 188	1,32%	0,34323	399,77	30 840,29	20 388	533 055	403 209	6 622 160
55	88 664	1 292	1,46%	0,33650	426,24	29 835,81	19 988	502 215	382 821	6 089 104
56	87 372	1 384	1,58%	0,32991	447,64	28 824,56	19 562	472 379	362 833	5 586 889
57	85 988	1 469	1,71%	0,32344	465,81	27 811,73	19 115	443 555	343 270	5 114 510

Age (x)	l_x	d_x	q_x	v^x	C_x	D_x	M_x	N_x	R_x	S_x
58	84 519	1 549	1,83%	0,31710	481,55	26 800,59	18 649	415 743	324 156	4 670 955
59	82 970	1 628	1,96%	0,31088	496,19	25 793,54	18 167	388 943	305 507	4 255 212
60	81 342	1 710	2,10%	0,30478	510,96	24 791,60	17 671	363 149	287 340	3 866 269
61	79 632	1 793	2,25%	0,29881	525,25	23 794,53	17 160	338 357	269 669	3 503 120
62	77 839	1 876	2,41%	0,29295	538,79	22 802,72	16 635	314 563	252 509	3 164 763
63	75 963	1 956	2,57%	0,28720	550,75	21 816,81	16 096	291 760	235 874	2 850 200
64	74 007	2 030	2,74%	0,28157	560,38	20 838,28	15 545	269 943	219 778	2 558 440
65	71 977	2 101	2,92%	0,27605	568,61	19 869,30	14 985	249 105	204 233	2 288 497
66	69 876	2 156	3,09%	0,27064	572,05	18 911,10	14 416	229 236	189 248	2 039 392
67	67 720	2 200	3,25%	0,26533	572,28	17 968,24	13 844	210 325	174 831	1 810 156
68	65 520	2 242	3,42%	0,26013	571,77	17 043,63	13 272	192 356	160 987	1 599 831
69	63 278	2 291	3,62%	0,25503	572,81	16 137,67	12 700	175 313	147 715	1 407 475
70	60 987	2 354	3,86%	0,25003	577,02	15 248,43	12 127	159 175	135 015	1 232 162
71	58 633	2 421	4,13%	0,24513	581,81	14 372,42	11 550	143 927	122 888	1 072 987
72	56 212	2 482	4,42%	0,24032	584,78	13 508,80	10 969	129 554	111 337	929 060
73	53 730	2 547	4,74%	0,23561	588,32	12 659,14	10 384	116 045	100 369	799 506
74	51 183	2 621	5,12%	0,23099	593,55	11 822,60	9 795	103 386	89 985	683 461
75	48 562	2 707	5,57%	0,22646	601,00	10 997,24	9 202	91 564	80 190	580 074
76	45 855	2 971	6,48%	0,22202	646,68	10 180,61	8 601	80 566	70 988	488 511
77	42 884	2 950	6,88%	0,21766	629,52	9 334,31	7 954	70 386	62 387	407 944
78	39 934	2 933	7,34%	0,21340	613,62	8 521,76	7 325	61 052	54 433	337 558
79	37 001	2 923	7,90%	0,20921	599,54	7 741,05	6 711	52 530	47 108	276 507
80	34 078	2 913	8,55%	0,20511	585,77	6 989,73	6 112	44 789	40 397	223 977
81	31 165	2 902	9,31%	0,20109	572,11	6 266,91	5 526	37 799	34 286	179 188
82	28 263	2 884	10,20%	0,19715	557,42	5 571,91	4 954	31 532	28 760	141 389
83	25 379	2 856	11,25%	0,19328	541,18	4 905,24	4 396	25 960	23 806	109 857
84	22 523	2 810	12,48%	0,18949	522,03	4 267,88	3 855	21 055	19 410	83 897
85	19 713	2 741	13,90%	0,18577	499,22	3 662,17	3 333	16 787	15 555	62 842
86	16 972	2 642	15,57%	0,18213	471,76	3 091,14	2 834	13 125	12 222	46 055
87	14 330	2 507	17,49%	0,17856	438,87	2 558,77	2 362	10 034	9 388	32 930
88	11 823	2 330	19,71%	0,17506	399,89	2 069,72	1 923	7 475	7 026	22 896
89	9 493	2 114	22,27%	0,17163	355,70	1 629,25	1 523	5 405	5 103	15 421
90	7 379	1 858	25,18%	0,16826	306,50	1 241,60	1 168	3 776	3 580	10 016
91	5 521	1 573	28,49%	0,16496	254,40	910,76	861	2 534	2 412	6 240
92	3 948	1 273	32,24%	0,16173	201,84	638,50	607	1 624	1 551	3 706
93	2 675	973	36,37%	0,15856	151,25	424,14	405	985	944	2 082
94	1 702	698	41,01%	0,15545	106,37	264,57	254	561	540	1 097
95	1 004	462	46,02%	0,15240	69,03	153,01	147	296	286	536
96	542	279	51,48%	0,14941	40,87	80,98	78	143	139	239
97	263	150	57,03%	0,14648	21,54	38,52	37	62	61	96
98	113	71	62,83%	0,14361	10,00	16,23	16	24	23	33
99	42	29	69,05%	0,14079	4,00	5,91	6	8	8	10
100	13	13	100,00%	0,13803	1,76	1,79	2	2	2	2

Female population mortality table

Age (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
0	100 000	416	0,42%	1,00000	407,84	100 000,00	21 669	3 994 640	1 615 205	121 326 835
1	99 584	27	0,03%	0,98039	25,95	97 631,37	21 261	3 894 640	1 593 536	117 332 195
2	99 557	17	0,02%	0,96117	16,02	95 691,08	21 235	3 797 009	1 572 274	113 437 554
3	99 540	2	0,00%	0,94232	1,85	93 798,77	21 219	3 701 318	1 551 039	109 640 545
4	99 538	12	0,01%	0,92385	10,87	91 957,73	21 217	3 607 519	1 529 820	105 939 227
5	99 526	11	0,01%	0,90573	9,77	90 143,76	21 206	3 515 562	1 508 603	102 331 708
6	99 515	10	0,01%	0,88797	8,71	88 366,47	21 197	3 425 418	1 487 396	98 816 147
7	99 505	8	0,01%	0,87056	6,83	86 625,09	21 188	3 337 051	1 466 200	95 390 729
8	99 497	6	0,01%	0,85349	5,02	84 919,73	21 181	3 250 426	1 445 012	92 053 677
9	99 491	6	0,01%	0,83676	4,92	83 249,62	21 176	3 165 506	1 423 830	88 803 251
10	99 485	5	0,01%	0,82035	4,02	81 612,35	21 171	3 082 257	1 402 654	85 637 745
11	99 480	6	0,01%	0,80426	4,73	80 008,09	21 167	3 000 645	1 381 483	82 555 488
12	99 474	7	0,01%	0,78849	5,41	78 434,57	21 162	2 920 636	1 360 316	79 554 843
13	99 467	9	0,01%	0,77303	6,82	76 891,23	21 157	2 842 202	1 339 154	76 634 207
14	99 458	11	0,01%	0,75788	8,17	75 376,73	21 150	2 765 311	1 317 996	73 792 005
15	99 447	14	0,01%	0,74301	10,20	73 890,59	21 142	2 689 934	1 296 846	71 026 695
16	99 433	15	0,02%	0,72845	10,71	72 431,55	21 132	2 616 043	1 275 704	68 336 761
17	99 418	14	0,01%	0,71416	9,80	71 000,61	21 121	2 543 612	1 254 572	65 720 717
18	99 404	15	0,02%	0,70016	10,30	69 598,64	21 111	2 472 611	1 233 451	63 177 106
19	99 389	14	0,01%	0,68643	9,42	68 223,67	21 101	2 403 012	1 212 340	60 704 495
20	99 375	15	0,02%	0,67297	9,90	66 876,53	21 092	2 334 789	1 191 239	58 301 482
21	99 360	16	0,02%	0,65978	10,35	65 555,33	21 082	2 267 912	1 170 147	55 966 693
22	99 344	18	0,02%	0,64684	11,41	64 259,58	21 071	2 202 357	1 149 066	53 698 781
23	99 326	19	0,02%	0,63416	11,81	62 988,17	21 060	2 138 097	1 127 994	51 496 424
24	99 307	21	0,02%	0,62172	12,80	61 741,30	21 048	2 075 109	1 106 934	49 358 327
25	99 286	23	0,02%	0,60953	13,74	60 517,88	21 035	2 013 368	1 085 886	47 283 217
26	99 263	25	0,03%	0,59758	14,65	59 317,51	21 022	1 952 850	1 064 851	45 269 849
27	99 238	29	0,03%	0,58586	16,66	58 139,78	21 007	1 893 533	1 043 829	43 316 999
28	99 209	31	0,03%	0,57437	17,46	56 983,13	20 990	1 835 393	1 022 822	41 423 467
29	99 178	35	0,04%	0,56311	19,32	55 848,35	20 973	1 778 410	1 001 832	39 588 074
30	99 143	38	0,04%	0,55207	20,57	54 733,96	20 954	1 722 561	980 859	37 809 664
31	99 105	41	0,04%	0,54125	21,76	53 640,18	20 933	1 667 827	959 906	36 087 103
32	99 064	43	0,04%	0,53063	22,37	52 566,66	20 911	1 614 187	938 973	34 419 276
33	99 021	45	0,05%	0,52023	22,95	51 513,57	20 889	1 561 620	918 061	32 805 089
34	98 976	49	0,05%	0,51003	24,50	50 480,55	20 866	1 510 107	897 173	31 243 468
35	98 927	54	0,05%	0,50003	26,47	49 466,23	20 841	1 459 626	876 307	29 733 361
36	98 873	61	0,06%	0,49022	29,32	48 469,83	20 815	1 410 160	855 465	28 273 735
37	98 812	68	0,07%	0,48061	32,04	47 490,13	20 786	1 361 690	834 650	26 863 575
38	98 744	77	0,08%	0,47119	35,57	46 526,91	20 754	1 314 200	813 865	25 501 885
39	98 667	88	0,09%	0,46195	39,85	45 579,05	20 718	1 267 673	793 111	24 187 684
40	98 579	100	0,10%	0,45289	44,40	44 645,48	20 678	1 222 094	772 393	22 920 011
41	98 479	114	0,12%	0,44401	49,62	43 725,68	20 634	1 177 449	751 715	21 697 917
42	98 365	130	0,13%	0,43530	55,48	42 818,69	20 584	1 133 723	731 081	20 520 468

Age (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
43	98 235	146	0,15%	0,42677	61,09	41 923,63	20 529	1 090 904	710 497	19 386 745
44	98 089	167	0,17%	0,41840	68,50	41 040,51	20 468	1 048 981	689 969	18 295 841
45	97 922	190	0,19%	0,41020	76,41	40 167,29	20 399	1 007 940	669 501	17 246 860
46	97 732	217	0,22%	0,40215	85,56	39 303,29	20 323	967 773	649 102	16 238 920
47	97 515	248	0,25%	0,39427	95,86	38 447,08	20 237	928 470	628 780	15 271 147
48	97 267	280	0,29%	0,38654	106,11	37 597,35	20 141	890 023	608 543	14 342 677
49	96 987	316	0,33%	0,37896	117,40	36 754,04	20 035	852 425	588 401	13 452 655
50	96 671	354	0,37%	0,37153	128,94	35 915,97	19 918	815 671	568 366	12 600 230
51	96 317	394	0,41%	0,36424	140,70	35 082,80	19 789	779 755	548 449	11 784 558
52	95 923	437	0,46%	0,35710	152,99	34 254,20	19 648	744 672	528 660	11 004 803
53	95 486	481	0,50%	0,35010	165,10	33 429,56	19 495	710 418	509 012	10 260 131
54	95 005	528	0,56%	0,34323	177,67	32 608,98	19 330	676 989	489 517	9 549 713
55	94 477	575	0,61%	0,33650	189,70	31 791,91	19 152	644 380	470 187	8 872 724
56	93 902	623	0,66%	0,32991	201,50	30 978,85	18 963	612 588	451 035	8 228 344
57	93 279	671	0,72%	0,32344	212,77	30 169,92	18 761	581 609	432 072	7 615 756
58	92 608	721	0,78%	0,31710	224,14	29 365,58	18 548	551 439	413 311	7 034 148
59	91 887	774	0,84%	0,31088	235,90	28 565,64	18 324	522 073	394 763	6 482 709
60	91 113	830	0,91%	0,30478	248,01	27 769,63	18 088	493 508	376 438	5 960 635
61	90 283	892	0,99%	0,29881	261,31	26 977,12	17 840	465 738	358 350	5 467 127
62	89 391	956	1,07%	0,29295	274,57	26 186,84	17 579	438 761	340 510	5 001 389
63	88 435	1 023	1,16%	0,28720	288,05	25 398,81	17 304	412 574	322 931	4 562 628
64	87 412	1 091	1,25%	0,28157	301,17	24 612,75	17 016	387 175	305 627	4 150 054
65	86 321	1 161	1,34%	0,27605	314,21	23 828,97	16 715	362 563	288 610	3 762 879
66	85 160	1 226	1,44%	0,27064	325,30	23 047,53	16 401	338 734	271 895	3 400 316
67	83 934	1 285	1,53%	0,26533	334,27	22 270,32	16 076	315 686	255 494	3 061 582
68	82 649	1 348	1,63%	0,26013	343,78	21 499,38	15 741	293 416	239 418	2 745 896
69	81 301	1 426	1,75%	0,25503	356,54	20 734,04	15 398	271 916	223 677	2 452 480
70	79 875	1 525	1,91%	0,25003	373,82	19 970,96	15 041	251 182	208 279	2 180 564
71	78 350	1 634	2,09%	0,24513	392,68	19 205,55	14 667	231 211	193 238	1 929 382
72	76 716	1 745	2,27%	0,24032	411,13	18 436,29	14 275	212 006	178 571	1 698 170
73	74 971	1 870	2,49%	0,23561	431,95	17 663,66	13 863	193 570	164 296	1 486 164
74	73 101	2 022	2,77%	0,23099	457,90	16 885,37	13 431	175 906	150 433	1 292 595
75	71 079	2 211	3,11%	0,22646	490,88	16 096,39	12 974	159 021	137 002	1 116 689
76	68 868	2 652	3,85%	0,22202	577,25	15 289,89	12 483	142 924	124 028	957 668
77	66 216	2 776	4,19%	0,21766	592,39	14 412,85	11 905	127 634	111 545	814 744
78	63 440	2 920	4,60%	0,21340	610,90	13 537,85	11 313	113 221	99 640	687 110
79	60 520	3 082	5,09%	0,20921	632,15	12 661,51	10 702	99 684	88 327	573 888
80	57 438	3 261	5,68%	0,20511	655,75	11 781,09	10 070	87 022	77 624	474 205
81	54 177	3 453	6,37%	0,20109	680,74	10 894,34	9 414	75 241	67 554	387 183
82	50 724	3 655	7,21%	0,19715	706,44	9 999,99	8 734	64 347	58 140	311 942
83	47 069	3 857	8,19%	0,19328	730,86	9 097,47	8 027	54 347	49 407	247 595
84	43 212	4 047	9,37%	0,18949	751,83	8 188,23	7 296	45 249	41 379	193 248
85	39 165	4 212	10,75%	0,18577	767,14	7 275,85	6 544	37 061	34 083	147 999
86	34 953	4 332	12,39%	0,18213	773,52	6 366,04	5 777	29 785	27 539	110 938

Age (x)	lx	dx	qx	vx	Cx	Dx	Mx	Nx	Rx	Sx
87	30 621	4 386	14,32%	0,17856	767,81	5 467,70	5 004	23 419	21 762	81 153
88	26 235	4 351	16,58%	0,17506	746,75	4 592,68	4 236	17 951	16 758	57 734
89	21 884	4 207	19,22%	0,17163	707,88	3 755,88	3 489	13 359	12 522	39 783
90	17 677	3 938	22,28%	0,16826	649,62	2 974,36	2 781	9 603	9 033	26 424
91	13 739	3 544	25,80%	0,16496	573,16	2 266,42	2 132	6 628	6 251	16 821
92	10 195	3 040	29,82%	0,16173	482,01	1 648,81	1 559	4 362	4 120	10 193
93	7 155	2 457	34,34%	0,15856	381,93	1 134,47	1 077	2 713	2 561	5 831
94	4 698	1 852	39,42%	0,15545	282,24	730,29	695	1 579	1 484	3 117
95	2 846	1 280	44,98%	0,15240	191,25	433,73	412	848	790	1 539
96	1 566	799	51,02%	0,14941	117,04	233,98	221	415	378	690
97	767	440	57,37%	0,14648	63,19	112,35	104	181	156	275
98	327	209	63,91%	0,14361	29,43	46,96	41	68	52	95
99	118	83	70,34%	0,14079	11,46	16,61	11	21	11	26
100	35	35	100,00%	0,13803	0,00	4,83	0	5	0	5

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