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INTERIOR-POINT ALGORITHMS FOR SYMMETRIC CONE HORIZONTAL LINEAR COMPLEMENTARITY PROBLEMS BASED ON A NEW CLASS OF ALGEBRAICALLY EQUIVALENT TRANSFORMATIONS

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Abstract. We introduce interior-point algorithms (IPAs) for solving $P_*(\kappa)$-horizontal linear complementarity problems over Cartesian product of symmetric cones. We generalize the primal-dual IPAs proposed recently by Illés et al. [21] to $P_*(\kappa)$-horizontal linear complementarity problems over Cartesian product of symmetric cones. In the algebraic equivalent transformation (AET) technique we use a modification of the class of AET functions proposed by Illés et al. [21]. In the literature, there are only few classes of functions for determination of search directions. The class of AET functions used in this paper differs from the other classes appeared in the literature. We prove that the proposed IPAs have the same complexity bound as the best known interior-point methods for solving these types of problems.
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1. Introduction

Interior-point algorithms (IPAs) provide an efficient tool for solving different optimization problems. We can read about theoretical results related to IPAs for solving linear programming (LP) problems in the monographs written by Roos, Terlaky and Vial [35], Wright [41] and Ye [42]. Applications of linear complementarity problems (LCPs) can be found in different areas, such as economics, logistics, engineering, see [7]. IPAs have been also extended LCPs, see [9, 10, 18–20, 22, 24]. Note that LCPs belong to the class of NP-complete problems [6]. In spite of this fact, Kojima et al. [24] showed that if the problem's matrix possesses the $P_*(\kappa)$-property, then IPAs for LCPs have polynomial iteration complexity in the size of the problem, bit length of the data and in the special parameter κ.

We can read about generalizations of IPAs to Cartesian symmetric cone linear complementarity problems (SCLCPs), as well [26, 27]. Cartesian symmetric cone horizontal linear complementarity problem (SCHLCP) is a more general problem class which has been proposed by Asadi et al. [3]. It should be mentioned that LP, convex quadratic optimization (CQO), LCPs, second-order cone optimization (SCO), symmetric cone optimization (SCO) and semidefinite programming (SDP) problems can be solved by using

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the SCHLCP framework. Mohammadi et al. [28] proposed an infeasible IPA taking full Nesterov-Todd steps for solving SCHLCPs. In [2], the authors presented an IPA for solving Cartesian SCHLCP which uses the search directions proposed in [8]. Later on, Asadi et al. [1] proposed a new IPA for solving Cartesian SCHLCP which is based on a positive-asymptotic barrier function. In [4], a feasible IPA for solving $P_*(\kappa)$-SCHLCP using a wide neighbourhood of the central path was defined.

The way we determine the search directions plays a crucial role in this paper. Note that there exist several approaches for defining search directions in case of IPAs. Peng et al. [34] introduced large-update IPAs for LP by using self-regular barriers. In [25], the authors provided a unified analysis of kernel-based IPAs for $P_*(\kappa)$-LCPs. Vieira [39] also proposed different IPAs for SCO problems that use kernel functions for determining search directions. 25 years ago, Tunçel and Todd [38] gave a reparametrization of the central path system. Karimi et al. [17] dealt with entropy-based search directions for LP. Later on, Darvay introduced a new technique for defining search directions for LP problems [8]. This method has been entitled later as algebraic equivalent transformation (AET) technique. In [32], different IPAs for LP, SCO and $P_*(\kappa)$- LCPs using the AET technique have been proposed. Haddou et al. [13] proposed a class of concave functions in the AET technique to define IPAs for solving monotone LCPs. However, they used other type of transformation of the central path system. In 2022, Illés et al. [21] introduced a new class of AET functions and they defined IPAs for $P_*(\kappa)$-LCPs. It needs to be mentioned that this new class differs from the class of concave functions proposed by Haddou et al. Moreover, there exists function belonging to the new class of AET functions proposed by Illés et al. [21], for which the corresponding kernel function is neither eligible, nor self-regular kernel function.

The aim of this paper is to generalize the algorithms presented in [21] to Cartesian SCHLCP possessing the $P_*(\kappa)$-property. We propose a modification of the class proposed in [21]. We also present the complexity analysis of the proposed IPAs and we provide some technical lemmas that will be useful in the analysis. In [21], the authors gave a relationship between the parameters appearing in the definition of the class of AET functions in order to prove the well-definedess of the IPAs. In this paper we prove the well-definedess of the IPAs without any relationship between the parameters. In this way, we provide a wider class of AET functions. We prove that the IPAs have the same complexity bound as the best known interior-point algorithms for solving these types of problems.

The paper is organized in the following way. Section 2 contains the Cartesian $P_*(\kappa)$-SCHLCP. In Section 3 we present the class of AET functions used in this paper and we give the generalization of the AET technique to Cartesian $P_*(\kappa)$-SCHLCP. In Section 4 the new IPAs are defined for Cartesian $P_*(\kappa)$-SCHLCPs. Section 5 is devoted to the complexity analysis of the proposed IPAs. In Section 6 we enumerate several concluding remarks and the future research plans. Note that the Appendix contains some results related to the theory of Euclidean Jordan algebras and symmetric cones.

2. Cartesian $P_*(\kappa)$-symmetric cone horizontal linear complementarity problem and the central path

In the Appendix we provide a more detailed presentation of the theory of Euclidean Jordan and symmetric cones. We denote by $\mathcal{V} := \mathcal{V}_1 \times \mathcal{V}_2 \times \cdots \times \mathcal{V}_m$ the Cartesian product space, where each space $\mathcal{V}_i$ is a Euclidean Jordan algebra. The corresponding cone of squares is denoted by $\mathcal{K} := \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_m$, where each $\mathcal{K}_i$ is the corresponding cone of squares of $\mathcal{V}_i$. Let $x = \left(x^{(1)}, x^{(2)}, \cdots, x^{(m)}\right)^T \in \mathcal{V}$ and $s = \left(s^{(1)}, s^{(2)}, \cdots, s^{(m)}\right)^T \in \mathcal{V}$.

The trace, the determinant and the minimal and maximal eigenvalues of the element x are given as

$$tr(x) = \sum_{i=1}^{m} tr(x^{(i)}), \quad \det(x) = \prod_{i=1}^{m} \det(x^{(i)}),$$
$$\lambda_{\min}(x) = \min_{1\le i\le m} \{\lambda_{\min}(x^{(i)})\}, \quad \lambda_{\max}(x) = \max_{1\le i\le m} \{\lambda_{\max}(x^{(i)})\}.$$

Furthermore,

$$x \circ s = \left(x^{(1)} \circ s^{(1)}, x^{(2)} \circ s^{(2)}, \cdots, x^{(m)} \circ s^{(m)}\right)^T, \quad \langle x, s\rangle = \sum_{i=1}^{m} \langle x^{(i)}, s^{(i)}\rangle.$$

The Lyapunov transformation and the quadratic representation of x is defined in the following way:

$$L(x) = \operatorname{diag}\left(L(x^{(1)}), L(x^{(2)}), \cdots, L(x^{(m)})\right),$$
$$P(x) = \operatorname{diag}\left(P(x^{(1)}), P(x^{(2)}), \cdots, P(x^{(m)})\right).$$

The Frobenius norm of x is given as $\|x\|_F = \left(\sum_{i=1}^{m} \left\|x^{(i)}\right\|_F^2\right)^{1/2}$.

In the Cartesian SCHLCP a vector pair $(x,s) \in \mathcal{V} \times \mathcal{V}$ should be found which satisfies

$$Qx + Rs = q, \quad \langle x, s\rangle = 0, \quad x \succeq_K 0, \quad s \succeq_K 0, \qquad (SCHLCP)$$

where $q \in \mathcal{V}$, $Q, R : \mathcal{V} \to \mathcal{V}$ are linear operators and $\mathcal{K}$ is the symmetric cone of squares of the Cartesian product space $\mathcal{V}$. Let $\kappa \ge 0$. We say that the pair (Q, R) possesses the $P_*(\kappa)$- property if for all $(x,s) \in \mathcal{V} \times \mathcal{V}$

$$Qx + Rs = 0 \ \text{ implies } \ (1+4\kappa) \sum_{i\in I_+} \langle x^{(i)}, s^{(i)}\rangle + \sum_{i\in I_-} \langle x^{(i)}, s^{(i)}\rangle \ge 0,$$

where $I_+ = \{i \ : \ \langle x^{(i)}, s^{(i)}\rangle > 0\}$ and $I_- = \{i \ : \ \langle x^{(i)}, s^{(i)}\rangle < 0\}$.

In case of IPAs the *interior-point condition* (IPC) is assumed, which means that there exists (x^0, s^0), such that

$$\begin{aligned} Qx^0 + Rs^0 &= q, \\ x^0 \succ_K 0,\ s^0 &\succ_K 0. \end{aligned} \qquad (IPC)$$

The initialization of the problem using self-dual embedding for conic convex programming and particular cases was discussed in [16, 23, 30, 31].

The central path system is characterized by

$$\begin{aligned} Qx + Rs = &\quad q, & x \succeq_K 0, \\ x \circ s = &\quad \mu e, & s \succeq_K 0, \end{aligned} \qquad (2.1)$$

where $\mu > 0$. Consider the subclass of Monteiro-Zhang family of search directions as

$$C(x,s) = \left\{u | u \text{ is invertible and } L(P(u)x)L(P(u)^{-1}s) = L(P(u)^{-1}s)L(P(u)x)\right\}.$$

The following lemma plays important role in the determination of the search directions.

Lemma 2.1. *(Lemma 28 in [36]) Let $u \in \operatorname{int}\mathcal{K}$. Then,*

$$x \circ s = \mu e \quad \Leftrightarrow \quad P(u)x \circ P(u)^{-1}s = \mu e.$$

Let $u \in C(x,s)$, $\widetilde{Q} = QP(u)^{-1}$, $\widetilde{R} = RP(u)$. By using Lemma 2.1, system (2.1) can be written as

$$\begin{aligned} \widetilde{Q}P(u)x + \widetilde{R}P(u)^{-1}s &= q, \qquad P(u)x \succeq_K 0, \\ P(u)x \circ P(u)^{-1}s &= \mu e, \qquad P(u)^{-1}s \succeq_K 0. \end{aligned} \tag{2.2}$$

System (2.2) has unique solution for each $\mu > 0$, if the (IPC) holds, see [3] and [27].

3. New class of AET functions for Cartesian $P_*(\kappa)$-SCHLCPs

In this section the generalization of the AET technique [8] to $P_*(\kappa)$-SCHLCP is presented by using [1, 33]. We give a modification of the class of AET functions proposed in Definition 2.4 of [21]. Instead of three conditions we will use two conditions in the complexity analysis of the IPAs.

Definition 3.1. Let $\varphi : (\xi, \infty) \to \mathbb{R}$ be a continuously differentiable, invertible function, such that $\varphi'(t) > 0$, $\forall t > \xi$, where $0 \le \xi < 1$. All functions φ satisfying the following two conditions belong to the new class of AET functions. There exist two positive real numbers $L_1 > 0$ and $L_2 > 0$, such that the inequalities

$$\left| \frac{\varphi(1) - \varphi(t^2)}{2t\varphi'(t^2)} \right| \le L_1 \left| 1 - t^2 \right|, \tag{$AET1$}$$

and

$$\begin{aligned} 4t^2(\varphi(1) - \varphi(t^2))\varphi'(t^2) \; - \; L_2\left(\varphi(1) - \varphi(t^2)\right)^2 &\le 4t^2(1-t^2)\left(\varphi'(t^2)\right)^2 \\ &\le 4t^2(\varphi(1) - \varphi(t^2))\varphi'(t^2) + \left(\varphi(1) - \varphi(t^2)\right)^2 \end{aligned} \tag{$AET2$}$$

hold for all $t > \xi$.

Corollary 3.1. *If condition ($AET2$) is satisfied, then there exists $L_3 > 0$, such that*

$$\left| 4t^2\varphi'(t^2)\left((1-t^2)\varphi'(t^2) - \varphi(1) + \varphi(t^2)\right) \right| \le L_3\left(\varphi(1) - \varphi(t^2)\right)^2, \tag{$AET2b$}$$

for all $t > \xi$.

Proof. Suppose that ($AET2$) holds and let $L_3 = \max\{1, L_2\}$. Then, the inequality ($AET2b$) is also satisfied.

Consider the continuously differentiable function $\varphi : (\xi, \infty) \to \mathbb{R}$ satisfying ($AET1$) and ($AET2$). Let $x = \sum_{i=1}^{r} \lambda_i(x)c_i$, where $\{c_1, \ldots, c_r\}$ is the corresponding Jordan frame. We define the function φ for the elements of the Euclidean Jordan algebra in the following way:

$$\varphi(x) := \varphi(\lambda_1(x))c_1 + \ldots + \varphi(\lambda_r(x))c_r. \tag{3.1}$$

In this way, (2.2) can be written as follows:

$$\begin{aligned} \widetilde{Q}P(u)x + \widetilde{R}P(u)^{-1}s &= q, \qquad P(u)x \succeq_K 0, \\ \varphi\left(\frac{P(u)x \circ P(u)^{-1}s}{\mu}\right) &= \varphi(e), \qquad P(u)^{-1}s \succeq_K 0. \end{aligned} \tag{3.2}$$

For the determination of the search directions we use the method given in [33, 40]. For the strictly feasible $x \in \operatorname{int} K$ and $s \in \operatorname{int} K$ our aim is to find the search directions $(\Delta x, \Delta s)$ that satisfy

$$\begin{aligned} \widetilde{Q}P(u)\Delta x + \widetilde{R}P(u)^{-1}\Delta s &= 0, \qquad P(u)x \succeq_K 0, \\ P(u)x \circ P(u)^{-1}\Delta s + P(u)^{-1}s \circ P(u)\Delta x &= a_\varphi, \qquad P(u)^{-1}s \succeq_K 0, \end{aligned} \tag{3.3}$$

where

$$a_\varphi = \mu \left(\varphi' \left(\frac{P(u)x \circ P(u)^{-1}s}{\mu} \right)^{-1} \right) \circ \left(\varphi(e) - \varphi\left(\frac{P(u)x \circ P(u)^{-1}s}{\mu} \right) \right),$$

where the φ does belong to the class given in Definition 3.1.

In the paper we consider the NT-scaling scheme. Let $u = w^{-\frac{1}{2}}$, where w is called the NT-scaling point of x and s:

$$w = P(x)^{\frac{1}{2}} \left(P(x)^{\frac{1}{2}} s \right)^{-\frac{1}{2}} = P(s)^{-\frac{1}{2}} \left(P(s)^{\frac{1}{2}} x \right)^{\frac{1}{2}}. \tag{3.4}$$

We use the following notations:

$$v := \frac{P(w)^{-\frac{1}{2}}x}{\sqrt{\mu}} = \frac{P(w)^{\frac{1}{2}}s}{\sqrt{\mu}}, \quad d_x := \frac{P(w)^{-\frac{1}{2}}\Delta x}{\sqrt{\mu}}, \quad d_s := \frac{P(w)^{\frac{1}{2}}\Delta s}{\sqrt{\mu}}. \tag{3.5}$$

Using (3.5) we obtain the scaled system

$$\begin{aligned} \sqrt{\mu} Q P(w)^{\frac{1}{2}} d_x + \sqrt{\mu} R P(w)^{-\frac{1}{2}} d_s &= \quad 0, \\ d_x + d_s &= \quad p_v, \end{aligned} \tag{3.6}$$

where

$$p_v = v^{-1} \circ (\varphi'(v \circ v))^{-1} \circ (\varphi(e) - \varphi(v \circ v)). \tag{3.7}$$

Let us introduce the following function: $f : (\xi, \infty) \to \mathbb{R}$:

$$f(t) = \frac{\varphi(1) - \varphi(t^2)}{t(\varphi'(t^2))}, \tag{3.8}$$

Using this notation the conditions presented in in Definition 3.1 can be written in the following form.

Proposition 3.1. *The conditions (AET1) and (AET2) can be formulated in the following equivalent form. There exist $L_1 > 0$ and $L_2 > 0$ such that the inequalities*

$$|f(t)| \le 2L_1 \left| 1 - t^2 \right|, \tag{AETa}$$

$$-L_2 \frac{f(t)^2}{4} \le 1 - t^2 - t f(t) \le \frac{f(t)^2}{4}, \tag{AETb}$$

hold for all $t > \xi$.

Corollary 3.2. *If condition (AETb) is satisfied, then there exists $L_3 > 0$ such that*

$$|1 - t^2 - t f(t)| \le L_3 \tag{AETb2}$$

holds for all $t > \xi$.

Proof. Suppose that (AETb) holds and let $L_3 = \max\{1, L_2\}$. Then the inequality (AETb2) is also satisfied.

It needs to be mentioned that most of the functions used in the literature belong to the new class of AET functions, see the AET functions used in [8, 11, 22]. The intervals on which the functions φ are defined are important in this approach. For example, $\varphi(t) = t$ does belong to this new class of AET functions only if it is defined on a (ξ, ∞) interval, where ξ is strictly positive.

The function $\varphi(t) = t^2 - t + \sqrt{t}$ is member of the new class of AET function and is the first AET function from the literature which has inflection point.

Haddou et al. [13] used another type of trasnformation of the central path system. Their class of concave functions differs from the class of AET functions used in this paper. For example, the function $\varphi(t) = t^2 - t + \sqrt{t}$ from our class is not member of the class of concave functions. Furthermore, $\varphi(t) = \log(1+t)$ does belong to the class of Haddou et al. [13], but it is not member of the class given in Definition 3.1.

The class of AET functions can be compared to the class of eligible kernel functions and self-regular functions, see [5, 34]. Lesaja et al. presented IPAs for $P_*(\kappa)$-LCPs over symmetric cones based on eligible kernel functions.

The general barrier function is given as

$$\Psi(x, s; \mu) = \Psi(v) = \sum_{j=1}^{m} \sum_{i=1}^{r_j} \psi(\lambda_i(v^{(j)})),$$

where $\psi : \mathbb{R}_+ \to \mathbb{R}_\oplus$ is the corresponding kernel function. The second equation of system (3.6) can be replaced by $d_x + d_s = -\nabla\Psi(v)$, see [34].

Theorem 3.1. *(Theorem 3.6.2 in [39]) Let $D \subset \mathbb{R}$ be an open set and $f : D \to \mathbb{R}$. Let $U = \{x \in \mathcal{V} : \lambda(x) \in D^r\}$ and $F : U \to \mathbb{R}$ defined as $F(x) = \sum_{i=1}^{r} f(\lambda_i(x))$, with $x = \sum_{i=1}^{r} \lambda_i(x) c_i \in \mathcal{V}$. If f is differentiable in D, then F is differentiable in U and $\nabla F(x) = D_x F(x) = \sum_{i=1}^{r} f'(\lambda_i(x)) c_i$.*

We can associate a kernel function [29] to several functions φ by

$$\psi(t) = \int_1^t \frac{\varphi(\bar{\tau}^2) - \varphi(1)}{\bar{\tau}\varphi'(\bar{\tau}^2)} d\bar{\tau}. \tag{3.9}$$

A detailed description about the comparison of the class of AET functions given in Definition 3.1 to the class of eligible kernel and self-regular functions is provided in [21]. It should be mentioned that the kernel function corresponding to the function $\varphi(t) = t^2 - t + \sqrt{t}$ is neither eligible, nor self-regular function. Furthermore, there are eligible kernel functions and self-regular ones for which the corresponding AET function does not belong to the class given in Definition 3.1.

4. New interior-point algorithm for solving Cartesian $P_*(\kappa)$-SCHLCPs

In this section we present the new IPAs that use functions belonging to the class given in Definition 3.1. The proximity measure to the central path is defined as

$$\delta(v) = \delta(x, s, \mu) := \frac{\|p_v\|_F}{2}. \tag{4.1}$$

The τ-neighbourhood of a fixed point on the central path is given by

$$\mathcal{N}(\tau, \mu) := \{(x, s) \in \mathcal{V} \times \mathcal{V} : Qx + Rs = q,\ x \succeq_K 0,\ s \succeq_K 0 :\ \delta(x, s, \mu) \le \tau\},$$

where $\mu > 0$ is fixed and τ is a threshold parameter.

We determine the search directions using system (3.6) with functions φ satisfying (*AET*1) and (*AET*2).

We can calculate the search directions Δx and Δs from

$$\Delta x = \sqrt{\mu} P(w)^{\frac{1}{2}} d_x, \quad \Delta s = \sqrt{\mu} P(w)^{-\frac{1}{2}} d_s. \tag{4.2}$$

Let $x^+ = x + \Delta x$ and $s^+ = s + \Delta s$ be the point after a full NT-step.

Our IPAs start with $(x^0, s^0) \in \mathcal{N}(\tau, \mu)$. In Figure 4.1 a whole class of IPAs is defined based on the new class of AET functions.

Algorithm 4.1 : IPAs for Cartesian $P_*(\kappa)$-SCHLCP using the new class of functions in the AET

Consider a function φ satisfying the conditions ($AET1$) and ($AET2$) of Definition 3.1. Let $\epsilon > 0$ be the accuracy parameter, $0 < \theta < 1$ the update parameter and τ the proximity parameter. Assume that for (x^0, s^0) the (IPC) holds such that $\delta(x^0, s^0; \mu^0) \le \tau$. Suppose that $\lambda_{\min}\left(\frac{x^0 \circ s^0}{\mu^0}\right) > \xi^2$.

begin

  $k := 0$;

  while $\langle x^k, s^k \rangle > \epsilon$ **do begin**

    compute w using (3.4);

    compute $(\Delta x^k, \Delta s^k)$ from system (3.6) using (4.2) with φ;

    belonging to the class of AET functions given in Definition 3.1;

    $x^{k+1} := x^k + \Delta x^k, \quad s^{k+1} := s^k + \Delta s^k, \quad \mu^{k+1} := (1-\theta)\mu^k$;

    k:=k+1;

  end

end.

The following section is devoted to the complexity analysis of the proposed IPAs.

5. Complexity analysis of the interior-point algorithms

In the first part of this section we provide technical lemmas that are necessary for the analysis of the IPAs. Let

$$q_v = d_x - d_s, \tag{5.1}$$

hence

$$d_x = \frac{p_v + q_v}{2}, \quad d_s = \frac{p_v - q_v}{2}, \quad d_x \circ d_s = \frac{p_v^2 - q_v^2}{4}. \tag{5.2}$$

In Lemma 5.1 we get an upper bound for $\|q_v\|_F$ in terms of $\|p_v\|_F$.

Lemma 5.1. *(Lemma 5.1 in [1]) We have $\|q_v\|_F \le \sqrt{1+4\kappa}\ \|p_v\|_F$.*

Let $x, s \in \operatorname{int}\mathcal{K}$, $\mu > 0$ and w be the scaling point of x and s. We have

$$\begin{aligned} x^+ &:= x + \Delta x = \sqrt{\mu} P(w)^{1/2}(v + d_x), \\ s^+ &:= s + \Delta s = \sqrt{\mu} P(w)^{-1/2}(v + d_s). \end{aligned} \tag{5.3}$$

Note that $x^+, s^+ \in \operatorname{int}\mathcal{K}$ if and only if $v + d_x, v + d_s \in \operatorname{int}\mathcal{K}$. This follows from the fact that $P(w)^{1/2}$ and $P(w)^{-1/2}$ are automorphisms of $\operatorname{int}\mathcal{K}$, see Proposition 6.1 part (ii) from Appendix. Using (5.3) we have

$$x^+ \circ s^+ = \mu(v + d_x) \circ (v + d_s). \tag{5.4}$$

The next lemma is a technical one which will be used when we prove the feasibility of the full-NT step.

Lemma 5.2 (Lemma 4.1 of [40]). *Let $x = x(0), s = s(0) \in int\,\mathcal{K}$ and for $0 \le \alpha \le \bar{\alpha}$, $x(\alpha) \circ s(\alpha) \in int\,\mathcal{K}$. Then, we have $x(\bar{\alpha}) \in int\,\mathcal{K}$ and $s(\bar{\alpha}) \in int\,\mathcal{K}$.*

In the following lemma the strict feasibility of the full NT-step is proven.

Lemma 5.3. *Let $x \succ_K 0$, $s \succ_K 0$ and $\delta := \delta(x,s;\mu) < \frac{1}{\sqrt{1+4\kappa}}$ and suppose that $\lambda_{\min}(v) > \xi$. For any function satisfying (AET2), after a full-NT step we have $x^+ \succ_K 0$ and $s^+ \succ_K 0$.*

Proof. For $0 \le \alpha \le 1$, let

$$v_x(\alpha) := v + \alpha d_x \quad \text{and} \quad v_s(\alpha) := v + \alpha d_s.$$

From the second equation of the system (3.6), we get

$$\begin{aligned} v_x(\alpha) \circ v_s(\alpha) &= v^2 + \alpha v \circ (d_x + d_s) + \alpha^2 (d_x \circ d_s) \qquad (5.5)\\ &= (1-\alpha)v^2 + \alpha\left(v^2 + v \circ p_v\right) + \alpha^2 \frac{p_v^2 - q_v^2}{4} \\ &= (1-\alpha)v^2 + \alpha\left(v^2 + v \circ p_v - e + \frac{p_v^2}{4}\right) + \alpha\left(e - (1-\alpha)\frac{p_v^2}{4} - \alpha\frac{q_v^2}{4}\right). \end{aligned}$$

The right hand side of AET2 of Proposition 3.1 yields $v^2 + v \circ p_v - e + \frac{p_v^2}{4} \in \mathcal{K}$. Using Lemma 5.1 we have:

$$\begin{aligned} \left\|(1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right\|_F &\le (1-\alpha)\left\|\frac{p_v^2}{4}\right\|_F + \alpha\left\|\frac{q_v^2}{4}\right\|_F \le (1-\alpha)\frac{\|p_v\|_F^2}{4} + \alpha\frac{\|q_v\|_F^2}{4} \\ &\le (1-\alpha)\frac{\|p_v\|_F^2}{4} + \alpha(1+4\kappa)\frac{\|p_v\|_F^2}{4} = ((1-\alpha) + \alpha(1+4\kappa))\delta^2 \\ &= (1+4\alpha\kappa)\delta^2 < (1+4\kappa)\delta^2. \qquad (5.6) \end{aligned}$$

Using this and the assumption $\delta < \frac{1}{\sqrt{1+4\kappa}}$ we obtain

$$\left\|(1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right\|_F < 1.$$

Using Lemma 6.1 we have

$$e - (1-\alpha)\frac{p_v^2}{4} - \alpha\frac{q_v^2}{4} \in \operatorname{int} \mathcal{K}. \qquad (5.7)$$

We know that $(1-\alpha)v^2 \in \operatorname{int} \mathcal{K}$. Using (5.5) we get that $v_x(\alpha) \circ v_s(\alpha) \in \operatorname{int} \mathcal{K}$, for $\alpha \in (0,1)$. If we substitute $\alpha = 1$ in Lemma 5.2, we obtain $v + d_x \in \operatorname{int} \mathcal{K}$ and $v + d_s \in \operatorname{int} \mathcal{K}$. From (5.3) and Proposition 6.1, part (ii), the lemma is proven. □

The following lemma will be used later in the complexity analysis of the IPA.

Lemma 5.4. *Let $\bar{f} : (\bar{d}, +\infty) \to \mathbb{R}_+$ be a function, where $\bar{d} > 0$ and $|\bar{f}(t)| \le \bar{k}\left|1 - t^2\right|$, for $t > \bar{d}$, where $\bar{k} > 0$. Assume that $v \in \mathcal{V}$. Then,*

$$\left\|\bar{f}(v)\right\|_F \le \bar{k}\left\|e - v^2\right\|_F.$$

Proof. Using Theorem 6.1 from Appendix, we assume that $v = \sum_{i=1}^r \lambda_i(v) c_i$. Besides, $\bar{f}(v) = \sum_{i=1}^r \bar{f}(\lambda_i(v)) c_i$. Then,

$$\left\|\bar{f}(v)\right\|_F = \sqrt{\sum_{i=1}^r \left(\bar{f}(\lambda_i(v))\right)^2} \le \bar{k}\sqrt{\sum_{i=1}^r \left(\lambda_i^2(e - v^2)\right)^2} = \bar{k}\left\|e - v^2\right\|_F.$$

□

Assume that x^+ and s^+ are the iterates after taking a full-NT step, and w^+ is their corresponding NT scaling point. The v-vector after a NT step is given by

$$\bar{v} := \frac{P(w^+)^{-1/2}x^+}{\sqrt{\mu}} \quad \left[= \frac{P(w^+)^{1/2}s^+}{\sqrt{\mu}}\right]. \tag{5.8}$$

Lemma 5.5 (Proposition 5.9.3 in [39]). *One has*

$$\bar{v} \sim \left(P(v+d_x)^{\frac{1}{2}}(v+d_s)\right)^{\frac{1}{2}}.$$

Lemma 5.6 gives an upper bound for the proximity measure after a full-NT step.

Lemma 5.6. *Let $(x,s) \in \mathcal{F}^+$ and suppose that $\delta := \delta(x,s;\mu) < \sqrt{\frac{1-\xi^2}{1+4\kappa}}$ and $\lambda_{\min}(v) > \xi$. For any function φ satisfying (AET1) and (AET2), we can say that after a NT-step we have $\lambda_{\min}(\bar{v}) > \xi$ and*

$$\delta(x^+,s^+;\mu) \leq L_1(L_3+2+4\kappa)\delta(x,s;\mu)^2,$$

where $L_1 > 0$, $L_2 > 0$ and $L_3 = \max\{1, L_2\}$.

Proof. The proof of Lemma 5.6 of [1] implies

$$\lambda_{\min}(\bar{v}) \geq \sqrt{1-(1+4\kappa)\delta^2}.$$

Using this and $\delta < \sqrt{\frac{1-\xi^2}{1+4\kappa}}$ we get $\lambda_{\min}(\bar{v}) > \xi$.

We use conditions ($AETa$) and ($AETb$) of Proposition 3.1. From (4.1) we have

$$\delta(x^+,s^+;\mu) := \frac{\|p_{\bar{v}}\|_F}{2} = \frac{\|f(\bar{v})\|_F}{2}, \tag{5.9}$$

where the function f is defined as (3.1) using (3.8). Using ($AETa$) of Proposition 3.1 and Lemma 5.4 we get

$$\delta(x^+,s^+;\mu) \leq L_1 \left\|e-\bar{v}^2\right\|_F. \tag{5.10}$$

Substituting $\alpha = 1$ in (5.5), we get

$$\left\|e-\bar{v}^2\right\|_F = \left\|e - v^2 - v \circ p_v - \frac{p_v^2}{4} + \frac{q_v^2}{4}\right\|_F. \tag{5.11}$$

From condition ($AETb$) of Proposition 3.1 we obtain that condition ($AETb2$) of Corollary 3.2 also holds. Using this, (5.11) and Lemmas 5.1 and 5.4 we get

$$\left\|e-\bar{v}^2\right\|_F \leq \left\|e-v^2-v\circ p_v\right\|_F + \left\|\frac{p_v^2}{4}\right\|_F + \left\|\frac{q_v^2}{4}\right\|_F \leq (2+L_3+4\kappa)\delta^2. \tag{5.12}$$

Using (5.10) and (5.12) we have

$$\delta(x^+,s^+;\mu) \leq L_1(L_3+2+4\kappa)\delta(x,s;\mu)^2,$$

and the lemma is proven. □

Lemma 5.7. *(Lemma 5.7 in [1]) Let d_x and d_s be the solutions of the system (3.6) with p_v defined as in (3.7), and $\delta := \delta(x,s;\mu)$. Then $\langle d_x, d_s\rangle \leq \delta^2$.*

We provide an upper bound for the duality gap after a full-NT step.

Lemma 5.8. *Let $\delta = \delta(x,s;\mu)$ and x^+ and s^+ be obtained after a full NT-step. For any function φ satisfying (AET2) of Definition 3.1 with $L_2 \in \mathbb{R}_+$, we have*

$$\langle x^+, s^+\rangle \le \mu\left(r + (L_2+1)\delta^2\right).$$

Proof. In the proof only the left hand side of ($AETb$) from Proposition 3.1 will be used, namely

$$tf(t) - L_2\frac{f(t)^2}{4} \le 1 - t^2, \quad t > \xi, \tag{5.13}$$

where $L_2 > 0$. Using Corollary 3.2 we have that condition ($AETb2$) is also satisfied. Using this, (5.4), (5.5) and (5.13) we get

$$\frac{1}{\mu}x^+ \circ s^+ = v^2 + v \circ p_v + d_x \circ d_s \preceq_K e + \frac{L_2}{4}p_v^2 + d_x \circ d_s. \tag{5.14}$$

Using (5.14) and Lemma 5.7 we have

$$\begin{aligned}\langle x^+, s^+\rangle &= \langle e, x^+ \circ s^+\rangle \le \mu\langle e, e + \frac{L_2}{4}p_v^2 + d_x \circ d_s\rangle &(5.15)\\ &\le \mu\left(r + L_2\delta^2 + \langle d_x, d_s\rangle\right) \le \mu\left(r + (L_2+1)\delta^2\right), &(5.16)\end{aligned}$$

and the proof is complete. □

The next lemma points out the effect of a μ-update on the proximity of the new iterates to the central path.

Lemma 5.9. *Suppose that $\delta := \delta(x,s,\mu) < \sqrt{\frac{1-\xi^2}{1+4\kappa}}$, $\lambda_{\min}(v) > \xi$ and $\mu^+ = (1-\theta)\mu$. Let $v^+ = \frac{\bar{v}}{\sqrt{1-\theta}}$ be the scaled vector v after the full-NT step and the μ-update. For any function φ satisfying (AET1) and (AET2) with $L_1 > 0$, $L_2 > 0$, we have $\lambda_{\min}\left(v^+\right) > \xi$ and*

$$\delta(x^+, s^+; \mu^+) \le \frac{L_1}{1-\theta}\left(\theta\sqrt{n} + (L_3 + 2 + 4\kappa)\delta^2\right),$$

where $L_3 = \max\{1, L_2\}$.

Proof. Using Lemma 5.6 we have $\lambda_{\min}(\bar{v}) > \xi$. From this and $0 < \theta < 1$ we obtain $\lambda_{\min}\left(v^+\right) > \xi$. We use the conditions ($AETa$) and ($AETb$) of Proposition 3.1. From (4.1) we get

$$\delta(x^+, s^+; \mu^+) = \frac{\left\|f\left(v^+\right)\right\|_F}{2}.$$

Using condition ($AETa$) of Proposition 3.1 and Lemma 5.4 , we have

$$\delta(x^+, s^+; \mu^+) \le L_1\left\|e - \left(v^+\right)^2\right\|_F. \tag{5.17}$$

Using (5.4) and (5.5) we have

$$\begin{aligned}\left\|e - (v^+)^2\right\|_F &= \left\|e - \frac{1}{1-\theta}\frac{x^+ \circ s^+}{\mu}\right\|_F = \left\|e - \frac{1}{1-\theta}\left(v^2 + v \circ p_v + \frac{p_v^2}{4} - \frac{q_v^2}{4}\right)\right\|_F\\ &= \frac{1}{1-\theta}\left\|-\theta e + e - v^2 - v \circ p_v - \frac{p_v^2}{4} + \frac{q_v^2}{4}\right\|_F. &(5.18)\end{aligned}$$

From (5.17), (5.18), condition ($AETb$) of Proposition 3.1 and Lemma 5.4 we obtain

$$\begin{aligned}\delta(x^+,s^+;\mu^+) &\leq \frac{L_1}{1-\theta}\left(\|\theta e\|_F + \left\|e-v^2-v\circ p_v\right\|_F + \left\|\frac{p_v^2}{4}\right\|_F + \left\|\frac{q_v^2}{4}\right\|_F\right)\\ &\leq \frac{L_1}{1-\theta}\left(\theta\sqrt{r}+(L+2+4\kappa)\delta^2\right), \end{aligned} \tag{5.19}$$

and the lemma is proven. □ □

We will set the values of the parameters θ and τ and we show that for these values the IPAs using the new class of AET functions are well defined.

Lemma 5.10. *Let $\varphi:(\xi,\infty)\to\mathbb{R}$ satisfying ($AET1$) and ($AET2$). Consider $L_4 = \max\{L_1,\frac{1}{4}\}$, $\tau = \frac{\sqrt{1-\xi^2}}{4L_4(L_3+2+4\kappa)}$, $\theta = \frac{\sqrt{1-\xi^2}}{16L_4^2(L_3+2+4\kappa)\sqrt{r}}$. Assume also that $\delta(x,s;\mu)\leq\tau$. Then, we have*

$$\delta(x^+,s^+;\mu^+)\leq\tau,$$

hence the IPAs defined in Figure 1 are well defined.

Proof. Using $\kappa\geq 0$, $L_4=\max\{L_1,\frac{1}{4}\}$ and $L_3=\max\{1,L_2\}$ we have $\tau=\frac{\sqrt{1-\xi^2}}{4L_4(L_3+2+4\kappa)} < \frac{1}{\sqrt{1+4\kappa}}$. Using this and the assumption conditions ($AET1$) and ($AET2$), from Lemma 5.3 we get that $(x^+,s^+)\in\mathcal{F}^+$.

Using Lemma 5.9 and $L_1\leq L_4$ we have

$$\delta(x^+,s^+;\mu^+) \leq \frac{L_4}{1-\theta}\left(\theta\sqrt{r}+(L_3+2+4\kappa)\delta^2\right). \tag{5.20}$$

From $\kappa\geq 0$, $r\geq 1$, $L_4=\max\{L_1,\frac{1}{4}\}$ and $L_3=\max\{1,L_2\}$ we conclude that $\frac{1}{1-\theta}\leq 2$ and $\theta=\frac{\sqrt{1-\xi^2}}{16L_4^2(L_3+2+4\kappa)\sqrt{r}}\leq\frac{1}{2}$. Using this and the values of τ and θ we have

$$\frac{L_4\theta\sqrt{r}}{1-\theta}\leq\frac{2L_4\sqrt{1-\xi^2}}{16L_4^2(L_3+2+4\kappa)}\leq\frac{1}{2}\tau. \tag{5.21}$$

Furthermore, using $\frac{1}{1-\theta}\leq 2$ and $\tau=\frac{1}{4L_4(L_3+2+4\kappa)}$ we obtain

$$\frac{L_4(L_3+2+4\kappa)\delta^2}{1-\theta}\leq 2L_4(L+2+4\kappa)\frac{1}{16L_4^2(L_3+2+4\kappa)^2}=\frac{1}{2}\tau. \tag{5.22}$$

From (5.20, (5.21) and (5.22) we obtain

$$\delta(x^+,s^+;\mu^+)\leq\tau. \tag{5.23}$$

□

The following lemma gives upper bound on the number of iterations.

Lemma 5.11. *Let $\varphi:(\xi,\infty)\to\mathbb{R}$ satisfying ($AET1$) and ($AET2$). Consider $L_4 = \max\{L_1,\frac{1}{4}\}$, $\tau=\frac{\sqrt{1-\xi^2}}{4L_4(L_3+2+4\kappa)}$, $L_3=\max\{L_2,1\}$ and $\theta=\frac{\sqrt{1-\xi^2}}{16L_4^2(L_3+2+4\kappa)\sqrt{r}}$. We assume that the pair (x^0,s^0) is strictly feasible, $\mu^0=\frac{\langle x^0,s^0\rangle}{r}$ and $\delta(x^0,s^0;\mu^0)\leq\tau$. Let x^k and s^k be the obtained iterates after k iterations. Then, for*

$$k\geq\left\lceil\frac{1}{\theta}\log\frac{\mu^0\left(r+\frac{L_2+1}{9}\right)}{\varepsilon}\right\rceil$$

we have $\langle x^k,s^k\rangle<\varepsilon$.

Proof. Using $L_3 = \max\{L_2, 1\}$, $\kappa \geq 0$, we have

$$\tau = \frac{1}{4L_4(L_3 + 2 + 4\kappa)} \leq \frac{1}{3}. \tag{5.24}$$

From (5.24) and Lemma 5.8 we have

$$\langle x^k, s^k \rangle \quad \leq \quad \mu^k \left(r + \frac{L_2 + 1}{9} \right) = (1-\theta)^k \mu^0 \left(r + \frac{L_2 + 1}{9} \right).$$

The condition $\langle x^k, s^k \rangle < \varepsilon$ holds if

$$(1-\theta)^k \mu^0 \left(r + \frac{L_2 + 1}{9} \right) < \varepsilon. \tag{5.25}$$

We take the logarithm of both sides of (5.25) and we get

$$k \log(1-\theta) + \log \left(\mu^0 \left(r + \frac{L_2 + 1}{9} \right) \right) < \log \varepsilon. \tag{5.26}$$

From $-\log(1-\theta) \geq \theta$ we have that (5.26) holds if

$$k\theta \geq \log \left(\mu^0 \left(r + \frac{L_2 + 1}{9} \right) \right) - \log \varepsilon = \log \frac{\mu^0 (r + \frac{L_2+1}{9})}{\varepsilon},$$

which proves the lemma. □

Theorem 5.1. *Let $\varphi : (\xi, \infty) \to \mathbb{R}$ satisfying (AET1) and (AET2). Consider $L_4 = \max\{L_1, \frac{1}{4}\}$, $\tau = \frac{\sqrt{1-\xi^2}}{4L_4(L_3+2+4\kappa)}$, $L_3 = \max\{L_2, 1\}$ and $\theta = \frac{\sqrt{1-\xi^2}}{16L_4^2(L_3+2+4\kappa)\sqrt{r}}$. We assume that the pair (x^0, s^0) is strictly feasible, $\mu^0 = \frac{\langle x^0, s^0 \rangle}{r}$ and $\delta(x^0, s^0; \mu^0) \leq \tau$. Then, the IPAs given in Figure 1 require no more than*

$$\left\lceil \frac{16L_4^2(L_3 + 2 + 4\kappa)}{\sqrt{1-\xi^2}} \sqrt{r} \log \frac{\mu^0 \left(r + \frac{L_2+1}{9} \right)}{\varepsilon} \right\rceil$$

interior-point iterations.

In the following section concluding remarks and further research plans are enumerated.

6. Conclusions and further research

We successfully generalized the full-NT step feasible IPA proposed in [21] to Cartesian $P_*(\kappa)$-horizontal linear complementarity problem. We also extend a modification of the new class of AET functions defined in [21] to the Cartesian $P_*(\kappa)$-SCHLCP framework by using the theory of Euclidean Jordan algebras and symmetric cones. It should be mentioned that the used class of AET functions differs from the existing classes to determine search directions in case of IPAs. We introduced new IPAs that use the AET functions belonging to the class given in Definition 3.1. We also presented the complexity analysis of the proposed IPAs and we proved that they have the same complexity bound as the best known interior-point methods for solving these types of problems. We proved the well-definedness of the IPAs without any relationship between the parameters L_1 and L_2. This provides a wider class of AET functions. As further research it would be interesting to extend the obtained results to non-symmetric cone optimization and to nonlinear complementarity problems over symmetric cones. Furthermore, it would worth giving a more general framework which could deal with problems where we do not assume the $P_*(\kappa)$-property of the pair (Q, R). This method would lead to analyse problems similar to general LCPs given in [19, 20].

Appendix

In this part some results related to the theory of Euclidean Jordan algebras and symmetric cones [12, 14, 37, 39] are presented.

Consider $\mathcal{V}$ as an n-dimensional vector space over $\mathbb{R}$ with the bilinear map $\circ : (x,y) \to x \circ y \in \mathcal{V}$. Then, $(\mathcal{V}, \circ)$ is said to be a Jordan algebra if for all $x,y \in \mathcal{V}$, we have $x \circ y = y \circ x$ and $x \circ (x^2 \circ y) = x^2 \circ (x \circ y)$, where $x^2 = x \circ x$. Note that $e \in \mathcal{V}$ is the identity element of $\mathcal{V}$ if and only if $e \circ x = x \circ e = x$, for all $x \in \mathcal{V}$. We call the element x invertible if there exists a unique element $\bar{x}$, such that $x \circ \bar{x} = e$ and $\bar{x}$ is a polynomial in x. We denote the inverse of x by x^{-1}.
If we have $\mathcal{V}$ with an identity element, then we call it Euclidean Jordan algebra if there exists a symmetric positive definite quadratic form $\bar{Q}$ on $\mathcal{V}$, for which $\bar{Q}(x \circ y, z) = \bar{Q}(x, y \circ z)$ is satisfied. Let $x \in \mathcal{V}$, Then, the Lyapunov transformation $L(x)$ is defined as $L(x)y := x \circ y$, for all $y \in \mathcal{V}$. The quadratic representation $P(x)$ of x is can be written as $P(x) := 2L(x)^2 - L(x^2)$, where $L(x)^2 = L(x)L(x)$. The degree of an element x, denoted by $deg(x)$, is the smallest integer r such that $\{e, x, \cdots, x^r\}$ is linearly dependent. The rank of $\mathcal{V}$ is denoted by $\operatorname{rank}(\mathcal{V})$ and is the largest $deg(x)$ for all $x \in \mathcal{V}$. We call a subset $\{c_1, c_2, \cdots, c_r\}$ of $\mathcal{V}$ ia Jordan frame if it is a complete system of orthogonal primitive idempotents. The following theorem plays important role in the theory of Euclidean Jordan algebras.

Theorem 6.1 (Theorem III.1.2 of [12])**.** *Suppose* $\operatorname{rank}(\mathcal{V}) = r$. *Then, for any x in $\mathcal{V}$ there exists a Jordan frame $c_1, \cdots, c_r$ and real numbers $\lambda_1, \cdots, \lambda_r$ such that $x = \sum_{i=1}^r \lambda_i c_i$.*

The numbers λ_i are named eigenvalues. Let $tr(x) = \sum_{i=1}^r \lambda_i$ and $\det(x) = \prod_{i=1}^r \lambda_i$. For any Euclidean Jordan algebra $\mathcal{V}$, we consider the corresponding cone of squares $\mathcal{K}(\mathcal{V}) := \{x^2 \ : \ x \in \mathcal{V}\}$. It can be proven that this is a symmetric cone, i.e. it is self-dual and homogeneous, see [12]. We also use

$$x \succeq_K 0 \quad \Leftrightarrow \quad x \in K \qquad \text{and} \qquad x \succ_K 0 \quad \Leftrightarrow \quad x \in \operatorname{int} K,$$

and

$$x \succeq_K s \quad \Leftrightarrow \quad x - s \succeq_K 0 \qquad \text{and} \qquad x \succ_K s \quad \Leftrightarrow \quad x - s \succ_K 0.$$

The inner product is given as $\langle x, y \rangle = tr(x \circ y)$. The induced Frobenius norm is

$$\|x\|_F = \langle x, x \rangle^{1/2} = \sqrt{tr(x^2)} = \sqrt{\sum_{i=1}^r \lambda_i^2(x)}. \tag{6.1}$$

We use the following lemmas in the complexity analysis of the IPAs.

Proposition 6.1. *The following statements hold:*
(i) $x \in \mathcal{V}$ is invertible if and only if $P(x)$ is invertible, in which case $P(x)^{-1} = P(x^{-1})$.
(ii) If $x \in \mathcal{V}$ is invertible, then $P(x)\mathcal{K} = \mathcal{K}$ and $P(x) int\, \mathcal{K} = int\, \mathcal{K}$.
(iii) If $x \in \mathcal{K}$, then $P(x)^{1/2} = P(x^{1/2})$.
(iv) If $x \in \mathcal{V}$, then $x \in \mathcal{K}$ ($x \succeq_{\mathcal{K}} 0$) if and only if $\lambda_i(x) \geq 0$ and $x \in int\, \mathcal{K}$ ($x \succ_{\mathcal{K}} 0$) if and only if $\lambda_i(x) > 0$, for all $i = 1, \cdots, r$.

Lemma 6.1 (Corollary 2.4 of [40])**.** *If $x \in \mathcal{V}$ and $\|x\|_F < 1$, then $e - x \in int\, \mathcal{K}$.*

Lemma 6.2 (Lemma 14 of [36])**.** *If $x, s \in \mathcal{V}$, then*

$$\lambda_{\min}(x+s) \geq \lambda_{\min}(x) + \lambda_{\min}(s) \geq \lambda_{\min}(x) - \|s\|_F.$$

Declarations

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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