Rotation of the age-varying parameters in multi-population mortality models

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Summary

In addition to the well-known Lee–Carter model, two versions of a multi-population mortality model, known as augmented common factor models, were fitted to Hungarian data in the present study. The two subpopulations considered in the analysis were men and women. When predicting mortality rates, it is important to not only predict the trend in mortality change (improvement) given that the age-specific coefficient of this time-varying parameter also changes over time. The phenomenon of this time dependence of the age pattern and consideration thereof in mortality projections are known in the literature as rotation. As a result of the present research, possible trajectories for the life expectancy of men and women in Hungary up to the year 2050 were determined by predicting rotated and nonrotated versions of three different mortality models.

Keywords: multi-population mortality models, modeling jointly, time-varying age patterns, rotation, forecasting life expectancy

A halandóság-javulás időben változó életkori mintázata multipopulációs modellekben

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Összefoglalás

Lee és Carter halandóság-előrejelző modellje több mint 30 éve népszerű módszer. E modell szerint a mortalitási ráták logaritmusa egy hosszú távú trend lineáris függvénye, amelynek korspecifikus együtthatója lehetővé teszi a halandóság változásának életkorok szerinti vizsgálatát. Az utóbbi évtizedekben számos kutató vállalkozott arra, hogy Lee és Carter modelljét továbbfejlessze, így az eredeti log-bilineáris modellnek ma már sokféle változata ismert. E továbbfejlesztett modellek közé tartoznak a multipopulációs halandósági modellek is, amelyek egy populáció alcsoportjainak (pl. egy régió országainak, egy ország területi egységeinek, férfiak és nők csoportjának) összefüggő elemzését és halandóság-előrejelzését teszik lehetővé.

A Lee–Carter modellcsaládba tartozó multipopulációs mortalitási modellek esetében a tény időszaki adatsorra illesztett (időtől, életkortól vagy születési évtől függő) paraméterek legalább egyike valamennyi alpopulációra nézve ugyanaz. E közös paraméter(ek) mellett szerepelnek még csoportspecifikus tényezők is, amelyek az alpopulációk sajátosságainak figyelembevételét teszik lehetővé a közös jellemzők mellett. Egymással szoros kapcsolatban álló és hasonló szocio-ökonómiai háttérrel rendelkező alpopulációk esetében indokolt lehet a multipopulációs halandósági modellek használata, amelyek legfőbb célja, hogy egy populáció alcsoportjainak halandóságát ne egymástól függetlenül vizsgáljuk.

A kutatás során 1960 és 2022 közötti magyar adatokat felhasználva három mortalitási modellt illesztettünk: a Lee–Carter modellt és ennek két multipopulációs változatát, a két- és háromfaktoros ACF (‚augmented common factor') modellt. A cél az volt, hogy férfiak és nők mortalitását összefüggő módon vizsgáljuk, és a modellben szereplő életkortól függő paraméterek közelmúltban megfigyelhető időbeli változását az előrejelzés során figyelembe vegyük. A szakirodalomban rotáció néven ismert az a jelenség, amely szerint a fejlett országokban a halandóság-javulás lassul a legfiatalabb korcsoportokban és gyorsul a legidősebb koréveket tekintve. A hosszabbodó várható élettartam

– a reprodukcióhoz szükségesnél alacsonyabb szintű termékenység mellett – hozzájárul a társadalom öregedéséhez, amely Magyarországon is egyre nagyobb kockázatot jelent a nyugdíjrendszer fenntarthatósága szempontjából.

A kutatás eredményeként a vizsgált halandósági modellek rotált és nem rotált változatainak előrejelzése révén meghatároztuk a várható élettartam lehetséges pályáját Magyarországon 2050-ig, férfiakra és nőkre külön-külön. Azt találtuk, hogy a Lee–Carter modellben szereplő korspecifikus együttható múltban megfigyelhető változása egyelőre még nem támasztja alá a rotáció szükségességét a rövid távú előrejelzés során. A nem rotált modellek közül a háromfaktoros ACF modell illeszkedése volt a legjobb és ebben az esetben sikerült egy olyan rotált változatot kialakítani, amely hosszú távon tükrözi azt, amit a rotációtól várunk: a legidősebb korcsoportok nagyobb mértékben járulnak hozzá a várható élettartam növekedéséhez a jövőben, mint a legfiatalabbak. Ugyanakkor a rotált, háromfaktoros ACF modell hosszú távon egyre gyorsabban növekvő várható élettartamot jelez előre, ami óvatosságra int az előrejelzés során.

Kulcsszavak: multipopulációs halandósági modellek, alpopulációk együttes modellezése, időben változó életkori jellemzők, rotáció, várható élettartam előrejelzése

Forewords

Forecasting demographic indicators is an important topic in economic, social and healthcare research. Many studies address the projection of mortality as a core part of this field. However, there is no universally accepted solution for forecasting mortality, thus models and methodologies need to be adapted to specific circumstances. Lívia's doctoral research focuses on studying the application of such mortality models to Hungarian data, which have not been widely used in the Hungarian practice yet.

The application of multi-population mortality models has already been the subject of an earlier study by Lívia, which addressed the regional projection of Hungarian mortality. The results of this research have also been applied by the author, as an employee of the Hungarian Central Statistical Office (HCSO), to further develop county population projections. Another study by Lívia deals with the projection of Hungarian mortality rates by cause of death, in addition to a third paper on excess mortality due to the COVID-19 epidemic. In the present paper, Lívia focuses on the age-dependent parameters of the improving mortality trend in Hungary, and considers their recent properties in predicting mortality rates. The results of this research will allow a fine-tuning of the mortality models applied.

As can be seen from the short list above, her research covers a wide range of mortality issues. However, Lívia not only tackles questions of research interest, but as an employee of the HCSO, she also develops predictive models on which future research can be built. Her diligence, thoroughness and the quality of her work mean that this task could not be in better hands.

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Lívia works on population projections at the Hungarian Central Statistical Office (HCSO), one of the pillars of which is mortality projections. The international literature describes a wide range of mortality models, and one of Lívia's main goals is to learn as much as possible about these models. In her previous work, she analyzed the selected models according to different disaggregations of Hungarian mortality data: she analyzed regional data and data disaggregated by cause of death. Her current research focuses on the analysis and projection of mortality and life expectancy for men and women. Among the selected models, her current work uses one that she has applied in practice for the first time. Lívia aims to broaden not only the aspect chosen, but also the range of models used in her doctoral research.

Lívia does a lot of work on the numerical programming of the mortality models, which adds significant value to the Hungarian population projections. Therefore, her work does not only involve compiling and analyzing the data, but also considerable programming efforts. Population projections are always prepared in several versions, representing calculations under different scenarios. This research will contribute to the optimistic version of the mortality projection, which Lívia will build on in her future work at the HCSO. Lívia's research is useful at her workplace and the work she has done is a niche project.

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Introduction

The mortality model of *Lee and Carter (1992)* has been a popular method for demographers, actuaries and researchers for more than 30 years. According to this model, the logarithm of mortality rates is a function of a long-term trend. The age-specific coefficient of the trend allows the change in mortality to be varied by age. The trend and the age-specific pattern of the mortality change can show large differences, for example by sex or geography. In recent decades, many researchers have attempted to improve the Lee–Carter model, for example by including additional factors, or by using new methods to fit the model and/or to predict the stochastic factor(s). The main aim of these developments was to improve the

fit of the original model and to make the forecast more accurate. As a result, many variants of the original log-bilinear model are now known.

These improved models include multi-population mortality models (see, e.g., *Villegas et al. 2017*), which allow the analysis and prediction of mortality for subgroups of a population (e.g., countries in a region, territorial units of a country, or groups of men and women) together. One of the main objectives of using multi-population mortality models is to study the mortality of subgroups of a population that are not independent of each other. In this type of model, which belongs to the Lee–Carter family, at least one of the fitted parameters (which are dependent on time, or age, or year of birth) is the same for all subpopulations. In addition to these common parameter(s), there are also group-specific factors that allow the characteristics of the subpopulations to be taken into account, in addition to the common features. For subpopulations that are closely related and have similar socioeconomic backgrounds, the use of multi-population mortality models may be justified.

The popularity of the Lee–Carter model is partly due to the fact that it is not a data-intensive method, requiring only a long time series of deaths and population size by age. Many examples for its application can be found in Hungarian research papers. *Baran et al. (2007)* fitted the original Lee–Carter model to Hungarian data for men and women. They also applied two- and three-factor versions of the model, following the idea suggested by *Booth–Maindonald–Smith (2002)*. The authors also considered the choice of the ideal length of the base period with respect to the change of political and economic regime in Hungary, which had an influence on mortality. *Májer–Kovács (2011)* applied the Lee–Carter model to predict life expectancy at old age, and to measure the associated uncertainty using Hungarian data, while also discussing the financial risks for the pension system. *Bajkó et al. (2015)* applied the Lee–Carter model to predict not only mortality rates but also fertility rates, and constructed a pension model. For validation purposes, they also made projections based on time series of different lengths for an already known period in order to compare the projected values with current data.

Vékás (2017) applied the notable models of the generalized age–period–cohort model family (see, e.g., *Villegas–Kaishev–Millossovich 2018*) to Hungarian data, which the author also described in a later paper (see *Vékás 2019*). The Lee–Carter model is also discussed in this framework. *Petneházi–Gáll (2019)* compared two methods for predicting mortality. The authors compared the results of the long short-term memory (LSTM) neural network with the original Lee–Carter prediction using data from different countries. *Gogola–Vékás (2020)* used the Lee–Carter model to predict the mortality rates for older age groups for their actuarial calculations using Hungarian and Czech data.

Obádovics–Tóth (2021, 2023) used a version of the Lee–Carter model modified by *Lee–Miller (2001)* to forecast mortality as part of their long-term population projection model. *Tóth (2021a, 2022a, 2022b)* analyzed the impact of the COVID-19 epidemic using the Lee–Miller version of the model to project the mortality of the period affected by the COVID-19 pandemic, i.e., to estimate how mortality would have developed in the absence of the epidemic and to determine the extent of excess mortality in Hungary. *Csiszár (2022)* provided estimates of mortality by cause of death not only for Hungary, and compared the Lee–Carter model with the P-splines method. *Szentkereszti–Vékás (2022)* also applied the LSTM neural network method to Hungarian data and fitted the Lee–Carter model for comparison. *Petneházi–Gáll (2023),* following *Baran et al. (2007),* used more recent data to investigate the possibility of applying the one- and multi-factor Lee–Carter model to Hungarian data.

Tóth (2021b) investigated the applicability of the product-ratio model developed by *Hyndman–Booth–Yasmeen (2013)* in Hungary, using data from the Visegrad Group countries. *Varga (2023)* fitted multi-population models from the Lee–Carter family to Hungarian regional data, allowing the joint analysis of mortality for subpopulations. The author found that the augmented common factor model and the augmented common factor model with common age effect showed a good fit. For some of the above literature, the Lee–Carter model is a mortality prediction model against which the results of modified versions or new methods are compared.

There are large differences in mortality between men and women in Hungary. However, there is a close relationship between the two subpopulations as they are exposed to the same influences, for example, they receive care from the same health care system. Nowadays, the lifestyles of men and women are also less different. Multi-population models are often used to predict mortality for men and women together (see, e.g., *Carter–Lee 1992; Li–Lee 2005*), however, country-specific, territorial applications are also widespread (see, e.g., *Danesi–Haberman–Millossovich 2015; Kleinow 2015; Enchev–Kleinow–Cairns 2017* and *Scognamiglio 2022*).

The present research examined multi-population mortality models that can be considered as modified and extended versions of the Lee–Carter model. The theoretical background of the selected mortality models is presented below. These models were fitted to Hungarian data for men and women to predict life expectancy up to 2050, which is also described below.

Theoretical background

The Poisson Lee–Carter model

Lee and Carter fitted a model to the mortality data of the USA using long time series in which the logarithm of age-specific mortality rates is a linear function of an unobserved time-varying parameter. The authors call this parameter the mortality index, which describes the long-term trend in mortality change. Lee and Carter used singular value decomposition (SVD) to fit the model. *Brouhns–Denuit–Vermunt (2002)* presented the Poisson Lee–Carter model with a nonadditive error term in the initial equation and used maximum likelihood to fit the model parameters.

Assuming a Poisson distribution of the number of deaths, the initial equation of the Lee–Carter model can be written as follows:

$$D_{xt} \sim Poisson(E^c_{xt}\, m_{xt}) \quad (1)$$

$$ln\, m_{xt} = \alpha_x + \beta_x \kappa_t \quad (2)$$

where m_{xt} is the central mortality rate, i.e., the number of deaths (D_{xt}) divided by the central population (E^c_{xt}), also called the central exposure by age x and year t. On the right side of Equation 2, α and β depend only on age and κ on time. κ_t denotes the mortality index. Its age-specific coefficient β_x expresses that the trend in mortality varies with age. α_x is the baseline shape of the mortality curve. The model includes two constraints to ensure that the solution to the initial equation is unique. Lee and Carter imposed the following constraints on β_x and κ_t:

$$\sum_x \beta_x = 1 \text{ and } \sum_t \kappa_t = 0. \quad (3)$$

As a result, α_x is the average of the log mortality rates by age over time. The right side of the initial equation of the model represents the unknown parameters, which were also determined in this study using the maximum likelihood method.

The projection of mortality rates is based on the prediction of the mortality index. In their analysis of US data, Lee and Carter found that the mortality index declined roughly linearly between 1900 and 1989, i.e., mortality was steadily improving (apart from the effect of the Spanish flu in 1918). They predicted the continuation of this trend using an ARIMA(0,1,0) with drift model:

$$\kappa_t = \delta + \kappa_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma^2_\kappa) \quad (4)$$

where δ denotes the drift parameter, and is white noise. *Brouhns–Denuit–Vermunt (2002)* fitted an ARIMA(0,1,1) model instead of ARIMA(0,1,0) to predict the mortality index:

$$\kappa_t = C + \kappa_{t-1} + \xi_t + \theta\xi_{t-1}, \quad \xi_t \sim N(0, \sigma^2_\kappa) \quad (5)$$

where the constant C refers to the average annual change in the mortality index κ_t, and ξ_t is the independent disturbance.

The augmented common factor model

The augmented common factor (ACF) model is a multi-population model described by *Li–Lee (2005)*. This model includes two mortality indices, the first of which does not differ between subpopulations, while the second stochastic factor is group-specific. The same applies to the coefficients of the two indices: while the first expresses the same age effects across groups, the second allows for variations between the subpopulations. *Li–Lee (2005)* fitted the model using the SVD method. *Li (2013)* and *Li–Tickle–Parr (2016)* assumed a Poisson distribution for the number of deaths and applied the maximum likelihood estimation. Assuming a Poisson distribution, the initial equation can be formulated as follows:

$$ln\, m_{xtj} = \alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)} \quad (6)$$

where $\beta_x^{(1)} \kappa_t^{(1)}$ is the common factor.[1] $\kappa_t^{(1)}$ describes the long-term common trend in mortality. The second index $\kappa_{tj}^{(2)}$ shows the short- and medium-term discrepancy from the historical trend by subpopulation j. The following constraints are required for the parameters: $\sum_x \beta_x^{(1)} = 1$, $\sum_t \kappa_t^{(1)} = 0$, $\sum_x \beta_{xj}^{(2)} = 1$ for $\forall\, j$, and $\sum_t \kappa_{tj}^{(2)} = 0$ for $\forall\, j$.

According to *Li–Lee (2005)*, independence of the $\kappa_t^{(1)}$ and $\kappa_{tj}^{(2)}$ indices, and independence of the additional factor between groups can be assumed since the latter index refers to random, periodic changes in different subpopulations. For example, the random walk with drift model can be used to predict mortality indices, and *Li–Lee (2005)* mentioned the possibility of using the AR(1) model for the group-specific index.

Li (2013) assumed a Poisson distribution for the number of deaths and fitted the ACF model using the maximum likelihood estimation. Another innovation of the author is the inclusion of more than one additional group-specific factor in the ACF model. Thus, the initial equation is:

$$ln\, m_{xtj} = \alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \sum_{i=2}^{n} \beta_{xj}^{(i)} \kappa_{tj}^{(i)}. \quad (7)$$

As noted by *Li (2013)*, the use of too many parameters should be avoided because if the third- or higher-order indices are irregular, then they are less suitable for prediction. Given the initial Equation 7, the following

[1] When only the parameters α_{xj}, $\beta_x^{(1)}$ and $\kappa_t^{(1)}$ appear on the right side of the equation, it is called the common factor (CF) model. In fact, Lee–Nault (1993) was the first to apply this model. In the CF model, the term α_{xj} is the only group-specific factor, and it may be appropriate if $\beta_x^{(1)}$ and $\kappa_t^{(1)}$ are very similar across subpopulations.

constraints are required: $\sum_x \beta_x^{(1)} = 1$, $\sum_t \kappa_t^{(1)} = 0$, $\sum_x \beta_{xj}^{(i)} = 1$ for $\forall$ *i, j,* and $\sum_t \kappa_{tj}^{(i)} = 0$ for $\forall$ *i, j.* Li (2013) used a random walk with drift model for the common mortality index, and proposed AR(p) models for the additional time-varying factors to predict mortality rates.

Rotation of the age-varying parameters

In developed countries, mortality improvements have been slowing down in younger age groups and accelerating in older age groups. *Li–Lee–Gerland (2013)* referred to this phenomenon as 'rotation'. For infants and young age groups, the rapid rate of improvement cannot be assumed to continue in the long term, as it would lead to unreasonably low mortality rates in these age groups. It is therefore reasonable to assume that the improvement in mortality for younger age groups will slow down and that the older age groups will contribute more to the increase in life expectancy than in the past. If the phenomenon of rotation is not taken into account, the results of the long-term projection become questionable: we may underestimate the number of elderly people and life expectancy as well. *Li–Gerland (2011)* introduced robust rotation for the parameter β_x of the Lee–Carter model. They called this model the Lee–Carter method with robust rotation. This is different from *Li–Lee–Gerland (2013),* who developed the Lee–Carter method extended with a rotation model.

Li–Lee–Gerland (2013) assumed that β_x values are converging to a certain level in the long run, which they called the ultimate β_x. The rotation starts at the forecast horizon when life expectancy at birth exceeds a certain value. *Li–Lee–Gerland (2013)* identified that this value is 80 years. The rotated time-dependent β_{xt} values were determined using a linear-weight function and a smooth-weight function. In their model, the rotation ends when the rotated β_{xt} values reach the ultimate β_x values. In summary, *Li–Lee–Gerland (2013)* devised a model in which β_{xt} for younger age groups gradually decreases over the projection period once life expectancy reaches a certain value. Since $\sum_x \beta_x = 1$, a decrease for younger age groups implies an increase for older age groups. *Li–Lee–Gerland (2013)* expect that the rotation will also occur in developing countries as they achieve higher life expectancy. According to *Li–Lee–Gerland (2013),* the Lee–Carter model may be applicable in the medium term with nonrotated β_x, as this parameter may be stable over shorter time periods.

The study by *Vékás (2018)* was the first in the Hungarian literature to present the phenomenon of rotation, following the methodology of *Li–Lee–Gerland (2013)*. *Vékás (2018)* showed the importance of rotation based on the data of 22 age groups between 1950 and 2015. The author found that the rotation of mortality improvement is weakly detectable for men, but significant for women in Hungary. Furthermore, *Vékás (2020)* studied the rotation for the member states of the European Union and demonstrated that the rotation of the age pattern is important for both sexes in 11 of the 28 member states.

In the literature, rotation is used for parameter $\beta_x^{(1)}{}_{(j)}$ in the Lee–Carter model and in a multi-population version of this model developed by *Li–Lee (2005)*. However, in multi-factor mortality models, there may also be multiple age patterns. In fact, α_x is also a parameter that is usually considered static in the prediction, but it also changes over time (its value decreasing with improving mortality).

In the present paper, we have attempted to account for the time dependence of the age-dependent parameters in the Lee–Carter, ACF and ACF with three factors models. We also use the term rotation for this purpose. The change in age-dependent parameter values over time can be estimated by the least squares method as follows:

$$\alpha_{xtj} = a_x + b_x t + \varepsilon_{xtj}, \qquad \varepsilon_{xtj} \sim N(0, \sigma^2) \quad (8)$$

and

$$\beta_{xt(j)}^{(i)} = c_x + d_x t + \varepsilon_{xt(j)}, \qquad \varepsilon_{xt(j)} \sim N(0, \sigma^2) \quad (9)$$

where the error terms ε_{xtj} and $\varepsilon_{xt(j)}$ are normally distributed. To estimate these equations, the mortality models must be fitted using base periods of different lengths in order to model the change in age-dependent parameters over time. Thus, in Equations 8–9, t is equal to the number of shortened base periods. In the present study, the future time-dependent values of α_{xtj} and $\beta_{xt(j)}^{(i)}$ were determined using parameters a_x, b_x, c_x and d_x. The sum of $\beta_{xt(j)}^{(i)}$ remains 1 in each year of the projection horizon, thus the original constraint of the mortality models is not violated.

Data and methodology

Table 1 summarizes the mortality models and their abbreviations used in the present study. The source of the age- and sex-specific mortality and population data for the models was the database of the Hungarian Central Statistical Office (HCSO), but the Human Mortality Database (HMD)[2] also provides the necessary data. Age-specific mortality and population data are available by year from 1950 onwards. The longest possible time series is preferred. However, the mortality models were fitted to a base period between 1960 and 2022. The mortality models were fitted using data by age expressed in year units, but the mortality data for those aged 90 years and over were considered as one group, as the un-

[2] Available at https://www.mortality.org/

Table 1 | The fitted mortality models based on Hungarian data

Literature	The names of the models	The initial equations	Constraints
Lee–Carter (1992), Brouhns–Denuit–Vermunt (2002)	Lee–Carter	$ln\, m_{xtj} = \alpha_{xj} + \beta_{xj}^{(1)} \kappa_{tj}^{(1)}$	$\sum_x \beta_{x(j)}^{(i)} = 1$ and $\sum_t \kappa_{t(j)}^{(i)} = 0$ for $\forall\ i, j$ $i = 1, 2, 3$
Li–Lee (2005), Li (2013)	ACF	$ln\, m_{xtj} = \alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)}$	
Li (2013)	ACF with three factors	$ln\, m_{xtj} = \alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)} + \beta_{xj}^{(3)} \kappa_{tj}^{(3)}$	

Note: The subscript *j* refers to the subpopulation, i.e., sex.

Source: Own editing

certainty of the estimation may increase for smaller groups. In addition, the number of persons aged 90 and over is not available by individual age group for the whole base period.

The population data for the 1980s and 1990s have been adjusted in our research. We used the adjusted total population data based on the results of the 1990 and 2001 censuses (see *HCSO 1990, 2001*). The age distribution of the adjusted data was set to match the age composition of the population as reported between two censuses. We used the central exposure for our calculations, thus we also needed the population on 1 January 2023, which was the population calculated from the 2011 census using the statistics of the vital events. The population on 1 January 2023, calculated from the latest census in October 2022, was also available during the analysis, but we did not use it to determine the mid-year population in 2022 for the purposes of consistency. As a further adjustment, the ratio of deaths at known ages to total deaths was used to distribute the number of deaths at an unknown age. This is consistent with the methodology of the HMD (see *Wilmoth et al. 2021*). Finally, the number of deaths was rounded to the nearest integer.

Lee–Carter (1992) assumed a Gaussian error term in the initial Equation 2. Following *Brillinger (1986)*, *Brouhns–Denuit–Vermunt (2002)* found it more realistic to assume that the number of deaths follows a Poisson distribution. In the present study, the mortality models were fitted using the maximum likelihood estimation with Newton–Raphson iteration method, and we assumed a Poisson distribution of the number of deaths. The numerical calculations were performed using the R software (R Core Team 2021). The iteration was repeated until the increase in the log-likelihood function was less than 10^{-6}.

The log-likelihood functions for the mortality models can be written as follows:

Poisson Lee–Carter

$$\ell_j(\theta) = \sum_{xt} \left\{ D_{xtj} \left(\alpha_{xj} + \beta_{xj}^{(1)} \kappa_{tj}^{(1)}\right) - E_{xtj}^{c} e^{\alpha_{xj} + \beta_{xj}^{(1)} \kappa_{tj}^{(1)}} \right\} + \mathrm{C} \tag{10}$$

ACF

$$\ell_j(\theta) = \sum_{xt} \left\{ D_{xtj} \left(\alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)}\right) - E_{xtj}^{c} e^{\alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)}} \right\} + \mathrm{C} \tag{11}$$

ACF with three factors

$$\ell_j(\theta) = \sum_{xt} \left\{ D_{xtj} \left(\alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)} + \beta_{xj}^{(3)} \kappa_{tj}^{(3)}\right) - E_{xtj}^{c} e^{\alpha_{xj} + \beta_x^{(1)} \kappa_t^{(1)} + \beta_{xj}^{(2)} \kappa_{tj}^{(2)} + \beta_{xj}^{(3)} \kappa_{tj}^{(3)}} \right\} + \mathrm{C} \tag{12}$$

where C is a constant.

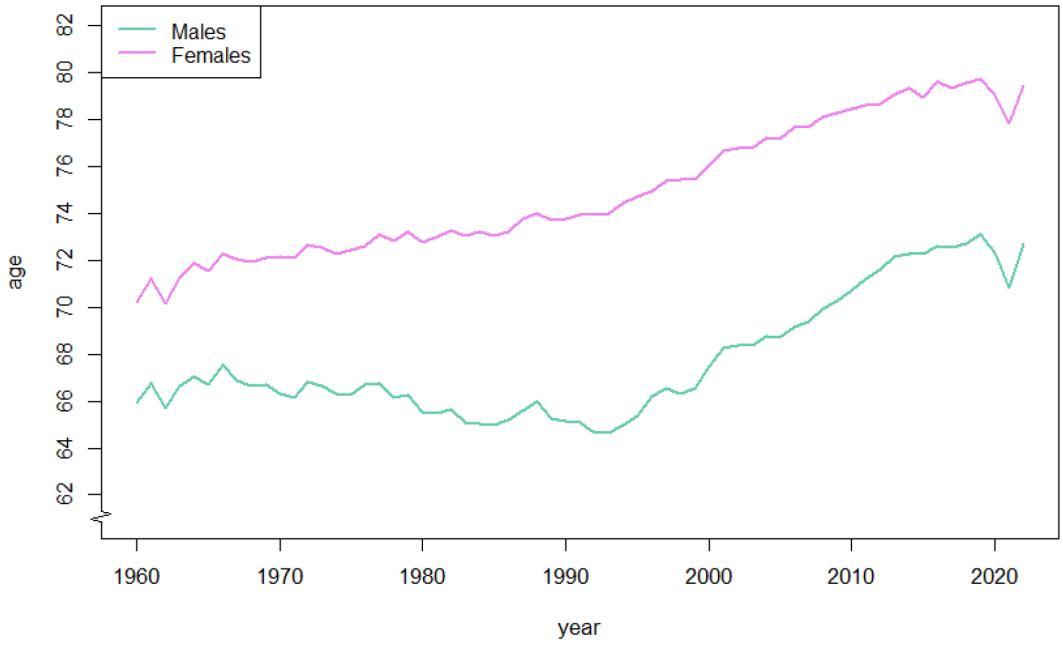

Figure 1 Life expectancy at birth for males and females between 1960 and 2022

Source: Own calculation

In the models, the sex-specific parameters α_{xj} are approximately equal to the average of the log mortality rates over time. However, one can also try a variant where α_{xj} is equal to the log mortality rates of the last period (or a selected year). In the case of the Lee–Carter model, the study by *Lee–Miller (2001)* suggests this approach. We applied the Lee–Miller modification to the ACF model with parameter re-adjustment. In fact, not only the parameters β_x or κ_t can be the same for subpopulations in multi-population mortality models with multiple factors (see, e.g., *Wen et al. 2021*). Thus, we also fitted the ACF model with a common parameter α_x. However, these two modified ACF models[3] were ignored due to the large fluctuations in the standardized residuals (see later) and were not used in predicting mortality rates.

Descriptive statistics

Figure 1 illustrates life expectancy at birth[4] for men and women in Hungary between 1960 and 2022. It shows that the difference between the two sexes was smallest in the early 1960s, when men's life expectancy at birth was only 4–5 years lower than that of women. Thereafter, the gap gradually widened, reaching a peak in the first half of the 1990s. The gap was widest between 1992 and 1995, when men's life expectancy at birth was 9.3 to 9.5 years lower than that of women. Since the second half of the 1990s, the gap has narrowed and the difference between male and female life expectancy at birth is 6–7 years today. The favorable life expectancy gap of the early 1960s has not been reached again during the past 60 years. However, from 1960 to the present, life expectancy has increased by years for both sexes. In 2022, life expectancy at birth was 72.7 years for men and 79.5 years for women. In recent decades, the two sexes have followed different paths: the increase in life expectancy at birth for men has not been continuous, unlike that for women. Life expectancy for men declined between 1960 and 1993 and was at its lowest in 1992–1993 (64.6 years). The years 2020–2021 are important because of the COVID-19 epidemic, the effects of which were still being felt in 2022. However, the negative effect of the epidemic on mortality was probably temporary.

Figure 2 shows the logarithm of the central mortality rates for men and women over the last 63 years. The gradual downward shift of the curve indicates an improvement in mortality. Again, however, it is important to highlight two periods: the early 1990s and the early 2000s. In the first half of the 1990s, partly as a result of the change of political and economic regime, mortality rates increased, especially for middle-aged people (especially men), as can be seen in *Figure 2*. The loss of life years among middle-aged women was compensated by improvements in mortality among the young and elderly, thus life expectancy for women, unlike for men, did

[3] The fitted parameters of the two models are presented in the online Appendix.
[4] In calculating the life tables, we basically followed the methodology of *Chiang (1968)* and Eurostat (see https://ec.europa.eu/eurostat/cache/metadata/Annexes/demo_mor_esms_an1.pdf). We chose 0.2 as the average fraction of the year lived by an infant, and the fraction of the last year of life was 0.5 for all other cohorts. In addition, the group of people aged 90 and over was considered as the oldest cohort.

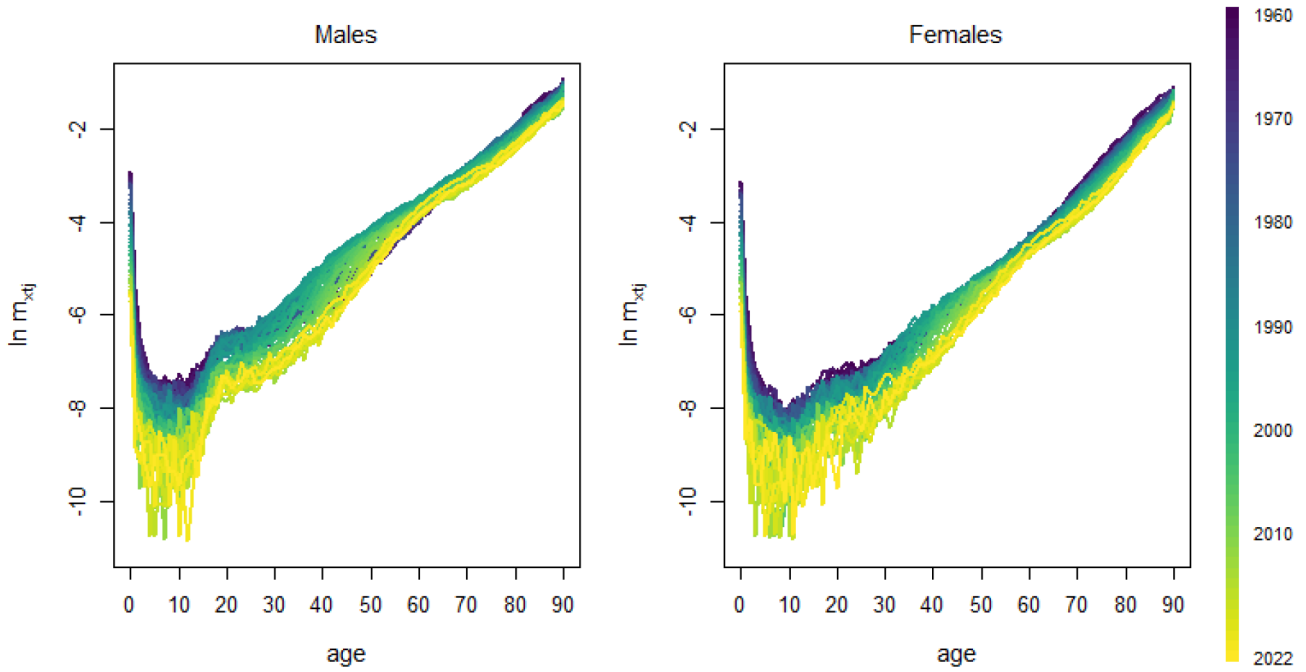

Figure 2 The logarithm of the central mortality rates between 1960 and 2022

Source: Own calculation

not decrease (see *Józan 2008* and *Bálint 2016*). In the early 1990s, the increase in mortality due to slowly progressive diseases was linked to the precariousness of life and to the increase in deaths from violent causes following the change of political and economic regime. The emergence and growing importance of slowly progressive diseases assumes a health-damaging lifestyle that had been in place for years or decades before the regime change, with alcohol consumption and smoking being highlighted as part of this. See *Józan (2008, 2012)*.

It should also be noted that, as a result of the COVID-19 epidemic, mortality was higher in older age groups in 2020–2021 than in 2019. Log mortality rates for women and men aged 57–59 and over were higher than in the last year before the epidemic. Comparing mortality at the beginning and the end of the observation period, there is a striking phenomenon among males. For men aged 52–66, the logarithm of the mortality rates observed in 2022 was higher than the level observed in the early 1960s. For younger age groups, there has been a clear improvement in mortality for both sexes over recent decades.

Results and discussion

The fitted parameters of the mortality models

Figures 3 and *4* show the values of the parameters of the mortality models fitted to data from 1960 to 2022. If one would like to examine mortality for men and women without considering national characteristics (trend and age effects), then one can rely on the results of the Lee–Carter model. The ACF and ACF with three factors are multi-population models that include common age effects and trends (for men and women together) in addition to sex-specific characteristics. For the multi-population models, the first step was to determine the common parameters that could be calculated from the national data, and then to estimate the sex-specific parameters using the common parameters determined in the first step.

The value of α_{xj} for all three models differs only slightly from the average of the log mortality rates over the base period, and is approximately the same. The trend in mortality change is described by the mortality indices $\kappa_t^{(1)}{}_{(j)}$, $\kappa_t^{(2)}{}_j$ and $\kappa_t^{(3)}{}_j$ for the Lee–Carter, ACF and ACF with three factors models. It can be seen that the additional factors modify the evolution of the first mortality index. In parallel, the shape of the age pattern(s) change(s). For almost all mortality indices, the negative impact of the COVID-19 epidemic on mortality is clearly visible, and it is also observed that the rate of improvement in mortality has already slowed down before 2020. Considering the ACF and ACF with three factors models, the values of the additional indices $\kappa_t^{(2)}{}_j$ and $\kappa_t^{(3)}{}_j$ were found to be predictable for both sexes. Further figures for the fitted parameters of the models are available in the online Appendix.

In order to develop a rotated version of the three models for forecasting purposes, it was necessary to fit the models to shorter base periods. The shortest base period was from 1960 to 2010 and the longest was from 1960 to 2022. For the Lee–Carter model, we found that

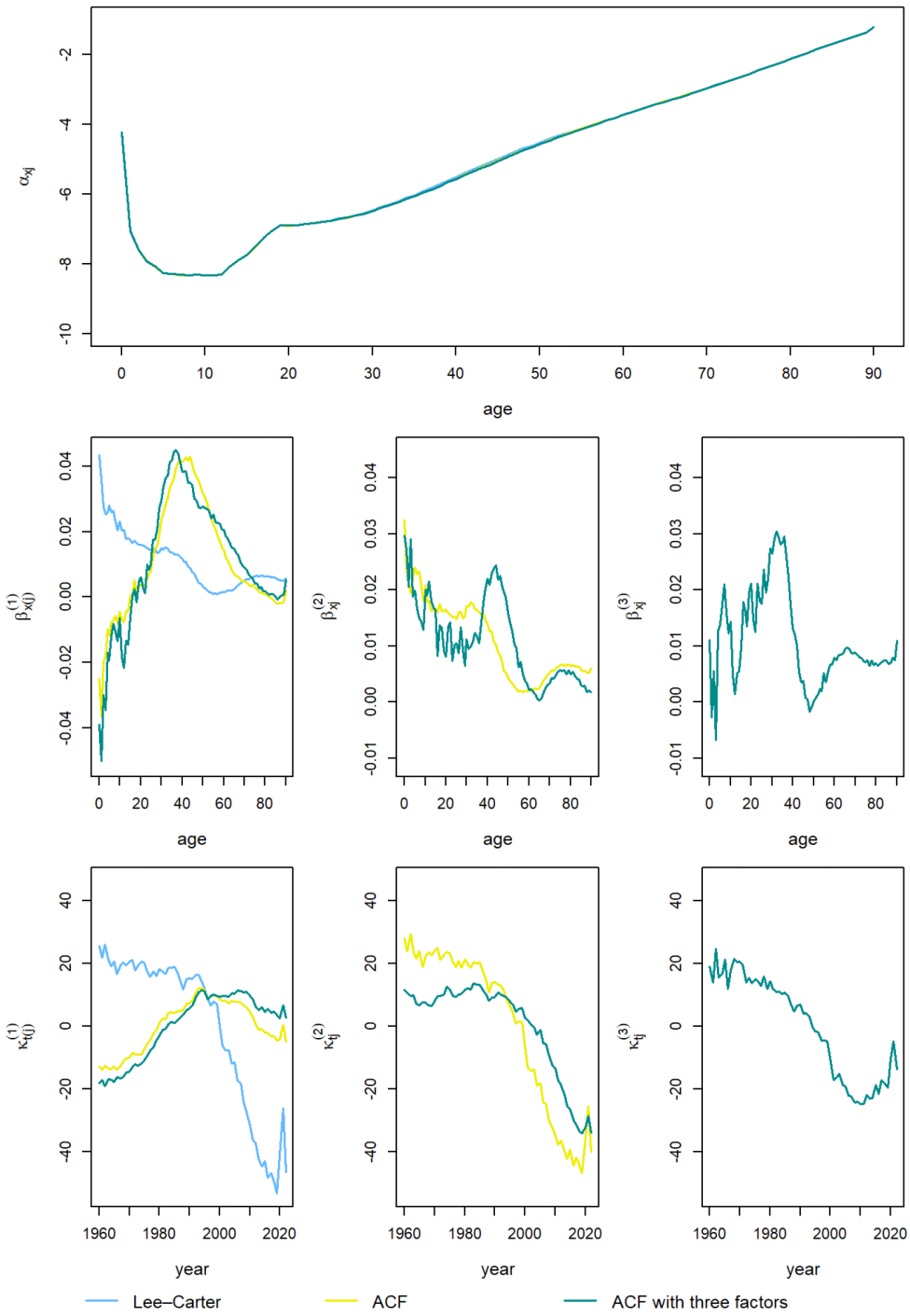

Figure 3 The fitted parameters of the mortality models for males

Source: Own calculation

there was considerable uncertainty in the choice of period length for parameter estimation in the case of males (if the model converged at all). This was not the case for the multi-factor models. Therefore, the rotated version of the Lee–Carter model for men was not used in this research. *Figure 5* shows the change in the parameter $\beta_x^{(1)}$ of the Lee–Carter model fitted jointly to the male and female data for different base periods. Further figures of the sex-specific changes in parameters α_{xj} and $\beta_{x(j)}^{(i)}$ of the other mortality models are available in the online Appendix. The graphs show that the change in the age-dependent parameters has slowed down over time.

Figure 5 confirms that the mortality improvement slows down at the youngest ages, as demonstrated by the decreasing β_x values. The same is true for the oldest ages, but this is the opposite of what would be expected from the phenomenon described as rotation. *Józan*

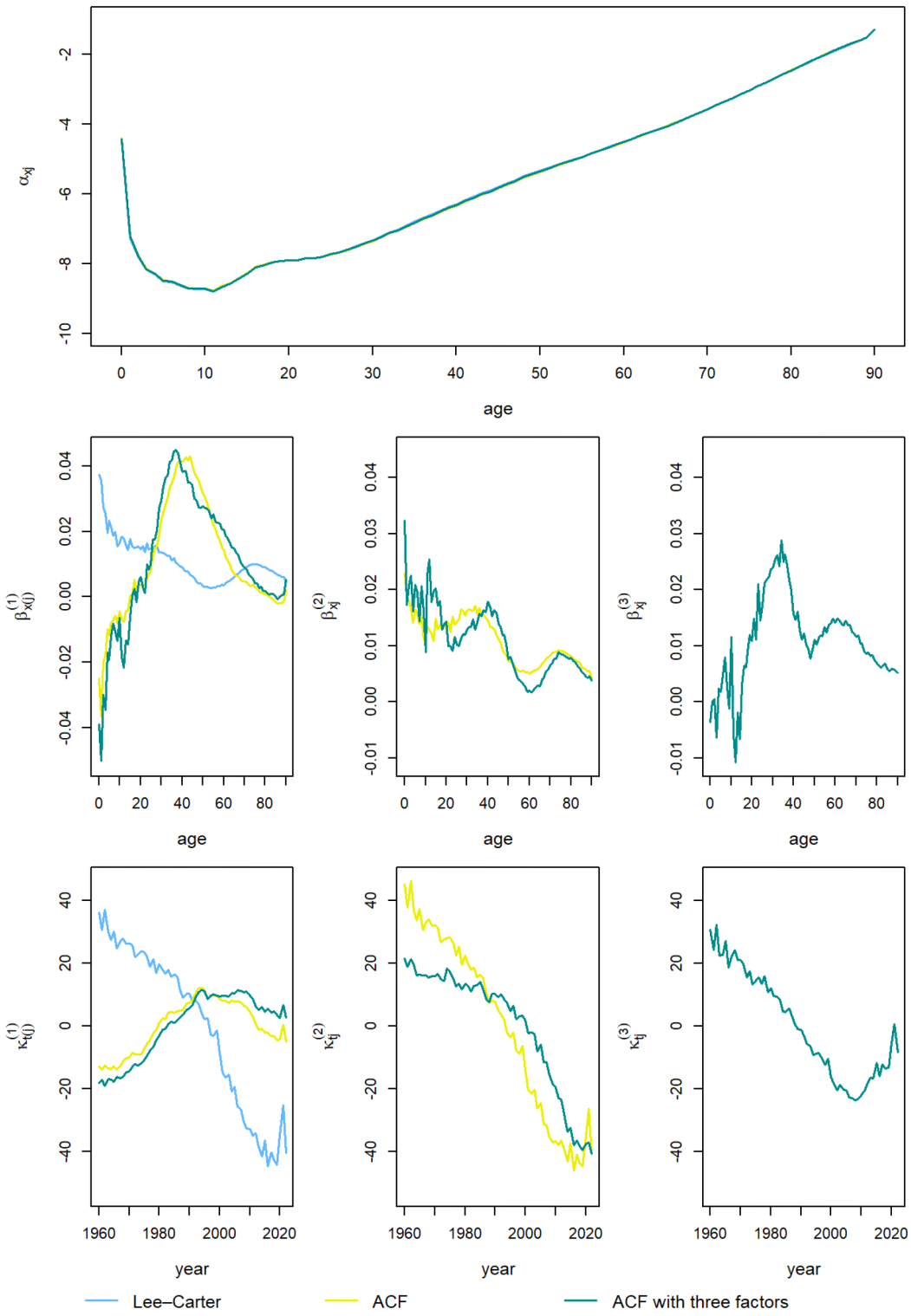

Figure 4 The fitted parameters of the mortality models for females

Source: Own calculation

(2008) also expected an acceleration of mortality improvement in Hungary in the 21st century for the oldest ages. However, taking into account the data of recent years, it can still be observed that the mortality improvement of the middle-aged is lagging behind and that this age group can still make a significant contribution to the increase in life expectancy in the short and/or medium term. As shown in *Figure 5*, an improvement in the mortality of the oldest age group can still be expected in Hungary.

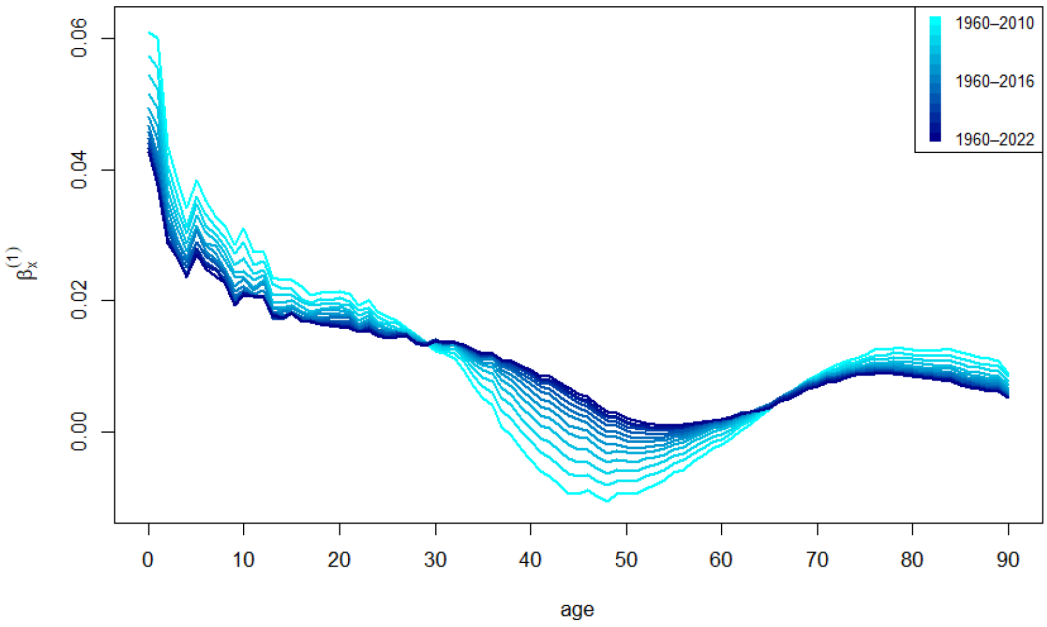

Figure 5 The shape of the age effect in different time periods for both sexes combined based on the Lee–Carter model

Source: Own calculation

The goodness of fit

There are several methods of in-sample analysis that can facilitate the choice between models. These include comparing the information criteria of the models, i.e., AIC and BIC (Akaike Information Criterion and Bayesian Information Criterion). These can be calculated as follows (see, e.g., *Villegas–Kaishev–Millossovich 2018*):

$$AIC = 2k - 2L, \tag{13}$$

$$BIC = k\,ln(n) - 2L \tag{14}$$

where k indicates the number of parameters, n refers to the number of observations, and L is the maximum value of the log-likelihood. The smaller the AIC or the BIC value, the better the model. Both the AIC and BIC values confirm that, for both men and women, the ACF model with three factors has the best goodness of fit among the three mortality models, while the Lee–Carter model has the worst (see *Table 2*).

In the present research, fitted mortality models can be considered as nested models where the more general model is the ACF with three factors with the most parameters. According to *Plat (2009)*, the likelihood ratio (LR) test is more appropriate for nested models for model selection than information criteria. For the LR test, the null hypothesis is that the nested model is the correct model against the more general model. The LR test statistic is calculated as:

$$\xi_{LR} = 2\,(L_{general} - L_{nested}) \tag{15}$$

where $L_{general}$ is the maximum log-likelihood value of the general model and L_{nested} is the log-likelihood of the nested model. Under the null hypothesis, ξ_{LR} has a χ^2 distribution with J degrees of freedom. J is the number of the additional estimated parameters in the general model in comparison with the nested model. The null hypothesis can be rejected if:

$$\xi_{LR} > \chi^2_{J,\alpha} \tag{16}$$

where α denotes the significance level. *Table 3* summarizes the results of the LR tests: the general model always outperforms the nested model.

Another way to compare different mortality models is to analyze the standardized residuals. The magnitude and heatmap of the residuals can be used to determine whether additional factors need to be included. The more random the order of the residuals, the better the goodness of fit of a model. The value of the standardized residuals can be calculated as follows (see, e.g., *Wen et al. 2021*):

$$Z_{xtj} = \frac{D_{xtj} - E^c_{xtj}\hat{m}_{xtj}}{\sqrt{E^c_{xtj}\hat{m}_{xtj}}} \tag{17}$$

where D_{xtj} is the number of deaths, E^c_{xtj} denotes the central exposure, and $\hat{m}_{xtj}$ is the estimated central mortality rate at age x, in year t, and in population j. The heatmaps and scatter plots show the residuals plotted against age, calendar year, and year of birth (see the online

Appendix).[5] An analysis of the magnitude and arrangement of the standardized residuals leads to the same conclusions as the AIC and BIC values. The ACF model with three factors seems to be the most appropriate of the three models, but the diagonal pattern suggests that the change (improvement) in mortality is not really independent of the year of birth. The cohort effect is not included in any of the three fitted models.

Table 2 | The mortality models' goodness of fit, 1960–2022

Sexes	Measures	Lee–Carter	ACF	ACF with three factors
Males	AIC	35,773,942	35,695,354	**35,690,129**
	BIC	35,775,559	35,697,982	**35,693,769**
Females	AIC	32,918,361	32,911,775	**32,907,238**
	BIC	32,919,978	32,914,403	**32,910,877**

Note: The most favorable values are highlighted in bold.

Source: Own calculation

Table 3 | The results of the likelihood ratio tests, 1960–2022

Null hypothesis	Likelihood ratio test statistic		Degrees of freedom	χ^2_j,α
	Males	Females		
Lee–Carter model against ACF	78,892	6,892	153	125.4
Lee–Carter model against ACF with three factors	84,420	11,732	307	267.4
ACF against ACF with three factors	5,528	4,840	153	125.4

Note: $\alpha = 0.05$

Source: Own calculation

The future life expectancy at birth

In order to forecast age-specific mortality rates up to 2050, it was necessary to forecast mortality indices, for which ARIMA models were chosen. In order to select appropriate ARIMA (p,d,q) models, we tested for the presence of unit roots[6] in the time series, and checked the autocorrelation and normality of the residuals[7] of the mortality models. On the basis of the unit root tests, we concluded that a first differencing was definitely necessary. To determine the parameters p and q, we considered the values 0, 1 and 2. In order to compare the different ARIMA models, we also calculated the AIC and BIC values,[8] which can also help in the selection process. The decision was also influenced by the width of the prediction interval. The statistics of the possible ARIMA models for the mortality indices are available in the online Appendix. *Table 4* shows which ARIMA models were chosen for the projection. In the projection, the values for the years 2020–2021 were treated as outliers and filtered out using a dummy variable. The figures in the online Appendix show the predicted values of the mortality indices and their 80% and 95% prediction intervals.

Table 4 | ARIMA models of the mortality indices

The mortality models	$\kappa_t^{(1)}{}_{(j)}$	$\kappa_t^{(2)}{}_j$	$\kappa_t^{(3)}{}_j$
Lee–Carter	ARIMA(0,1,1) with drift	–	–
ACF	ARIMA(0,1,0) with drift	ARIMA(0,1,1) with drift	–
ACF with three factors	ARIMA(0,1,0) with drift	ARIMA(0,1,1) with drift	ARIMA(0,1,1) with drift

Note: The ARIMA models do not differ by sex.

Source: Own editing

To forecast the rotated versions of the three mortality models, it was also necessary to predict the age-varying parameters, which were determined as described above (see Equations 8 and 9). For the rotation of the age-varying parameters, the changes observed since 2015 were used. However, we cannot assume that recent trends will continue in the long term, thus this may be a weakness of the rotation method we used. Since two of the fitted models (the ACF and the ACF with three factors) have multiple β_x coefficients, it was possible to rotate several parameters. We first considered the case where all age patterns in all models were rotated. This approach proved to be inappropriate, especially in the case of three factors because life expectancy was not increasing in the long run, but rather decreasing. Finally, we adopted the solution of rotating only $\beta_x^{(1)}{}_{(j)}$ in addition to α_{xj}.

Using the selected ARIMA models and rotated age-dependent parameters, we projected age-specific mortality rates up to 2050, and we calculated life expectancy for men and women. *Figure 6* shows the results of different rotated and nonrotated model variants. Life expec-

[5] We applied the *StMoMo* package of *Villegas–Kaishev–Millossovich (2018)* to prepare these figures.
[6] For this, we used augmented Dickey–Fuller (ADF) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) unit root tests.
[7] The following tests were used: Ljung–Box Q-statistic, Breusch–Godfrey test, and Shapiro–Wilk test.
[8] These can be calculated as follows: $AIC = T\ ln(sum\ of\ squared\ residuals) + 2n$ and $BIC = T\ ln(sum\ of\ squared\ residuals) + n\ ln(T)$ where n is the number of estimated parameters (p + q + possible constant term), and T is the number of usable observations (see, e.g., *Enders 2014*).

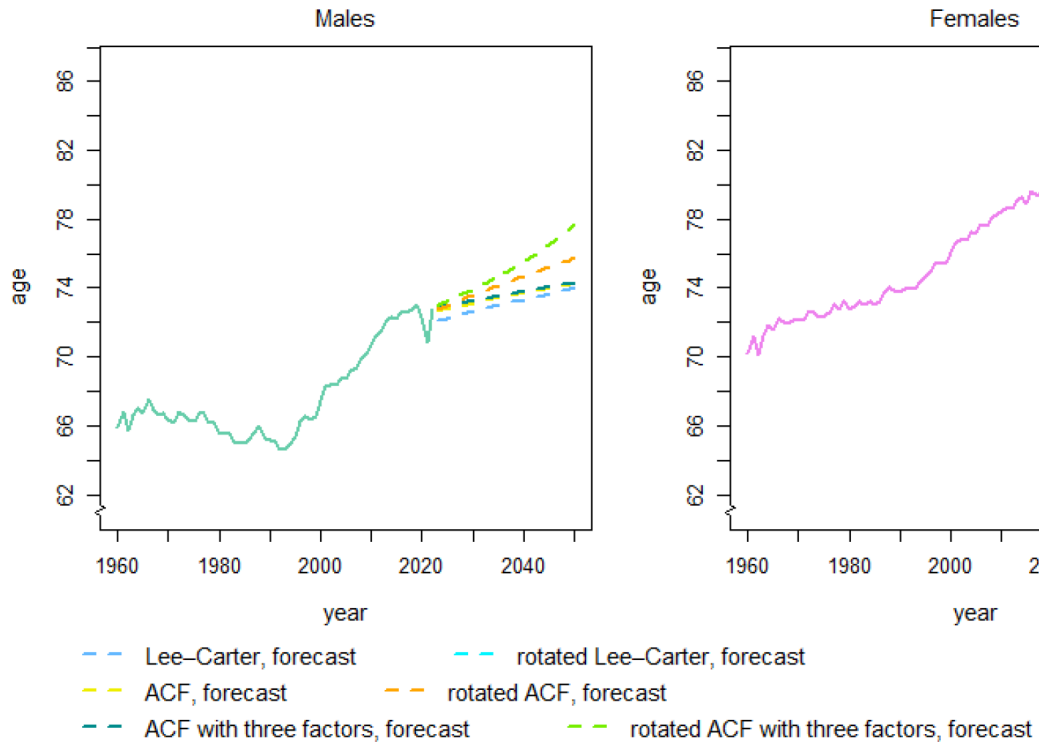

Figure 6 Life expectancy at birth for males and females between 1960 and 2050

Source: Own calculation

tancy at birth is also summarized in the online Appendix. It was found that the rotated models can predict higher life expectancy from the first year of projection than the nonrotated models. Among the models, the nonrotated Lee–Carter model is the most pessimistic for both men and women, predicting 74 and 81.8 years respectively in 2050. According to this model, the gap between the two sexes in life expectancy at birth will widen. The nonrotated ACF model and the nonrotated ACF model with three factors forecast very similar life expectancies. These are 74.3 years for men and 82.6 and 82.2 years for women in 2050. These models also predict a greater difference between the two sexes than is typical today. Of the rotated models, the three-factor ACF model is the most optimistic, predicting 77.7 years for men and 86.3 years for women in 2050. According to this model, the largest difference in life expectancy between the two sexes is expected in the last year of the projection.

One important question regarding rotation is which age groups contribute more to the increase in life expectancy in the long run. On the basis of *Figure 7*, it could be predicted that the rotated Lee–Carter model would improve mortality in middle age to a greater extent. *Figure 7* shows the log mortality rates in 2050 based on the different models for men and women separately. As the rotated Lee–Carter model was only fitted for women, conclusions could be drawn based on this projection. It can be observed that the rotated Lee–Carter model does indeed lead to improvements in the mortality of middle-aged people compared to the nonrotated version, and that this leads to a significant improvement by 2050. However, for the youngest and oldest age groups, mortality is higher for the rotated Lee–Carter model than for the nonrotated version. Our rotation method can only be a short-term solution for projections based on the Lee–Carter model.

The rotated ACF model improves mortality in almost all age groups except those aged 60 and over. Thus, almost all age groups will contribute to an increased life expectancy in 2050 based on the rotated ACF model compared to the nonrotated version. For the three-factor ACF model, we can observe what we originally expected from the rotation. In the youngest age groups, there is no significant difference between the rotated and nonrotated versions, while all other age groups show a more favorable projection, especially the group aged 90 and over. According to this model, life expectancy will be higher in the long run until 2050 at an increasingly faster rate (see *Figure 6*).

Coherence in multi-population models

If mortality models without common parameters (e.g., the Lee–Carter model) are fitted separately by subpopulation, this may lead to divergent mortality rates between groups in the projection. For groups with similar socioeconomic backgrounds, it is generally assumed that observed past mortality differences between them, which are also due to biological endowments, will not change

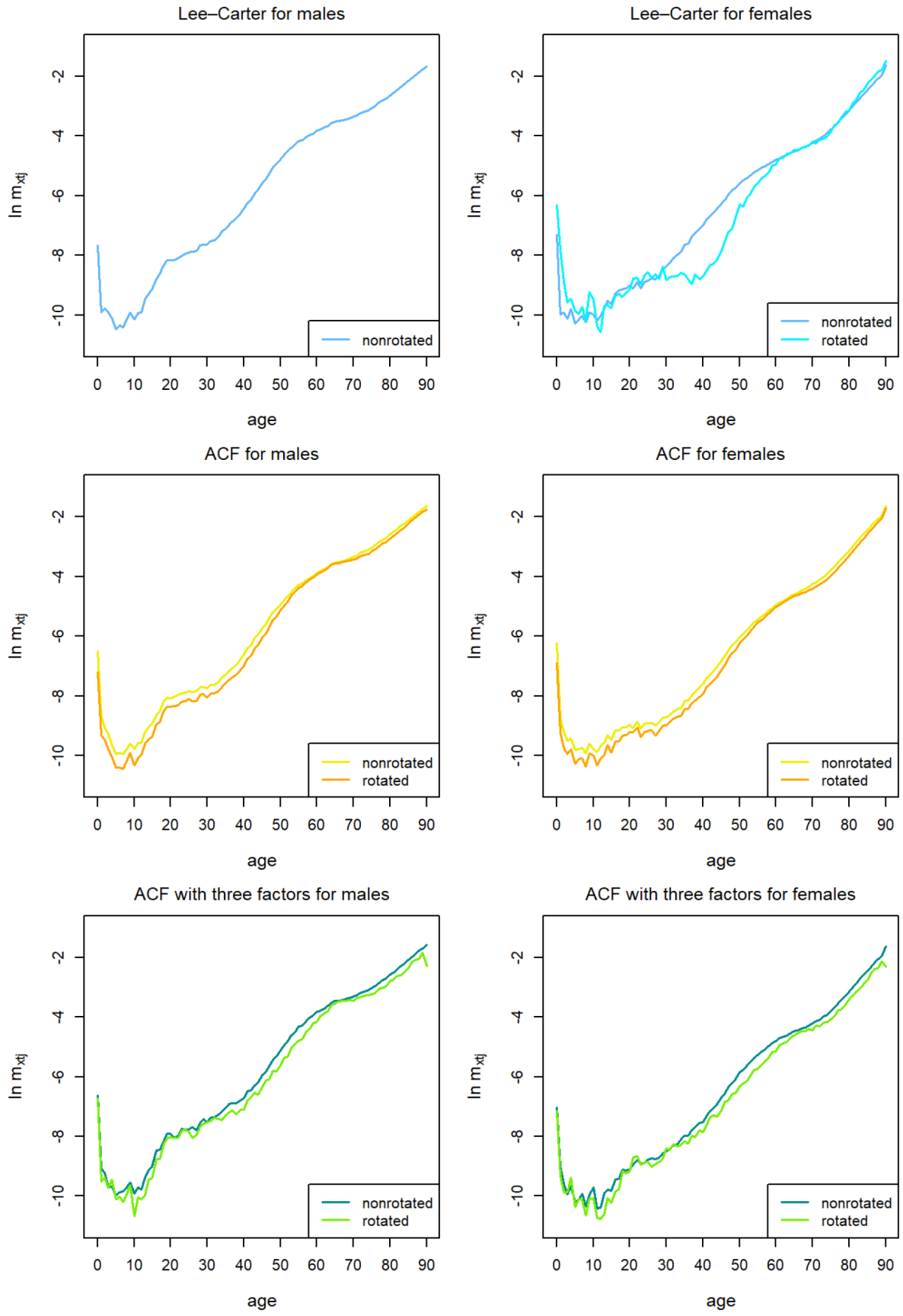

Figure 7 The logarithm of the central mortality rates in 2050 based on different mortality models

Source: Own calculation

in the future. For example, in developed countries, infant mortality rates for women are usually lower than those for men (this is also the case in Hungary), and this trend is expected to continue in the projection. However, predicting mortality rates for subpopulations independently may lead to a crossover effect. If it is true that a multi-population mortality model can be used to avoid the crossover effect, then the model is coherent in the sense described by *Li–Lee (2005)*.

By comparing the projected age-specific mortality rates of the subpopulations, it is possible to analyze how the mortality difference varies over the projection period and whether it remains constant. To investigate this, we calculated the ratio of the mortality rates for each age group and for each year of the projection: female mortality rates were divided by male mortality rates. The values of the female-to-male ratios are available in the online Appendix. From this perspective, it could be concluded that infant mortality is higher for women than for men throughout the projection period according to both the Lee–Carter and the ACF models. However, this seems unrealistic. Of the two multi-population models, the ACF model with three factors performs well in this respect, and the same was found for the oldest age group (those aged 90 and over): female mortality does not increase relative to male mortality. This is what we would expect on the basis of the current data. Looking at the future evolution of the female-to-male ratio, the Lee–Carter model shows less change in the ratio in the long run. For the two multi-population models there are several age groups where the ratio does not remain constant.

Conclusions

In this study, we fitted three mortality models to Hungarian data from 1960 to 2022: the Lee–Carter model and its two multi-population variants, the two-factor and three-factor ACF models. The objective was to model mortality for men and women jointly and to take into account the time dependence of the age-varying coefficients of the mortality improvement trend in the forecast. In the literature, this phenomenon is known as rotation, whereby mortality improvement is slowing down in the youngest age groups and accelerating in the oldest age groups in developed countries. In Hungary, it is reasonable to assume that the reserves of decreasing mortality of infants and young age groups are slowly diminishing, and that the period of faster improvement in the life expectancy of the elderly is approaching, i.e., that their life expectancy will be significantly extended. Modelling the future evolution of life expectancy is an important issue for policy makers, as pension systems are challenged by population ageing and longevity risk.

Rotation is usually associated with the Lee–Carter model. The time dependence of the age pattern of the Lee–Carter model does not yet confirm the need for rotation in short-term projections. However, we have constructed rotated versions of selected mortality models and projected age-specific mortality rates up to 2050. We found that among the nonrotated models, the three-factor ACF model had the best fit to the data, and the female-to-male ratios of infant mortality rates and old-age mortality rates were found to be appropriate, which is not the case for the other two models in this study. With the three-factor model, we have succeeded in developing a rotation version that reflects what is expected from rotation in the long run: the oldest age groups contribute more to the increase in life expectancy in the future than the youngest ones. However, the rotated three-factor ACF model predicts an increasingly rapid improvement in life expectancy in the long run, suggesting caution in forecasting.

In the case of the Lee–Carter model, the rotation method we used may only be suitable in the short term: it leads to an increasing improvement in mortality in middle age, which is unsustainable in the long term. It was also seen that in the case of men, fitting the Lee–Carter model to different base periods is not stable in terms of the evolution of the age pattern. *Li–Lee–Gerland (2013)* address the rotation of a single age pattern. In our study, we constructed rotated versions of multi-factor mortality models, and the baseline shape of mortality is also a rotated term in our models. The latter may be a suitable alternative to the *Lee–Miller (2001)* method to avoid jump-off bias. In multi-factor mortality models, the age-varying coefficients of the mortality indices are more volatile, so that the estimation of their year-to-year change is subject to uncertainty. It is important to account for rotation in projections, as the rate of improvement in mortality does indeed vary between different age groups. Further research should be devoted to constructing rotated versions of other multi-population models and to developing a method of rotation that can be applied in both the short and long term.

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Online Appendix

https://github.com/LiviaVarga/Rotation-of-the-age-varying-parameters-in-multi-population-mortality-models