MODELLING PREFERENCE TIES AND EQUAL TREATMENT POLICY

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ABSTRACT

The college admission problem (CAP) has been studied extensively in the last 65 years by mathematicians, computer scientists and economists following the seminal paper of Gale and Shapley (1962). Their basic algorithm, the so called deferred acceptance mechanism always returns a student optimal stable matching in linear time, and it is indeed widely used in practice. However, there can be some special features which may require significant adjustments on this algorithm, or the usage of other techniques, in order to satisfy all the objectives of the decision maker. The college admissions problem with ties and equal treatment policy is solvable with an extension of the Gale and Shapley algorithm, but, if there are further constraints, such as lower quotas, there exist no efficient way to find a stable solution. Both of these features are present in the Hungarian higher education matching scheme and a simple heuristic is used to compute the cutoff scores. Integer programming is a robust technique that can provide optimal solutions even when we have multiple requirements. In this paper we develop and test a new IP formulation for finding stable solutions for CAP with ties and equal treatment policy. This formulation is more general than the previously studied ones, and it has better performance, as we demonstrate with simulations, mostly because of its pure binary nature.

INTRODUCTION

The so-called Gale-Shapley algorithm (Gale and Shapley 1962) became widely used in the past decades for solving various kinds of two-sided matching problems under preferences, such as resident allocation to hospitals, college admissions (CAP in the following) and school choice. This algorithm was also the core mechanism studied in the corresponding theoretical research, see a comprehensive overview on the algorithmic (Manlove 2013) and game theoretical aspects (Roth and Sotomayor 1990) of this topic. However, in many practical applications there may be special features which make the problem more challenging to solve. For instance, in the Hungarian higher education admission scheme these special features are the presence of ties, lower quotas, common quotas and paired applications, where each of the latter three features makes the problem NP-hard to solve (Biró et al. 2010).

In a recent paper we formulated integer linear programmes to tackle these special issues one by one (Ágoston et al. 2016). One important finding of that paper was that even an NP-hard problem, such as the college admission with lower quotas, can be possible to solve with IP techniques for really large instances if we use some clever preprocessing heuristics. However, some of our formulations turned out to be inefficient to solve large instances, and we have not considered the cases when multiple special features are present at the same time. In this paper we give a new formulation for the special feature of ties with equal treatment property and we show that this new formulation, based only on binary variables, is easier to solve for large instances than the other natural formulation with cutoff scores. This will give us a chance to tackle also the combined case with IP technique, where both ties and lower quotas are present, which is relevant in the Hungarian application.

In CAP, students give their strict preferences over colleges (or programmes) where they apply. Universities rank the applicants according to their scores. Scores are usually based on secondary school grades and entrance exams. It may happen that two (or more) applicants have the same score for a college, so a *tie* may occur in the ranking of the university. There are many ways around the world, how the ties are handled see (Biró and Kiselgof 2015). In most countries the ties are broken in some way. In Spain the scoring method is very fine, so no tie may occur. In Ireland a random number is generated for each student and preferences of universities are determined by considering both the scores and the generated random number. In Turkey the ties are broken according to the age of the students. However, in some other countries the equal treatment policy is used, which means that the applicants with

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the same score have to be treated equally, either all of them are accepted or all of them are rejected. In Hungary the college quotas cannot be exceeded, so the last group of students with the same score whose admission would lead to the violation of the quota is always rejected. In Chile the equal treatment policy is more permissive, college quotas can be exceeded with the last group of students with equal score. The Gale-Shapley algorithm can be extended for finding student-optimal and student-pessimal stable solutions in case of ties for both the Hungarian policy (Biró 2008) and for the Chilean policy (Biró and Kiselgof 2015).

In (Ágoston et al. 2016) we formulated an IP for finding stable matchings for the Hungarian policy with using the cutoff scores as variables. This formulation turned out to be inefficient when solving large instances. In this paper we propose a new formulation using only binary variables on the applications. We show that with an appropriate objective function this IP can be used for finding the student-optimal stable solution. We also give techniques to reduce the size of the problem and thus speed up the solution. Our hope is that by building on this new formulation we will also be able to solve the combined case with ties and lower quotas for realistic instances.

Related works

In the classical assignment problem the two sides of the market has to be assigned in order to maximize or minimize the total utility or cost, respectively. Both LP solutions and efficient algorithms have been used for solving this problem, e.g. the so-called Hungarian method (Kuhn 1955). In the corresponding stable matching problems, which is called as stable marriage problem in the one-to-one case and college admissions problem in the many-to-one case, we build our LP formulations on the assignment problem, but we need to add constraints that provide stability with respect to the agents' submitted preferences.

The stable marriage problem was first investigated by Vande Vate (1989) as an LP problem. He formulated an LP and he proved the integrality property, i.e. that that extreme points of the feasibility set are all integers and they correspond to the set of stable matchings. Rothblum (1992) gets the same result for a more general problem. He defines another stability constraint which can be generalized to CAP. Baïou and Balinski (2000) investigate this formulation for CAP, and they show that this formulation allows fractional solutions to be extreme points, but they propose an alternative formulation which satisfies the integrality property. The formulation given in (Baïou and Balinski 2000) is a pure binary problem, i.e. all of the decision variables are binary. Also in this paper we can find a different formulation for CAP which satisfies the integrality property. However this second formulation has a large number of constraints.

In case of ties, the literature mostly focuses on the concept of weak stability, where the rejection of a student is considered fair if the quota of the college is filled with students of greater or the same score. Here, the problem of finding a maximum size weakly stable matching is NP-hard (Manlove 2013), but still this solution concept is used e.g. in the resident allocation in Scotland. IP technique was developed for solving this problem in Irving and Manlove (2009).

The concept of stable matching with ties and equal treatment property corresponding to the Hungarian policy was defined in (Biró 2008) and studied in (Biró and Kiselgof 2015) and (Fleiner and Jankó 2014). A mixed integer LP (MILP) with the cutoff scores being the variables was formulated in (Ágoston et al. 2016)

PRELIMINARIES

In the college admissions problem (CAP) let $A = \{a_1, \dots a_n\}$ be the set of applicants and let $C = \{c_1, \dots, c_m\}$ be the set of colleges. Let u_j denote the upper quota of college c_j. Regarding the preferences and priorities, let r_{ij} denote the rank of college c_j in a_i's preference list, meaning that a_i prefers c_j to c_k if $r_{ij} < r_{ik}$. For the sake of simplicity, an applicant's most preferred college gets rank 1, the second gets rank 2, and so on. The maximum of r_{ij} is denoted by $\overline{r}$. Let s_{ij} be an integer representing the score of a_i at college c_j, meaning that a_i has priority over a_k at college c_j if $s_{ij} > s_{kj}$. In the classical CAP by Gale and Shapley the scores of the students are different at every college, so the rankings by the universities are strict. We denote the set of applications by E. A *matching* M is a subset of applications, such that every student is allocated to at most one college and the number of allocated students at a college is less than or equal to its quota. A college is said to be *saturated* in a matching if its quota is filled. A matching is *stable* if for any unselected application either the student is allocated to a better college of her preference or the college filled its quota with better students. In the classical CAP a stable matching can be described with a set of cutoff scores $\overline{t} = [t_1, \dots t_m]$, where each student is allocated to the best college in her list where she achieves the cutoff score. A natural choice for such a set of cutoff scores for a stable matching is when t_j is the lowest score of the allocated students at c_j if c_j is saturated and zero otherwise. The relation of the set of cutoff scores and stable matchings for the classical CAP was studied in details in (Azavedo and Leshno 2016).

If ties are allowed in the scores of the students and we use the equal treatment policy with no quota violation then stability of a matching can be defined through cutoff scores. A set of cutoff scores is *H-stable* if no college with positive cutoff score can decrease its cutoff score without violating its quota in the corresponding matching. A matching is H-stable if it corresponds to a H-stable set of cutoff scores. Note that if no ties occur then this stability definition is equivalent to the Gale-Shapley stability for CAP. Let us refer to the CAP with ties as CAPT, and the above described model as H-CAPT, that is the college admission problem with ties and equal treatment policy with higher score-stability. Biró and Kiselgof (2015) proved that the natural extension of the Gale-Shapley student proposing algorithm always produces a student-optimal H-stable solution, which corresponds to a set of minimal cutoff scores, i.e. there is no other H-stable set of cutoff scores where even one college could have a lower cutoff score. Note that this also applies then in this case the number of admitted students is as high as possible and that every student gets the best possible place of her preference. However, we shall also mention that this mechanism does not remain strategy-proof, as in the classical model.

INTEGER PROGRAMMING FORMULATIONS

When describing a linear programme for CAP and H-CAPT, we introduce binary variables $x_{ij} \in \{0,1\}$ for each application by applicant a_i to college c_j, as a characteristic function of the matching, where $x_{ij} = 1$ corresponds to the case when a_i is assigned to c_j. The feasibility of a matching can be ensured with the following sets of constraints.

$$\sum_{j:(a_i,c_j)\in E} x_{ij} \leq 1 , \qquad \forall a_i \in A \tag{1}$$

$$\sum_{i:(a_i,c_j)\in E} x_{ij} \leq u_j , \qquad \forall c_j \in C \tag{2}$$

For CAP, the stability can be provided with the following set of constraints (see for example Baïou and Balinski 2000).

$$\left(\sum_{k:r_{ik}\leq r_{ij}} x_{ik}\right) \cdot u_j + \sum_{h:(a_h,c_j)\in E, s_{hj}\geq s_{ij}} x_{hj} \geq u_j \qquad \forall (a_i,c_j) \in E \tag{3}$$

Constraints (1) and (2) are called feasibility constraints and constraints (3) are called stability constraints. The feasible set S_s contains all nonnegative solutions which satisfies constraints (1), (2) and (3). Integer points of S_s corresponds to stable matching for CAP, and vica versa. However, note that this formulation does not have the integrality property, since S_s may have non-integer extreme points, as illustrated in (Baïou and Balinski 2000). Objective function is arbitrary in this case, but if we minimize or maximise the sum of the ranks of the allocated students then we can obtain the student-optimal and student-pessimal solutions, respectively.

If we consider the possibility of ties in the rankings (CAPT) then constraints (3) ensures only the so-called weak stability condition of the matching, where the rejection of an application can be explained with the saturation of the quota with students at least as good as the student concerned. Example 1 describes why a weakly stable solution for CAPT is not necessary H-stable as well.

Example 1: We have one college and two applicants, who have the same score. The upper limit for the college is one. In this case none of the applicants can be assigned to the college for H-CAPT, but according to constraints (1), (2) and (3) it is a feasible solution (and thus weakly stable for CAPT) if we allocate one of them to the college.

In case of H-CAPT, the college quota will not necessarily be full in a H-stable matching, even if there are more than enough first applications submitted to the college, as we have also seen in Example 1. So stability constraints (3) are not appropriate in case of ties. We formulated another model in (Ágoston et al. 2016), where the cutoff scores are the main variables, but that model is not a pure binary model and according to our simulations the running times of the solver for that model are high even for a small instances.

In our new formulation that we investigate in this paper, we keep feasible constraints (1) and (2) and change (3) to

$$\sum_{k:r_{ik}\leq r_{ij}} x_{ik} \geq x_{hj} \quad \forall (a_i,c_j),(a_h,c_j) \in E, s_{ij} \geq s_{hj} \tag{4}$$

Intuitively the above constraint means that if a student a_h is allocated to college c_j then every student a_i, who has a score at c_j at least as high as a_h has, must also be allocated to c_j or to a better college of her preference. Let S_p contain all nonnegative solutions that satisfy constraints (1), (2) and (4).

Example 2 shows the difference between S_s and S_p for CAP.

Example 2: We have two applicants, a_1 and a_2, and three colleges ,c_1,c_2 and c_3. The first applicant's preferences are $c_1 \succ c_2 \succ c_3$; the second applicant's preferences are $c_3 \succ c_1$. The first applicant' scores for the three colleges are $(1;2;2)$, the scores of the second applicant are $(2;1;1)$. The quotas are 2 for c_1 and 1 for c_2. The set S_s contains only one point: $x_{11} = 1$ and $x_{23} = 1$, whilst S_p has many extremal points:

$$\begin{pmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{23} \\ x_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{23} \\ x_{21} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

Our first remark is that constraints (3) do not yet ensure the stability of a matching, since the empty matching is also a feasible solution. With regard to CAP, the integer solutions in S_p correspond to the so-called *envy-free matchings* defined in (Wu and Roth 2016). In such a matching no student has a *justified envy* over another student, meaning that there exist no two students a_i and a_h, and a college c_j such that a_i is matched to c_j, but a_h has a higher score there than a_i and she is matched to a less preferred college (or no college at all). In other words, a matching is envy-free for an instance of CAP if and only if it is a stable matching for the adjusted quotas, where the quota of each college is equal to the number of students assigned there in the matching.

The same statement holds for H-CAPT, the integer solutions in S_p correspond to H-stable matchings with regard to adjusted quotas, where the quota of each college is equal to the number of students assigned there in the matching. Let us call these matchings *H-envy-free matchings*, that is envy-free matchings with regard to the equal treatment property. In this definition a student would also feel justified envy if she is no assigned to a college and any better of her preference, whilst a student with the same score is admitted there.

In the following two propositions we summarize the connections between the extreme points of S_p, the envy-free matchings, and the matchings induced by cutoff scores.

Proposition 3: For an instance of CAP, the set of envy-free matchings are the integer extreme points of S_p and these are also the feasible matchings that can be induced by a set of cutoff scores.

Proof: We have already seen that the envy-free matchings are the integer extreme points of S_p, and vice versa. Now we suppose that M is an envy-free matching and we show that it can be induced by a set of cutoff scores. Indeed, if we choose the score of the weakest admitted student at each college to be the cutoff score of that college then M is induced by these cutoff scores. In the other direction, let t be a set of cutoff scores and let M be the induced feasible matching (which does not violate the quotas of the colleges). It is easy to see that M is an envy-free matching by definition, since the students admitted to a more preferred college from a student a_i's perspective have all higher scores there than a_i has. ■

Proposition 4: For an instance of CAPT, the set of H-envy-free matchings are the integer extreme points of S_p and these are also the feasible matchings that can be induced by a set of cutoff scores.

Proof: The proof is the same as above. ■

The structure of envy-free matchings for CAP has been studied in (Wu and Roth 2016). In particular they showed that the student-optimal stable matching for the instance of CAP is also student-optimal in the set of envy-free matchings. Regarding CAPT, we know that there exists a student-optimal H-stable matching, obtained by the extended Gale-Shapley algorithm (Biró and Kiselgof 2015). In the following proposition we show that this matching is also optimal for the set of H-envy-free matchings.

Proposition 5: For an instance of CAPT, the student-optimal H-stable matching is also optimal for the students in the set of H-envy-free matchings.

Proof: Suppose for a contradiction that there exists a matching, where at least one student gets a better college than in the student-optimal H-stable matching, M_s, and suppose also that it is not Pareto-dominated with any other such matchings. Let the corresponding set of cutoff scores be $\bar{t}$. This matching cannot be a H-stable matching, as that would contradict with the student-optimality of M_s in the set of H-stable matchings, thus there must exist at least one college, c_j, where we can increase the cutoff score by admitting more students, but without violating the quota. Let this new set of cutoff scores be denoted by $\bar{t}'$. The induced matching by $\bar{t}'$ is feasible, and by Proposition 4 it is also H-envy-free. Moreover, every new student admitted at c_j improved their assignment compared to M and nobody else received a worse college, so M was not Pareto optimal among the set of H-envy-free matchings, a contradiction. ■

Now, we describe how one can obtain the student-optimal H-stable matching using an appropriate objective function.

Lemma 6: Let we set objective function as

$$\max \sum_{k=1}^{\bar{r}} \alpha_i \sum_{r_{ij}=k} x_{ij} \ , \qquad (5)$$

where $\alpha_1 > \alpha_2 > \cdots > \alpha_{\bar{r}}$. The optimal integer solution of (5) subject to constraints (1), (2), (4) and $x_{ij} \geq 0$ is the student-optimal H-stable matching.

Proof: From Proposition 4 we know that any feasible solution corresponds to a H-envy-free matching. Proposition 5 implies that the student-optimal H-stable matching, M_s is also optimal in the set of H-envy-free matchings. Therefore there is no other H-envy-free matching where even one student can get a better college, so the objective function (5) must be maximized for M_s. ■

NUMERICAL RESULTS

In this section we investigate how our proposed IP model behaves numerically. We replicate some results from (Ágoston et al. 2016) and extend it with new results.

For the numerical modeling we used a desktop computer with 2.33 GHz Intel Pentium processor and 6 GB RAM. Operating system is Windows 7 Enterprise.

We used the GLPK software (version 4.55 version) for solving IP problems. We kept the default parameter setting except where we explicitly mention it.

We used randomly generated samples for testing the IP models: we had n applicants and k colleges, each applicant choosing five colleges uniformly at random without replacement. So there are about $\frac{n}{k}$ first place applications at each college. We fixed the quotas at $\frac{n}{2k}$, so every quota is expected to be full. Scores are integers distributed randomly between 0 and $\bar{s}$.

First we consider the case where there are no ties ($\bar{s}$ is quite large, e.g. 10000).

Table I shows how large problems can be solved with the 'Stable Cutoff Scores IP model' (SCS-IP) defined in (Ágoston et al. 2016), where the cutoff scores are the main variables. Table II shows the running times for the basic IP model, called 'Basic College Admission IP' (BCA-IP), with objective function (5) and constraints (1), (2) and (3). We can see that considerably larger

TABLE I: Running times of SCS-IP for CAP

n	k	running time (sec)
20	10	0.2
40	10	4.0
60	10	81.3
80	10	1443.8

TABLE II: Running times of BCA-IP for CAP

n	k	running time (sec)
100	10	0.0
500	20	0.3
2500	30	28.9
7500	40	588.7

TABLE IV: Summarizing results for the speeding-up process for CAP instances. The 'OEF-IP' column shows the running time for the OEF-IP model, the same as in Table III. The '#filt.assign' columns shows how many applications can be filtered out. The 'filtered' column shows the running time of the solver for OEF-IP after filtering.

n	k	OEF-IP	#filt.assig.	filtered
160	20	1032.2	461	3.5
200	20	4868.6	573	12.7
240	20	> 3600	669	28.9
280	20	> 3600	784	45.6
320	20	> 3600	899	31.6
360	20	> 3600	987	52.3
400	20	> 3600	1083	171.6

problems can be solved with BCA-IP than with SCS-IP. However, the BCA-IP model cannot be simply extended for CAPT instances.

Table III shows the running times for our new IP model, called 'Optimal Envy-Free IP model' (OEF-IP), where we maximize (5) subject to constraints (1), (2) and (4). The running times are larger for OEF-IP than for BCA-IP, but lower than for SCS-IP. We would like to emphasize that for H-CAPT we cannot use BCA-IP, so OEP-IP remains the best approach for that.

Speeding up the solution by filtering

As we see in Table III our new formulation, OEF-IP, performs better than its alternative, but the difference is still very small. However, we have some tools that can speed up the solution, as we describe below.

Among the applications there are many which are surely not possible to accept. There are known techniques in the literature that can filter out impossible pairs (e.g. defined in (Irving and Manlove 2009), and applied in (Kwanashie and Manlove 2014)). We propose another way, which uses the known properties of the simplex method. We check each variable, one by one, by setting $x_{ij} = 1$ and solving the LP relaxation of the problem. If there is no feasible solution then we know that variable x_{ij} has to be zero.

When we checked all the variables once and could set at least one to be zero then we repeat this process for the remaining variables. We stop this filtering process when no variable can be set to be zero in a round, thus after checking each variable at most m times (where m is the number of applications). With the following instance of CAPT in Example 7 we illustrate why multiple rounds may be useful in this filtering process.

Example 7: We have 3 colleges (c_1, c_2 and c_3) and three applicants (a_1, a_2 and a_3). All the three applicants have score of 3 to c_1, score of 2 to c_2 and score of 1 to c_3. Applicant a_1 submitted application only to c_3; the other two submitted to all the three colleges, their preference orders are the same: $c_1 \succ c_2 \succ c_3$. The quota is 1 for all the three colleges. In the first round we first investigate whether $x_{1,3}$ can be one, and we find that indeed there exists such a feasible solution for the LP, namely $x_{1,3} = 1$, $x_{2,1} = 0.5$, $x_{2,2} = 0.5$, $x_{3,1} = 0.5$ and $x_{3,2} = 0.5$. However, when we check $x_{2,1}$, we see that it is not possible to set it 1, since in this case $x_{3,1}$ have to be 1 as well and we would exceed the quota for c_1. Analogously it will be clear that $x_{3,1}$ has to be zero as well. But if both $x_{2,1}$ and $x_{3,1}$ are equal to 0 then $x_{1,3}$ cannot be 1 either, so in the second round of our filtering process we can also set $x_{1,3}$ to be 0.

TABLE III: Running times of OEP-IP for CAP

n	k	running time
80	20	23,0
120	20	322.5
160	20	1032.2
200	20	4868.6

In our first simulation we simply investigate how much the running time decreases by using the filtering process (without deleting redundant constraints) for CAP instances. The running time decreases dramatically, as seen in Table IV, column 'filtered'.

Multiple rounds of the filtering process turned out to the useful indeed. For example, in case of the last row of Table IV in the second iteration we filter out another 90 variables and for this reason the running time decreases to 2.0 sec.

We now turn our attention to the H-CAPT model. We decrease $\overline{s}$ to 10, and then to 5. As we see in Table V and Table VI the running times for solving such problems are higher. This is because if there are no ties then the optimal value of the LP relaxation problem equals with the optimal value of the IP problem more often. Therefore it is more likely that we find an integer solution to the problem quickly, and in this case we can bound all the open branches. However, if there are ties, then the optimal solution of IP is significantly smaller than the optimal solution of the LP relaxation (see 'Adm. appl.' column in Table V and Table VI) resulting in longer branch-and-bound processes.

TABLE V: Running times and the number of admitted applicants for H-CAPT instances with $\overline{s} = 10$, $k = 20$, using OEF-IP with filtering.

n	without filt.	#filt.assig.	with filt.	Adm. appl.
80	78.0			30
120	1600.4			45
160	6226.7	532	0.1	56
200	> 3600	648	0.2	75

TABLE VI: Running times and the number of admitted applicants for H-CAPT instances with $\overline{s} = 5$, $k = 20$ using OEF-IP with filtering.

n	without filt.	#filt.assig.	with filt.	Adm. appl.
80	16.5			13
120	201.6			18
160	1082.4	602	0.1	24
200	> 3600	731	0.2	30

Perhaps at first sight it may seem strange that running times in Table VI are smaller than running times in Table V. However, as the number of applicants with the same score increases, we have more and more applications that we can filter out, which decreases the number of nodes in the branch-and-bound tree. As we can see, solving the case for filtered problem takes less time in case of $\overline{s} = 5$ than in case of $\overline{s} = 10$.

CONCLUSIONS

In this paper we presented a new IP formulation for college admission problem with ties under the equal treatment policy, a case present in the Hungarian higher education admission scheme. Our new IP formulation is binary, which turned out to be significantly easier to solve than our previous formulation (Ágoston et al. 2016) with integer variables. The constraints in the problem do not ensure that every (integer) solution of the problem will correspond to a stable allocation, but with an appropriate objective function the student-optimal stable solution can be obtained, as we proved. We also presented methods on how to speed-up the solution of our new formulation. Although we were able to solve much larger instances than before, it still requires further research to solve the H-CAPT problem for a real instance of the Hungarian scheme with around 100 thousands applicants.

Furthermore, we shall also investigate how one can define a fair solution for the case when besides the issue of ties the lower quotas are also present, as it occurs in Hungary. For this case with multiple special features, where one of the features (lower quotas) makes the problem NP-hard, the usage of IP technique can be especially useful. Note that in our previous work (Ágoston et al. 2016) we were indeed able to solve the feature of lower quotas successfully with IP techniques after an efficient filtering process. Our most important future plan is to formulate a combined IP for solving the H-CAPT problem with lower quotas, and develop filtering techniques that can make our approach work for large instances.

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