

RESEARCH ARTICLE



Margin requirements based on a stochastic correlation model

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Abstract

We demonstrate that margin requirements of central counterparties show a significantly different behavior when calculated with a portfolio-wise treatment instead of taking the weighted sum of the margin requirements of the components without accounting for their correlation structures. This is shown via simulating trajectories of a joint stochastic volatility–stochastic correlation model. Results indicate that an unnecessarily large overmargin requirement is set by regulators when the applied risk measure is not calculated via a portfolio-wise treatment. Finally, accounting for the correlation structure of the assets during the margining process would not lead to an overly prudent method, nor would it cause greater procyclicality.

KEYWORDS

central counterparty, EMIR regulation, initial margin, procyclicality

JEL CLASSIFICATION

G15, G17, G18

1 | INTRODUCTION

Central counterparties (CCPs), also known as systematically important payments systems, are key members of the financial market infrastructure. As their defining role, they take over counterparty risk from other market participants through the process of novation by becoming the seller of each buyer and buyer of each seller (Hancock et al., 2016). During the financial crises of 2007/2008, CCPs proved to be crisis-resistant; even the defaults of the largest clearing members, like that of Lehman Brothers, were efficiently handled by the CCPs (Gregory, 2014). As a consequence, the importance of CCPs has recently increased (Domanski et al., 2015); this importance is also reflected in the growing number of related scientific studies (Berndsen, 2021). Moreover, the increased interest is also identifiable by the changing regulatory environment around the world, for example, in the USA, the Dodd–Frank Act (Dodd–Frank Wall Street Reform and Consumer Protection Act; The United States of America, 2010) was enacted in 2010, whereas in the European Union the European Market Infrastructure Regulation (EMIR) regulation (European Union, 2012) was enacted in 2012, both concerning the regulation of CCPs. These regulations came into force in accordance with the proposition of the G20 in 2009. This proposal incentivized that by 2012, other than a few exceptions, all standardized over-the-counter (OTC) transactions through financial institutions should be cleared by CCPs (Financial Stability Board, 2009). These few exceptions are clarified, for example, in Doyle et al. (2016): such an exception can be granted if

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the transaction's aim is to hedge commercial risk (Gregory, 2014) in the framework of EMIR, or if the trade is a foreign exchange transaction (J. C. Hull, 2018) in the framework of the Dodd–Frank Act.

In particular, EMIR has been extended by a Technical Standard (European Union, 2013), and they require that a multilevel guarantee system has to be operated by the CCPs to efficiently manage a possible default of one or more of its clearing members (Berndsen, 2021). As stated in Article 53 (European Union, 2013), the guarantee system is obliged to meet the so-called “Cover 2 rule,” with an intent to sufficiently cover the losses resulting from the default of the two clearing members with the largest exposures toward the CCP. This guarantee system is a structured default management process. It has three main components: (1) initial margin (“defaulter pay”), aimed to cover the losses under normal market conditions; (2) default fund (mutualized), aimed to cover losses under stressed market conditions (Goldman & Shen, 2020); and finally (3) skin-in-the-game, which is 25% of the CCP's own capital (European Union, 2012, 2013). The order of usage of the guarantee system's elements is termed as the default waterfall.

In this study we follow the European jurisdiction and so we examine the value of initial margin requirements adopting EMIR regulation. In this European framework we will extend the literature by comparing two margin requirements of a given portfolio, either incorporating or disregarding the effect of diversification. These two main margin calculation methods will be referred to as *Portfolio treatment* and *Individual treatment*.

- *Portfolio treatment*: According to this method we solely monitor the value of the whole portfolio without tracking the individual prices of the components to obtain the final margin of the portfolio.
- *Individual treatment*: According to this method we individually calculate the required margin of each component and weight their respective contribution to the portfolio value to obtain the final margin of the portfolio.

We will demonstrate via a comprehensive simulation framework that, similarly to the effect of risk mitigation, on average the margin requirement is significantly lower when *Portfolio treatment* is applied, counterexamples occurring very rarely and being of negligible magnitude. We also show that applying *Portfolio treatment* does not entail inferior anti-procyclicality (APC) values.

On the other hand, the current EMIR regulation is rather strict regarding portfolio margin calculations. Margin reduction based on the effect of correlation can only be taken into account if the correlation is significant and persistent, and this correlation can be supported economically. Also the size of any reduction is limited European Union (2013) Article 27 (Heckinger et al., 2016). On the basis of the findings of this study we claim that failure to incorporate interasset dependency structure into the method results in the withdrawal of a significant amount of liquidity from the market. Moreover, in some unique cases the *Individual treatment* does not set a larger margin requirement after the open positions' of the clearing member. Finally, based on our simulations we also assert that by fully following the *Portfolio treatment* we can still pass the back-test.

The paper is organized as follows. In Section 2 we review the corresponding literature. In Section 3 we mathematically formulate the modeling of assets. In Section 4 we present and discuss the results. In Section 5 we conclude.

2 | LITERATURE REVIEW

Despite that the main goal of the CCPs is to reduce counterparty risk through the process of novation, it increases liquidity risk and systemic risk (King et al., 2020) as the margin and collateral setting methods of CCPs have procyclical effects (Glaser & Panz, 2016), being even more notable during stressed market conditions (Wong & Zhang, 2021). According to Financial Stability Board (2009) and Hannoun (2010), procyclicality is defined as the tendency that a financial asset tends to move in line with the economic trends. That said, asset volatility and correlation between the assets increase under financial stress, especially if there is a downward trend in the market. In the case of CCPs this means that, by applying a risk-sensitive margin, the margin requirement will grow during stressed periods—as the variance of the underlying risk factor grows—forcing the clearing member to post additional collateral. Liquidity risk arises because larger margin requirements entail larger collateral for the clearing members, with a restriction that the collateral is only cash or highly liquid assets (Carter & Cole, 2018). Besides, in most cases, the value of the collateral decreases in a stressed situation (European Systemic Risk Board, 2017), but at the same time the applied haircut increases, overall leading to an increase in the additional required collateral. The coverage of the collateral can be achieved either by the sale of illiquid assets, or by withdrawing funding from other firms. This causes a funding

liquidity shortage that might exacerbate the distress (Glasserman & Wu, 2018). On the one hand, this spillover effect has been proven theoretically, for example, He et al. (2017) have shown in a structured credit risk model that the margin requirement has a significant effect on the funding liquidity. On the other hand, it has also been asserted using empirical data (Miglietta et al., 2015) on the Italian MTS repo market, that initial margins have a significantly positive effect on the cost of funding. Moreover, Glaser and Panz (2016) analyzed the bond market to ascertain that margin substantially affects systemic illiquidity, while Bakoush et al. (2020, 2019) identified that the margin requirements on the OTC markets cause a liquidity shortage in the interbank market, indicating the manner the stress can spillover. It has also been discussed that during events when funding illiquidity gets coupled with market illiquidity (Brunnermeier & Pedersen, 2009; Heller & Vause, 2012), or in case there is a correlation between asset returns and funding cost (Valderrama, 2010), then liquidity risk can easily transform into systemic risk. On top of this, an important source of systemic risk is associated with the liquidity issues faced by the CCP itself under events when several clearing members default simultaneously. In this case the stress in the market is so large that the prefunded resources are insufficient to cover the losses, causing liquidity problems for the CCP, and finally resulting in a systemwide instability (Domanski et al., 2015).

The aforementioned concerns about initial margin rendered the topic of procyclicality an important area in the literature on the margin calculation, studied by many researchers, for example, Murphy et al. (2014), Goldman and Shen (2020), Brunnermeier and Pedersen (2009), and Glasserman and Wu (2018). The central banks (Longworth, 2010) and policy makers also identified the importance of this topic, and in the EMIR regulation three methods are being introduced—from which the CCPs can freely choose—to handle the procyclicality of the initial margins:

- (a) apply an additional 25% buffer that can be exhausted in case the margin would increase significantly;
- (b) take into account with a 25% weight a historical stressed observation during the margin calculation;
- (c) apply a floor value based on a 10-year look-back period (see Article 28 of European Union, 2013).

To address the problem of the increasing liquidity risk and systemic risk, regulators require the CCPs to apply an anti-procyclical margin instead of a purely risk-sensitive one. In essence, margins should be higher in normal times as their values would depend only on a risk measure, serving as a buffer in case of a stress event (Glasserman & Wu, 2018). Optimizing or defining a trade-off between risk-sensitivity and procyclicality has been analyzed by several authors, for example, Murphy et al. (2016), Berlinger et al. (2019a, 2019b), Goldman and Shen (2020, 2017), Raykov (2014, 2018), and Tambucci (2014). The main takeaway of this literature is that neither a totally risk-sensitive nor a totally anti-procyclical margin methodology is optimal, instead this optimal value lies somewhere in between. Murphy et al. (2016) and Maruyama and Cerezetti (2019) have emphasized the importance of regulating the desired outcome of the margin requirements rather than regulating the tools and input parameters of the margining model. In subsequent research, Murphy and Vause (2021) pointed out that policy makers should target a certain level of procyclicality, and margin models should be recalibrated accordingly. A cost-to-benefit analysis was conducted in their study to assess the performance difference among procyclicality mitigating methods. Raykov (2018) and Goldman and Shen (2020) pointed out a very important aspect of the margin calculation from procyclicality point of view, meaning that there is a trade-off between the default fund and the initial margin, so the lower the initial margin, the higher the default fund *ceteris paribus*. Hence, taking into account procyclicality issues only on a margin level is not correct, since it can happen that the mutualized layer of the guarantee system will increase if we keep the margin level stable, which can consequently have a negative effect on the liquidity. Gurrola-Perez (2020) goes beyond, as the authors state that due to procyclicality being a systemic issue, it is insufficient to handle it on the level of the initial margin. That said, the changes in the portfolio composition, the incentives, and the behavior of the system participants should be taken into account, among others.

There is a shortage of studies concerning procyclicality phenomenon using real historical margin data that are set by CCPs. For example, Park and Abruzzo (2016) analyzed the Chicago Mercantile Exchange and Intercontinental Exchange. They observe that margin requirements tended to move along with the volatility if the latter was rising, however no such effect was discerned when volatility was on a decline. The remaining studies mostly examined margin requirements via the means of simulations that utilize either historical asset prices or even simulated ones (Goldman & Shen, 2017; Murphy et al., 2016; Murphy & Vause, 2021; Zhang, 2019). In a few studies, the framework rather focuses on theoretical modeling (e.g., Berlinger et al., 2019a; O'Neill & Vause, 2018; Raykov, 2014, 2018). No matter what the applied method is, the pivotal part of the margin model is a risk measure, or an alternative method that can capture the underlying risk of the asset's or portfolio's change in the value. In most cases—in practice and in academic studies as

well—the calculation of the initial margin relies on the Value at Risk (VaR) risk measure (Vicente et al., 2015). The input parameters of VaR are strictly regulated by the EMIR (European Union, 2012), with the inclusion of the look-back period, the liquidation period, and the significance level. The most crucial point of the VaR calculation is the accurate estimation of the underlying uncertainty process, namely, the volatility. In practice, for the purpose of volatility estimation, usually the historical volatility is applied (Heckinger et al., 2016), by quantifying volatility either with equally weighted or exponentially weighted moving average weighted standard deviation. Some scientific papers also use generalized autoregressive conditional heteroscedastic (GARCH) models (e.g., Glasserman & Wu, 2018; Goldman & Shen, 2017, 2020; Li et al., 2021; Murphy et al., 2016).

Regarding the mathematical formulation of the procyclicality of the initial margin, the generally applied measures follow the recommendations of Murphy et al. (2014, 2016) and European Securities and Markets Authority (2018a, 2018b). These measures are called APC measures, and it can be classified as short-term measures or long-term measures. The main goal of the short-term measures is to analyze the short-term changes of the margin when the market is under a stressed condition, while the long-term measures capture a long-term economic cycle (Zhang, 2019). As an example of the short-term measure, we will apply the *n-day measure*, defined as the cumulated margin change over an *n*-day period (Murphy et al., 2014), if the cumulated margin value is positive, otherwise the value of this measure is zero. We will consider this indicator by taking the difference between the maximal and minimal daily required margins throughout the *n*-day period to be able to quantify the worst additional liquidity need. The length of the *n*-day period will be 30 days also based on (Murphy et al., 2014). Besides the *n*-day measure, we are going to apply another short-term measure (European Securities and Markets Authority, 2018b), that is, the *standard deviation* of the margin. This will be measured on a 1-year time horizon, to be consistent with the look-back period of the VaR model. The long-term APC measure will be the *Peak-to-trough measure*, which is the ratio of the largest and smallest margin values throughout a certain period. For this APC measure we will consider a time period of 10 years, assuming that such a time period is sufficiently long to cover economic booms and recessions as well.

As for this present study, we are going to calculate a historical VaR model, and being compliant with the regulations, we apply a 99% significance level, a 250-day look-back period, and a liquidation period consisting of 2 days. In particular, we handle procyclicality according to Article 28/b of European Union (2013). We focus on the impact of the correlation on the value of the initial margin requirement and also on the procyclicality measures. We aim to fill a substantial gap in the literature by assessing the magnitude of overmargining when all the rules of the regulators are prudently implemented. As a consequence, we can also decide whether our particular margin method satisfies the property of subadditivity. This implies that a method neglecting the correlation structure between assets is not necessarily the most effective one to handle risk. Moreover, Vicente et al. (2015) stated that by not taking into account the diversification effect on portfolio level by CCPs to reduce the risk of a potential underestimation of the margin, does not necessarily lead to a reduction in the systemic risk. For example, the clearing members themselves are not incentivized to eliminate their own risk via diversification or applying a hedging strategy. In the literature the effect of the correlation is not often analyzed. Li et al. (2021) have analyzed the effect of correlation change during changing market environment, and its effect on the margin. On the basis of the above, we extend the literature by carefully modeling the correlation structure of the assets, and assess its effect on the margin requirements and procyclicality.

3 | JOINT MODELING OF ASSET PRICES AND DETERMINATION OF MARGIN

3.1 | Theoretical background

In this section we discuss the joint modeling of asset prices. Thereafter we detail the fitting procedure and the simulation technique which we apply in Section 4 for our calculations. Our main goal is to obtain suitable individual asset price modeling and it is also of crucial importance to capture the dependency structure between different assets. We demonstrate differences in applying margin calculation methods on a *Portfolio treatment* versus on an *Individual treatment*, thus we enable the dependency structure to be dynamic and the correlation between assets is not fixed for the entire time-span.

That said, we apply a stochastic volatility–stochastic correlation asset structure model following the lines of Driessen et al. (2013) and Márkus and Kumar (2021).

We are given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$. We assume that the price of asset i ($i = 1, 2$), denoted by S_i , follows an Itô process with respect to this filtered probability space with expected return μ_i and with stochastic diffusion ϕ_i :

$$dS_i = \mu_i S_i dt + \phi_i S_i dW_i, \quad i = 1, 2. \quad (1)$$

Further, let us give an adapted process $\rho(t)$ with $|\rho(t)| \leq 1$ on the filtered probability space. The modeling framework assumes that the pairwise correlation between the two Wiener processes is derived from this $\rho(t)$ process in the following regard: $d[W_i, W_j]_t = \rho(t)dt$, where by $d[\cdot, \cdot]_t$ we denote the quadratic covariation between the two processes at time t . We call the unique $\rho(t)$ process the stochastic correlation process to express the changing correlation between the drivers of the assets.

We further tailor Equation (1) to consider the Hull–White *stochastic volatility* (SV) model (see Chesney & Scott, 1989; J. Hull & White, 1987; Taylor, 1982). Each asset price follows a diffusion with a random volatility process:

$$dS(t) = \mu S(t)dt + V(t)S(t)dW_S(t), \quad (2)$$

where $\ln V(t)^2$ is a mean-reverting process

$$d \ln V(t)^2 = \kappa(\theta - \ln V(t)^2)dt + \sigma_V dW_V(t) \quad (3)$$

and W_S and W_V are the independent Wiener processes.

This SV model is a popular alternative to the discrete GARCH-type model which rather models the volatilities stochastically. As claimed in Jacquier et al. (2002) and in Kim et al. (1998) and as also stated in Kastner and Frühwirth-Schnatter (2014) the observed log-returns ($y(t) = \ln S(t) - \ln S(t-1)$) of the SV model in Equations (2) and (3) can be discretized as

$$y(t) = \mu + e^{\frac{h_t}{2}} \epsilon(t), \quad (4)$$

$$h(t) = \alpha + \beta(h(t-1) - \alpha) + \sigma_V \eta(t), \quad (5)$$

where the i.i.d standard normal innovations $\epsilon(t)$ and $\eta(t)$ are independent for all t .

We use the stochvol R package of Kastner (2019) to estimate the parameters of the model in Equations (4) and (5) and to simulate trajectories. Moreover, the package provides model residuals that can be treated as realizations of the $W_S(t)$ Wiener process for the considered S assets, and with these residuals we can also estimate the parameters of $\rho(t)$ correlation process.

We now specify the choice of $\rho(t)$ correlation process. We apply the framework of Van Emmerich (2006) who chose a mean-reverting process for the stochastic correlation as follows:

$$d\rho(t) = \kappa(\theta - \rho(t))dt + \sqrt{1 - \rho^2(t)} dW_\rho(t). \quad (6)$$

As for estimating the model parameters of Equation (6) we use the stationary Fokker–Planck equation of this process. As shown in Van Emmerich (2006) this equation is of the following form:

$$p(x) = c \left(\frac{1-x}{1+x} \right)^{-\kappa\theta} (1-x^2)^{\kappa-1} \quad (7)$$

with c such that $\int_{-1}^1 p(x) dx = 1$ holds.

3.2 | Estimation with historical prices

As pointed out earlier we consider two different margin calculation methodologies (*Individual treatment* and *Portfolio treatment*) by considering portfolios that contain two assets. Namely, we set up a portfolio which consists of an asset

representing the Financial Times Stock Exchange 250 (FTSE250) index and an asset representing the British Pound/US Dollar (GBP/USD) currency of equal weights on the first day, keeping the number of assets as fixed for the remaining time. For estimations we consider the time frame January 2, 1990–April 20, 2021 using daily observations. First, we estimate model parameters for the two historical asset price processes. We start by individually estimating the parameters of Equations (4) and (5). Next, we proceed by estimating the parameters of the $\rho(t)$ empirical stochastic correlation process. To do so, we invoke the findings of Teng et al. (2016) that applied a sliding window technique to estimate historical correlation. This sliding window technique is based on calculating the Pearson correlation of the two variables for the previous n observations. As outlined we use daily observations and we consider the 30-day method, meaning that for each estimation the last 30 observations for the two variables are used. Again, we apply this calculation method for the readily available model residuals that are treated as realizations of the Wiener processes in Equation (2).

To make estimation to the corresponding parameters of $\rho(t)$ in Equation (6) viable, we first express the empirical density function of the correlations. The applied procedure implies that these historical observations are representative of the stationary distribution of the mean-reverting $\rho(t)$ process and so the empirical density function can be expressed in the form of Equation (7). Thus we proceed by fitting a polynomial in software R and so to calibrate the κ_ρ and θ_ρ parameters to historical data points.

Figure 1 reveals that an accurate fit can be achieved using the aforementioned steps.

Moreover, we want to confirm the goodness of the overall modeling choice. We do so by simulating process trajectories with the estimated parameters. We can simulate $\rho(t)$ using the estimated parameters of Equation (6), and as a next step, we can simulate two Wiener processes that are correlated with each other in accordance with the values of the simulated $\rho(t)$ process. Next, we can use Equations (4) and (5) and their estimated parameters to obtain simulated log-returns for the three assets.

In Figure 2, we show in two time scales the estimated historical correlation process between the Wiener processes of the FTSE250 and the GBP/USD prices applying the SV model estimation. On top of this, we also show the simulated stochastic correlation processes in two equivalent time scales. The estimated $\rho(t)$ process values exhibit mean-reverting property, thus corroborating the choice of modeling as proposed by Equation (6). Having a careful look at Figure 2 and comparing the estimated and simulated process values in both time scales, one can discern that a good representation of the empirical correlation process can be achieved via simulation.

All estimated parameters can be seen in Table 1. To provide further reassurance of the goodness of estimation, we also plot the historical asset prices against the simulated ones. In Figure 3 we can see this comparison which reveals that simulated trajectories are akin to the historical ones in terms of trend, volatility, and covered range using equally long time steps for the simulations.

The simulations were all done using the estimated parameters shown in Table 1. Nonetheless, estimation is based on an underlying method, thus there is always a selection bias that makes the accurate characterization of model

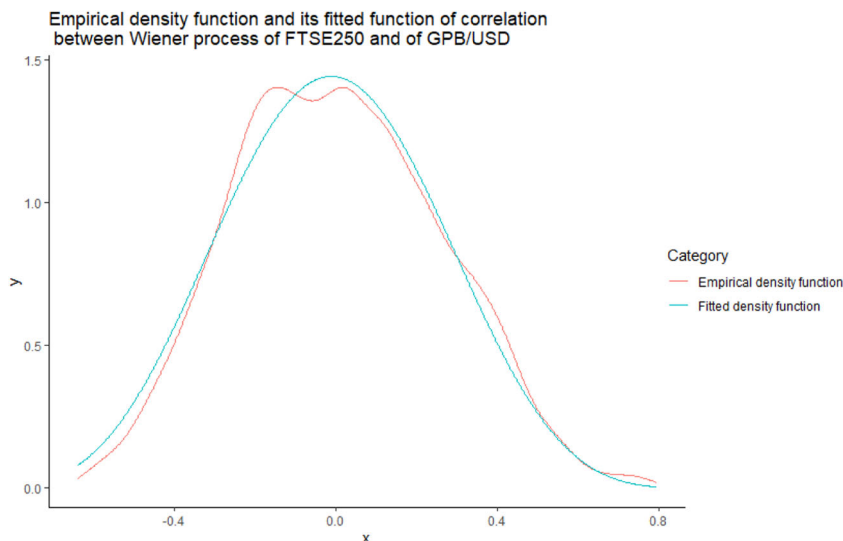


FIGURE 1 Empirical density function of the correlation between $W_{\text{GBP/USD}}$ and W_{FTSE250} and the fitted the $p(x)$ function using the nonlinear least-squares method. FTSE, Financial Times Stock Exchange; GBP, British Pound; SV, stochastic volatility; USD, US Dollar. [Color figure can be viewed at wileyonlinelibrary.com]

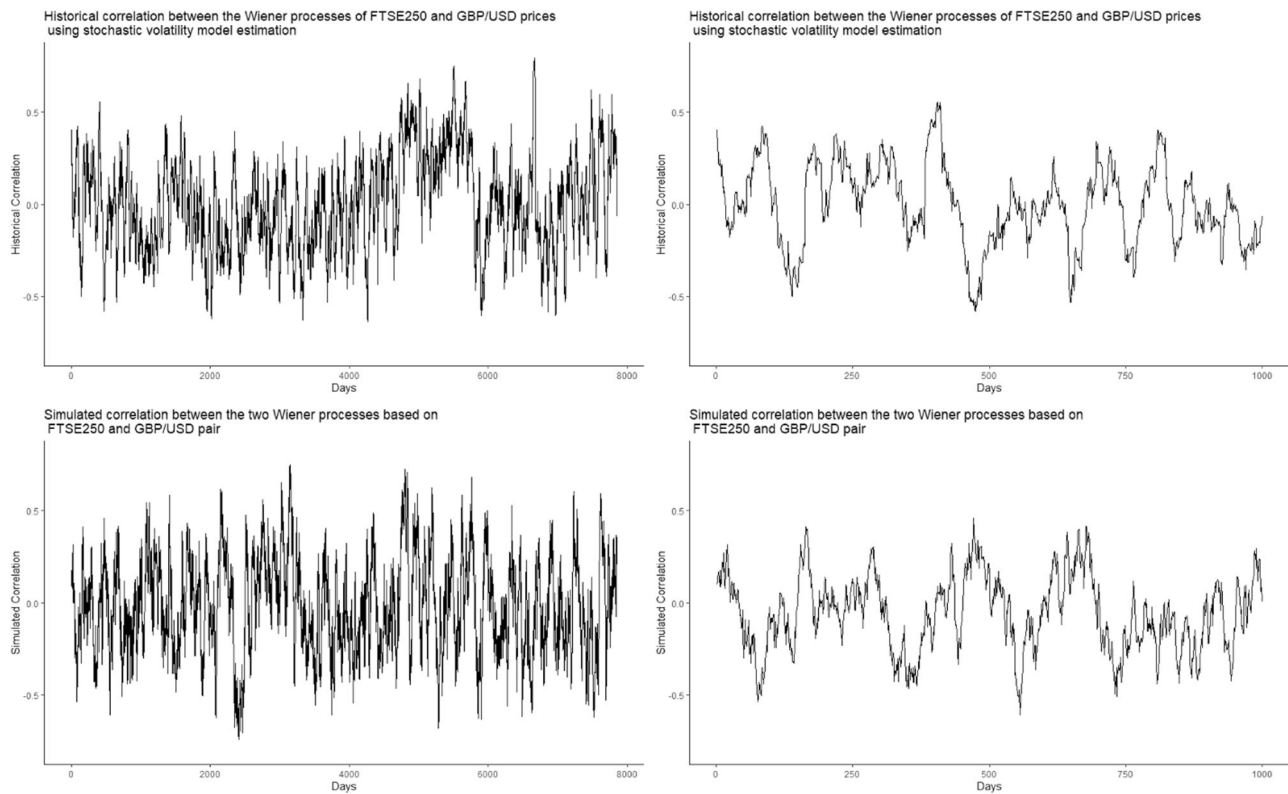


FIGURE 2 Graphs at the top show the estimate of the $\rho(t)$ stochastic correlation process of the two Wiener processes corresponding to the SV model fits of the historical FTSE250 and GBP/USD processes. A 30-day sliding window technique is applied and the time period used for the SV model fit is January 2, 1990–April 20, 2021. The graphs at the bottom show simulated trajectories of the $\rho(t)$ correlation process of the two Wiener processes using the estimated κ and θ parameters from Table 1. The left graphs of both rows show these process values for 7857 days, whereas the right graphs show only the first 1000 days of these $\rho(t)$ processes. FTSE, Financial Times Stock Exchange; GBP, British Pound; SV, stochastic volatility; USD, US Dollar.

TABLE 1 The first column of this table shows the estimated parameters using the technique discussed in the section

| Parameter | Estimation | Minimum | Maximum |
|------------------------------|----------------|----------|---------|
| $\alpha_{\text{GBP USD}}$ | -10.476 | -12.5 | -9.5 |
| $\beta_{\text{GBP USD}}$ | 0.9811 | 0.52 | 0.993 |
| $\sigma_{V, \text{GBP USD}}$ | 0.1263 | 0 | 0.24 |
| $\mu_{\text{GBP USD}}$ | -1.82924E - 05 | -0.00007 | 0.00007 |
| α_{FTSE250} | -9.934 | -12 | -9 |
| β_{FTSE250} | 0.971 | 0.52 | 0.993 |
| $\sigma_{V, \text{FTSE250}}$ | 0.252 | 0.01 | 0.49 |
| μ_{FTSE250} | 0.0002676 | -0.00006 | 0.00054 |
| κ | 6.68583 | 1.337 | 33.429 |
| θ | -0.00981 | -0.5 | 0.5 |

Note: The second column shows the minimum value for the given parameter, whereas the third one shows the maximum value for the parameter. Abbreviations: FTSE, Financial Times Stock Exchange; GBP, British Pound; USD, US Dollar.

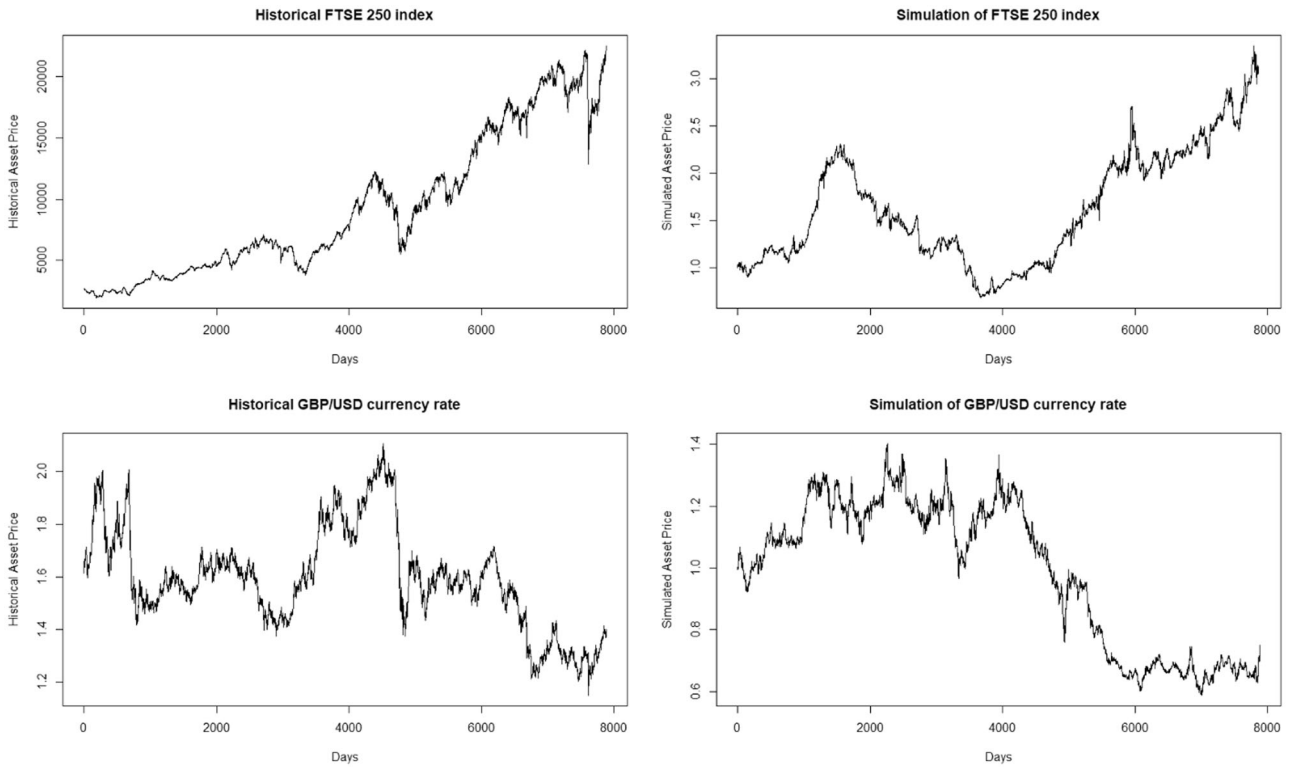


FIGURE 3 Historical asset prices on the left and simulated asset prices on the right. The graphs correspond to FTSE250 at the top and to GBP/USD at the bottom. The parameters used for this simulation were taken from Table 1. FTSE, Financial Times Stock Exchange; GBP, British Pound; USD, US Dollar.

parameters not viable. On the basis of this and also on the goal to provide an overall framework, we will simulate processes with a wide range of parameters.

The main attempt of Section 3.3 is to demonstrate that certain properties always hold independently of the choice of parameters.

To that end we assign an interval for each parameter, and we only simulate trajectories with parameters that lie within their corresponding interval. These intervals are meant to cover a set of parameters that produce reasonable asset trajectories. That said, within this set, we still consider the possibility of outlier trajectories that might occur every so often, but we stay within the range of reality. The range for all considered parameters is also shown in Table 1. We now provide some individual confirmation for the choice of the interval of the individual parameter.

The α , β , and σ_V parameters all correspond to the $h(t)$ latent SV process. We could get as low as 0 with the σ_V parameter whilst fixing the remainder parameters and still obtain reasonable trajectories, nonetheless to avoid extremely volatile processes we fix the upper limit of σ_V of both the currency and the index process, restricting the currency process to generally less volatile trajectories. The β parameter of the discussed model is estimated in the literature close to 1 (see, e.g., Kastner, 2019), slightly lower than this upper boundary in the parameter range. The choice of this study reflects this by considering β parameters that come near to 1, still we provide sufficient flexibility by setting the lower boundary to 0.52. The α parameter has been carefully selected, had we dipped below the lower boundary we would have obtained extremely nonvolatile processes, had we exceeded the upper boundary we would have obtained extremely volatile processes.

The μ parameters correspond to the drift of the assets or in other words indicate the mean of the log-returns, the currency process is symmetric in this regard, whereas we consider more simulations with positive drifts for the index process. Clearly one needs to be careful to choose financially sensible values for drifts as estimation techniques for drift can be unreliable. Moving out of the chosen range leads to obtaining yearly returns that contradict empirical observations even under crisis or blooming periods.

Finally, the stochastic correlation parameters κ and θ from Equation (6) are chosen in a rather flexible way. We let the long-term correlation of the two Wiener processes to be as low as -0.5 and be as high as 0.5 . The mean-reversion

parameter corresponding to the speed of readjustment is also chosen in a widespread range, the fastest speed is 25 times the slowest speed.

3.3 | Determination of margin

In accordance with the EMIR regulation, we apply margin calculation methodology based on the (b) part of Article 28 of European Union (2013), namely to take into account with a 25% weight a historical stressed observation during the margin calculation. To reflect this condition, we calculate a weighted average of two VaR values, one covering the previous 10 years of observations with a 25% weight, while the other one is based on the previous 1 year of observations, with a 75% weight.

Denoting by *Marg* the proposed margin requirement the corresponding formula is given as

$$Marg = 75\% \cdot PC_{1y} + 25\% \cdot PC_{10y}, \quad (8)$$

where

$$PC_{10y} = \max(\text{abs}(\min(VaR_{0.01}(R_{1y})_{10y})), (\max(VaR_{0.99}(R_{1y})_{10y}))) \quad (9)$$

and

$$PC_{1y} = \max((VaR_{0.99}(R_{1y})), (VaR_{0.01}(R_{1y}))). \quad (10)$$

R_{1y} denotes the series of log-returns of the given asset for the past 1 year, $VaR_{\alpha}(\hat{x})$ denotes the Value at Risk at α percentile calculated for the \hat{x} vector, and $VaR_{\alpha}(\hat{x})_t$ denotes the vector of calculated Value at Risks for the previous t time period, where each VaR is calculated for the equally long \hat{x} vector. In the above case, for instance, $VaR_{0.01}(R_{1y})_{10y}$ denotes the vector of Value at Risks, where each VaR is calculated for the previous 1 year of log-returns, and this vector contains these calculated VaRs for the previous 10 years. It is important to mention related to Equations (9) and (10) that we consider both the 1% percentile and 99% VaR value, since CCPs have the same amount of short and long positions, so they face the downside and upside risk as well.

4 | RESULTS AND DISCUSSION

Using the methodology discussed in Section 3.3, for a given time point, to calculate the *Margin* according to Equation (8) we need observations covering 11 years of time. We have 30 years of data for the two assets, however to expand this analysis for a more comprehensive picture we will also use simulated prices. Using this simulation we will be able to test how much the results depend on the choice of parameters. For each simulation we carry out four margin-related calculations, and three APC measures related calculations. The APC measures will be the *n-day measure*, the *standard deviation of the margins*, and the *Peak-to-trough ratio*. The four margin-related calculations are as follows:

1. *Ratio of the initial margins*: It is the ratio of the initial margins calculated with *Individual treatment* and with *Portfolio treatment*.
2. *Ratio of the overmargining values*: Overmargining is calculated as the proportion of margin requirement for a given day and the absolute logarithmic change of underlying's value on that given day. This captures the extent to which the calculated margin covers the actually required margin for the given day. The ratio is the overmargining value of the *Individual treatment* and the *Portfolio treatment*.
3. *Back-test of the margin for Individual treatment*: The main goal of the back-test is to quantify the ratio of the insufficient margin requirements based on *Individual treatment*. As for insufficient margin requirement, for each day we calculate a proportion using the previous 250 days when the required margin failed to sufficiently cover the change in the underlying value. As long as this value is below 1%, the applied model can be regarded as an adept one, since the VaR model was calibrated on a 99% significance level.
4. *Back-test of the margin for Portfolio treatment*: This is an analogous calculation to 3, the only difference is that this is applied based on *Portfolio treatment*.

Regarding the *Ratio of the initial margins*, in case there is a ratio below 1, we find occurrence of violating the subadditivity property. In general we do not expect a subadditive margin, because VaR itself is not subadditive (Acerbi, 2002), meaning that the following is not necessarily satisfied:

$$\text{VaR}(X + Y) \leq \text{VaR}(X) + \text{VaR}(Y), \quad (11)$$

where X and Y are the two random variables. The issue of VaR and lack of subadditivity has been thoroughly discussed in Danielsson et al. (2013), where the authors showed that under certain requirements such that asset returns are multivariate regularly varying VaR is subadditive in the tail region, but they also claim that this asymptotic result may not hold in practice, violating the subadditivity of VaR.

We present the results of the comparisons in Figures 4–8 and A1–A8. In particular, Figure 4 reveals the daily *Ratio of initial margins* applying the two margin calculation methods to the historical data set. The figure shows that for almost all historical days the margin requirement was higher when calculated on an *Individual treatment* than on a *Portfolio treatment*, as the value of the ratio tended to be above 1. The minimal value of this ratio is 0.967 and the average value is 1.155. It indicates that calculating the required margin on an *Individual treatment* leads to a substantial increase in the margin requirement compared with calculating it on a *Portfolio treatment*. Besides, the lower than 1 minimal values also show examples when the subadditivity property is violated. A possible explanation for this violation, on top of the general lack of subadditivity of VaR, is that based on Equations (8)–(10), the final margin requirement is not even a linear transformation of the VaR values, leading to path-dependent margin values. Interestingly, as Figure 4 shows the time period when the margin requirements calculated on a *Portfolio treatment* exceeded the requirements calculated on an *Individual treatment* were around the onset of COVID-19 pandemic. The emergence of COVID-19 caused a crisis on the financial markets, and as for this particular crisis the requirement of *Portfolio treatment* was more protective than that of *Individual treatment*.

Figure 5 reveals the daily comparison of the discussed APC measures, calculated for the two margin calculation methods on the historical data set. Regarding APC measures, we see that the daily standard deviation of margins calculated over the previous 250 days has a few spikes when it deviates to a rather high level. On average its value is 1.3023 indicating that the calculation on *Individual treatment* leads to more volatile margin requirements. On the other hand, the Peak-to-trough ratio of margins calculated on *Individual* and *Portfolio treatment* is for most days below 1, indicating that calculating the margin on a *Portfolio treatment* on average leads to higher Peak-to-trough values. Finally, the graph corresponding to the difference in 30-day measures between margins calculated on *Individual* and *Portfolio treatment* does not reveal any spectacular pattern. Its average value is 0.006%, indicating that there is no significant difference in 30-day measures when calculating the requirement on an *Individual* or on a *Portfolio treatment*. To sum up, based on the historical results we can argue that margin calculation on an *Individual treatment* tends to lead to a greater amount of margin requirement, still the APC measures do not emphatically show that any of the calculation methods is more anti-procyclical than the other one.

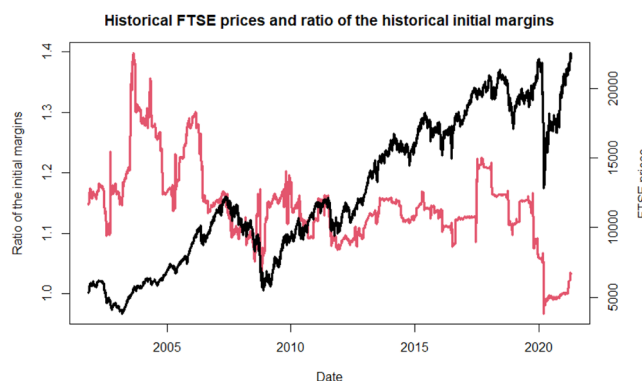


FIGURE 4 The historical daily FTSE prices in black and the daily ratio of historical initial margins of the considered portfolio (consisting of an asset representing the FTSE index and an asset representing the GBP/USD currency of equal weights on the first day, keeping the number of assets as fixed for the remaining time) in red. The daily ratio is calculated as the ratio of margin requirement based on *Individual treatment* to margin requirement based on *Portfolio treatment*. The covered time period is November 2001–April 2021. FTSE, Financial Times Stock Exchange; GBP, British Pound; USD, US Dollar. [Color figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

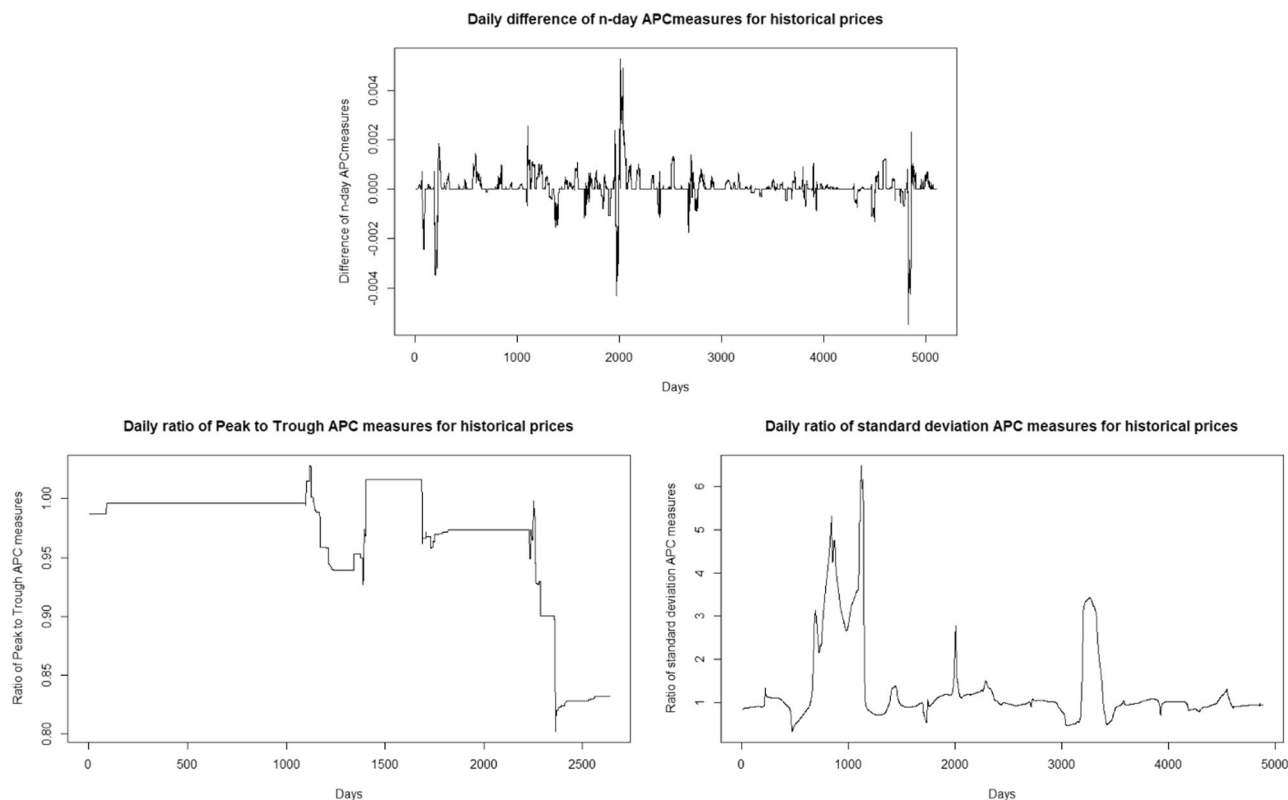


FIGURE 5 Different comparisons of margin requirements based on *Individual* and *Portfolio treatment* from the viewpoint of the APC measures. For all graphs we display values for which we used historical FTSE and GBP/USD values covering the time range from January 1, 1990 to April 20, 2021 in accordance with the data set used for estimation. From left to right and from top to bottom we display the difference between n -day APC measure based on *Individual treatment* and n -day APC measure based on *Portfolio treatment*; the ratio of Peak-to-trough measure based on *Individual treatment* to Peak-to-trough measure based on *Portfolio treatment* and the ratio of margin's standard deviation based on *Individual treatment* to margin's standard deviation based on *Portfolio treatment*. APC, anti-procyclicality; FTSE, Financial Times Stock Exchange; GBP, British Pound; USD, US Dollar.

Nonetheless, the previous consideration only used the historical data set, and so lacks extensiveness to draw conclusion. To get a better picture of the discussed topic we consider a range of simulations. Thus, we now discuss the results corresponding to the simulations based on Figures 6–8 and A1–A8. For each figure we fix all but two parameters from Table 1, and with the two varying parameters we aim to cover asset trajectories that result in reasonable asset prices. Thus, the range of parameters for simulation is chosen to reflect those values that do not extensively violate any of the stylized facts. Most importantly we excluded parameter values that result in exploding or extremely nonvolatile (almost deterministic) simulated asset prices. We cover the two varying grids with 25–25 parameters, overall showing the results on each graph to 625 simulations each covering 30 years of data.

Figures 6 and 7 in this section and Figures A1–A8 in Appendix A follow a similar structure, meaning that for all parameter sets (henceforth: fixtures) we present four margin-related and three APC-related graphs in two separate figures. Figures that contain the four margin-related contour maps, visualize in the upper left corner the ratio of the average margin requirements calculated on *Individual* and on *Portfolio treatment*. All these graphs unequivocally show that on average margin requirement calculated on *Individual treatment* is substantially higher. For each fixture we can only see a few particular parameter combinations for which the corresponding simulation gave a ratio lower than one; this ratio never dips below 0.95. The ratio on all five corresponding graphs has a mean value of around 1.2–1.25. Thus, based on this comprehensive simulation we can assess that there is a 20%–25% difference between the two calculation methods; the individual method allocates a 20%–25% higher margin in the long term. The few counterexamples when the ratio is below 1 occur infrequently, still it again verifies that the applied margin calculation method is not subadditive.

The remaining contour maps help us to decide on the necessity of the higher-margin requirement. We present the ratio of average overmargining in the upper right corner, which aims to determine the ratio of the extents to which the

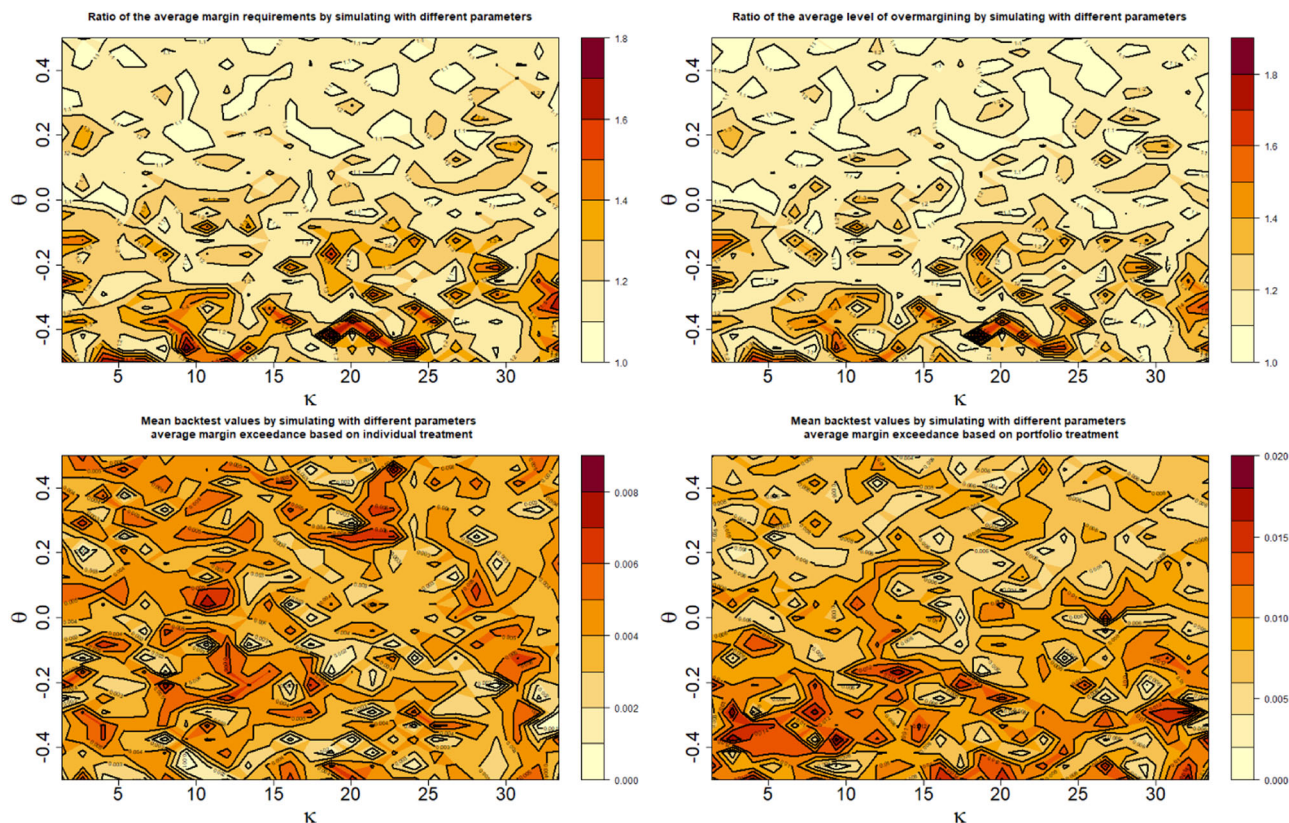


FIGURE 6 Different comparisons of margin requirements using *Individual* and *Portfolio treatment* following the four points outlined at the beginning of Section 4. For all graphs we display values that are based on simulations using 30 years of data by selecting the θ and κ parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the ratio of average margin requirements based on *Individual treatment* to average margin requirements based on *Portfolio treatment*; the ratio of average overmargining based on *Individual treatment* to average overmargining based on *Portfolio treatment*; the proportional average of days with insufficient margin requirement based on *Individual treatment* and the proportional average of days with insufficient margin requirement based on *Portfolio treatment*. For all graphs quantities are calculated with a daily frequency and the average is taken over these daily values. [Color figure can be viewed at wileyonlinelibrary.com]

two methods overallocate margins. These graphs reveal that for all fixtures the individual method leads to a higher level of overmargining. In other words, the *Individual treatment* margin method allocated a significantly (around 25%–35%) higher level of unused margin. To deem the *Portfolio treatment* an acceptable method, we need to assess its prudence with a back-test. Thus, we perform an appropriate back-test for the two calculation methods. We calculate the percentage of days when the set margin failed to sufficiently cover the change in the portfolio's value. Comparing the bottom left and bottom right graphs in figures with four contour maps, we assess that *Individual treatment* requirement proved to be insufficient only on average around 0.6% of the days, whereas *Portfolio treatment* was insufficient on average around 0.8%–1% of the days. This suggests that the generally higher requirement set via *Individual treatment* is not absolutely unnecessary, it indeed provides an occasional additional shield when the *Portfolio treatment* fails to fully cover the change in the portfolio value. Nonetheless, as discussed the average margin exceedance based on *Portfolio treatment* also hovers around 1%, being slightly less than 1% on average. Thus, we obtained reassurance that for both methods the 99% based margin calculation efficiently covers changes for at least 99% of the days. We continue the discussion by separately analyzing the back-test results for the parameter pairs. Our goal is to find any existing pattern on the graphs and to unveil whether the parameter combinations with a higher or lower level of inefficient coverage are linked to cases when crises are more frequent. One can argue that during market crashes correlations become high and the volatility of individual asset prices increases. In the considered joint stochastic volatility–stochastic correlation framework the parameters that affect the likelihood of such market crashes are θ , α_1 , α_2 , σ_1 , and σ_2 . If θ is high, then the stochastic drivers of the individual assets move in the same direction, and so during a crisis period the individual asset breakdowns get transmitted among the assets. Regarding α_1 , α_2 , σ_1 , and σ_2 , as mirrored

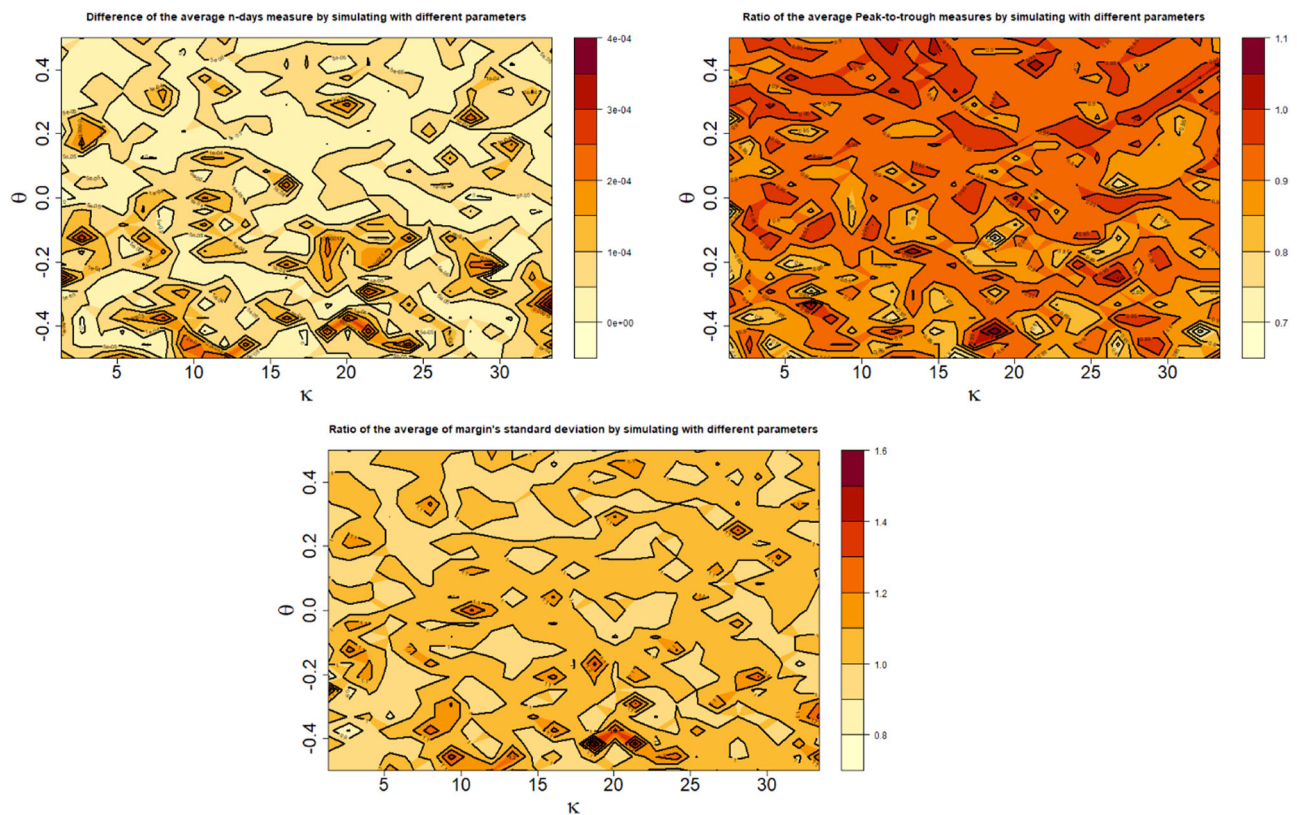


FIGURE 7 Different comparisons of anti-procyclical measures using *Individual* and *Portfolio treatment*. For all graphs we display values that are based on simulations using 30 years of data by selecting the θ and κ parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the difference between average n -day APC measures based on *Individual treatment* and average n -day APC measures based on *Portfolio treatment*; the ratio of average Peak-to-trough measures based on *Individual treatment* to average Peak-to-trough measures based on *Portfolio treatment* and the ratio of average margin's standard deviation based on *Individual treatment* to average margin's standard deviation based on *Portfolio treatment*. For all graphs APC values are calculated with a daily frequency and the average is taken over these daily values. APC, anti-procyclical. [Color figure can be viewed at wileyonlinelibrary.com]

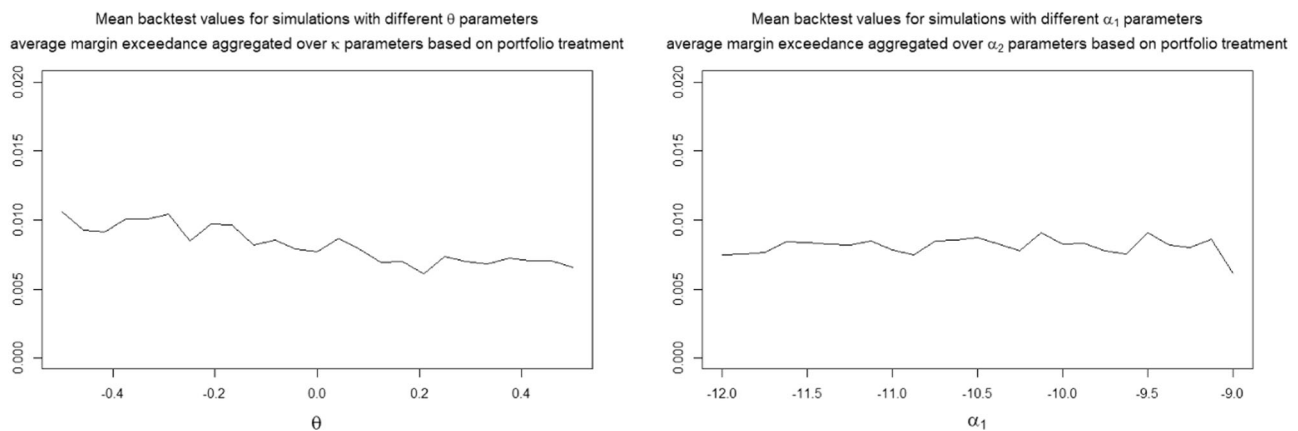


FIGURE 8 The mean back-test values for the *Portfolio treatment*. For the graph on the left, we display values that are based on simulations using 30 years of data by selecting the θ parameter from the axis and fixing all parameters other than κ as “Estimation” from Table 1. For all given θ values we average the ratios obtained by choosing different κ parameters from the range of κ according to Table 1. For the graph on the right, we display values that are based on simulations using 30 years of data by selecting the α_1 parameter from the axis and fixing all parameters other than α_2 as “Estimation” from Table 1. For all given α_1 values we average the ratios obtained by choosing different α_2 parameters from the range of α_2 according to Table 1. For both graphs quantities are calculated with a daily frequency and the average is taken over these daily values.

by Equations (4)–(6), the likelihood of sudden changes in prices increases as these parameter values get higher. Thus we more carefully inspect the bottom right graphs of Figures 6, A1, and A5. None of the corresponding contour maps shows a tendency of a higher failure of back-tests for *Portfolio treatment* as we move in the direction when shocks are more frequent. Therefore we can assert that the *Portfolio treatment* succeed to be as adequate under market crashes as during normal periods.

In Figure 8 we more carefully investigate whether the *Portfolio treatment* passes the back-test by aggregating the back-test values of the contour maps by fixing one parameter in the (θ, κ) and (α_1, α_2) pairs and varying the other parameter. The corresponding graphs show that the exceedance rate is acceptable as it consistently stays around or slightly below 1%, reinforcing the accuracy of *Portfolio treatment*. Regardless of the above arguments, the primary task of the margin is to cover losses under normal market conditions, whereas the default fund should cover the losses in case of “extreme but plausible” market conditions (European Union, 2012). Thus, we can state that the obtained results for *Portfolio treatment* are in accordance with the goal of the regulation and we cannot dismiss the *Portfolio treatment* by the means of this study.

Next, we examine the contour map figures pertaining to the three APC measures. In the upper left corner the n -day measure can be seen applying a 30-day interval length. The difference between the average 30-day measures of the *Individual* and the *Portfolio treatment* is greater than 0 for most parameter combinations, we can see only a few exceptions when the value dips below 0. This shows that the average 30-day APC measure is likely to be higher when margin requirement is set according to the *Individual treatment* method. The corresponding contour maps to Peak-to-trough measures are located at the top right corner of the APC measure figures, and one can observe that the ratio is quite close to 1, there is a slight tendency not to reach 1 for most parameter combinations. Values slightly less than 1 indicate that the denominator of the ratio is more often greater, meaning that the *Portfolio treatment* margin requirement is likely to have a higher corresponding Peak-to-trough value than that of the *Individual treatment* margin requirement. As for standard deviations, we see on the corresponding contour maps that the ratio of the average of margins' standard deviation is around 1, and for most fixtures these ratios are slightly higher than 1. This indicates that there is no major difference in terms of daily volatility between the two calculation methods, still ratios are likely to be higher than 1, meaning that the standard deviation of the margin of the *Individual treatment* is in general larger. To sum up, based on the three APC measures we cannot resolutely claim that any of the two calculation methods leads to less anti-procyclical margin requirements. There is no emphatic accordance between the corresponding graphs that would clearly indicate such an outcome.

We collate all aforementioned results, to assert that calculating margins on a *Portfolio treatment* fulfills its primary goal, that is, the sufficient coverage in the change of the assets' values. The higher level of untapped margin in case of the *Individual treatment* implies an unnecessary requirement toward the clearing members, making their operation unfavorable. Although the higher-margin level indicates that the CCP initially takes away a larger amount of liquid assets from the clearing members; what really matters from liquidity management point of view is not the absolute level of margin, instead the frequency and extent of changes in margins over time. This can be best measured by the volatility of margin requirements, as one of the APC measures. It tends to be smaller when using the *Portfolio treatment*, indicating that it does not withdraw more liquidity from the market than the *Individual treatment*. Finally, if we take into account all of the APC measures, none of the treatments are inferior compared with the other from a procyclicality point of view. To sum up, the *Portfolio treatment* requires a substantially lower amount of funds from the clearing members to set aside for daily margins, it is moderately superior from a daily liquidity management point of view, besides it comfortably passes the back-test as well. Thus, based on the comprehensive framework of this study, we claim that the *Portfolio treatment* method is a more efficient margin calculation method than the *Individual treatment* method.

5 | CONCLUSION

In this paper we contributed to the growing literature concerning the margin requirements of CCPs. We compared two margin calculation methodologies of potential portfolios of the clearing members, one ignoring possible correlation among the individual assets (*Individual treatment*), the other one taking this into account (*Portfolio treatment*). The results reveal that even though for the vast majority of days the *Individual treatment* requires a higher-margin level on the overall open positions, still there are examples when the requirement of *Portfolio treatment* exceeds the another one. Although these few examples are rare, they are important from a regulatory point of view because CCPs are systematically important payment systems, for which it is important to handle extreme and rare cases to as not to increase systemic risk. Moreover, we have shown on a historical data set that such a rare case happened during the onset of COVID-19 pandemic when a shock hit the financial market. As the main contribution of this study, based on a

comprehensive simulation framework, we can claim that on average using the *Individual treatment* we require a 20%–25% higher margin, putting under too much distress to the clearing members. Meanwhile, the back-tests show that using the *Portfolio treatment* we satisfy the main objective of the requirement, only exceeding the actual margin requirement, with the absolute price changes, in fewer than 1% of the days. This shows the adequacy of the *Portfolio treatment*. Moreover, the APC measures reveal discordant results, because the corresponding findings do not indicate that any of the treatments are less anti-procyclical. Nevertheless, the results showed that the volatility of the margin tends to be larger for the *Individual treatment*, indicating that the daily liquidity management of the clearing members is more strenuous when *Individual treatment* is applied. In sum, we obtain that the *Portfolio treatment* is no less prudent than the *Individual treatment*.

These results are important from a regulatory point of view. In light of the above, we could substantially reduce the burden on the clearing members without affecting the efficiency of the clearing houses in terms of applying adept margin methods.

We acknowledge the limitations of this study that opens up future research opportunities. A more comprehensive identification of the circumstances when *Portfolio treatment* gives a larger margin requirement than the *Individual treatment*, and how their relationship changes through an economic cycle can both be further investigated. A future study can also investigate portfolios containing more than two assets to see how the results of this study are affected by considering a wider range of assets.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in MarginRequirements_StochasticCorrelation at https://github.com/davidzoltanszabo/MarginRequirements_StochasticCorrelation, see Szabó (2022).

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APPENDIX A

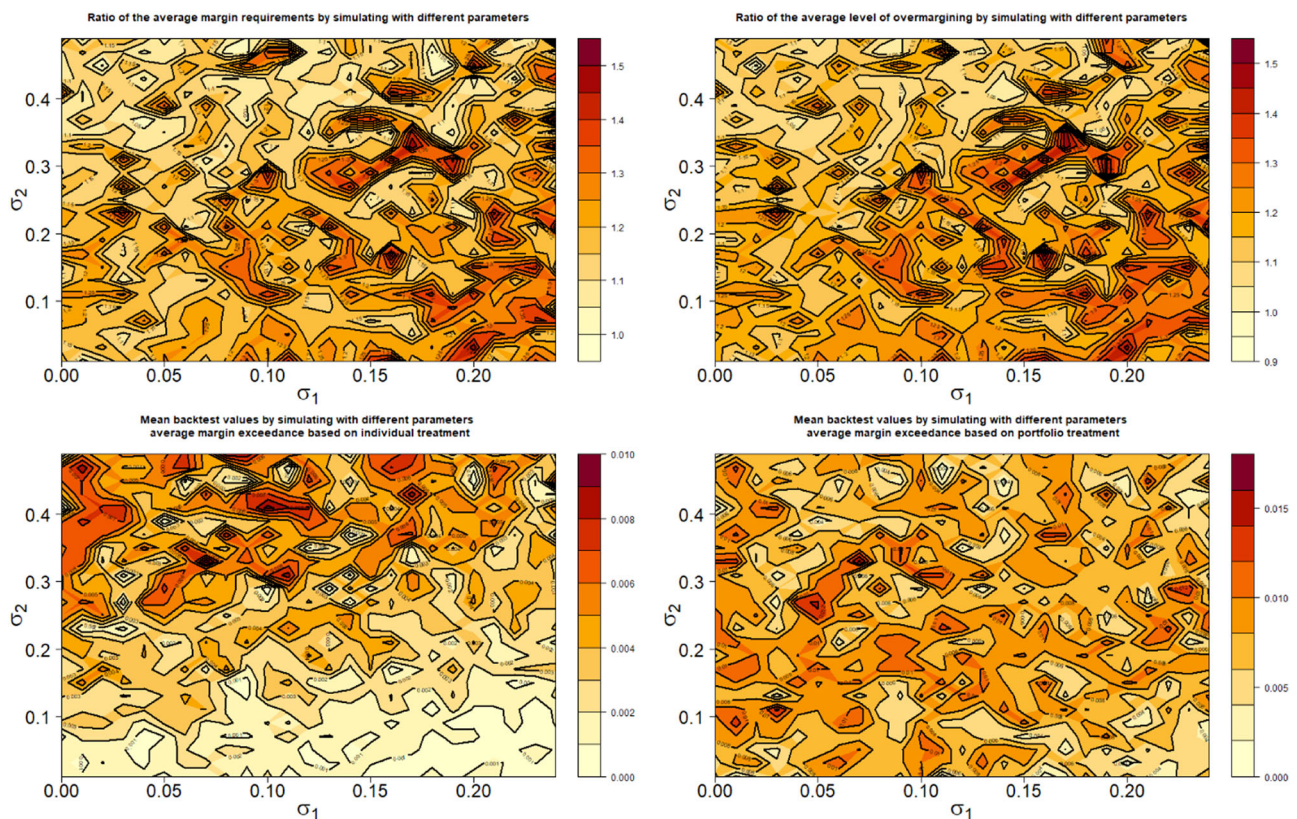


FIGURE A1 Different comparisons of margin requirements using *Individual* and *Portfolio* treatment following the four points outlined at the beginning of Section 4. For all graphs we display values that are based on simulations using 30 years of data by selecting the σ_1 and σ_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the ratio of average margin requirements based on *Individual* treatment to average margin requirements based on *Portfolio* treatment; the ratio of average overmargining based on *Individual* treatment to average overmargining based on *Portfolio* treatment; the proportional average of days with insufficient margin requirement based on *Individual* treatment and the proportional average of days with insufficient margin requirement based on *Portfolio* treatment. For all graphs quantities are calculated with a daily frequency and the average is taken over these daily values. [Color figure can be viewed at [wileyonlinelibrary.com](https://onlinelibrary.wiley.com)]

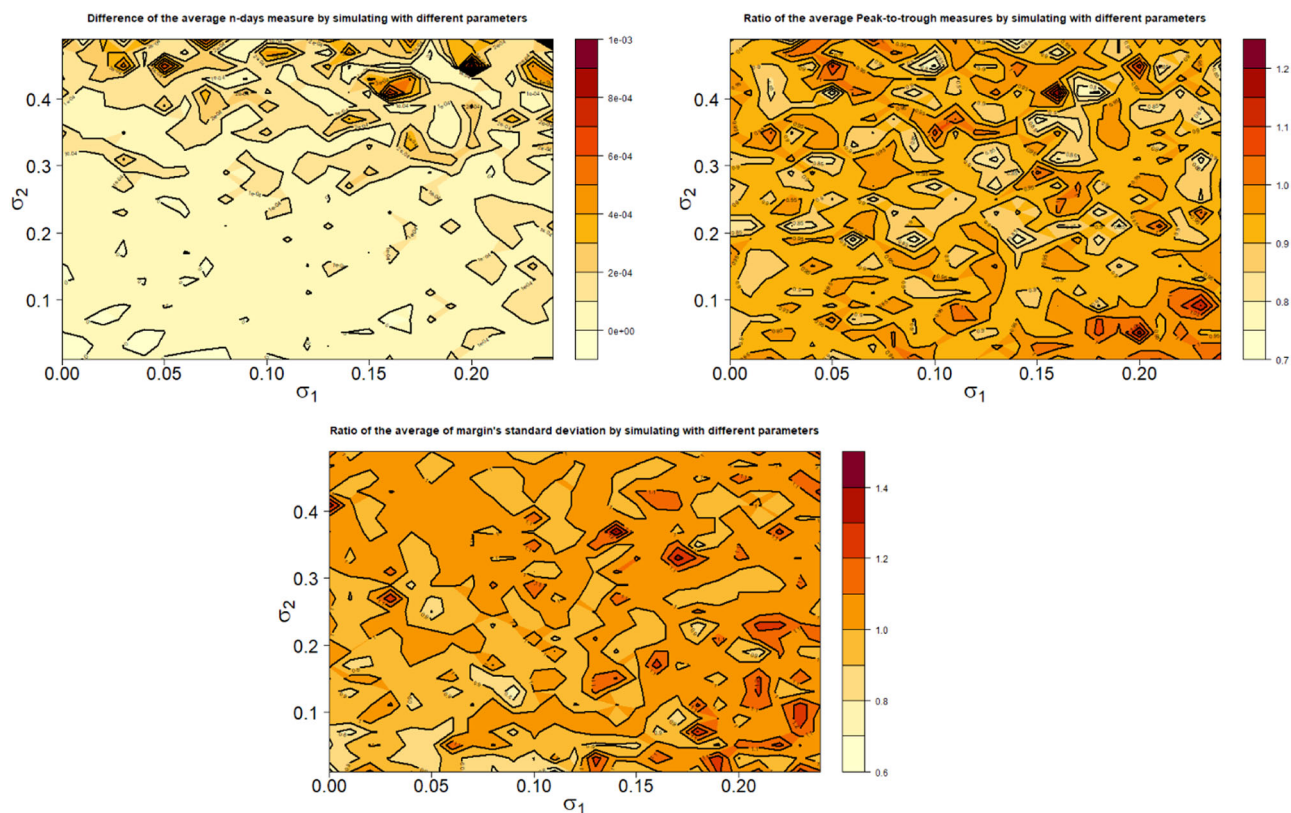


FIGURE A2 Different comparisons of anti-procyclical measures using *Individual* and *Portfolio treatment*. For all graphs we display values that are based on simulations using 30 years of data by selecting the σ_1 and σ_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the difference between average n -day APC measures based on *Individual treatment* and average n -day APC measures based on *Portfolio treatment*; the ratio of average Peak-to-trough measures based on *Individual treatment* to average Peak-to-trough measures based on *Portfolio treatment* and the ratio of average margin's standard deviation based on *Individual treatment* to average margin's standard deviation based on *Portfolio treatment*. For all graphs APC values are calculated with a daily frequency and the average is taken over these daily values. APC, anti-procyclicality. [Color figure can be viewed at wileyonlinelibrary.com]

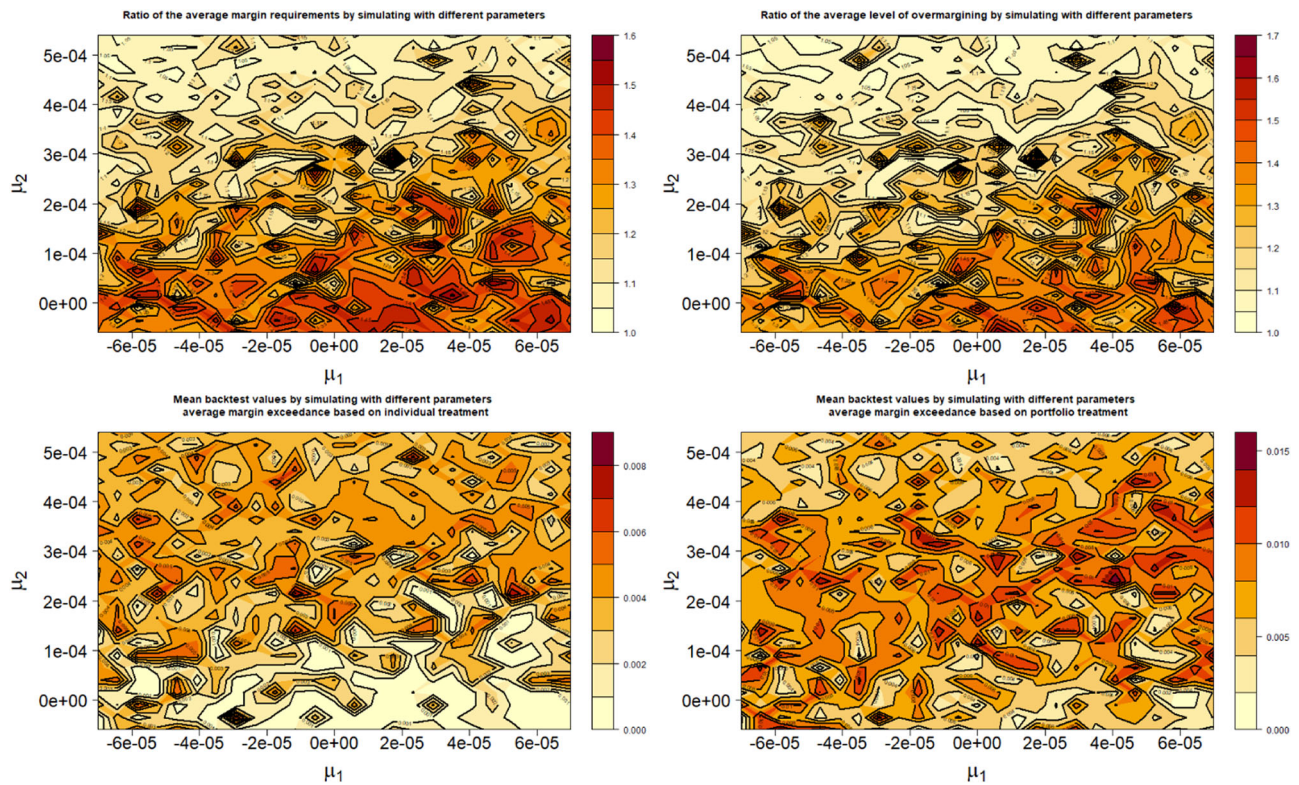


FIGURE A3 Different comparisons of margin requirements using *Individual* and *Portfolio* treatment following the four points outlined at the beginning of Section 4. For all graphs we display values that are based on simulations using 30 years of data by selecting the μ_1 and μ_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the ratio of average margin requirements based on *Individual treatment* to average margin requirements based on *Portfolio treatment*; the ratio of average overmargin based on *Individual treatment* to average overmargin based on *Portfolio treatment*; the proportional average of days with insufficient margin requirement based on *Individual treatment* and the proportional average of days with insufficient margin requirement based on *Portfolio treatment*. For all graphs quantities are calculated with a daily frequency and the average is taken over these daily values. [Color figure can be viewed at wileyonlinelibrary.com]

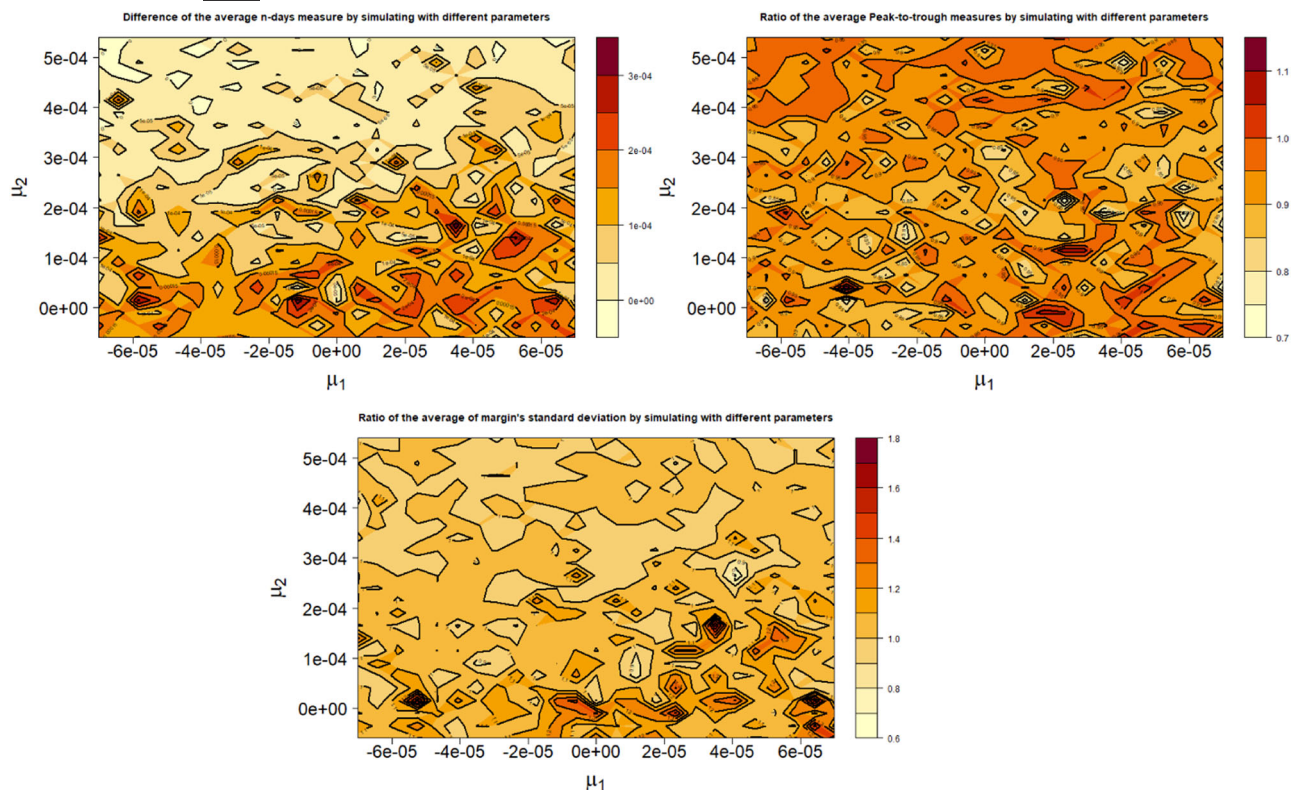


FIGURE A4 Different comparisons of anti-procyclical measures using *Individual* and *Portfolio treatment*. For all graphs we display values that are based on simulations using 30 years of data by selecting the μ_1 and μ_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the difference between average n -day APC measures based on *Individual treatment* and average n -day APC measures based on *Portfolio treatment*; the ratio of average Peak-to-trough measures based on *Individual treatment* to average Peak-to-trough measures based on *Portfolio treatment* and the ratio of average margin's standard deviation based on *Individual treatment* to average margin's standard deviation based on *Portfolio treatment*. For all graphs APC values are calculated with a daily frequency and the average is taken over these daily values. APC, anti-procyclicality. [Color figure can be viewed at wileyonlinelibrary.com]

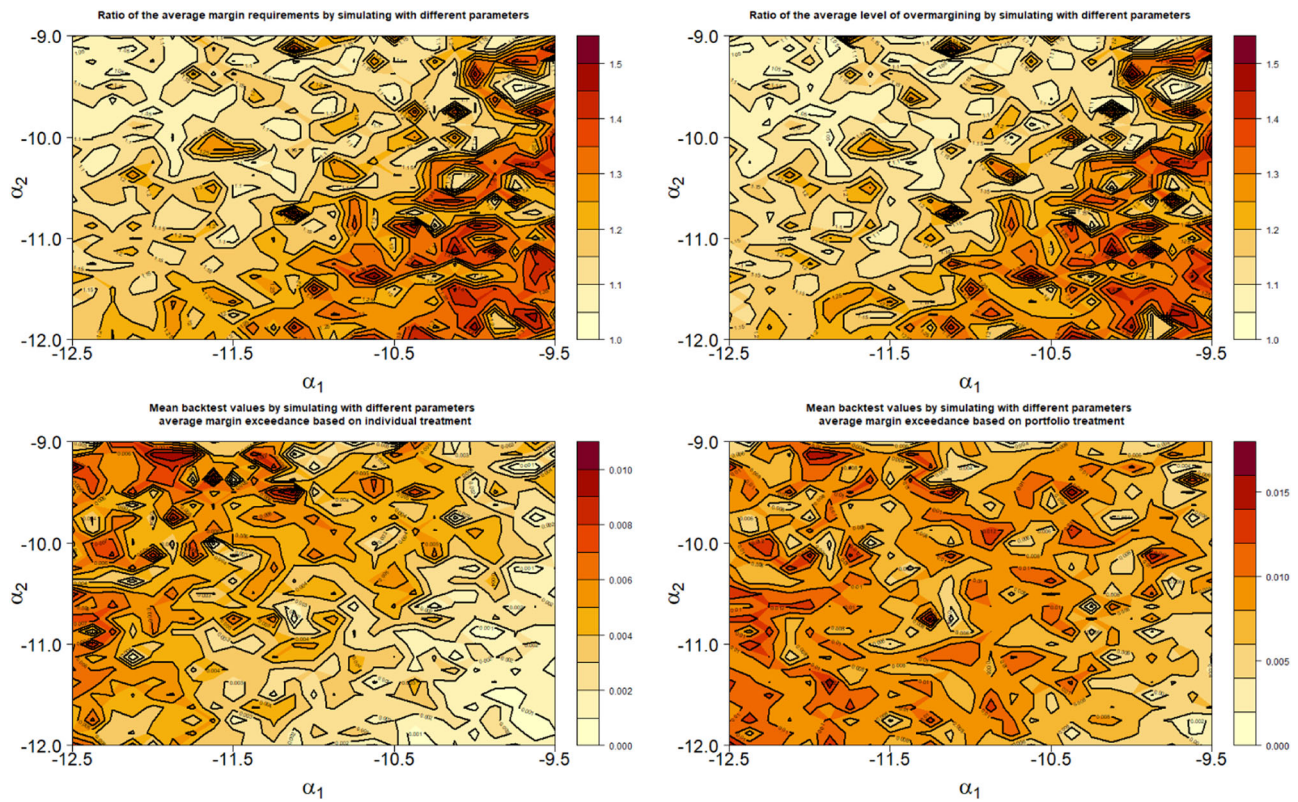


FIGURE A5 Different comparisons of margin requirements using *Individual* and *Portfolio treatment* following the four points outlined at the beginning of Section 4. For all graphs we display values that are based on simulations using 30 years of data by selecting the α_1 and α_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the ratio of average margin requirements based on *Individual treatment* to average margin requirements based on *Portfolio treatment*; the ratio of average overmargining based on *Individual treatment* to average overmargining based on *Portfolio treatment*; the proportional average of days with insufficient margin requirement based on *Individual treatment* and the proportional average of days with insufficient margin requirement based on *Portfolio treatment*. For all graphs quantities are calculated with a daily frequency and the average is taken over these daily values. [Color figure can be viewed at wileyonlinelibrary.com]

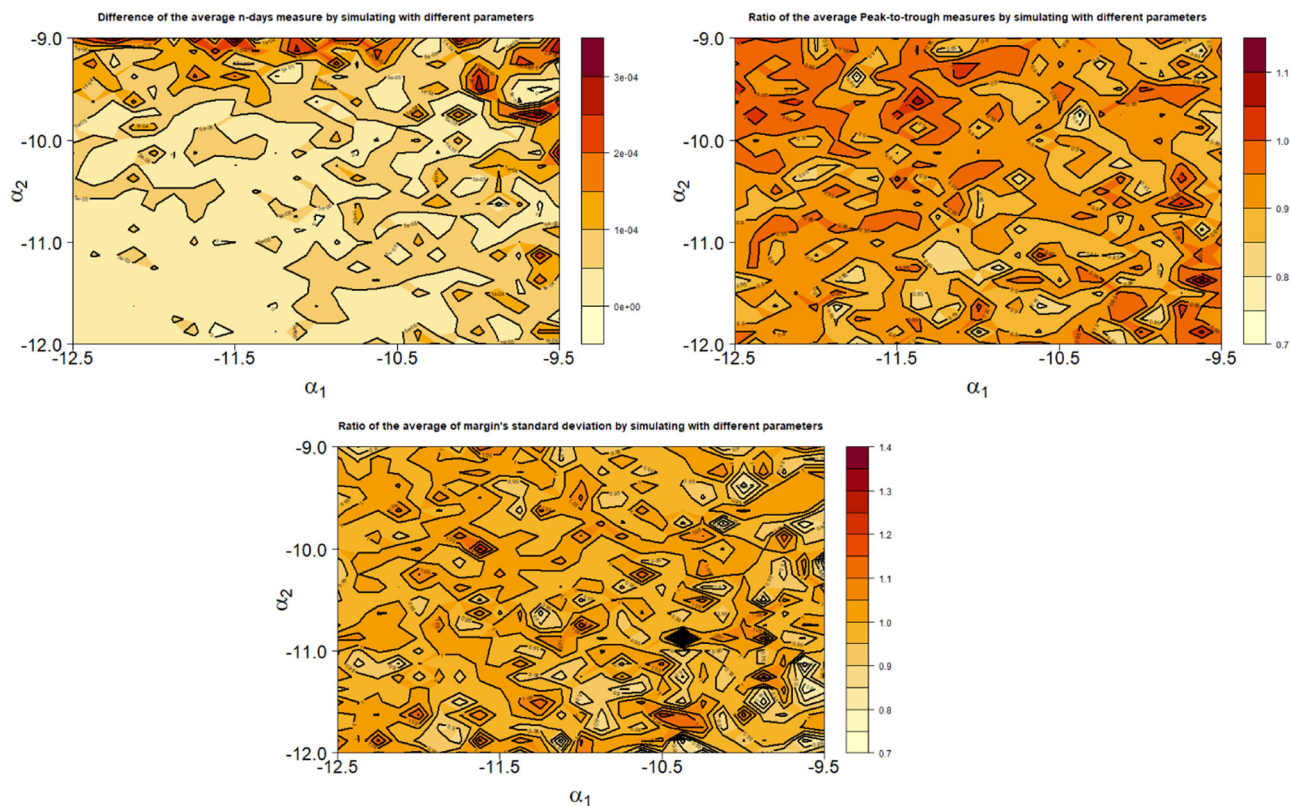


FIGURE A6 Different comparisons of anti-procyclical measures using *Individual* and *Portfolio treatment*. For all graphs we display values that are based on simulations using 30 years of data by selecting the α_1 and α_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the difference between average n -day APC measures based on *Individual treatment* and average n -day APC measures based on *Portfolio treatment*; the ratio of average Peak-to-trough measures based on *Individual treatment* to average Peak-to-trough measures based on *Portfolio treatment* and the ratio of average margin's standard deviation based on *Individual treatment* to average margin's standard deviation based on *Portfolio treatment*. For all graphs APC values are calculated with a daily frequency and the average is taken over these daily values. APC, anti-procyclicality. [Color figure can be viewed at wileyonlinelibrary.com]

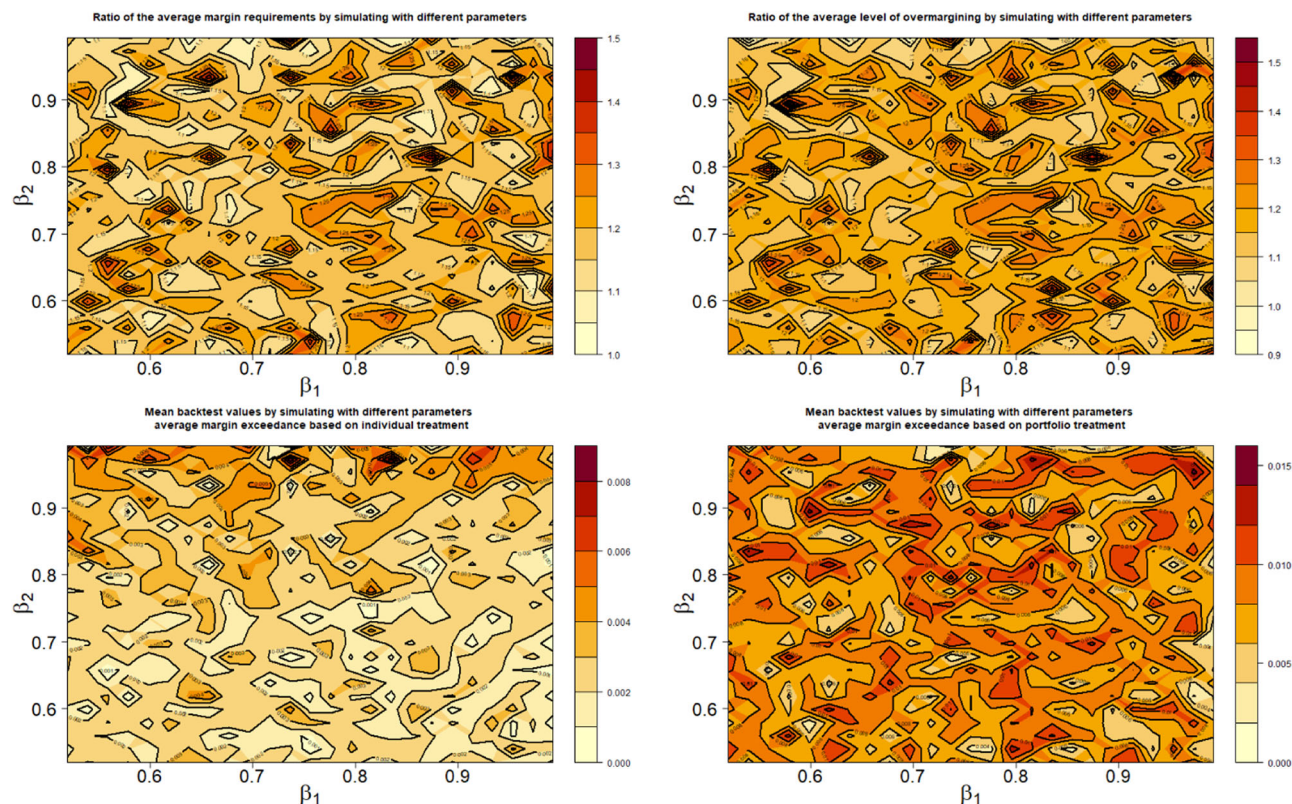


FIGURE A7 Different comparisons of margin requirements using *Individual* and *Portfolio treatment* following the four points outlined at the beginning of Section 4. For all graphs we display values that are based on simulations using 30 years of data by selecting the β_1 and β_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the ratio of average margin requirements based on *Individual treatment* to average margin requirements based on *Portfolio treatment*; the ratio of average overmargining based on *Individual treatment* to average overmargining based on *Portfolio treatment*; the proportional average of days with insufficient margin requirement based on *Individual treatment* and the proportional average of days with insufficient margin requirement based on *Portfolio treatment*. For all graphs quantities are calculated with a daily frequency and the average is taken over these daily values. [Color figure can be viewed at wileyonlinelibrary.com]

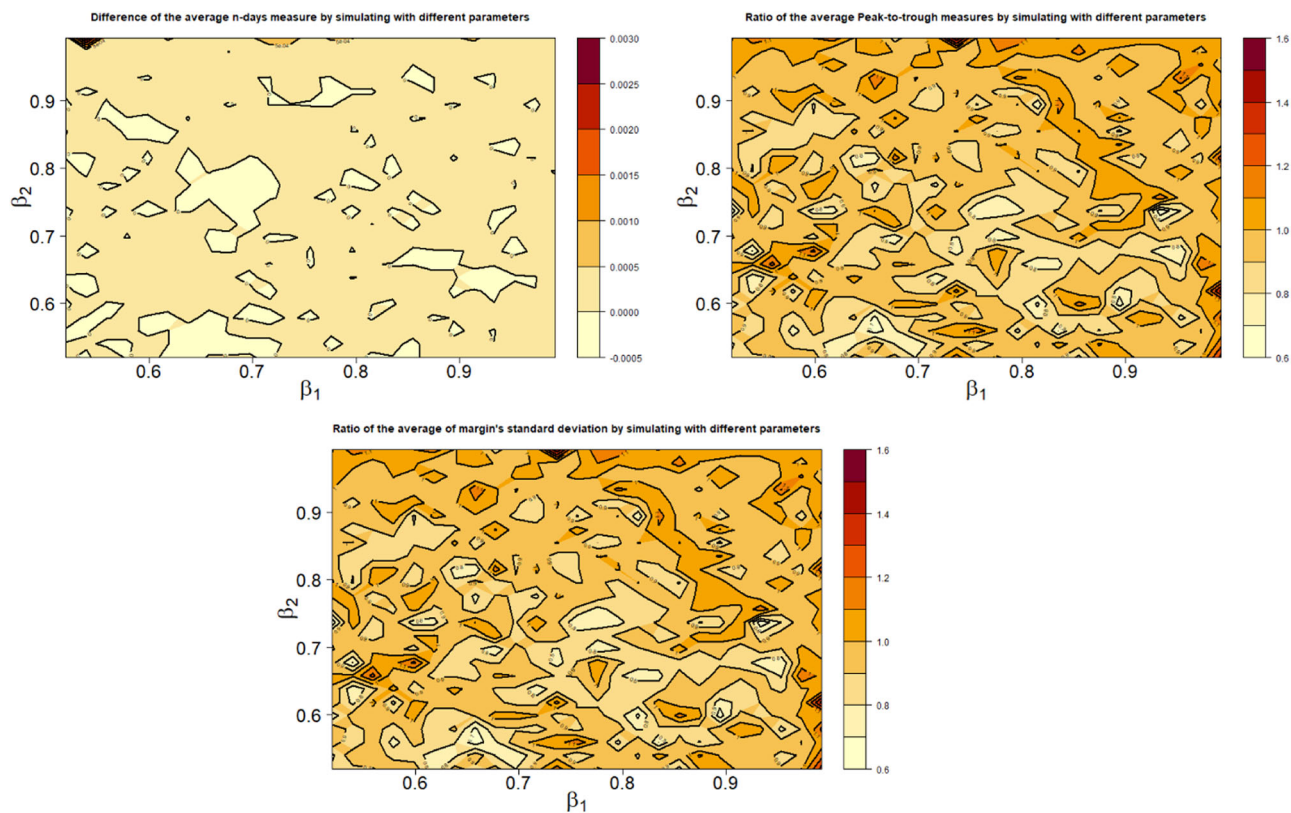


FIGURE A8 Different comparisons of anti-procyclical measures using *Individual* and *Portfolio treatment*. For all graphs we display values that are based on simulations using 30 years of data by selecting the β_1 and β_2 parameters from the axes and fixing all other parameters from Table 1. From left to right and from top to bottom we display the difference between average n -day APC measures based on *Individual treatment* and average n -day APC measures based on *Portfolio treatment*; the ratio of average Peak-to-trough measures based on *Individual treatment* to average Peak-to-trough measures based on *Portfolio treatment* and the ratio of average margin's standard deviation based on *Individual treatment* to average margin's standard deviation based on *Portfolio treatment*. For all graphs APC values are calculated with a daily frequency and the average is taken over these daily values. APC, anti-procyclicity. [Color figure can be viewed at wileyonlinelibrary.com]