Central European Journal of Operations Research (2024) 32:131–154
https://doi.org/10.1007/s10100-023-00865-1

What is the economically and socially optimised child support?

Introducing a child welfare optimization approach

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Accepted: 26 May 2023 / Published online: 24 June 2023

Abstract
The study is based on the principle that children with divorced parents should have the same standard of living as their parents, both parents. Access to education, nourishment and extracurricular activities should be supported at the same level after divorce as before to ensure undisrupted development of the children. This study approaches the sensitive topic of child support payment objectively, adopting well-known techniques from operations research, with the goal to provide a guide to determine "optimised child support payment". The proposed guide for decision makers aims to mitigate the probability of rising inequality in the separated family and through it in the society as well as reducing the risk of single-parent units slipping into poverty. Using simulations based on German child support data, a dynamic, globally applicable child support payment model is proposed. The objective function is to minimize the differences in the equivalised income of the separated family units, thereby, reducing risk of poverty and the financial stress for the custodial partner. The proposed child support payment scheme is well-defined and child focused and it potentially could address a serious real-world problem, the increasing risk of poverty after divorce for the child and the custodial parent.

Keywords Child support · Child penalty · Equivalised income · Equivalence scales · Income inequality · Single-parent household · Spousal support

'After all Children should not have to save up for their parents But parents for their children.'
2 Corinthians 12, 14.

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1 Introduction

Traditionally monthly child support payments are made by the noncustodial parent (i.e., the parent who does not live with the child). While some form of compensation for the expenses of the custodial or caretaker parent are implemented in most countries, the legal environment and enforcement of child support payments vary significantly around the world, even within the European Union (European Parliament 2014). Szalma and Rékai (2019) found that the paid amounts are rather arbitrary depending on the noncustodial parent's propensity to pay in Hungary where there are no calculators only some principles in the family law. In Germany in the largest economy of the EU, there is a well-established framework based on the so-called '*Düsseldorfer* Tabelle' (DT from hereafter) while similar system is by and large absent in a number of EU states (European Parliament 2014). Varga (2022) analysed the effects of child support, without spousal support, recommendation based on the German official table to compare differences in the standard of living in separated families after divorce. She shows that in extreme cases, the noncustodial parent has almost 9 times higher representative income than her/his child(ren) and this income difference often results in high(er) risk of poverty for the child(ren).

The objective of this study to offer a welfare enhancing solution for determining child support instead, which could reduce over time the risk of single-parent families slipping into poverty and improve living conditions of millions of children whose parents are divorced. Furthermore, a general model can be easily implemented across Europe with customized inputs. A simple and objective child support model could also reduce the courts burden and confusion in dealing with international divorces and payments across countries.

The optimization process focuses on ensuring (maximizing) the welfare of the child(ren) after divorce, with the binding constraint that the financial obligation from the child support should not jeopardize the well-being of the noncustodial parent. Practically, this objective is achieved by minimizing the differences in the equivalised income across parental units. The analysis is based on the simulation of the income and financial situation of over 1000 German parent pairs, and comparing the proposed optimised child support payment with the current official table for both child and spousal support. In this paper, the German system is used for the simulation and comparison for transparency, but the results and insights are not country specific, easily adoptable across Europe. The analysis and optimisation are made according to the OECD concept of equivalised income, which makes the incomes of households with different household compositions comparable. While the study is not a traditional operations research problem, it has all the required elements (Fores and Krarup 2013): (A) multiple decision makers (parents, judges, law-makers), (B) the institute of obligatory child support as system, (C) conditions, various inputs to be considered, such as income, wealth, employment of the parents, and age of the child(ren), (D) a dynamic optimization process (optimizing differences in economic welfare of the child and his/her caretaker and the child payment provider unit to derive optimised child support payment).

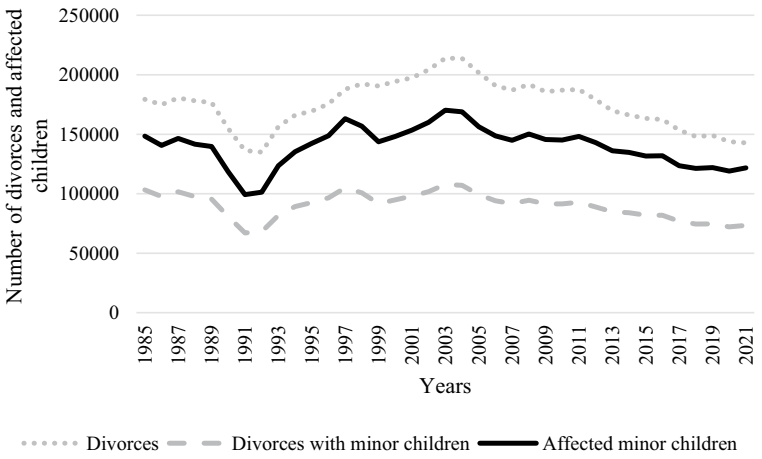

Fig. 1 German divorce data from 1985 to 2021. *Data source*: (FSOG 2022a)

Table 1 Families and family members with minor (under age of 18) children living in the family by living arrangement in 2021

	Married couples (%)	Cohabiting couples (%)	Lone parents (%)	Total (%)
Families	70	12	18	100
Family members	76	11	13	100

Data source: (FSOG 2022a)

The significance of the problem is best illustrated by the number of people affected. The Federal Statistical Office of Germany (hereafter, FSOG) publishes data about the number of marriages, divorces, and the affected minor (age under 18) children since 1985. This (FSOG 2022a) dataset for the period of 1985 to 2021 was used in the paper and visualized in Figure 1 which shows the total number of divorces, the number of divorces with minor children, and the number of minor children affected by divorces. In Germany, more than 100,000 (in some years nearly 150,000) new children have been affected by divorce every year in recent decades. This means that more than 100,000 children's parents divorce every year, and since those whose parents divorced in earlier years remain affected (until they grow up), the total number of affected children in a given year is several times higher than 100,000.

Table 1 shows the distribution of family types with minor children in Germany for the year 2021. The first row shows the proportion of married couples, cohabiting couples and lone parent families among families. The second row shows the same distribution but in terms of the proportion of family members. As shown in Table 1, in 12% of families the parents are unmarried, about one seventh of two-parent families. If we understand divorce in a broader sense, not only as the

break-up of a marriage but also as the separation of cohabiting couples, we must also take into account the number of children affected by the 'divorce' of unmarried parents when we want to get a sense of the significance of the problem. When we talk about optimal child support, it should apply equally to them. The obligation to pay child support should be extended also to those children who were born into cohabiting relationships.

In Germany alone, the number of children involved in maintenance is therefore in the millions. The case of Germany is likely to be applicable to other EU counries, and beyond with similar jurisdictional, social and religious-cultural setting for market economy system.

2 Review of relevant literature

2.1 Representative income: the concept of equivalised income

To be able to meaningfully compare the standard of living in different types of households, OECD suggests the use of equivalised income. 'The equivalised disposable income is the total income of a household, after tax and other deductions, that is available for spending or saving, divided by the number of household members converted into equalised adults' (Eurostat 2022a).

$$Equivalised\ disposable\ income\ of\ the\ household = \frac{Total\ disposable\ income\ of\ the\ household}{Equalised\ number\ of\ adults}$$

Both the Eurostat and the OECD statistics currently use the above measure, but there are several alternative methods to determine the equivalised number of adults in a household. Eurostat calculates the equivalised adults according to the so-called 'OECD-modified' (or OECD2) equivalence scales. In this method, the weight of the first adult is 1, and any other member of the family aged over 14 years is 0.5, while a child (aged under 14) weighs 0.3. Recent OECD publications apply a so-called square root (SR) scale. According to it, the age of any member of the household does not count, only the size of the household. The equalised number of adults is the square root of the number of all members independently of age.

$$Equalised\ number\ of\ adults = \sqrt{Number\ of\ all\ members}$$

Overall, the equivalised income of a household is the total household income divided by the square root of the number of persons living in the household (OECD 2022).

Table 2 presents the two alternative methods, the modified OECD and the square root methods, with one adult in the household and 0–4 children (aged under 14).

Hansen et al. (2006) compared the poverty among households and used the modified OECD scale. Varga (2022) calculated both methods to compare the effect of child support and received comparable results but preferred the SR method for

Table 2 Equalised adults in households with one adult

Household size	Per-capita income	OECD2	SR	Household income
1 adult	1.00	1.00	1.00	1.00
1 adult, 1 child	2.00	1.30	1.41	1.00
1 adult, 2 children	3.00	1.60	1.73	1.00
1 adult, 3 children	4.00	1.90	2.00	1.00
1 adult, 4 children	5.00	2.20	2.24	1.00

Source: author's calculation based on (OECD 2022)

Table 3 Statistics of inequality in Germany in 2020

	Average	Household composed of 1 adult	Household composed of 1 adult with dependent children
People at risk of poverty	22.5%	34.8%	46.7%
S80/S20 (age less than 65 years)	4.99	n/a	n/a

Data source: Eurostat (2022b), Eurostat (2022c)

analytics because of its simplicity. (In addition, the use of square root method in the optimisation model likely result in an underestimation—conservative valuation—of the child support since this method is favourable for the noncustodial parent.) Hereinafter referred to as this paper uses the square root method.

2.2 Measure of income inequality

The concept of equivalised income is used to determine: (1) the risk of poverty rate (share of people whose equivalised disposable income is below the at-risk-of-poverty threshold) and also the (2) income quintile share ratio (S80/S20 gives the income gap between the richest and poorest 20% of the society).

The poverty line is set at 60% of the national median equivalised disposable income (after social transfers) according to the Eurostat statistics (Eurostat 2022a), but other publications use a more conservative 50% median value cut-off. In the sample of Hansen et al. (2006) the different levels of the threshold gave similar results, both show that in Germany (compared with Norway) the social policy favoured couples against lone parents. The current study uses both thresholds (50 and 60%) as well to compare custodial and noncustodial parent's risk of poverty and implied that noncustodial parents are favoured against custodial ones and their children in Germany which is why a better solution is needed according to my calculation.

From Table 3 in 2020 the risk of poverty rate in Germany was 22.5% on average, but the two most endangered types of households' values were 46.7% (single-parent households) and 34.8% (for one adult households).

In Table 3 the S80/S20 ratio characterizes the distribution of income in Germany in 2020 among the population aged under 65 (in the 'divorce story' with children it seemed relevant). The S80/S20 income share ratio presents that the richest quintile of German society had around 5 times more equivalised disposable income than the poorest 20% had. Unfortunately, there are no available data for each household type.

2.3 The childbearing's effect on the disposable income

2.3.1 Correlation between spouses' earnings before and after having children

Gonalons-Pons et al. (2021) examined the elements of correlation between spouses' earnings and show that the increasing earnings homogamy (labeled as assortative mating) is only the starting point when we search for the relationship between spousal's earnings, as the correlation changes between marriage and parenthood and after parenthood. In their US data source from 1965 to 2015, correlation was higher before parenthood and lower after parenthood, but both correlations increased over the period. Pre-birth data were between 0.2 and 0.4, while the post-birth correlation went from negative to positive, in 2015 was around 0.2. Nieuwenhuis et al. (2017) used data from Luxembourg Income Study and found that the correlation between spouses' earnings was negative (between − 0.2 and 0) in Germany from 1973 to 2013 but has been increasing since 1990.

2.3.2 Child penalty

The gender wage gap means that females who are similar in every important aspect earn less than their male counterparts in the same job category. The motherhood wage gap means that mothers earn less than other similarly educated women without children, supported by extensive research (e.g., Grimshaw and Rubery 2015; Angelov et al. 2016; Lovász et al. 2019; Cukrowska-Torzewska and Lovász 2020). In general, the extant research on the economic impact of childbearing shows that the parenthood gap is not symmetric: mothers suffer from the parenthood penalty meanwhile fathers are awarded a parenthood premium (and this increases further the gender gap) so the child penalty is a motherhood wage gap. For Germany Gangl and Ziefle (2009) found that the wage penalty for motherhood was 16%–18% per child, which was the highest result among the investigated countries (its value was 9–16% in the USA or a13% in Britain) in the 50th and 60th decades.

Cukrowska-Torzewska and Lovász (2020) found that only the gender gap, excluding the child penalty for mothers, was 24% in Germany in the 2000's year. Kleven et al. (2019) determined 61% in Germany as the long-run child penalty in earnings for women who have their first child between the ages of 20 and 45, which was the highest rate among the six investigated countries (the second-highest value was 51% in Austria). Using again German data, Feldhof (2021) found that the average

Table 4 Child benefits in Germany

	Child benefit per capita in euro
1st and 2nd child	219
3rd child	225
From 4th child	250

Source: Bundesregierung (2022)

long-run child penalty is 63%: 'the effect childbirth … women earn 63% less of their potential income compared to men' (pp. 72). In Germany, this high child penalty may be since 40% of mothers with children aged 0–14 work only part-time (Cukrowska-Torzewska et al. 2020).

2.3.3 Child benefit

Since childbearing causes a higher risk of poverty, the national social welfare policies often contain child tax allowances. However, these tax credits help only those whose taxable income is in the upper percentiles, where the tax benefit is economically significant. To reduce the poverty and income instability among children, the governments usually pay child benefits too which are independent of the taxable income (or have an upper limit of income) and depend on the number of children. Since in my calculation net incomes are used the German tax allowance does not count but the extent of child benefit can be seen in Table 4.

2.3.4 Child and spousal support

The custodial parent takes care of the child in kind (that is the main reason for earning less since they can work less hours) while the noncustodial parent is expected to be responsible for the financial support of their child. The most important financial asset which can moderate the risk of poverty in single-parent families is a well-defined child support from the noncustodial parent (Monostori 2019). In the US there are more methods to determine child support. The 'Percentage Income' model (applied in 8 states) is based only on the noncustodial parent's income, while the two other models, the 'Income Shares' and the 'Melson Formula', calculate both the custodial and the non-custodial parents' incomes (National Conference of State Legislatures 2020). In Germany, the recommendation for the payment amounts of child support is prescribed in the DT, shown in Table 5, where child support is explicitly based on the noncustodial unit. However, DT also incorporates spousal support payment which depends on the custodial parent's income as well. Overall, the DT model is a complex system, where the combined child and spousal support considers both parents' income.

Interpreting Table 5 for a noncustodial parent with a chargeable net income of €4500 euros a month, (s)he should pay €601 for a child aged 18 and €658.5 for a

Table 5 Child Support Payment (in euros) per capita in case of two dependents in 2022

Chargeable net income		For the first and second child, depends on the age of the child (years)				For the third child—depends on the age of the child (years)				From the fourth child—depends on the age of the child (years)			
From	To	0–5	6–11	12–17	From 18*	0–5	6–11	12–17	From 18*	0–5	6–11	12–17	From 18*
1	1900	286.5	345.5	423.5	350	283.5	342.5	420.5	344	271	330	408	319
1901	2300	306.5	368.5	450.5	379	303.5	365.5	447.5	373	291	353	435	348
2301	2700	326.5	391.5	477.5	407	323.5	388.5	474.5	401	311	376	462	376
2701	3100	346.5	414.5	503.5	436	343.5	411.5	500.5	430	331	399	488	405
3101	3500	366.5	436.5	530.5	464	363.5	433.5	527.5	458	351	421	515	433
3501	3900	397.5	473.5	573.5	510	394.5	470.5	570.5	504	382	458	558	479
3901	4300	429.5	509.5	615.5	555	426.5	506.5	612.5	549	414	494	600	524
4301	4700	461.5	546.5	658.5	601	458.5	543.5	655.5	595	446	531	643	570
4701	5100	492.5	582.5	701.5	646	489.5	579.5	698.5	640	477	567	686	615
5101	5500	524.5	618.5	743.5	692	521.5	615.5	740.5	686	509	603	728	661
5501	6200	556.5	655.5	786.5	737	553.5	652.5	783.5	731	541	640	771	706
6201	7000	587.5	691.5	829.5	783	584.5	688.5	826.5	777	572	676	814	752
7001	8000	619.5	728.5	871.5	828	616.5	725.5	868.5	822	604	713	856	797
8001	9500	651.5	764.5	914.5	874	648.5	761.5	911.5	868	636	749	899	843
9501	11,000	682.5	800.5	956.5	919	679.5	797.5	953.5	913	679.5	797.5	953.5	913

*The entitlement of a child aged 18 or over to child support is conditional

Source: Düsseldorfer Tabelle (2022)

child aged 13 if there are no more children and no spousal obligation (so the sum of the dependents is two). If there is a third (younger) child in this family, the non-custodial parent must pay according to a lower income bracket: €555 for the child aged 18 and €615.5 for the child aged 13. The child support payment for the third (youngest) child is determined by the second block (columns 7–10): €506.5 for a child whose age is between 6 and 11 (if still no spousal support). In the case of four children, the 'discount' is -2, so the noncustodial parent must pay according to a lower income bracket by 2. From the example above, the noncustodial parent should pay €510 for the child aged 18 and €573.5 for the child aged 13 if the two siblings are younger. The third child received support according to the second block, and the fourth child along the third block (columns 11–14). If the custodial parent is also entitled to maintenance, the number of dependents and the discount increase by one. Both parents can be entitled for spousal support which is the 45% of the difference between their chargeable income after deducting the child support payment and the work-related costs.

In addition, there is a maximum level of child and spousal support depending on the net chargeable income (NCI) of the parents (i.e., where NCI = net income − 5% of the net income). If this could not cover the sum of the calculated support and the minimum living expenses (in 2022: 1.160 euros), (s)he only must pay the difference between the chargeable net income and the minimum living expenses. In the case when the difference is negative, no payment is obligated for the non-custodial parent. However, if the custodial parent has sufficient income to cover the calculated child support (s)he needs to pay spousal support.

2.4 The distribution of income

Over the past century or more, many attempts have been made to describe the distribution of income. Kleiber and Kotz (2003) give a detailed description of the family of distribution functions, including definitions, history, inequality measurement, characterization and empirical results: Pareto (pp. 59–106), lognormal (pp. 107–146), gamma-type (e.g. log-gamma and Weibull, pp. 147–182) and beta-type (e.g. Dagum, Singh-Maddala, beta-2 and generalized beta-2; pp. 183–234) functions are presented. A relatively new approach is the double Pareto-lognormal distribution to describe income. Using grouped data from ten countries, Hajargasht and Griffiths (2013) compared with the performance of the generalized beta-2 distribution. Their results show that both distributions provide a good fit. There is no consensus on which function best describes the distribution of income in general, but it seems certain to be left-skewed and fat-tailed. Many different models are being used and tested for their goodness of fit, but so far there is no single winning solution in the literature. Boccanfuso et al. (2008) examine the income distribution assumptions of poverty analysis for CGE micro-simulation modeling. After presenting various widely used income distribution functions (including the lognormal, beta, gamma

Table 6 The simulation inputs for net incomes

	Distribution	Mean	Deviation	Correlation 1 (scenario 1)	Correlation 2 (scenario 2)
Non-custodial (Y_{NC})	Normal	6450 EUR	3000 EUR	0.3	0.0
Custodial (Y_C)	Normal	2515.5 EUR	1170 EUR		

Source: author's model parameters

distributions), and testing by graphical and statistical approach, they conclude that no single solution can be called the best one to apply to all cases for poverty analysis.

3 Methods

In this section, under Sect. 3.1, dataset derived using simulation and the income adjustments are made based on the 2022 DT table. These results are analysed in Sect. 3.2 with discussion of the poverty traps of the current system and finally the optimised child support system is proposed in Sect. 3.3.

3.1 Dataset by simulation

Since there is limited data available on the financials of separated families and relevant child and spousal support payments, the study uses simulated data. Over 1000 parent couples' number of children and earnings were generated from simulation (described below). Száz (2018) (pp. 332) Based on the simulated numbers for income and number of children, the "status quo" child support obligation is determined according to DT framework and finally the equivalised incomes of the households.

3.1.1 Net income of the parents

In designing the simulation, it was important to show the effect of having children on the income of the two types of parents: the child penalty on the custodial parent and the correlation between the ex-spousals. The income of both types of parents was generated with correlated normal distributions with different means and variances and the lognormality of the income distribution of the whole population was tested at the end of the calculations (see the test results in Sects. 3.1.3. by graphical and 3.2.4. by statistical approach).

First, the monthly net income of the noncustodial parent (Y_{NC}) was defined. It was set as independent and normally distributed random variables, with expected value of €6450 (the mean of the upper value of the first and the last income category from the DT) and their deviation at €3000 euros. The aim of this interval was to test the

effect of the whole range of the German guideline.[1] Second, the net income of custodial parents (Y_C) was defined similarly as the noncustodial parent, using two alternative scenarios as presented in Table 6. These normally distributed random variables' mean and the deviation was deducted by 61% (the amount of child penalty according to Kleven et al. 2019) so their expected value was set at 2515.5 euros, while their deviation was at 1170 euros. In the first scenario, these data were set to correlate (the coefficient was 0.3) with the net income of the Ex partner's (non-custodial parent's) income while in the second scenario the correlation coefficient was 0.

3.1.2 Size of the families

Initially, 1432 sample families were simulated, with zero or one child in four age categories, namely ages 0–5, 6–11, 12–17 and over 18 years old. Since the average number of children in Germany was 1.58 in 2021 (FSOG 2022b), the expected values of the binomial distribution in the four age categories were set at $1.58/4 = 0.395$. The adult child (age 18+) was supposed to be a pupil in the analysis (otherwise, the amount of child support would diverge from Table 5).

The further calculation supposed that every family has one custodial and one noncustodial parent, all children in the family live with the custodial parent, and the noncustodial parent lives alone. A new marriage or new children would modify both the disposable income and the number of equalised adults in the households. The paper focuses only on the financial effects of the divorce and does not deal with the question of re-marriage, new partners, or children in the future. These kinds of changes would need further calculations. For simplicity and tractability of the results, this analysis ignores these opportunities and assumes only one adult in each household.

3.1.3 The final simulated dataset

In this section, the simulated dataset is systematically cleaned to ensure real world representativeness of the sample and focusing on the households with child(ren).

In the **first scenario** for 22 cases the non-custodial parent's income became negative. To address these outlier effects, the sample is symmetrically truncated excluding these 22 lowest and the 22 highest income cases. Similarly, in 13 cases the custodial parent's income became negative so these cases and symmetrically the highest 13 income were also eliminated from the sample. In 167 of the simulated families, there were zero children in all four age groups. To focus on relevant families with children, these 167 cases were also dropped, so that the final sample consisted of 1195 cases. Income data of the final sample from the first scenario are summarized in Table 7.

[1] The initial value of the simulation may seem high at first, but in the final data set the mean equivalised income per capita was in line with the current German value.

Table 7 Simulation 1: Net income summary of final dataset

First scenario	Mean	Deviation	Correlation	Child penalty
Non-custodial (Y_{NC})	6349.9 EUR	2722.0 EUR	0.23	–
Custodial (Y_C)	2551.5 EUR	1078.4 EUR		59.82%

Source: author's own calculation

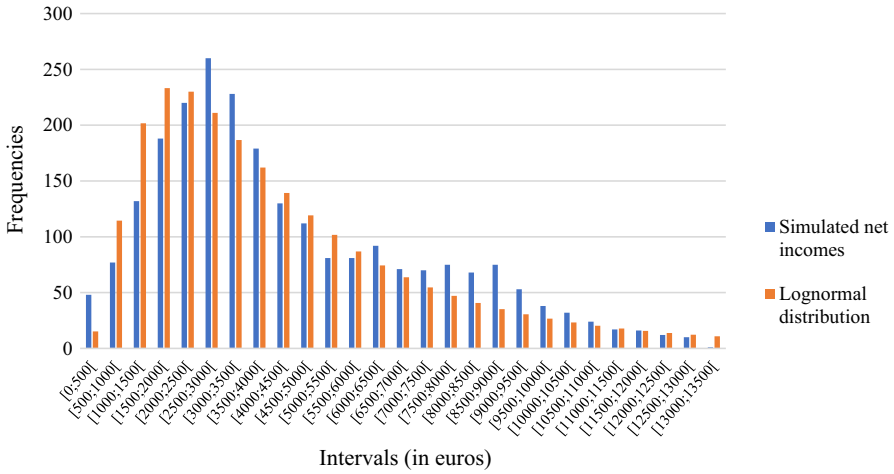

Fig. 2 The lognormal distribution and the distribution of income in the simulated dataset of the first scenario when the correlation between the ex-spousal's income is 0.23. *Source* author's own calculation

I graphically compared the distribution of net income in the sample to a lognormal distribution. The skewed to the left (and fat-tailed) distribution is clearly visible in the pattern as shown in Figure 2.

In the **second scenario**, with zero correlation between parents' income, similar datacleaning is performed. There were 23 cases where the non-custodial parent's income became negative so these cases and symmetrically the highest 23 income were eliminated from the sample. In 14 cases the custodial parent's income became negative so these cases and symmetrically the highest 14 income were also eliminated from the sample. Among these simulated families, there were 0 children in each category in 168 cases. These families were eliminated, so remained 1190. Income data of the final sample from the second scenario are summarized in Table 8.

I graphically compared the distribution of net income in the sample to a lognormal distribution. The characteristic left-skew (and fat-tailed) is clearly visible in the pattern as shown in Figure 3.

Table 8 Simulation 2: Net income summary of final dataset

Second scenario	Mean	Deviation	Correlation	Child penalty
Non-custodial (Y_{NC})	6388.5 EUR	2739.9 EUR	− 0.04	–
Custodial (Y_C)	2498.2 EUR	1100.3 EUR		60.90%

Source: author's own calculation

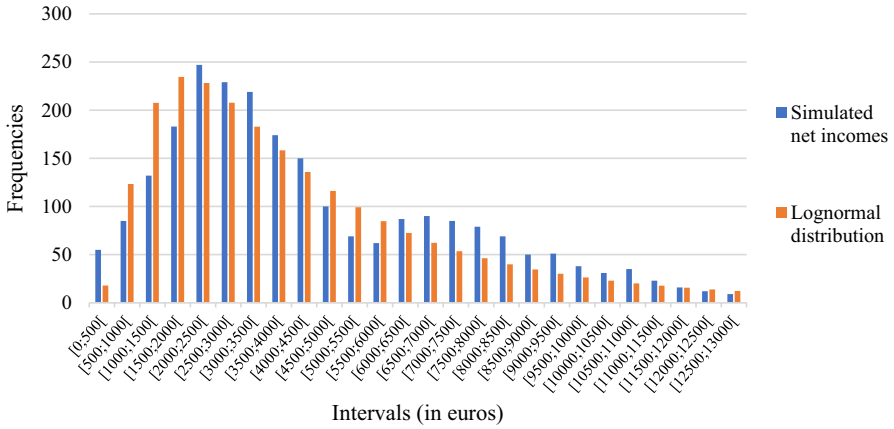

Fig. 3 The lognormal distribution and the distribution of income in the simulated dataset of the second scenario when the correlation between the ex-spousal's income was −0.04. *Source* author's own calculation

3.1.4 Child and spousal support

For determining the child support, the chargeable net income (Y_{NC}^{ch}, Y_C^{ch}) are needed which came from the net income by reducing it by 5%. (This 5% is the countable cost related to work that is deductible according to the guideline.)

$$Y_{NC}^{ch} = 0.95 Y_{NC} \tag{1}$$

$$Y_C^{ch} = 0.95 Y_C \tag{2}$$

The child and spousal support payments were determined from the chargeable net incomes according to chapter 2.3.4. Hereafter *ChS* indicates the child support while *SpS* the spousal support paid by the noncustodial. The latter can also be negative if the noncustodial receives it. (Both supports can also be zero.)

The net income of the noncustodial parent was modified by the child and spousal supports to determine the disposable income (Y_{NC}^{d}).

Table 9 The ratio of the non-custodial and custodial parents' equalised income in the three sample

Support payment	Child*	Child + Spousal	Child + Spousal
Correlation between spousal's income	n.a. (0)*	0.23	− 0.04
Highest equivalised income ratio (RoE) in the sample	8.8*	2.9	4.4
Average equivalised income ratio (RoE) in the sample	2.2*	1.6	1.6
Fraction of custodial parents with less equivalised income in the sample	83.2%*	97.2%	97.0%

Source: *Varga (2022) and author's own calculation

$$Y_{NC}^{d} = Y_{NC} - ChS - SpS \tag{3}$$

The custodial parent's income was also modified by the child and spousal supports complemented by the child benefit (hereafter *ChB*) to determine the disposable income (Y_{C}^{d}). The *ChB* depends on the number of children (n) according to Table 4.

$$Y_{C}^{d} = Y_{C} + ChS + SpS + ChB(n) \tag{4}$$

3.1.5 Equivalised income of households

Since the noncustodial parent is assumed to live alone, his/her equivalised income (Y_{NC}^{e}) is simply his/her disposable income (Table 9).

$$Y_{NC}^{e} = Y_{NC}^{d} \tag{5}$$

In the case of custodial parent's household, to determine the equivalised income the disposable income was divided by the square root of the total number of household members (i.e. $(1 + n)$: one custodial parent plus the number of children).

$$Y_{C}^{e} = Y_{C}^{d} / \sqrt{1 + n} \tag{6}$$

Hereafter referred to as the ratio of the non-custodial and custodial parents' equalised income is denoted by *RoE*.

$$RoE = Y_{NC}^{e} / Y_{C}^{e} \tag{7}$$

3.2 Income distribution in the simulated sample with current guideline

3.2.1 The ratio of the noncustodial and custodial parents' equivalised income in the sample

In the simplified model of Varga (2022), excluding spousal support, the non-custodial parent's household could have had 8.8 times higher equivalised income than the custodial one with child support according to DT. In 83.2% of the cases, the

single-parent's household had less equivalised income than the non-custodial parent. The average ratio of equivalised incomes (RoE) was 2.2.

In the current sample calculating with both the child and spousal supports, in the most extreme situation, the non-custodial parent's household has only 2.9 (in first scenario) or 4.4 (in second scenario) times higher equivalised income than the custodial has. At the same time in 97.2% (first) or 97.0% (second scenario) of the cases the single-parent's household has less equivalised income than the non-custodial parent since the spousal support may also be charged to him/her. The average ratio (RoE) was reduced to 1.6 in both scenarios. These data are summarized in Table 9.

Now the effect of spousal support is visible: (1) the average gap and the highest gap between the equivalised incomes reduced. However, (2) there is an adverse effect, the proportion of cases with the custodial parent has less income became higher, increased from 83.2% to higher than 97%.

For the first and second scenarios Figs. 4 and 5 show the support payments in the sample paid by the non-custodial (i.e., the value is positive when (s)he pays it and negative when (s)he receives it), ordering the cases by increasing ratio of equivalised income (*RoE*). The most prominent part when despite having less equivalised income (i.e., where the ratio is more than 1), the custodial parent must pay spousal support to the non-custodial (i.e., where the spousal support is negative). It occurs in 13.6% (first scenario) or 14.2% (second scenario) of the cases.

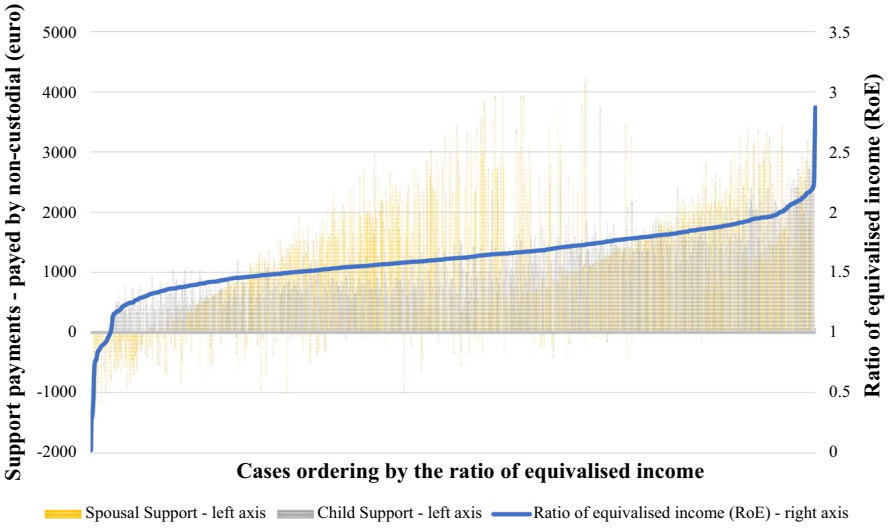

Fig. 4 The non-custodial parent's child and spousal support payments according to DT and the ratio of the noncustodial and custodial parents' equivalised income in the generated sample when the correlation between the ex-spousal's income is 0.23. *Source* author's own calculation

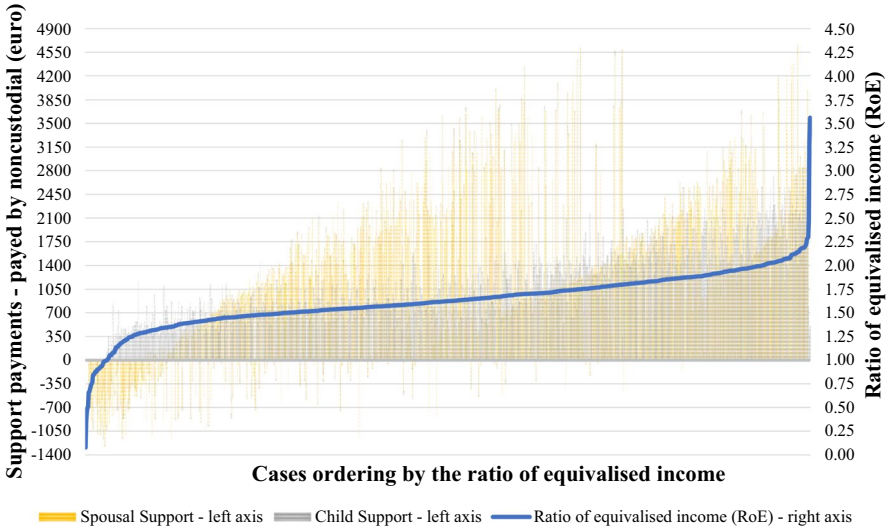

Fig. 5 The noncustodial parent's child and spousal support payments according to DT and the ratio of the noncustodial and custodial parents' equivalised income in the generated sample when the correlation between the ex-spousal's income is -0.04. *Source* author's own calculation

3.2.2 People and their equivalised income in the lowest and highest income quintile in the samples

To be able to establish the quintiles for the population, from the 1195 final sample (using Scenario 1) we expand the sample for each household member. After assigning the equivalised income for every member of the sample (first scenario: 1195 custodial + 1195 non-custodial + 2146 children = 4536 people; second scenario: 1190 custodial + 1190 non-custodial + 2145 children = 4525 people) and sorting in ascending order the highest and lowest income quintiles can be characterized. Tables 10 and 11 summarizes the quintiles' data for the two scenarios. The average equivalised income in the richest quintile is around €5000, 74–79% of these people is noncustodial parent. The average equivalised income in the poorest quintile is around €1400, 89–91% of these people is custodial parent or children. The average income is 3.33–3.63 times higher in the highest income category (see the value of S80/S20).

3.2.3 The risk of poverty in the samples

The risk of poverty in the samples for households defined at different percentages of median income is summarized in Tables 12 and 13. The risk of poverty for the

custodial parent and children are at least 3 times higher than for the non-custodial by both poverty line and in both scenarios.

Subsection 3.2 has shown that under the current DT system: (1) the equivalised income of the custodial parent and the children is lower in the most cases (97%); (2) in the lowest income quintile their proportion is around 90% while in the highest income category only 21–26%; and (3) their risk of poverty is 3 times higher than the noncustodial parent. Obviously, if we would like to equalise their standards of living the determination of child and spousal support needs reform.

3.2.4 The lognormality of incomes in the samples

I performed Kolmogorov-Smirnov test for the lognormality of equalised disposable incomes after payment of child and spousal support and receipt of child benefit for the following null hypotheses.

A) H0_1: The sample follows a Log-normal distribution in scenario 1.

Ha_1: The sample does not follow a Log-normal distribution in scenario 1.

B) H0_2: The sample follows a Log-normal distribution in scenario 2.

Table 10 The highest and lowest income quintiles in the first scenario ($\rho = 0.23$) with Düsseldorf Table

	Mean equivalised income (euro)	Fraction of non-custodials (%)	Fraction of custodials and children (%)
S80	5001.3	74.1	25.9
S20	1377.8	10.7	89.3
S80/S20	3.63	–	

Source: author's own calculation

Table 11 The highest and lowest income quintiles in the second scenario ($\rho = -0.04$) with Düsseldorf Table

	Mean equalised income (euro)	Fraction of non-custodials (%)	Fraction of custodials and children (%)
S80	4911.7	78.9	21.1
S20	1475.9	8.6	91.4
S80/S20	3.33	–	

Source: author's own calculation

Ha_2: The sample does not follow a Log-normal distribution in scenario 2.
The results of the tests are in Tables 14 and 15.

Table 12 The risk of poverty in the first scenario ($\rho = 0.23$) with Düsseldorf Table

Poverty line	Whole sample (%)	Non-custodials (%)	Custodials and children (%)
50% of median	7	3	9
60% of median	14	5	17

Source: author's own calculation

Table 13 The risk of poverty in the second scenario ($\rho = -0.04$) with Düsseldorf Table

Poverty line	Whole sample (%)	Non-custodials (%)	Custodials and children (%)
50% of median	5	2	6
60% of median	10	4	12

Source: author's own calculation

Table 14 The test statistics of lognormality in the first scenario ($\rho = 0.23$)

D	0.014
p-value (Two-tailed)	0.311
alpha	0.05

Source: author's own calculation

Table 15 The test statistics of lognormality in the second scenario ($\rho = -0.04$)

D	0.028
p-value (Two-tailed)	0.002
alpha	0.01

Source: author's own calculation

In the first scenario (see Table 14), as the computed p-value is greater than the significance level alpha=0.05, one cannot reject the null hypothesis H0_1: the distribution of equivalised disposable income can be considered lognormal in scenario 1.

In the second scenario (see Table 15), as the computed p-value is lower than even the significance level alpha=0.01, one should reject the null hypothesis H0_2, and accept the alternative hypothesis Ha_2: the distribution of equivalised disposable income cannot be considered lognormal in scenario 2. Nevertheless, the optimised support proposed in the next subsection will also be analysed in this scenario, but I will only consider its results in a limited way.

3.3 Optimization of the child and spousal support: suggestion for a new guideline

The purpose of spousal maintenance is to ensure that there is no gap between the ex-spouses' income after divorce. However, this is not what we saw in the two scenarios. These results motivated the following optimization model which combines the child and spousal supports and searches for the optimised support (OpS) where the standard of living, measured by equivalised income, is the same for the separated parts of a family (i.e. $Y^e_{NC} = Y^e_C$). From this principle the optimised function can be written as formula (8):

$$\left| Y^{SR}_{NC} - Y^{SR}_{C} \right| \rightarrow_{ChS} min \tag{8}$$

In this case the optimised support can be determined by formula (9):

$$OpS = \frac{\sqrt{n+1} \cdot Y_{NC} - Y_C - ChB(n)}{1 + \sqrt{n+1}} \tag{9}$$

Now the children and their parents have the same equivalised income. It means that the ratio of equivalised incomes is 1 for each separated family. The fraction of noncustodial, custodial parents and their children in the highest and lowest income quintile and the households' risk of poverty are discussed in the next section.

4 Discussion of the proposed child support payment model

In this section, we compare the outputs from the proposed model with the current DT model implications. In each section, first, the proposed model outputs are presented and subsequently, the relative improvement in comparison with DT model results are discussed.

4.1 Inequality of income distribution (S80/S20 rate) in the samples with optimised support

Tables 16 and 17 summarizes the equivalised incomes for the top and bottom quintiles for the two scenarios with the proposed optimised child support system. Since the average family size is different across the top and bottom quintile, the fraction of noncustodial and the fraction of custodial parents and children are different in the top and bottom quintiles. As expected in our simulation, the average number of children is less in the highest equivalised income quintile, than in the lowest equivalised income quintile.

With the proposed child support model, the median equivalised income increased by approximately 31–33%, the mean equivalised income increased by around 21% in the society. The mean equivalised income increased by approximately 4% in the highest income quintile, and by 30–32% in the lowest income quintile. The ratio of average equivalised income of top and bottom quintile reduced by 20–21% which means a

Table 16 The highest and lowest income quintiles in the first scenario ($\rho = 0.23$) with optimised support

	Mean equivalised income (euro)	Fraction of Non-custodials (%)	Fraction of Custodials and children (%)
S80	5193.4	28.4	71.6
S20	1797.4	25.2	74.8
S80/S20	2.9	–	

Source: author's own calculation

Table 17 The highest and lowest income quintiles in the second scenario ($\rho = -0.04$) with optimised support

	Mean equivalised income	Fraction of non-custodials (%)	Fraction of custodials and children (%)
S80	5139.1	28.4	71.6
S20	1953.3	25.1	74.9
S80/S20	2.6	–	

Source: author's own calculation

Table 18 Changes in the mean equivalised income in the top and bottom quintile and the income quintile share ratio using the optimised support instead of DT

	First scenario ($\rho = 0.23$) (%)	Second scenario ($\rho = -0.04$) (%)
Median equivalised income	+30.93	+33.36
Mean equivalised income	+20.91	+21.10
S80 mean equivalised income	+3.84	+4.63
S20 mean equivalised income	+30.46	+32.35
S80/S20	− 20.40	− 20.94

Source: own calculation

narrowing of the social gap. The optimised support's effect on equivalised incomes is summarized in Table 18.

4.2 Risk of poverty in the samples with optimised support

Section 4.1. has shown the median income of the sample increased under the optimised support which means that the poverty line went up at the same time. Overall, the risk of poverty within the sample did not change, only the proportions between custodial and noncustodial parents levelled out. To compare the impact of DT and the optimised child support, I used the median wage in Germany in 2021 (as 2022 is not yet available) to determine the poverty threshold (FSOG 2022c) and examined what percentage of people in the samples live below this poverty line. Overall, in the first, more relevant, scenario the proportion of people at the risk of poverty has fallen by around 60%, and among custodial parents and their dependent children, the reduction is closer to 70%. In the second scenario the non-custodial parent's risk of poverty also decreased when the poverty line was set at 60% (as usual in Eurostat data), however when the poverty

Table 19 Change in the risk of poverty rate using the optimised support instead of the DT in the first scenario ($\rho = 0.23$)

When the poverty line is at	Whole sample (%)	Non-custodials (%)	Custodials (%)
50% of median	− 66	− 5	− 72
60% of median	− 61	0	− 67

Source: author's own calculation

Table 20 Change in the risk of poverty rate using the optimised support instead of the DT in the second scenario ($\rho = -0.04$)

When the poverty line is at	Whole sample (%)	Non-custodials (%)	Custodials (%)
50% of median	− 54	+5	− 61
60% of median	− 65	− 28	− 71

Source: author's own calculation

line is at 50% of the median there is a minimal (5%) increase. Detailed data are given in Tables 19 and 20.

4.3 Incentive effects: the sensitivity of the child support with respect to the earnings

Schaubert (2022) was the first who investigated the causal effects of child support obligations on the parents after divorce in a microsimulated model. Her research question was the impact of having a new child. In my paper I deal with the impact of increasing earnings by one euro.

First the change of child support was determined if the earnings increase by one unit. From (9) the partial derivatives of the optimised child support payment function with respect to the noncustodial (10) and the custodial (11) parents' earnings are:

$$\frac{\partial ChS}{\partial Y_{NC}} = \frac{\sqrt{n+1}}{1+\sqrt{n+1}} \tag{10}$$

$$\frac{\partial ChS}{\partial Y_{C}} = -\frac{1}{1+\sqrt{n+1}} \tag{11}$$

The change in child support depends on the number of children and independent of the current income level. Table 21 presents the change in the disposable income and the equivalised income if a parent earns one more unit depending on the number of children.

Using Eqs. (10) and (11) if the custodial parent earns one more euro the received child support will be reduced by 41 to 31 cents depending on the number of children, so the disposable income will increase by 59 to 69 cents and equivalised income of the household will increase by 41 to 31 cents. If the noncustodial parent earns one

Table 21 Change (in euro) of the optimised support payment and the disposable income if a parent's earning increases by one euro

Number of children	+1 euro for the custodial parent			+1 euro for the non-custodial parent	
	Optimised support	Custodial parent's disposable income	Custodial parent's equalised income	Optimised support	Non-custodial parent's equalised (=disposable) income
1	−0.41	+0.59	+0.41	+0.59	+0.41
2	−0.37	+0.63	+0.37	+0.63	+0.37
3	−0.33	+0.67	+0.33	+0.67	+0.33
4	−0.31	+0.69	+0.31	+0.69	+0.31

Source: author's own calculation

more euro the paid child support will increase by 59 to 69 cents depending on the number of children, so the (equivalised) disposable income will increase by 41 to 31 cents. For any number of children, the change of incomes will be the same for the households. It means that the incentive effects would be the same for the custodial and noncustodial parents to earn one more euro. Indeed, it is a straight consequence of our optimization principle: the optimised child support is there where the standard of living is the same for the separated parts of a family.

5 Conclusion

The provision in the Düsseldorf Table, according to which in addition to child support the parent may also be obliged to pay spousal support, is clearly intended to ensure that the ex-partner does not suffer financially from the divorce. In comparison, we have seen in the sample that the (1) equivalised income of the custodial parents and children is in 97% of cases less than that of the noncustodial parent, yet in 14% of cases the custodial parent pays spousal support to the ex-partner. (2) in the lowest income quintile proportion of the custodial parents and children is around 90% while in the highest income category only 21–26%. (3) their risk of poverty is 3 times higher than the noncustodial parent.

The application of the proposed optimised child support, which equalises incomes and risks in the separated families, would reduce income inequality in society (measured by S80/S20 quintile share ratio) by 20% and the risk of poverty by approximately 60%.

The starting point of my model was that the standard of living in a separated household had been the same for all members. After divorce, a fair system should ensure that moving forward no parties economic situation deteriorates, especially not to the point of falling into poverty. The model presented here provides a mathematical solution to facilitate fairness in establishing child support globally. Besides its fairness the suggestion is efficient in the sense that both parents have the same incentive to earn one more unit. The analyses of incentives on new marriage (partner) or

additional child requires further research, since the paper's calculation supposed that every family has one custodial and one noncustodial parent, all children in the family live with the custodial parent, and the noncustodial parent lives alone. Further family members would alter the model inputs, such as disposable income and the number of equalised adults in the households. Overall the study aims to encourage further research in this area, as the determination of child support is critical in our world with the increasing divorce rates worldwide (European Parliament 2014).

Funding Open access funding provided by Corvinus University of Budapest. The author did not receive support from any other organization for the submitted work. The author has no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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