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What is the child-related compensational pension system good for and what is not?

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Abstract

There is increasing attention to the sustainability and fairness of the pay-as-you-go pension system as a consequence of the aging society and the imbalance between the old and the young generation's number. In this system, the pension depends only on the previous contribution, which indirectly punishes childbearing. The purpose of this article is to compare the effect of the present Hungarian regulation to a possible child-related compensational pension scheme, where the amount of pension takes into account the childbearing time. The evaluation of the pension systems is based on the lifespan utility of representative agents (with or without children) and the economic effects of the possible pension reform.

We built up a dynamic general equilibrium model in an overlapping generations framework (calibrated on the basis of Hungarian data) to investigate the effects of our pension reform proposal. As a result we receive that such a pension system could increase the utility of the consumer who has children by 0.2149% percent, but decrease the steady state utility of childless consumer by 0,0130% percent. The amount of children and the time spent with children increase slightly, but these positive elements that could have raised the output does not compensate the negative effect of the decreasing work-related efforts, so the output falls.

Keywords: Computable General Equilibrium Models, OLG model, Public Pension, Retirement Policies

JEL classification: C68, D15, H55, J26

1 Introduction

In several European countries, a pure pay-as-you-go (from now on PAYG) pension system is still in place, or at least the pension system has a strong pay-as-you-go pillar, for example, in Hungary, Slovakia, Czechia. But the sustainability and the fairness of these pension systems are questionable

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if the country's population has had a negative demographic dynamic for a longer time: if the population is aging (low fertility coupled with longer life expectancies) and/or if there is an unsustainable population pyramid. In Hungary, these demographic problems appear at once. In twenty years a numerous generation will retire and will leave behind relatively low number of children.

The PAYG pension system can not be *fair* for certain groups, among others to agents who have children along with such demographic trends. The amount of the pension benefits depends on the working years and the average wage during the lifetime, and thus, indirectly, childbearing is punished in a PAYG pension system. In Hungary, the long-term lifetime cost connected to childbearing weights typically on mothers, and these costs may be severe for them in retirement age. The current calculation method of pension favors the childless persons.

In this paper, we introduce a new approach to a possible child-related pension system. We called this reform idea a *compensational pension system*, in which the pensions compensate the forgone labor time, which is a result of childbearing: the time and efforts the parents spent on bringing up children. Our goal is to build up a dynamic general equilibrium model in an overlapping generations framework, which is able to evaluate the effect of a possible pension calculation modification on the representative agents' lifespan utility, the pension system, and the economy as a whole. The benchmark is the present Hungarian PAYG system. The contribution of our paper to the literature is the analysis of this new pension reform's economic and welfare effects in an overlapping generations framework based on Hungarian data.

The paper is organized as follows: section 2 gives a comprehensive picture of the Hungarian pension system and the demographic situation of Hungary. Section 3 presents facts about childbearing's effect on pensions and possible pension reforms. We introduce our OLG model in detail in Section 4, while section 5 documents the data and the calibration. Section 6 shows the effects of pension reform (compensational pension system) based on the model, and finally section 7 concludes the subject.

2 The Pension System and the Demographic Situation of Hungary

2.1 The Hungarian Pension System

The basic requirements for a pension system maintained by the state are to assure sufficient income for the old-aged during the retirement years and maintain the system's financial sustainability (which means that the operation of the pension system burden the state budget not too much). The one type of pension system is the pay-as-you-go (PAYG) pension system, which builds on the linkage between generations. The young and the middle-aged working population pays the revenues of the system (the contribution rate), and the older generation receives the expenditures (the pension benefit) (Lee (2016)). Individuals who have reached the retirement age and have sufficient service time are eligible for pensions. A defined-benefit type of PAYG system provides the eligible persons' guaranteed income for retired ages (Banyár (2020)).

In Hungary, there has been a pure pay-as-you-go state pension system since 2011, which is mandatory for all persons (Vékás (2021)). The Hungarian PAYG system is of the defined benefit type with an

earnings-related public pension. The gradually increasing statutory retirement age reaches the age of 65 in January 2022 for both men and women. Those who have reached the age of 65 and have paid the necessary number of years of insurance contributions (who have a minimum of 20 years of service/contributory time) are eligible for an *old-age pension*. For women, there is an option to be eligible for pension before the standard retirement age if they have a minimum of 40 years of an eligibility period (we called this type of pension "*Women 40*"). The service time can be any period of work (at least 32 years of gainful activity are needed) or childcare years (maximum eight years)¹. We do not cover here the other retirement opportunities (Hungarian Ministry for National Economy. Central Administration of National Pension Insurance (2017), European Commission (2021)).

The amount of the pension depends mainly on the average wage and the number of service years. The average wage is determined on the basis of the earnings subject to pension contributions between January 1, 1988, and the date of retirement. As of January 2012, the pension benefits are indexed to inflation according to the CPI. But the exact calculation of the pension depends on many different parameters of contributions' payment and the pension benefit. The first step is that wages without tax and contribution rate need to calculate for every year during the service time. Then the net earnings have to be valorised by the valorization multiplier² (up to one year before the retirement age), and thereafter, it has to be averaged for the years of gainful activity. After it, a progressive reduction has to be applied for higher levels of average valorised net wages. Finally, the average of these adjusted earnings is multiplied by a rate related to the number of service years obtained. This pension multiplier rate is 80 percent for 40 service years. The rate of pension contribution (which is 10%) and the social contribution rate (which is 18,5% from July 2020) together are the sources of the pension's financing in Hungary (the Pension Insurance Fund) (Hungarian Ministry for National Economy. Central Administration of National Pension Insurance (2017), European Commission (2021)).

In the past decade, many factors have changed in Hungary, which helps to improve the sustainability of the pension system. The most important one is that the average number of pension beneficiaries has decreased gradually since 2000 from 3100 thousand to 2500 thousand, thanks to the raising retirement age limit and the tightening approving resolutions during the 2010s. Consequently, now in 2021, 26% of the Hungarian population is a pensioner on average (Hungarian Central Statistical Office (2021)³). As a percentage of the average net nominal earnings, the monthly nominal amount of pension benefits has followed a drastic negative trend in the past seven years (from 67,3% to 49,7% between 2014 and 2020). The amount of pensions, benefits, annuities, and other provisions paid as a percentage of the GDP also decreased from 11,6% to 8,6% between 2012 and 2020 (Hungarian Central Statistical Office (2021)).

2.2 The demographic tendencies of Hungary

The PAYG system can perform well if the population's movement is in balance. But suppose the population is aging or there is an imbalance in the population pyramid. These demographic changes undermine the PAYG pension systems' balance because fewer contributors may have to finance an increasing number of old pensioners, and the inter- and intra-generational fairness may hurt. In these cases, the state has to find efficient responses to the serious demographic process.

¹The eligibility period for women is reduced by one year after every child raised in the household for women raising five or more children with a maximum reduction of 7 years.

²The valorisation multiplier is calculated as the growth of nationwide net average earnings.

³HCSO is the abbreviation of the Hungarian Central Statistical Office.

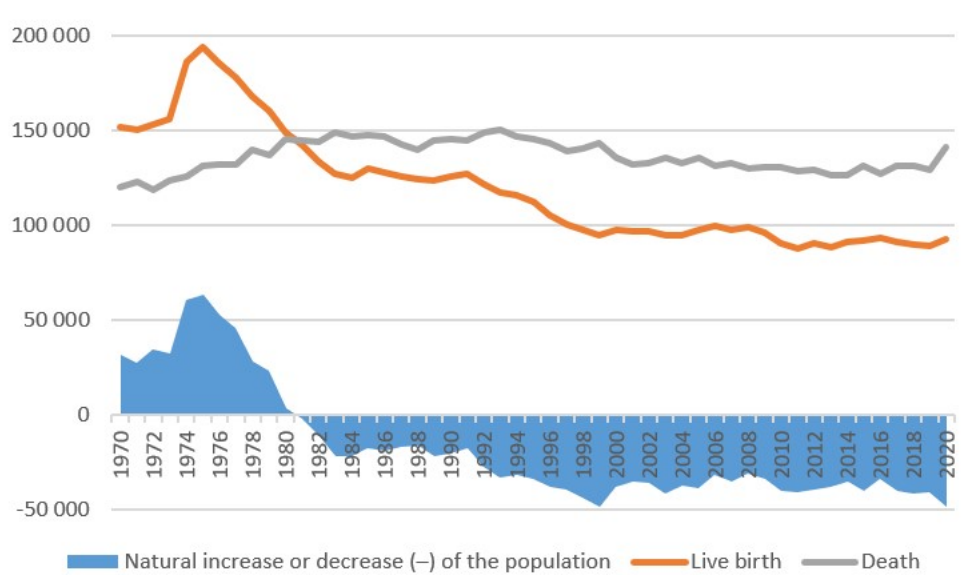


Figure 1: Natural increase or decrease of the population

Source: Own graph based on data of Hungarian Central Statistical Office (2019), Hungarian Central Statistical Office (2021) (downloaded on October 1, 2021)

In terms of fiscal stability, Hungary's pension system faces challenges due to unfavorable demographic changes ahead. In Hungary, there has been a stable population decrease for 40 years, which negative tendency is unique across Europe (see Figure 1). The number of live birth showed a substantial drop during the 90s (from 126000 to around 97000), and once again a stronger fall at the end of the 2000s (from 99000 to around 90000). Since 2010 the number of live birth has been around 90000 (Figure 1).

Meanwhile, the life expectancy at birth has been increasing persistently since 1993 until now if we ignore the Covid-effect of 2020 (Hungarian Central Statistical Office (2019)). This index raised more than five years between 1999 and 2019 (Hungarian Central Statistical Office (2021)). Thanks to these demographic phenomena, the old-age dependency ratio ⁴ is rising constantly; it was 20% in 1990, 25,1% in 2013 and 30,3% in 2020 (Hungarian Central Statistical Office (2021)). The extent of the population ageing in Hungary was a bit favorable than the level of the EU average (32%) in 2020 (Eurostat (2021)) measured by the old-age dependency ratio.

In Hungary, the population dynamics in the past 60 years have led to other serious problems for today. There is a significant imbalance between the size of the different generations thanks to the presence of the Ratkó-children and Ratkó-grandchildren ⁵. But additional salient values in the younger generation we can not experience (see Figure 2). For comparison, some actual data about to Hungarian population represented by Figure 2: on January 1, 2021, the number of people aged 65 and 45 was 144.495 and 181.551, but the exact data for aged 15 was only 98.892, for aged ten was 90.906, and for five-year-old was 92.694 (Hungarian Central Statistical Office (2021)).

⁴Old-age dependency ratio measures the ratio of the population 65 years or over to population 15 to 64 years.

⁵The Ratkó-children were born for the most part between 1953 and 1956 and the Ratkó-grandchildren were born en masse between 1974 and 1979.

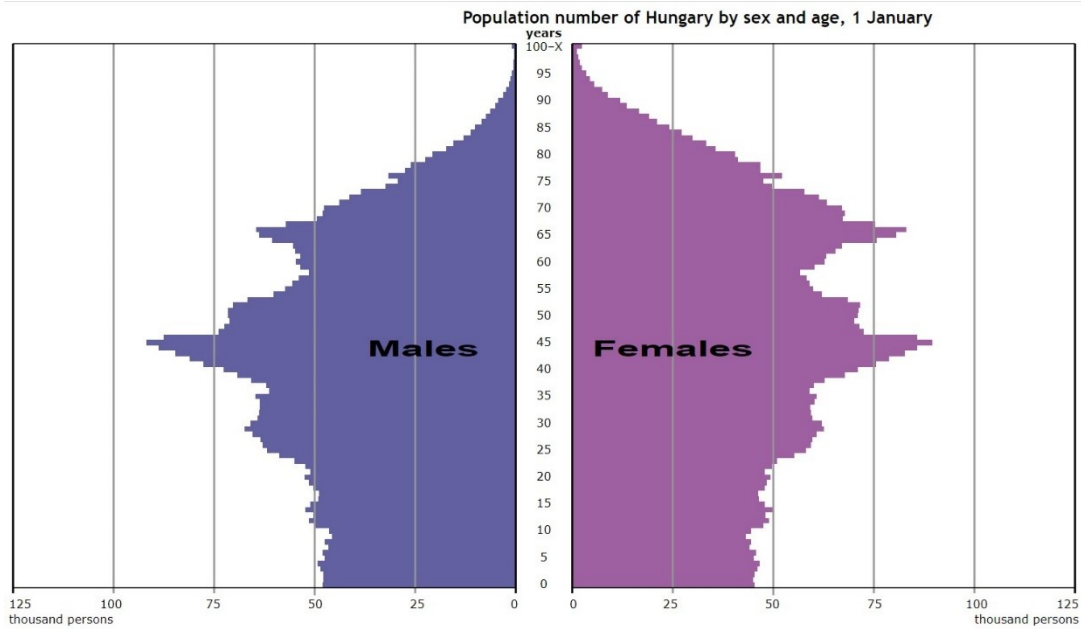


Figure 2: Population pyramid of Hungary, January 1, 2021

Source: Interactive population pyramids of Hungarian Central Statistical Office (2021) (downloaded on October 1, 2021)

Consequently, many childless people and low average child numbers are characteristic of the large Ratkó-grandchildren contrast to the earlier generations. So we can state today that a large number of Ratkó great-grandchildren have been missed (Kapitány and Spéder (2017)). The Ratkó-grandchildren are of working age now and can help to finance the vast number of Ratkó-pensioners. But in 20-25 years later, when the Ratkó-grandchildren will retire, there will be a sustainability gap associated with the fluctuation of the generation size.

3 The childbearing's effect on pensions

3.1 The PAYG pension system's problems related to childbearing

The judgment of a given pension system usually happens along with the following aspects: (i.) sustainability, i.e., the long-run equilibrium between the incomes and the outcomes of the system; (ii.) appropriateness, i.e., the quality of life of the pensioners; (iii.) justice, i.e., the equilibrium between the contribution and the benefit of the different individuals and groups (Vékás (2021)). We highlight here only such viewpoints that what kind of contradictions the traditional PAYG pension system shows related to childbearing.

The PAYG system is highly vulnerable and *not sustainable* if the population is decreasing. It means that the contribution payment capacity of the working generation is lower than the pension promises

for the pensioners (see details in Bajkó et al. (2015), Németh et al. (2020), Freudenberg et al. (2016)).

The system is *inconsistent* because the active, working, and insured population contributes to the sustainment of the PAYG pension system not only with money (pay the contribution rate) but with childbearing (provide the future contributor capacity). (Gál (2017), Banyár (2021))

The computational method of the pensions is *unfair* because the amount of the pension benefits depend on the working years (service time) and the average wages during the lifetime; thus, indirectly, childbearing is punished in a PAYG pension system. Suppose a family brings up more children or uses more effort to rearing them. In that case, the mothers can typically earn fewer lifetime earnings and are entitled to less pension during retirement ages (Regős (2015), Benda (2020), Mihályi (2019)). The cost is especially high, if the mothers are out of the labour market for a long time.

According to Banyár (2021), the present pension system is furthermore *counter-incentive* because it motivates depopulation. For individuals, it is economically irrational to raise children (Mihályi (2019), Banyár (2021)). From now on, we concentrate on the character of the PAYG system that women who have children are at a disadvantage compared to those who are childless (Grimshaw and Rubery (2015)).

3.2 The motherhood pay gap

The Grimshaw (Grimshaw and Rubery (2015)) definition of the motherhood pay gap is that it measures the pay gap between mothers and non-mothers. The relevant literature refers to this wage gap as the "motherhood penalty" (Gough and Noonan (2013)). In many European countries, empirical research confirms that the value of the gap increases as the number of children a woman increases. (Grimshaw and Rubery (2015)).

Ample evidence suggests that motherhood has long-term lifetime costs on the labour market; as a consequence, employed mothers earn less than women without children (Budig and England (2001), Gangl (2009), Grimshaw and Rubery (2015)). Childbearing may disturb women's career opportunities, and the mothers may not go ahead in the pay hierarchy at the same rate. In addition, the human capital depreciates for years after a child, which generates lower lifetime earnings also (Bartus et al. (2013)). According to Meurs et al. (Meurs et al. (2010)), the pay gap between women with and without children is completely explained by human capital differences (Meurs et al. (2010), Wilde et al. (2010)). In sum, because the parents deal with their children, they may be left out of several social interactions and learning processes, which lead to higher wages and career advancement during their lifespan.

The wage penalty for motherhood is significant in many cases among high-income countries, estimated at about 10% and 18% per child for Germany and England (Gangl (2009)) or about 10% per child for Hungary (Cukrowska-Torzewska and Lovász (2020)). Gangl (2009) pointed out that mothers in Europe are more strongly penalized than in the U.S. labor market. Cukrowska-Torzewska et al. (Cukrowska-Torzewska and Lovász (2020)) found the highest motherhood penalties in Eastern European countries among 26 European countries. In these countries, the states provide long leaves, low childcare availability under age 3, and mothers have preferences for within-family care.

In Hungary, the long-term lifetime cost connected to childbearing appears typically by mothers (Makay (2018)). This negative effect on a family's lifetime earning is not offset by the fatherhood wage premium in Hungary (Cukrowska-Torzewska and Lovász (2020)). According to a survey based

on labour market data between 1997-2005, the graduated mothers stay at home with one child 3,2 years, with two children 4,6 years and with three children 6,3 years on average in Hungary; these values for low-skilled women are respectively the following: 4,2, 5,9 and 7,9 years (Bálint and Köllő (2007)). According to Makay (Makay (2018)), up to now, long-lasting childcare has been accepted by children under three years old.

If the mothers stay at home longer with children, it leads to lower work experience and more significant wage gaps. Although there is a negative, detectable correlation between the length of the paid maternity leave and the motherhood penalty, the extent of the wage penalty depends on many other factors also (Buligescu et al. (2009)). In Portugal, there is a relatively short period of leave compared to Hungary. Still, the gaps between the wages of mothers and childless women are almost the same (the raw gap is -0,094 for Portugal and -0,1 for Hungary) (Cukrowska-Torzewska and Lovász (2020) page 16). According to Cukrowska-Torzewska and Lovász (2020) the explanation behind this fact is sought in the higher women's employment in the case of Portugal, but in the case of Hungary, the long maternity leaves and the difficult family-work reconciliation are the explanatory factors.

We can experience significant differences in the extent of the motherhood wage gap according to the mother's educational attainment, age, profession, or income profile also. The wage trajectories diverge sharply after having children for highly educated women, but this phenomenon is not valid for low-skilled women. It follows that the lifetime costs of childbearing, especially early childbearing, are especially high for highly educated women. In other words, delaying childbirth mitigated the motherhood wage gap for mothers with high qualifications (Wilde et al. (2010), Landivar (2020)). These costs may be severe in retirement age if the pension calculation is related to working years and average lifetime wages.

3.3 Pension reforms proposal related to childbearing

There are several different pension system proposals, in which the amount of pension depends on the childbearing or the number of children as well. It is justified, particularly in Hungary, because the long-lasting childcare, the motherhood pay gap, and the PAYG pension system regulations together decrease the expected pensions of women with children.

According to one proposal, the pension calculation should have a so-called child-to-parent based element ("C2P"). It means that the parents would be entitled to an extra pension from their working children's contribution payments. This reform's aim would be to alleviate the two different deficits (population and financial deficit) at the same time (Giday and Szegő (2018), Giday and Szegő (2020)).

Other authors suggest a type of child-related pension system, in which the amount of pension would depend on the number of children the pensioners brought up (among others Demény (2016), Sinn (2005), Hyzl et al. (2004), Regős (2015)). Regős (2015) got a result from his OLG model that such a pension reform could increase fertility also (although the output and consumption per capita and the welfare of the society would decrease).

Banyár (2021) has an idea about a pension system funded with human capital. The pension of agents with children depends on the human capital of the raised, new, working generation. The human capital would be repaid by interest to pensioners who have children. But agents with no or few children even receive a traditionally funded pension.

This paper introduces a new approach to a possible child-related pension system connected to the literature. We called this reform idea a compensational pension system, in which the pensions compensate the time and efforts the parents spent on bringing up children. Agents with children can get an extra amount of pension compared to childless agents, depending on the long-term lifetime cost of their childbearing efforts.

4 Model Settings

Our goal is to build up a dynamic general equilibrium model in an overlapping generations framework (calibrated on the basis of Hungarian data), which is able to evaluate the effect of the newly proposed compensational pension system' effect on the representative agents' lifespan utility, the pension system and the economy as a whole. The benchmark is the present Hungarian PAYG system.

We connect to the OLG literature, which focuses on the welfare and the fertility effects of the PAYG pension system. Typically the main focus of these papers is to quantify and derive the pension system's negative effect on fertility (for example van Groezen et al. (2003), Holler (2007), Regős (2015)). Furthermore, some Hungarian papers model the modification of current pension system with child-related elements as well (Banyár (2021), Regős (2015)).

In our model, output and capital accumulation, and fertility are all endogenous, and we consider heterogeneous agents according to the number of children. Our artificial economy consists of five economic agents: three generations of consumers – young, middle-aged and senior agents –, a representative firm and a fiscal policy decision-maker.

There are many similarities between the three generations of consumers: all of them seek to maximize their utility subject to a budget constraint, all of them receive some income, all of them purchase goods and services. But there are crucial differences as well. The young consumer may or may not have children, if she has children, then she cares for her children, and obtains only wage-income, the middle aged agent receives income from her previous period savings as well, and the senior agent is eligible for pension. A further difference between the young and the middle-aged consumer is that during her employment as a young agent, the consumer accumulates some experience, and as a middle-aged consumer, only the additional effort comes as a cost in terms of utility. Since our main focus is on the number of children in our model, we distinguish the consumer with children from the consumer who has no children.

For the shake of simplicity and an extremely strong focus on the behavior of consumers, we keep the behavior of other agents as simple as possible. The representative firm uses labor and capital to produce its output in a profit-maximizing way, and the fiscal policy maker collects tax revenue to finance its expenditures that includes the pension as well.

The agents of the artificial economy carry out transactions in four markets: in the market for goods and services, in two factor markets and in an asset market.

4.1 Demography

Three decision making generation is living together at every period: the young Y , the middle aged M and the old O . Let $N_{t,Y}$, $N_{t,M}$ and $N_{t,O}$ denote the number of agents in these group respectively, thus the number of the decision making population is given by

$$N_t = N_{t,Y} + N_{t,M} + N_{t,O}$$

for all t .

We assume that a constant n fraction of the young group has children, the average number of children for the child-raising agent is ch_t , γ_1 and γ_2 represent the mortality rate of the young and the middle aged group respectively. Within such terms, we can express the number of all groups for period $t + 1$ in terms of $N_{t,Y}$ as

$$\begin{aligned} N_{t,Y} &= n \cdot ch_{t-1} N_{t-1,Y} \\ N_{t,M} &= (1 - \gamma_1) N_{t-1,Y} \\ N_{t,O} &= (1 - \gamma_2) N_{t-1,M} = (1 - \gamma_2) (1 - \gamma_1) N_{t-2,Y} \\ &= (1 - \gamma_1) (1 - \gamma_2) \frac{1}{n \cdot ch_{t-2}} N_{t-1,Y} \end{aligned}$$

These formulas can be used to determine the two ratios that we use while detrending the aggregate variables.

$$\begin{aligned} \frac{N_{t,M}}{N_{t,Y}} &= \frac{(1 - \gamma_1)}{n \cdot ch_{t-1}} \\ \frac{N_{t,O}}{N_{t,Y}} &= (1 - \gamma_1) (1 - \gamma_2) \frac{1}{n^2 \cdot ch_{t-1} ch_{t-2}} \end{aligned}$$

The following figure illustrates the dynamics of the population. The young population in period t becomes middle aged in period $t + 1$, those who were middle-aged at period t will be senior agent in period $t + 1$ and the children of period t form the young group in period $t + 1$. Unfortunately, there is mortality, so the number of agents in a group at a given period is always lower than it was the number of agents of a one generation younger group at the previous period.

4.2 The Consumer with no Children

The consumer who does not raise children seeks to maximize her life-cycle utility subject to the time series of budget constraints. Her utility depends on the consumption path $c_{t,Y,NCH}$, $c_{t+1,M,NCH}$, $c_{t+2,O,NCH}$ and the effort and skills she is willing to demonstrate in her workplace $l_{t,Y,NCH}$, $l_{t+1,M,NCH}$. Formally:

$$\begin{aligned} U_{NCH} &= \frac{c_{t,Y,NCH}^{1-\sigma_{NCH}}}{1-\sigma_{NCH}} - \Psi_{NCH} \frac{l_{t,Y,NCH}^{1+\eta_{NCH}}}{1+\eta_{NCH}} \\ &+ (1 - \gamma_1) \beta \left(\frac{c_{t+1,M,NCH}^{1-\sigma_{NCH}}}{1-\sigma_{NCH}} - \Psi_{NCH} \frac{(l_{t+1,M,NCH} - b \cdot l_{t,Y,NCH})^{1+\eta_{NCH}}}{1+\eta_{NCH}} \right) \\ &+ (1 - \gamma_1) (1 - \gamma_2) \beta^2 \frac{c_{t+2,O,NCH}^{1-\sigma_{NCH}}}{1-\sigma_{NCH}} \end{aligned}$$

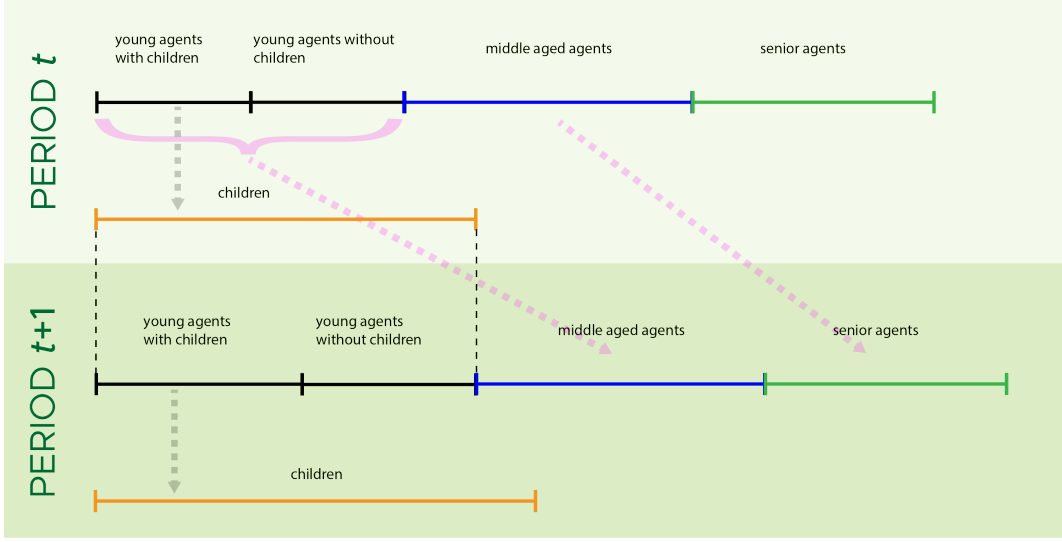


Figure 3: The dynamics of the population over time in our artificial economy.

where γ_1 and γ_2 are the mortality rates between the young and middle aged and the middle aged and senior groups respectively.

By the expression $l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH}$ we wanted to display in the utility function that during her first active period the consumer had already accumulated some experience, and during the second active period, only acquiring additional skills incurs some additional cost.

The budget constraints that affect the whole utility maximizing problem express, that the consumer receives some income - during the first period this income is only the labor income, in the second period this is complemented by the return on previous savings, and in the senior period she receives pension - and spends her income on purchasing goods and services and buying assets. The constraints are written as

$$\begin{aligned}
 (1 - \tau_{L,t}) w_t l_{t,Y,NCH} + profit_{t,Y,NCH} &= c_{t,Y,NCH} + s_{t+1,Y,NCH} \\
 (1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,NCH} \\
 + (1 + r_{t+1}) s_{t+1,Y,NCH} + profit_{t+1,M,NCH} &= c_{t+1,M,NCH} + s_{t+1,M,NCH} \\
 pension_{t+2,NCH} + (1 + r_{t+2}) s_{t+1,M,NCH} + profit_{t+2,O,NCH} &= c_{t+2,O,NCH}
 \end{aligned}$$

where τ_L is the tax rate, social security contribution included.

Solving this optimization problem leads us to the following behavioral equations

$$\begin{aligned}
 \Psi_{NCH} l_{t,Y,NCH}^\eta &= (1 - \gamma_1) \beta \Psi_{NCH} (l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH})^\eta h \\
 &\quad + c_{t,Y,NCH}^{-\sigma} (1 - \tau_{L,t}) w_t \\
 \Psi_{NCH} (l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH})^\eta &= c_{t+1,M,NCH}^{-\sigma} (1 - \tau_{L,t+1}) w_{t+1} \\
 c_{t,Y,NCH}^{-\sigma} &= (1 - \gamma_1) \beta c_{t+1,M,NCH}^{-\sigma} (1 + r_{t+1}) \\
 c_{t+1,M,NCH}^{-\sigma} &= (1 - \gamma_2) \beta c_{t+2,O,NCH}^{-\sigma} (1 + r_{t+2}) \\
 (1 - \tau_{L,t}) w_t l_{t,Y,NCH} + profit_{t,Y,NCH} &= c_{t,Y,NCH} + s_{t+1,Y,NCH}
 \end{aligned}$$

$$\begin{aligned}
& (1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,NCH} \\
& + (1 + r_{t+1}) s_{t+1,Y,NCH} + profit_{t+1,M,NCH} = c_{t+1,M,NCH} + s_{t+1,M,NCH} \\
& \quad pension_{t+2,NCH} \\
& + (1 + r_{t+2}) s_{t+1,M,NCH} + profit_{t+2,O,NCH} = c_{t+2,O,NCH}
\end{aligned}$$

At given prices and exogenous variables these equations set the optimal values of the following endogenous variables: $c_{t,Y,NCH}$, $c_{t+1,M,NCH}$, $c_{t+2,O,NCH}$, $l_{t,Y,NCH}$, $l_{t+1,M,NCH}$, $s_{t+1,Y}$ and $s_{t+2,M}$.

4.3 The Consumer with Children

Although there are similarities – both agents seek to maximize their life-cycle utility subject to budget constraints –, the problem of the consumer with children differs in many ways from that of the consumer without any children.

The pure existence of children affects the utility of the agent. Any increase in the number of children raises the utility, although the additional unit of children causes the value of the objective function to increase less and less, so the number of children is displayed in the utility function, but the marginal utility is decreasing.

Having children requires two types of expenditures (or at least under our extremely simple model settings, it requires only two types of expenditures): goods and services and time. Time appears in the utility function as a leisure decreasing factor, the extra spending on goods and services affects the first period budget constraint.

In light of the above considerations the objective function of the agent is written as

$$\begin{aligned}
U_{CH} &= \frac{c_{t,Y,CH}^{1-\sigma_{CH}}}{1-\sigma_{CH}} + \frac{ch_t^{1-\nu}}{1-\nu} - \Psi_{CH} \frac{(l_{t,Y,CH} + twch_t)^{1+\eta_{CH}}}{1+\eta_{CH}} \\
&+ (1-\gamma_1) \beta \left(\frac{c_{t+1,M,CH}^{1-\sigma_{CH}}}{1-\sigma_{CH}} - \Psi_{CH} \frac{(l_{t+1,M,CH} - b \cdot l_{t,Y,CH})^{1+\eta_{CH}}}{1+\eta_{CH}} \right) \\
&+ (1-\gamma_1)(1-\gamma_2) \beta^2 \frac{c_{t+2,O,CH}^{1-\sigma_{CH}}}{1-\sigma_{CH}}
\end{aligned}$$

where ch_t is the number of children and $twch_t$ indicates the amount of time spent with children. This is the time, the young-aged population used for childbearing totally. It is rather reasonable to assume, that this latter variable depends on the number of children. We assume the following functional relationship:

$$twch_t = a_{CH} ch_t^\gamma$$

The consumer is forced to solve her problem subject to the following budget constraints

$$\begin{aligned}
(1 - \tau_{L,t}) w_t l_{t,Y,CH} + profit_{t,Y,CH} &= (1 + x \cdot ch_t) c_{t,Y,CH} + s_{t+1,Y,CH} \\
(1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,CH} \\
+ (1 + r_{t+1}) s_{t+1,Y,CH} + profit_{t+1,M,CH} &= c_{t+1,M,CH} + s_{t+1,M,CH} \\
pension_{t+2,CH} + (1 + r_{t+2}) s_{t+1,M,CH} + profit_{t,O,CH} &= c_{t+2,O,CH}
\end{aligned}$$

where x represents the ratio of the consumption of a child to the consumption of a young adult.

Under such conditions the behavioral equations of a consumer who raises children are

$$\begin{aligned}
\Psi_{CH}(l_{t,Y,CH} + twch_t)^\eta &= (1 - \gamma_1) \beta \Psi_{CH}(l_{t+1,M,CH} - h \cdot l_{t,Y,CH})^\eta h \\
&\quad + \frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} (1 - \tau_{L,t}) w_t \\
\Psi_{CH}(l_{t+1,M,CH} - h \cdot l_{t,Y,CH})^\eta &= c_{t+1,M,CH}^{-\sigma} (1 - \tau_{L,t+1}) w_{t+1} \\
ch_t^{-\nu} &= \frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} x \cdot c_{t,Y,CH} \\
&\quad + \Psi_{CH}(l_{t,Y,CH} + twch_t)^\eta \gamma \cdot a_{CH} ch_t^{\gamma-1} \\
\frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} &= (1 - \gamma_1) \beta c_{t+1,M,CH}^{-\sigma} (1 + r_{t+1}) \\
c_{t+1,M,CH}^{-\sigma} &= (1 - \gamma_2) \beta c_{t+2,O,CH}^{-\sigma} (1 + r_{t+2}) \\
(1 - \tau_{L,t}) w_t l_{t,Y,CH} + profit_{t,Y,CH} &= (1 + x \cdot ch_t) c_{t,Y,CH} + s_{t+1,Y,CH} \\
(1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,CH} & \\
+ (1 + r_{t+1}) s_{t+1,Y,CH} + profit_{t+1,M,CH} &= c_{t+1,M,CH} + s_{t+1,M,CH} \\
pension_{t+2,CH} & \\
+ (1 + r_{t+2}) s_{t+1,M,CH} + profit_{t+2,O,CH} &= c_{t+2,O,CH} \\
twch_t &= a_{CH} ch_t^\gamma
\end{aligned}$$

These 9 equations set the optimal value for the following nine endogenous variables: $c_{t,Y,CH}$, $c_{t+1,M,CH}$, $c_{t+2,O,CH}$, $l_{t,Y,CH}$, $l_{t+1,M,CH}$, ch_t , $twch_t$, $s_{t+1,Y,CH}$ and $s_{t+2,M,CH}$

4.4 The Representative Firm

In this artificial economy the representative firm combines capital and labor to produce its output. The technological process behind this transformation is represented by a Cobb-Douglas production function

$$Y_t = a_Y K_{t,Y}^{\alpha_Y} L_{t,Y}^{1-\alpha_Y}$$

Since the firm is the owner of capital thus it is the firm's responsibility to purchase additional capital and replace the worn-out factors. The law of motion equation for capital

$$I_t = K_{t+1} - (1 - \delta) K_t$$

The firm uses labor and capital up to the point where the output and the amount of factors maximize its profit, where the profit function is given as

$$\text{PROFIT}_t = Y_t - I_t - w_t L_t$$

It is worth to mention here that a certain fraction of the profit goes to young economic operators, a certain fraction to middle-aged agents and the senior agents also receive a certain fraction of the profit.

$$\text{profit}_{t,Y,NCH} = \frac{h_{Y,NCH}}{N_{t,Y,NCH}} \text{PROFIT}_t$$

$$\begin{aligned}
\text{profit}_{t,M,NCH} &= \frac{h_{M,NCH}}{N_{t,M,NCH}} \text{PROFIT}_t \\
\text{profit}_{t,O,NCH} &= \frac{h_{O,NCH}}{N_{t,O,NCH}} \text{PROFIT}_t \\
\text{profit}_{t,Y,CH} &= \frac{h_{Y,CH}}{N_{t,Y,CH}} \text{PROFIT}_t \\
\text{profit}_{t,M,CH} &= \frac{h_{M,CH}}{N_{t,M,CH}} \text{PROFIT}_t \\
\text{profit}_{t,O,CH} &= \frac{h_{O,CH}}{N_{t,O,CH}} \text{PROFIT}_t
\end{aligned}$$

where $h_{Y,NCH} + h_{M,NCH} + h_{O,NCH} + h_{Y,CH} + h_{M,CH} + h_{O,CH} = 1$

By solving the profit maximization problem subject to the production function and the investment function we are able to express the formulas that drive the behavior of the representative firm

$$\begin{aligned}
K_t &= \alpha \frac{Y_t}{r_t^K} \\
L_t &= (1 - \alpha) \frac{Y_t}{w_t} \\
r_{t+1}^K + (1 - \delta) &= 1 + r_{t+1} \\
I_t &= K_{t+1} - (1 - \delta) K_t \\
Y_t &= a_Y K_{t,Y}^{\alpha_Y} L_{t,Y}^{1-\alpha_Y} \\
\text{PROFIT}_t &= Y_t - I_t - w_t L_t
\end{aligned}$$

4.5 The Fiscal Policy Decision Maker

The fiscal policy decision maker collects taxes and social security contributions, accumulates debt, spends on goods and services, provides the elderly with pensions and pays interest on outstanding debt. Thus this agent budget constraint takes the following form:

$$\begin{aligned}
\tau_{t,L} w_t L_t + D_{t+1} &= G_t + N_{t,O,CH} \text{pension}_{t,O,CH} + N_{t,O,NCH} \text{pension}_{t,O,NCH} \\
&+ (1 + r_t) D_t
\end{aligned}$$

4.6 Market clearing conditions

Equilibrium occurs in the market for goods and services. The total output Y_t equals the sum of the total consumption, investment and government spending. Formally

$$\begin{aligned}
Y_t &= n \cdot N_{t,Y} (1 + x \cdot ch_t) c_{t,Y,CH} + (1 - n) N_{t,Y} c_{t,Y,NCH} \\
&+ n \cdot N_{t,M} c_{t,M,CH} + (1 - n) N_{t,M} c_{t,M,NCH} \\
&+ n \cdot N_{t,O} c_{t,O,CH} + (1 - n) N_{t,O} c_{t,O,NCH} \\
&+ I_t + G_t
\end{aligned}$$

In the input markets the demand for the input equals the total supply of that factor. Four types of agents supply these kind of factors - aka labor and capital - thus these equilibrium conditions are given by

$$L_t = n \cdot N_{t,Y} l_{t,Y,CH} + (1 - n) N_{t,Y} l_{t,Y,NCH} \\ + n \cdot N_{t,M} l_{t,M,CH} + (1 - n) \cdot N_{t,M} l_{t,M,NCH}$$

Finally, in the financial asset market the supply of those assets equals the demand for those assets, so the total debt of the social security system is financed by the savings of the consumers. Formally

$$D_{t+1} = n \cdot N_{t,Y} s_{t+1,Y,CH} + (1 - n) N_{t,Y} s_{t+1,Y,NCH} \\ + n \cdot N_{t,M} s_{t+1,M,CH} + (1 - n) \cdot N_{t,M} s_{t+1,M,NCH}$$

4.7 The Model

The model consists of behavioral equations and market clearing conditions. We list the equations of the formal model in Appendix A. These 29 formulas determine the path of the following 29 variables: $y_t, i_t, l_t, k_{t+1}, c_{t,Y,CH}, c_{t,M,CH}, c_{t,O,CH}, c_{t,Y,NCH}, c_{t,M,NCH}, c_{t,O,NCH}, s_{t,Y,CH}, s_{t,M,CH}, s_{t,Y,NCH}, s_{t,M,NCH}, l_{t,Y,CH}, l_{t,M,CH}, l_{t,Y,NCH}, l_{t,M,NCH}, profit_t, profit_{t,Y,CH}, profit_{t,M,CH}, profit_{t,O,CH}, profit_{t,Y,NCH}, profit_{t,M,NCH}, profit_{t,O,NCH}, w_t, r_t^K, 1 + r_t$ and d_{t+1} .

5 Data and Calibration

There are several standard parameters in our model that we can take from the modeling literature, but most of the parameters are calibrated on Hungarian data from HCSO and from the national accounts data of Eurostat. We also relied heavily on the European Commission's 2018 Aging Report, especially on the appendix related to the Hungarian pension projections European Commission (2018).

To get the targeted paths of the consumption per GDP, investment per GDP, government spending per GDP, and employment data, we used the following parameters

Parameter	Value	Source
σ	2	Standard in the literature.
η	0.76	Standard in the literature
β	0.3314	To get the 1.05 ²⁰ real interest rate in steady state.

δ	0.8319	To obtain the targeted capita-output ratio.
a	1	Standard in the literature.
a_{CH}	0.46	To get the tartgeted labor-time with children ratio
γ	0.2	To get the tartgeted labor-time with children ratio
γ_1	0.00127	Table 6.2.1. Hungarian Demographic Yearbook (2019)
γ_2	0.0141	Table 6.2.1. Hungarian Demographic Yearbook (2019)
b	0.8	To obtain the targeted labor market structure.
$h_{Y,NCH}$	0.1666	We assume that the all groups receive equal share from the profit of the firm
$h_{Y,CH}$	0.1666	We assume that the all groups receive equal share from the profit of the firm
$h_{M,NCH}$	0.1666	We assume that the all groups receive equal share from the profit of the firm
$h_{M,CH}$	0.1666	We assume that the all groups receive equal share from the profit of the firm
$h_{O,NCH}$	0.1666	We assume that the all groups receive equal share from the profit of the firm
$h_{O,CH}$	0.1666	We assume that the all groups receive equal share from the profit of the firm
n	0.4	Table 1.2.8. Hungarian Demographic Yearbook (2019)
ν	1.5	

$\Psi_{Y,CH}$	0.3744	Table 14.1.1.13. (HCSO (2021))
$\Psi_{M,CH}$	0.9525	Table 14.1.1.13. (HCSO (2021))
$\Psi_{Y,NCH}$	0.2825	Table 14.1.1.13. (HCSO (2021))
$\Psi_{M,NCH}$	0.62	Table 14.1.1.13. (HCSO (2021))
τ	0.5752	Actual Hungarian regulations
x	0.5	Gál and Márton (2019)

6 Results

Before we present our result a disclaimer alert: we have used a calibrated version of the model, so – according to Fabio Canova Canova (2007) – we undertook a "computational experiment" rather than creating an environment where we were able to test the parameters on observed data. So we used "economic" rather than "statistic" criteria to get the parameters.

The following table summarizes the effect of an introduction of the compensation scheme, that is of changing the algorithm behind the pension computation from

$$\text{pension}_{i,t} = \nu \frac{w_{t-2} l_{Y,i,t-2} + w_{t-1} l_{M,i,t-1}}{2}$$

where $i = CH, NCH$, to

$$\text{pension}_{t,CH} = \nu \frac{w_{t-2} (l_{Y,CH,t-2} + twch_{t-2}) + w_{t-1} l_{M,CH,t-1}}{2}$$

while there is no change in the computation of the pension of the agent who does not raise children.

Here we have displayed only the change in the main aggregates, the complete list can be found in Appendix B.

Macroaggregate	Change in the variable
Per capita output	-0.0665%
Pension of the consumer who has no children	+0.3651%
Pension of the consumer who has children	+9.7887%

Time spent with the children	+0.0443%
Number of children	+0.2218%
Total amount of labor	-0.2921%
Amount of capital	+0.5581%
Wage	+0.2257%
Rental rate of capital	-0.4004%

The main result of our computational experiment is that although the introduction of a compensation scheme slightly increases the number of children and the time spent with children, but these positive elements that could have raised the output does not compensate the negative effect of the decreasing work-related efforts, so the output falls.

In our economy, the additional income at her third life stage increases the present value of the life-cycle income of the agent who has children and motivates her to raise her expenditures on goods and services she values according to her utility function: consumption, children, and leisure. Since the compensation depends on the time spent with children, this scheme increases the marginal benefit of having children that raises the number of children this agent intends to have. But the increasing number of children leads to an increase in the time spent with children, which the consumer considers as an additional cost while determining the optimal amount of labor supply. Her labor supply falls. The increase in her consumption and the decrease in her labor supply cause her savings to decrease.

Since the falling labor supply leads to an increase in the real wage, the consumer who has no children increases her work-related efforts securing a higher per-period income, which enables a larger first period consumption and a larger first period savings relative to the benchmark. The focus of the change in the pension-computation algorithm is on the other agent thus there is just a minimal change in her life-cycle income and this is behind the result that the change in income is mostly reflected in the increase of the savings. The higher savings enables the consumer to finance larger capital accumulation. The amount of capital provided by the consumers increases, but the demand for it decreases (since the aggregate work-related effort falls), which leads to a decrease in the real rental rate of capital.

The most important element is the utility of the consumers. We have used

$$\begin{aligned}
U_{NCH} &= \frac{c_{t,Y,NCH}^{1-\sigma_{NCH}}}{1-\sigma_{NCH}} - \Psi_{NCH} \frac{l_{t,Y,NCH}^{1+\eta_{NCH}}}{1+\eta_{NCH}} \\
&+ (1-\gamma_1) \beta \left(\frac{c_{t+1,M,NCH}^{1-\sigma_{NCH}}}{1-\sigma_{NCH}} - \Psi_{NCH} \frac{(l_{t+1,M,NCH} - b \cdot l_{t,Y,NCH})^{1+\eta_{NCH}}}{1+\eta_{NCH}} \right) \\
&+ (1-\gamma_1)(1-\gamma_2) \beta^2 \frac{c_{t+2,O,NCH}^{1-\sigma_{NCH}}}{1-\sigma_{NCH}}
\end{aligned}$$

for the consumer who has no children and

$$\begin{aligned}
U_{CH} = & \frac{c_{t,Y,CH}^{1-\sigma_{CH}}}{1-\sigma_{CH}} + \Phi \frac{ch_t^{1-\nu}}{1-\nu} - \Psi_{CH} \frac{(l_{t,Y,CH} + twch_t)^{1+\eta_{CH}}}{1+\eta_{CH}} \\
& + (1-\gamma_1)\beta \left(\frac{c_{t+1,M,CH}^{1-\sigma_{CH}}}{1-\sigma_{CH}} - \Psi_{CH} \frac{(l_{t+1,M,CH} - b \cdot l_{t,Y,CH})^{1+\eta_{CH}}}{1+\eta_{CH}} \right) \\
& + (1-\gamma_1)(1-\gamma_2)\beta^2 \frac{c_{t+2,O,CH}^{1-\sigma_{CH}}}{1-\sigma_{CH}}
\end{aligned}$$

for a consumer who raises some.

Based on our computational experiment the introduction of the compensation scheme decreases the steady state utility of the consumer who does not have children by 0,0130% percent, but increases the utility of the consumer who has children by 0.2149% percent.

7 Conclusion

There is increasing attention to the sustainability and fairness of the pay-as-you-go pension system as a consequence of the aging society and the imbalance between the old and the young generation's number. In Hungary, these demographic problems appear at once. In twenty years, numerous generations will retire and will leave behind a relatively low number of children.

The PAYG pension system can not be fair to agents who have children along with such demographic trends. In this system, the pension depends only on the previous contribution, so it indirectly punishes childbearing. As we see, in Hungary, the long-term lifetime cost connected to childbearing, the so-called motherhood pay gap is significant and causes severe costs in retirement age. Because of this fact, many different child-related pension reform proposals were introduced in the Hungarian pension literature. In this paper, we presented a very new, child-related compensational pension scheme, in which the pensions compensate the forgone labor time and efforts which is a result of childbearing.

We built up a dynamic general equilibrium model in an overlapping generations framework (calibrated on the basis of Hungarian data) to investigate the effects of our pension reform proposal. We compared the effects of the present PAYG system to the child-related compensational pension schemes, where the amount of pension takes into account the childbearing time. The evaluation of the pension systems were based on the lifespan utility of representative agents (with or without children) and the economic effects of the pension reform.

Our reform proposal reached its primary goal. The pension system recommended by us could increase the utility of the consumer who has children but decrease to a lesser extent the steady state utility of childless consumers. The compensational pension system would have additional positive effects on the amount of children and the time spent with children, but these positive elements that could have raised the output does not compensate the negative effect of the decreasing work-related efforts, so overall the output would fall.

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A The Formal Model

Our artificial economy is driven by the following behavioral equations and market clearing conditions:

$$\begin{aligned}
\Psi_{NCH} l_{t,Y,NCH}^\eta &= (1 - \gamma_1) \beta \Psi_{NCH} (l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH})^\eta h \\
&\quad + c_{t,Y,NCH}^{-\sigma} (1 - \tau_{L,t}) w_t \\
\Psi_{NCH} (l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH})^\eta &= c_{t+1,M,NCH}^{-\sigma} (1 - \tau_{L,t+1}) w_{t+1} \\
c_{t,Y,NCH}^{-\sigma} &= (1 - \gamma_1) \beta c_{t+1,M,NCH}^{-\sigma} (1 + r_{t+1}) \\
c_{t+1,M,NCH}^{-\sigma} &= (1 - \gamma_2) \beta c_{t+2,O,NCH}^{-\sigma} (1 + r_{t+2}) \\
(1 - \tau_{L,t}) w_t l_{t,Y,NCH} & \\
+ profit_{t,Y,NCH} &= c_{t,Y,NCH} + s_{t+1,Y,NCH} \\
(1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,NCH} & \\
+ (1 + r_{t+1}) s_{t+1,Y,NCH} + profit_{t+1,M,NCH} &= c_{t+1,M,NCH} + s_{t+1,M,NCH} \\
pension_{t+2,NCH} + (1 + r_{t+2}) s_{t+1,M,NCH} + profit_{t+2,O,NCH} &= c_{t+2,O,NCH} \\
\Psi_{CH} (l_{t,Y,CH} + twch_t)^\eta &= (1 - \gamma_1) \beta \Psi_{CH} (l_{t+1,M,CH} - h \cdot l_{t,Y,CH})^\eta h \\
&\quad + \frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} (1 - \tau_{L,t}) w_t \\
\Psi_{CH} (l_{t+1,M,CH} - h \cdot l_{t,Y,CH})^\eta &= c_{t+1,M,CH}^{-\sigma} (1 - \tau_{L,t+1}) w_{t+1} \\
ch_t^{-\nu} &= \frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} x \cdot c_{t,Y,CH} \\
&\quad + \Psi_{CH} (l_{t,Y,CH} + twch_t)^\eta \gamma \cdot a_{CH} ch_t^{\gamma-1} \\
\frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} &= (1 - \gamma_1) \beta c_{t+1,M,CH}^{-\sigma} (1 + r_{t+1}) \\
c_{t+1,M,CH}^{-\sigma} &= (1 - \gamma_2) \beta c_{t+2,O,CH}^{-\sigma} (1 + r_{t+2}) \\
(1 - \tau_{L,t}) w_t l_{t,Y,CH} & \\
+ profit_{t,Y,CH} &= (1 + x \cdot ch_t) c_{t,Y,CH} + s_{t+1,Y,CH} \\
(1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,CH} & \\
+ (1 + r_{t+1}) s_{t+1,Y,CH} + profit_{t+1,M,CH} &= c_{t+1,M,CH} + s_{t+1,M,CH} \\
pension_{t+2,CH} + (1 + r_{t+2}) s_{t+1,M,CH} + profit_{t,O,CH} &= c_{t+2,O,CH} \\
twch_t &= a_{CH} ch_t^\gamma \\
K_t &= \alpha \frac{Y_t}{r_t^K} \\
L_t &= (1 - \alpha) \frac{Y_t}{w_t} \\
r_{t+1}^K + (1 - \delta) &= 1 + r_{t+1} \\
I_t &= K_{t+1} - (1 - \delta) K_t
\end{aligned}$$

$$\begin{aligned}
Y_t &= a_Y K_{t,Y}^{\alpha_Y} L_{t,Y}^{1-\alpha_Y} \\
PROFIT_t &= Y_t - I_t - w_t L_t
\end{aligned}$$

$$\begin{aligned}
\text{profit}_{t,Y,NCH} &= \frac{h_{Y,NCH}}{N_{t,Y,NCH}} \text{PROFIT}_t \\
\text{profit}_{t,M,NCH} &= \frac{h_{M,NCH}}{N_{t,M,NCH}} \text{PROFIT}_t \\
\text{profit}_{t,O,NCH} &= \frac{h_{O,NCH}}{N_{t,O,NCH}} \text{PROFIT}_t \\
\text{profit}_{t,Y,CH} &= \frac{h_{Y,CH}}{N_{t,Y,CH}} \text{PROFIT}_t \\
\text{profit}_{t,M,CH} &= \frac{h_{M,CH}}{N_{t,M,CH}} \text{PROFIT}_t \\
\text{profit}_{t,O,CH} &= \frac{h_{O,CH}}{N_{t,O,CH}} \text{PROFIT}_t \\
\tau_{t,L} w_{t,L} L_t + D_{t+1} &= G_t + N_{t,O,CH} \text{pension}_{t,O,CH} + N_{t,O,NCH} \text{pension}_{t,O,NCH} \\
&\quad + (1 + r_t) D_t \\
Y_t &= n \cdot N_{t,Y} (1 + x \cdot ch_t) c_{t,Y,CH} + (1 - n) N_{t,Y} c_{t,Y,NCH} \\
&\quad + n \cdot N_{t,M} c_{t,M,CH} + (1 - n) N_{t,M} c_{t,M,NCH} \\
&\quad + n \cdot N_{t,O} c_{t,O,CH} + (1 - n) N_{t,O} c_{t,M,NCH} \\
&\quad + I_t + G_t \\
L_t &= n \cdot N_{t,Y} l_{t,Y,CH} + (1 - n) N_{t,Y} l_{t,Y,NCH} \\
&\quad + n \cdot N_{t,M} l_{t,M,CH} + (1 - n) \cdot N_{t,M} l_{t,M,NCH} \\
D_{t+1} &= n \cdot N_{t,Y} s_{t+1,Y,CH} + (1 - n) N_{t,Y} s_{t+1,Y,NCH} \\
&\quad + n \cdot N_{t,M} s_{t+1,M,CH} + (1 - n) \cdot N_{t,M} s_{t+1,M,NCH}
\end{aligned}$$

Since there is an element, that changes over time (the number of agents) we must detrend all formulas that include the number of some agent group or some aggregate variable.

The detrended model

$$\begin{aligned}
\Psi_{NCH} l_{t,Y,NCH}^\eta &= (1 - \gamma_1) \beta \Psi_{NCH} (l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH})^\eta h \\
&\quad + c_{t,Y,NCH}^{-\sigma} (1 - \tau_{L,t}) w_t \\
\Psi_{NCH} (l_{t+1,M,NCH} - h \cdot l_{t,Y,NCH})^\eta &= c_{t+1,M,NCH}^{-\sigma} (1 - \tau_{L,t+1}) w_{t+1} \\
c_{t,Y,NCH}^{-\sigma} &= (1 - \gamma_1) \beta c_{t+1,M,NCH}^{-\sigma} (1 + r_{t+1}) \\
c_{t+1,M,NCH}^{-\sigma} &= (1 - \gamma_2) \beta c_{t+2,O,NCH}^{-\sigma} (1 + r_{t+2}) \\
(1 - \tau_{L,t}) w_t l_{t,Y,NCH} & \\
+ \text{profit}_{t,Y,NCH} &= c_{t,Y,NCH} + s_{t+1,Y,NCH} \\
(1 - \tau_{L,t+1}) w_{t+1} l_{t+1,M,NCH} & \\
+ (1 + r_{t+1}) s_{t+1,Y,NCH} + \text{profit}_{t+1,M,NCH} &= c_{t+1,M,NCH} + s_{t+1,M,NCH} \\
\text{pension}_{t+2,NCH} + (1 + r_{t+2}) s_{t+1,M,NCH} + \text{profit}_{t+2,O,NCH} &= c_{t+2,O,NCH} \\
\Psi_{CH} (l_{t,Y,CH} + twch_t)^\eta &= (1 - \gamma_1) \beta \Psi_{CH} (l_{t+1,M,CH} - h \cdot l_{t,Y,CH})^\eta h \\
&\quad + \frac{1}{1 + x \cdot ch_t} c_{t,Y,CH}^{-\sigma} (1 - \tau_{L,t}) w_t \\
\Psi_{CH} (l_{t+1,M,CH} - h \cdot l_{t,Y,CH})^\eta &= c_{t+1,M,CH}^{-\sigma} (1 - \tau_{L,t+1}) w_{t+1}
\end{aligned}$$

$$\begin{aligned}
ch_t^{-\nu} &= \frac{1}{1+x \cdot ch_t} c_{t,Y,CH}^{-\sigma} x \cdot c_{t,Y,CH} \\
&\quad + \Psi_{CH} (l_{t,Y,CH} + twch_t)^n \gamma \cdot a_{CH} ch_t^{\gamma-1} \\
\frac{1}{1+x \cdot ch_t} c_{t,Y,CH}^{-\sigma} &= (1-\gamma_1) \beta c_{t+1,M,CH}^{-\sigma} (1+r_{t+1}) \\
c_{t+1,M,CH}^{-\sigma} &= (1-\gamma_2) \beta c_{t+2,O,CH}^{-\sigma} (1+r_{t+2}) \\
(1-\tau_{L,t}) w_t l_{t,Y,CH} & \\
+ profit_{t,Y,CH} &= (1+x \cdot ch_t) c_{t,Y,CH} + s_{t+1,Y,CH} \\
(1-\tau_{L,t+1}) w_{t+1} l_{t+1,M,CH} & \\
+(1+r_{t+1}) s_{t+1,Y,CH} + profit_{t+1,M,CH} &= c_{t+1,M,CH} + s_{t+1,M,CH} \\
pension_{t+2,CH} + (1+r_{t+2}) s_{t+1,M,CH} + profit_{t,O,CH} &= c_{t+2,O,CH} \\
twch_t &= a_{CH} ch_t^\gamma \\
k_t \frac{1}{n \cdot ch_{t-1}} &= \alpha \frac{y_t}{r_t^K} \\
l_t &= (1-\alpha) \frac{y_t}{w_t} \\
i_t &= k_{t+1} - (1-\delta) k_t \frac{1}{n \cdot ch_{t-1}} \\
r_{t+1}^K + (1-\delta) &= 1 + r_{t+1} \\
y_t &= a_Y \left(k_t \frac{1}{n \cdot ch_{t-1}} \right)^{\alpha_Y} l_{t,Y}^{1-\alpha_Y} \\
profit_t &= y_t - i_t - w_t l_t \\
profit_{t,Y,NCH} &= \frac{h_{Y,NCH}}{1-n} profit_t \\
profit_{t,M,NCH} &= \frac{h_{M,NCH} n \cdot ch_{t-1}}{(1-n)(1-\gamma_1)} profit_t \\
profit_{t,O,NCH} &= \frac{h_{O,NCH} n^2 \cdot ch_{t-1} ch_{t-2}}{(1-n)(1-\gamma_1)(1-\gamma_2)} profit_t \\
profit_{t,Y,CH} &= \frac{h_{Y,CH}}{n} profit_t \\
profit_{t,M,CH} &= \frac{h_{M,CH} n \cdot ch_{t-1}}{n(1-\gamma_1)} profit_t \\
profit_{t,O,CH} &= \frac{h_{O,CH} n^2 \cdot ch_{t-1} ch_{t-2}}{n(1-\gamma_1)(1-\gamma_2)} profit_t
\end{aligned}$$

$$\begin{aligned}
\tau_{t,L} w_{t,L} l_t + d_{t+1} &= g_t + n(1-\gamma_1)(1-\gamma_2) \frac{1}{n^2 \cdot ch_{t-1} ch_{t-2}} pension_{t,O,CH} \\
&\quad + (1-n)(1-\gamma_1)(1-\gamma_2) \frac{1}{n^2 \cdot ch_{t-1} ch_{t-2}} pension_{t,O,NCH} \\
&\quad + (1+r_t) d_t \frac{1}{n \cdot ch_{t-1}} \\
y_t &= n(1+x \cdot ch_t) c_{t,Y,CH} + (1-n) c_{t,Y,NCH}
\end{aligned}$$

$$\begin{aligned}
& + n \cdot \frac{(1-\gamma_1)}{n \cdot ch_{t-1}} c_{t,M,CH} + (1-n) \frac{(1-\gamma_1)}{n \cdot ch_{t-1}} c_{t,M,NCH} \\
& + n \cdot (1-\gamma_1)(1-\gamma_2) \frac{1}{n^2 \cdot ch_{t-1} ch_{t-2}} c_{t,O,CH} \\
& + (1-n)(1-\gamma_1)(1-\gamma_2) \frac{1}{n^2 \cdot ch_{t-1} ch_{t-2}} c_{t,M,NCH} \\
& + i_t + g_t \\
l_t & = n \cdot l_{t,Y,CH} + (1-n) l_{t,Y,NCH} \\
& + n \cdot \frac{(1-\gamma_1)}{n \cdot ch_{t-1}} l_{t,M,CH} + (1-n) \cdot \frac{(1-\gamma_1)}{n \cdot ch_{t-1}} l_{t,M,NCH} \\
d_{t+1} & = n \cdot s_{t+1,Y,CH} + (1-n) s_{t+1,Y,NCH} \\
& + n \cdot \frac{(1-\gamma_1)}{n \cdot ch_{t-1}} s_{t+1,M,CH} + (1-n) \cdot \frac{(1-\gamma_1)}{n \cdot ch_{t-1}} s_{t+1,M,NCH}
\end{aligned}$$

A.1 Steady State

Along balanced growth path the transformed economy is in steady state and all endogenous (de-trended) variables are constant. Formally it means that the t indexes do not include further information they can be eliminated.

$$\begin{aligned}
\Psi_{NCH} l_{Y,NCH}^\eta & = (1-\gamma_1) \beta \Psi_{NCH} (l_{M,NCH} - h \cdot l_{Y,NCH})^\eta h \\
& + c_{Y,NCH}^{-\sigma} (1-\tau_L) w \\
\Psi_{NCH} (l_{M,NCH} - h \cdot l_{Y,NCH})^\eta & = c_{M,NCH}^{-\sigma} (1-\tau_L) w \\
c_{Y,NCH}^{-\sigma} & = (1-\gamma_1) \beta c_{M,NCH}^{-\sigma} (1+r) \\
c_{M,NCH}^{-\sigma} & = (1-\gamma_2) \beta c_{O,NCH}^{-\sigma} (1+r) \\
(1-\tau_L) w l_{Y,NCH} & \\
+ profit_{Y,NCH} & = c_{Y,NCH} + s_{Y,NCH} \\
(1-\tau_L) w l_{M,NCH} & \\
+ (1+r) s_{Y,NCH} + profit_{M,NCH} & = c_{M,NCH} + s_{M,NCH} \\
pension_{NCH} + (1+r) s_{M,NCH} + profit_{O,NCH} & = c_{O,NCH} \\
\Psi_{CH} (l_{Y,CH} + twch)^\eta & = (1-\gamma_1) \beta \Psi_{CH} (l_{M,CH} - h \cdot l_{Y,CH})^\eta h \\
& + \frac{1}{1+x \cdot ch} c_{Y,CH}^{-\sigma} (1-\tau_L) w \\
\Psi_{CH} (l_{M,CH} - h \cdot l_{Y,CH})^\eta & = c_{M,CH}^{-\sigma} (1-\tau_L) w \\
ch^{-\nu} & = \frac{1}{1+x \cdot ch} c_{Y,CH}^{-\sigma} x \cdot c_{Y,CH} \\
& + \Psi_{CH} (l_{Y,CH} + twch)^\eta \gamma \cdot a_{CH} ch^{\gamma-1} \\
\frac{1}{1+x \cdot ch} c_{Y,CH}^{-\sigma} & = (1-\gamma_1) \beta c_{M,CH}^{-\sigma} (1+r) \\
c_{M,CH}^{-\sigma} & = (1-\gamma_2) \beta c_{O,CH}^{-\sigma} (1+r) \\
(1-\tau_L) w l_{Y,CH} &
\end{aligned}$$

$$\begin{aligned}
& +profit_{Y,CH} = (1 + x \cdot ch) c_{Y,CH} + s_{Y,CH} \\
& (1 - \tau_L) wl_{M,CH} \\
& + (1 + r) s_{Y,CH} + profit_{M,CH} = c_{M,CH} + s_{M,CH} \\
& twch = a_{CH} ch^\gamma \\
& k \frac{1}{n \cdot ch} = \alpha \frac{y}{r^K} \\
& l = (1 - \alpha) \frac{y}{w} \\
& i = \left(1 - \frac{(1 - \delta)}{n \cdot ch} \right) k \\
& r^K + (1 - \delta) = 1 + r \\
& y = a_Y \left(k \frac{1}{n \cdot ch} \right)^{\alpha_Y} l_Y^{1 - \alpha_Y} \\
& profit = y - i - wl \\
& profit_{Y,NCH} = \frac{h_{Y,NCH}}{1 - n} profit \\
& profit_{M,NCH} = \frac{h_{M,NCH} n \cdot ch}{(1 - n)(1 - \gamma_1)} profit \\
& profit_{O,NCH} = \frac{h_{O,NCH} n^2 \cdot ch^2}{(1 - n)(1 - \gamma_1)(1 - \gamma_2)} profit \\
& profit_{Y,CH} = \frac{h_{Y,CH}}{n} profit \\
& profit_{M,CH} = \frac{h_{M,CH} n \cdot ch}{n(1 - \gamma_1)} profit \\
& profit_{O,CH} = \frac{h_{O,CH} n^2 \cdot ch^2}{n(1 - \gamma_1)(1 - \gamma_2)} profit
\end{aligned}$$

$$\begin{aligned}
\tau_L wl + d &= g + n(1 - \gamma_1)(1 - \gamma_2) \frac{1}{n^2 \cdot ch^2} pension_{O,CH} + (1 - n)(1 - \gamma_1)(1 - \gamma_2) \frac{1}{n^2 \cdot ch^2} pension_{O,NCH} \\
& + (1 + r) d \frac{1}{n \cdot ch} \\
y &= n(1 + x \cdot ch) c_{Y,CH} + (1 - n) c_{Y,NCH} \\
& + n \cdot \frac{(1 - \gamma_1)}{n \cdot ch} c_{M,CH} + (1 - n) \frac{(1 - \gamma_1)}{n \cdot ch} c_{M,NCH} \\
& + n \cdot (1 - \gamma_1)(1 - \gamma_2) \frac{1}{n^2 \cdot ch^2} c_{O,CH} + (1 - n)(1 - \gamma_1)(1 - \gamma_2) \frac{1}{n^2 \cdot ch^2} c_{M,NCH} \\
& + i + g \\
l &= n \cdot l_{Y,CH} + (1 - n) l_{Y,NCH} \\
& + n \cdot \frac{(1 - \gamma_1)}{n \cdot ch} l_{M,CH} + (1 - n) \cdot \frac{(1 - \gamma_1)}{n \cdot ch} l_{M,NCH} \\
d &= n \cdot s_{Y,CH} + (1 - n) s_{Y,NCH} \\
& + n \cdot \frac{(1 - \gamma_1)}{n \cdot ch} s_{M,CH} + (1 - n) \cdot \frac{(1 - \gamma_1)}{n \cdot ch} s_{M,NCH}
\end{aligned}$$

B The effect of the introduction of the compensation scheme

Macroaggregate	Change in the variable
y	-0.066614249%
$c_{Y,CH}$	0.238377814%
$c_{M,CH}$	0.09431076%
$c_{O,CH}$	-0.093856192%
$c_{Y,NCH}$	0.124756844%
$c_{M,NCH}$	-0.063384304%
$c_{O,NCH}$	-0.251129584%
$l_{Y,CH}$	-0.40736412%
$l_{M,CH}$	-0.283474487%
$l_{M,NCH}$	0.188039379%
l	-0.292097329%
k	0.558188429%
$s_{Y,NCH}$	-115.8699502%
$s_{M,NCH}$	-2.573550129%
$s_{Y,CH}$	-3.819298071%
$s_{M,CH}$	-5.642102318%
w	0.225774364%
r^K	-0.400441978%
R	-0.375397491%

<i>ch</i>		0.221819069%
<i>invest</i>		0.612349406%
<i>profit</i>		-0.33220628%
<i>twch</i>		0.044269198%
<i>d</i>		-5.263157895%
<i>pension_{CH}</i>		9.788725957%
<i>pension_{NCH}</i>		0.365160941%
<i>U_{NCH}</i>	<i>U_{NCH}</i>	-0.013030228%
<i>U_{CH}</i>	<i>U_{CH}</i>	0.21492738%
