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An aspirational perspective on the negative risk-return relationship

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ABSTRACT

The existence of a negative risk-return relationship challenges the conventional wisdom of finance, which typically assumes a positive correlation between risk and return. Reference-dependent preferences, motivated by prospect theory, offer a possible explanation for this negative risk-return relation. However, as we demonstrate in this paper prospect theory does not provide a general explanation for this puzzle. We show that the expected utility theory with an aspiration level can effectively account for this phenomenon.

1. Introduction

A fundamental principle in finance is the positive risk-return relationship. Investments with higher levels of risk are generally expected to yield higher returns than those with lower risk. The key assumption in most asset pricing models that underlies this principle is a utility function of the decision makers that is globally concave, which, as a consequence, implies a risk-averse behavior for investors. However, empirical evidence challenges this conventional view and suggests that an opposite phenomenon might actually occur in certain situations. This means that high-risk investments may not necessarily yield higher average returns, and low-risk investments could outperform high-risk ones over time. For instance, a study conducted by Ang et al. (2006) examined high-risk stocks, as measured by their idiosyncratic volatility, and found that they earned very low average returns. Additionally, Baker et al. (2011) reported that low-volatility or low-risk stocks consistently outperformed high-volatility and high-risk stocks. Even when using risk measures that do not directly rely on returns, similar conclusions emerge. Diether et al. (2002) found that the dispersion of analysts' opinions, a risk measure, negatively predicted future returns. Similarly, the cash-flow volatility or age, used as proxies for risk (Zhang, 2006), were also found to negatively predict future returns. While the majority of the existing literature has primarily focused on security markets, empirical findings by Bowman (1980) suggests a negative correlation between business risk and return across companies within many industries.

The empirical evidence presented poses a challenge to traditional finance theories, specifically the Capital Asset Pricing Model (CAPM). The CAPM suggests a positive relationship between risk and return. The existence of this challenge is significant for understanding financial markets, as it contradicts the fundamental principles of risk compensation. In conventional financial thinking, investors are anticipated to receive rewards for assuming additional risk.

Bhootra and Hur (2015) suggest that the negative risk-return relationship is only observed in stocks with unrealized capital losses and not present for capital gains stocks. Similarly, Wang et al. (2017) present evidence that the negative risk-return relationship is much more pronounced among stocks with a low capital gains overhang, and the relationship is positive among stocks in which investors face prior gains.

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With the assumption of concave utility functions of expected utility theory, where investors are uniformly risk-averse, it is difficult to provide a compelling explanation for these empirical findings. One might argue that this anomaly may simply be the consequence of an irrational preference towards risky stocks of the investors due to some behavioral biases. For example, overconfidence bias (see Daniel et al., 1998 or Barber and Odean, 2001) or high sentiment (see Duxbury and Wang, 2023) might lead to an increase in the demand for high-risk stocks causing these stocks to be overpriced and as a consequence to have a low expected return. This reasoning may partially explain the negative risk-return relationship, however, it is hard to believe that this bias-induced irrationality is present across the board, including institutional investors as well. A telling argument against this is that institutional investors, who supposedly are not as susceptible to behavioral biases as individual investors, could and would take advantage of the price impact of the irrational demand for risk of individual investors. Yet, as Baker et al. (2011) argues, institutional investors' mandate to outperform some fixed benchmarks limits the arbitrage activity in certain stocks, especially low-beta and low-volatility stocks, therefore, the low-risk anomaly is likely to persist.

Behavioral models based on prospect theory, introduced by Kahneman and Tversky (1979), have been proposed to explain the observed anomaly of the negative risk-return relationship. According to prospect theory's S-shaped utility function, decision-makers are assumed to exhibit risk-seeking behavior when faced with losses relative to a reference point and risk-averse behavior when they have prior gains compared to the reference point. This suggests that decision-makers may display both risk-seeking and risk-averse tendencies in different regions, depending on whether they are experiencing losses or gains. This contrasting attitude towards risk could potentially explain the negative risk-return relationship. Specifically, investors who have unrealized losses may exhibit risk-seeking behavior, leading them to prefer stocks with high idiosyncratic volatility. Consequently, stocks with unrealized capital losses could become overvalued. On the other hand, stocks with unrealized capital gains might be undervalued due to the preference for risk-averse choices, providing a rationale for the negative risk-return relationship among stocks with unrealized losses but not among those with unrealized gains (as discussed in Bhootra and Hur, 2015 or Wang et al., 2017). While this explanation appears compelling, it may not hold up well in dynamic settings, as acknowledged by Wang et al. (2017). Additionally, even in static cases, the results may not be consistent, as this article will demonstrate.

The various explanations put forth in these studies rely on prospect theory as their foundational framework. However, a noteworthy limitation of these explanations is that they assume payoffs to be either strictly positive or strictly negative. This oversimplified assumption might not fully capture the complexity of financial decision-making scenarios, where both positive and negative prospects can coexist. For instance, consider a high-risk investment with the potential for significant returns; it involves both the possibility of a substantial gain and the risk of a considerable loss.

When confronted with such mixed prospects, it becomes evident that existing models based on prospect theory encounter challenges in effectively explaining observed behavior. To accommodate the empirical data, these models often require unrealistic restrictions to be imposed on their parameters, artificially constraining the representation of decision-making processes. Alternatively, if these restrictions are not applied, the models fail to deliver the predictions one would expect based on real-world evidence. This limitation indicates that while prospect theory can serve as a motivation for developing models that address specific empirical results, it might not be a fully normative framework for decision-making in scenarios involving mixed prospects.

2. Prospect theory and the negative risk-return puzzle

To see this more clearly, consider the following simple case. Let $s_1, \dots, s_n$ be n mutually exclusive events with associated probabilities $p_1, \dots, p_n$ with $\sum_{i=1}^{n} p_i = 1$. Assume that the (monetary) consequence of s_i is x_i for $i = 1, \dots, n$. Thus, a lottery $L(x, p)$ can be defined as a pair of vectors x and p.

In line with the main assumptions of prospect theory, let us assume that the decision-maker evaluates the outcomes in a reference-dependent manner. Namely, the payoffs of different outcomes are evaluated relative to a reference point (x_r), that is, instead of x_i, the decision maker considers $\tilde{x}_i \equiv x_i - x_r$ as the prospects' payoffs. Furthermore, to differentiate between losses and gains, it is assumed that the strictly increasing value function $v(x)$ based on which the decision maker evaluates the outcomes is convex in losses and concave in gains. Lastly, loss aversion is captured by the difference in the steepness of the value function around the reference point, being steeper for losses than for gains. An often-used value function that satisfies these assumptions is

$$v(x) = \begin{cases} \tilde{x}^{\alpha} & \text{if} \quad \tilde{x} \geq 0 \\ -\lambda \tilde{x}^{\beta} & \text{if} \quad \tilde{x} < 0 \end{cases} \tag{1}$$

where $\alpha, \beta > 0$ and $\lambda > 1$. According to the empirical results $\alpha = \beta = 0.88$ and $\lambda = 2.25$ (see Tversky and Kahneman, 1992).

Regarding the probability weighting property of prospect theory, an overvaluation of small probabilities and an undervaluation of large probabilities is assumed to be prevalent in decision-making. Formally, it is assumed that $\pi(p) > p$ for small values of p and $\pi(p) < p$ for large values of p, while $\pi(0) = 0$ and $\pi(1) = 1$. One common example of a probability weighting function in prospect theory is the $\pi(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}$, with $\gamma = 0.61$ for gains and $\gamma = 0.69$ for losses according to empirical estimates (see Wu and Gonzales, 1996 or Prelec, 1998).

Now, let us consider a simple example in which to facilitate a clear explanation, we consider only two states of the world, denoted as s_1 and s_2, with associated monetary outcomes of x_1 and x_2, respectively. We will refer to s_1 as a *bad* state, representing an unfortunate outcome, and s_2 as a *good* state, representing a favorable outcome.

In this example, we will concentrate on two crucial components of prospect theory, namely, reference dependence and gain–loss asymmetry. For simplicity and clarity, we will set aside the complexities that may arise from the probability weighting aspects of the theory. Specifically, we will assume that $\pi(p_1) = p_1$ and $\pi(p_2) = p_2$. This assumption, however, does not weaken our argument in any way; in fact, if we were to incorporate the probability weighting property of prospect theory, it would only further strengthen our conclusions. Nevertheless, for the sake of a focused examination and to avoid unnecessary complexity, we will exclude this aspect from our analysis.

Finally, in line with prospect theory, we assume that the utility of a lottery is calculated as

$$\sum_i \pi(p_i)v(\tilde{x}_i). \tag{2}$$

To keep things simple, assume that $v(\cdot)$ is a money metric value function, $\lambda = 1$ (no loss-aversion present), and $\alpha = \beta = 0.88$ as suggested by the empirical research mentioned above.[1]

To demonstrate our main point, consider the following simple example. Let $\tilde{x}_1 = 2$ occur with probability 0.9 and $\tilde{x}_2 = -10$ with 0.1. According to prospect theory, the utility of this lottery is 0.9. Another lottery with higher volatility and the same probabilities that would produce the same expected payoff is $\tilde{x}_1 = 3$ and $\tilde{x}_2 = -19$. Prospect theory suggests that this lottery yields a value level of 1.03. That is, increasing the volatility in the gain domain increases the value of the lottery contrary to what one would expect according to the theory and the empirical evidence. Now, consider another example with $\tilde{x}_1 = -2$ and $\tilde{x}_2 = 10$ with $p_1 = 0.9$ and $p_2 = 0.1$. This lottery leads to a value level of -0.9, while the lottery with the same probabilities but with an increased volatility of $\tilde{x}_1 = -3$ and $\tilde{x}_2 = 19$ yields -1.03. Again, this example suggests the opposite of what one might expect (see the following table).

	Loss domain		Gain domain	
	Low volatility	High volatility	Low volatility	High volatility
$p = 0.1,\ x_1$	10	19	−10	−19
$p = 0.9,\ x_2$	−2	−3	2	3
Expected value	−0.8	−0.8	0.8	0.8
Value of the prospect	−0.9	−1.03	0.9	1.03

These examples suggest that the increased volatility in the loss domain decreases the value of a lottery, while in the gain domain results in higher utility. This contradicts the predictions of prospect theory and goes against the empirical findings documented in both Bhootra and Hur (2015) and Wang et al. (2017), that suggest that greater volatility in the loss (gain) domain corresponds to an increase (decrease) in the value of a stock. It is worth noting, though, that both Bhootra and Hur (2015) and Wang et al. (2017) are built upon the particularly strict assumption that all outcomes under consideration are either positive or negative. The aforementioned example serves as evidence of the limitations associated with this assumption in their studies. Consequently, a more nuanced approach should be adopted to account for situations where outcomes can differ from one another in this regard.

3. Expected utility with an aspiration level

In the following, we present a simple model that offers an alternative explanation for the negative risk-return puzzle. Our approach is based on the standard expected utility theory by von Neumann and Morgenstern (1944) combined with the assumption that aspiration levels play an important role in decision-making.[2]

Building upon the works of Lopes and Oden (1999), Diecidue and van de Ven (2008) and Rieger (2010), we assume that investors have specific aspiration levels for their payoffs, considering payoffs above this level as successes and those below as failures.[3] A key aspect of our model is that falling short of their aspiration level has a negative impact on investors' utility. This could be because payoffs below the aspiration level are considered failures, leading to feelings of regret over their decisions or even the potential humiliation they might experience if their performance becomes widely known.[4]

Specifically, let us assume that the decision-maker is presented with a risky choice, which is represented by a lottery, as discussed in the previous section. According to the expected utility theory, her overall utility is equal to the sum of the utilities of each outcome weighted by the probabilities of the respective outcomes. This is expressed as $\sum_{i=1}^{n} p_i u(x_i)$, where x_i (for $i = 1, \dots, n$) is an outcome that occurs with probability p_i, and $u(\cdot)$ is a real-valued function defined on the set of outcomes that maps the outcomes to the utility derived from the total wealth resulting from the wealth change induced by x_i. We assume that this utility is influenced by the existence of an aspiration level in a way that if the realized payoff is short of that level, the decision-maker perceives it as a loss that decreases her expected utility. More formally, we assume that the utility function of the decision-maker is given by

$$U(L) = \sum_{i=1}^{n} p_i u(x_i) - \lambda 1_{0,\bar{x}}(x_i) \tag{3}$$

[1] Note, that our argument is applicable to any set of values for α and β, even if they are not identical.

[2] Several other papers, such as those by Gebka (2019), Bakó and Neszveda (2020), and Kuhle (2022), argue that it is unnecessary to assume irrationality to explain certain "puzzling" financial phenomena.

[3] The pursuit of an aspiration level is a recurring theme in empirical studies. For example, Allen et al. (2017) found that marathon runners tend to establish an aspiration level they are determined to achieve, showing little concern for the degree of deviation once they fall short. Similarly, in finance, particularly in accounting, companies appear to establish an earnings aspiration level, going to great lengths to avoid falling short. This dynamic results in a significant discontinuity in the distribution of earnings at the aspiration level (Byzalov and Basu, 2019). Additionally, the introduction of an aspiration level has been proposed as a means to address the equity premium puzzle in asset pricing (Levy and Levy, 2009).

[4] For example, as Zeisberger (2022) demonstrated in a series of experiments, decision-makers appear to be willing to accept suboptimal choices in order to reduce the probability of encountering failures.

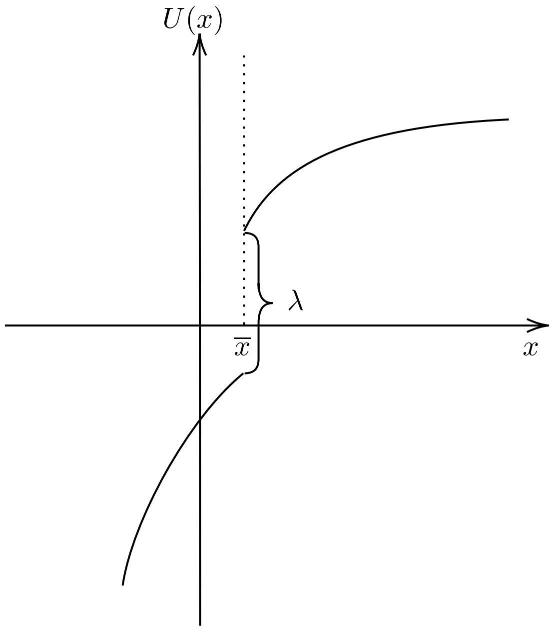

Fig. 1. Utility of lotteries with aspiration level $\bar{x}$.

where $\lambda > 0$ denotes the disutility she experiences if her payoff does not reach her aspiration level of $\bar{x}$, while $1_{0,\bar{x}}(x_i)$ is the indicator function that maps to 1 if $x_i < \bar{x}$ and to zero otherwise. Thus, the utility function $U(L)$ is discontinuous at the aspiration level, that is, a slight decrease in the outcome at this level results in a discrete drop in utility, as illustrated in Fig. 1.[5] This approach contrasts with prospect theory on several fronts. First, it significantly diverges from prospect theory's loss aversion, which magnifies all losses. In contrast, in this approach, there exists a singular substantial fixed loss at the aspiration level, without further penalization for additional losses beyond that point. Second, the decision-maker exhibits risk aversion both above and below the aspiration level when all outcomes fall above or below it, whereas prospect theory suggests an asymmetry in these scenarios. Third, unlike prospect theory, this approach does not incorporate probability weighting.

Applying this framework to the examples presented in the previous section and setting the decision maker's aspiration level to zero, the model suggests a behavior that aligns with the empirical findings in the literature. Moreover, as we will demonstrate in the next section, this model also provides a comprehensive explanation for the negative risk-return phenomenon in a broad and general context.[6]

4. The model

Consider a two-period model as, for example, in Bordalo et al. (2013). There is a mass of identical investors normalized to 1 who derive utility from consumption in periods 0 and 1. The utility of investors is represented by a positive, increasing utility function defined over consumption-vectors (c_0, c_1), where c_t denotes the consumption of period $t = 0, 1$.

In period 0 it is assumed that each investor receives an endowment of e_0 of the consumption good and one unit of each available asset, denoted by j, where $j = 1, \dots, J$. For the sake of tractability, units are defined such that the prices of each asset are equal to 1. It is assumed that in period 1, investors receive a payoff of x_j on asset j. The expected payoff of asset j is denoted by μ_j and the standard deviation of it is σ_j. For simplicity, we assume that there is no time discounting between the two periods.

The expected utility of an investor who engages in trading h_j amount of asset $j = 1, \dots, J$ at $t = 0$ is expressed as follows

$$u(c_0) + E(u(c_1)) - E(\lambda \sum_j (h_j + 1) 1_{(0,\bar{x}_j)}(x_j)) \tag{4}$$

where $u(\cdot)$ represents the investor's consumption utility; $h_j + 1$ denotes the amount of asset j held in period $t = 1$ after selling or buying h_j of it in period $t = 0$; $\lambda > 0$ signifies the disutility experienced by the investor if asset j with a payoff of x_j fails to reach the aspiration level $\bar{x}_j$; and finally $1_{(0,\bar{x}_j)}(x_j)$ is the indicator function, taking the value of 1 if $0 < x_j < \bar{x}_j$ and 0 otherwise. To

[5] For further details, see e.g. Levy and Levy (2009).

[6] Both prospect theory and expected utility theory with an aspiration level stand apart from other behavioral models that assume irrationality based on context-dependent preferences. This distinction includes works by e.g. Bordalo et al. (2013), Kőszegi and Szeidl (2013), Bakó et al. (2018), and Dezső et al. (2021).

elaborate, the first term reflects the investor's utility derived from consumption in the initial period. The second term accounts for the expected utility stemming from consumption in the subsequent period. The third term captures the expected disutility experienced by the investor due to investments falling short of their aspiration level. This is calculated as the sum of disutilities across all assets, considering the investor's specified disutility parameter λ and the indicator function based on the aspiration level $\bar{x}_j$ for each asset j.

The investor's problem at $t = 0$ can be given as

$$\max_{h_1,\ldots,h_J} u(c_0) + E(u(c_1)) - E(\lambda \sum_j (h_j + 1) 1_{(0,\bar{x}_j)}(x_j)) \tag{5}$$

subject to

$$\begin{cases} 0 \le c_0 \le e_0 - \sum_j h_j \\ 0 \le c_1 \le \sum_j (h_j + 1) x_j \end{cases}$$

Since the investor's expected utility function is increasing in both c_0 and c_1, it follows that both constraints will be strictly satisfied. Consequently, the problem can be reformulated as follows

$$\max_{h_1,\ldots,h_J} u(e_0 - \sum_j h_j) + E(u(\sum_j (h_j + 1) x_j)) - E(\lambda \sum_j (h_j + 1) 1_{(0,\bar{x}_j)}(x_j)) \tag{6}$$

From the first-order conditions, we have that the following holds for every j

$$-u'(c_0) + E(u'(c_1) x_j) - \lambda E(1_{(0,\bar{x}_j)}(x_j)) = 0 \tag{7}$$

Considering that the second derivative of the expected utility function with respect to h_j equals to $u''(c_0) + E(u''(c_1) x_j^2)$, which is always negative for any well behaving $u(\cdot)$, the solution to (7) will indeed correspond to a utility-maximizing decision.

Rearranging (7) and dividing both sides by $u'(c_0)$ we get

$$E\left(\frac{u'(c_1)}{u'(c_0)} x_j\right) - \frac{\lambda E(1_{(0,\bar{x}_j)}(x_j))}{u'(c_0)} = 1 \tag{8}$$

One can think of $\frac{u'(c_1)}{u'(c_0)}$ as the stochastic discount factor, henceforth denoted by m. Thus (8) can be written as

$$E(m x_j) - \frac{\lambda E(1_{(0,\bar{x}_j)}(x_j))}{u'(c_0)} = 1 \tag{9}$$

Consider the conditions given by (9) for asset j and the risk-free asset. Given that condition (9) holds for any asset, it follows that it also holds for the risk-free asset, for which the payoff is denoted by x_f. Subtracting them from each other, we get that

$$E(m x_j^e) - \frac{\lambda E(1_{(0,\bar{x}_j)}(x_j))}{u'(c_0)} + \frac{\lambda E(1_{(0,\bar{x_f})}(x_f))}{u'(c_0)} = 0 \tag{10}$$

where $x_j^e = x_j - x_f$ denotes the excess payoff of asset j over the risk-free return. (10) can be rewritten as

$$E(m)E(x_j^e) + cov(m, x_j^e) - \frac{\lambda E(1_{(0,\bar{x}_j)}(x_j))}{u'(c_0)} + \frac{\lambda E(1_{(0,\bar{x_f})}(x_f))}{u'(c_0)} = 0 \tag{11}$$

which yields

$$E(x_j^e) = -\frac{cov(m, x_j^e)}{E(m)} + \frac{\lambda E(1_{(0,\bar{x}_j)}(x_j))}{E(m)u'(c_0)} - \frac{\lambda E(1_{(0,\bar{x_f})}(x_f))}{E(m)u'(c_0)} \tag{12}$$

According to (12), the expected excess revenue from asset j is determined by three components. Not surprisingly, the covariance between the stochastic discount factor and the excess revenue does affect the expected excess return, which is a well-established result of the stochastic discount factor approach. Furthermore, the second part of the right-hand side in (12) suggests that the probability of failure, $E(1_{(0,\bar{x})}(x_j))$, impacts the expected excess revenue as well. Increasing the probability of failure of an asset leads to a higher expected payoff for that asset. Finally, the third part of the expression depends only on the probability of failure of the risk-free asset; therefore, this part is a constant and is the same for all assets.

Eq. (12) offers a potential new explanation for the negative risk-return puzzle. To see this, consider the probability of failure captured by the numerator in the second part of the right-hand side of Eq. (12). By definition, this failure can be determined by

$$E(1_{(0,\bar{x})}(x_j)) = P\left(\frac{x_j - \mu_j}{\sigma_j} < \frac{\bar{x}_j - \mu_j}{\sigma_j}\right) = G_j\left(\frac{\bar{x}_j - \mu_j}{\sigma_j}\right) \tag{13}$$

where $G_j(\cdot)$ is the cumulative distribution function of $\frac{x_j - \mu_j}{\sigma_j}$.

Thus, (12) can be rewritten as

$$E(x_j^e) = -\frac{cov(m, x_j^e)}{E(m)} + \frac{\lambda}{E(m)u'(c_0)}\left(G_j\left(\frac{\bar{x}_j - \mu_j}{\sigma_j}\right) - E(1_{(0,\bar{x_f})}(x_f))\right) \tag{14}$$

Taking the first derivative of (14) with respect to σ_j, we obtain that

$$\frac{\partial E(x_j^e)}{\partial \sigma_j} = -\frac{1}{E(m)}\frac{\partial cov(m, x_j)}{\partial \sigma_j} - \frac{\lambda}{E(m)u'(c_0)} g_j\left(\frac{\bar{x}_j - \mu_j}{\sigma_j}\right)\left(\frac{\bar{x}_j - \mu_j}{\sigma_j^2}\right) \tag{15}$$

where $g_j\left(\frac{\bar{x}_j-\mu_j}{\sigma_j}\right)$ stands for the probability density function of $\frac{x_j-\mu_j}{\sigma_j}$, i.e. $g_j(\cdot) \equiv G_j'(\cdot)$. Notice that the sign of (15) depends on two terms. Since the first part of the expression captures the well-known effects of the standard theory, we focus our attention on the second term only. The sign of this term depends on the sign of $\bar{x}_j - \mu_j$, since all other terms in the expression are positive. Thus, $\frac{\partial E(x_j^e)}{\partial \sigma_j}$ may be negative if $\bar{x}_j - \mu_j > 0$ and positive otherwise.

This result indicates that when the standard deviation increases, the expected return decreases if the investor's aspiration level is higher than the expected payoff of the asset. On the contrary, the expected return increases if the aspiration level is lower than the expected payoff. In essence, this implies that the positive risk-return relationship, where higher risk is typically associated with higher returns, may hold if the investor's aspiration level is lower than the expected return. In these situations, investors may become more risk-averse, seeking to avoid potential losses and aiming for more conservative investments. However, if the investor's aspiration level is higher than the expected return, the positive risk-return relationship may not hold. In fact, the relationship may be reversed, resulting in a negative risk-return association. In such cases, investors may be willing to take on higher risk for the potential of achieving higher returns. In essence, when an investor sets a highly ambitious financial target, they may feel compelled to increase their chances of success by assuming greater investment risks.

In summary, the presence or absence of the positive risk-return relationship is influenced by the relative positioning of the investor's aspiration level compared to the expected returns. When the aspiration level is lower, the traditional positive relationship holds, but if the aspiration level exceeds the expected returns, the risk-return relationship may be violated, leading to a negative correlation between risk and return.

5. Conclusion

There is substantial empirical evidence supporting a negative risk-return relationship in financial markets. Several reference-point-based models have been proposed to explain this phenomenon, drawing on principles from prospect theory. However, these existing explanations often focus solely on positive payoffs or negative payoffs, which restricts their ability to comprehensively account for the observed relationship. One limitation of prospect theory-based models is their inability to fully explain the negative risk-return relationship when considering prospects with both positive and negative payoffs. As a result, these models may not adequately capture the complexities of investor behavior when faced with various risk levels and potential outcomes.

In this paper, we propose a simple alternative model of risky decisions that incorporates expected utility theory with the introduction of an aspiration level. Decision makers are sensitive to this aspiration level, such that falling short of it results in disutility, reflecting their dissatisfaction with not achieving their desired payoff. This emotional response influences their perception of the investment outcome. In other words, the aspiration level serves as a psychological benchmark against which investors evaluate their investment outcomes.

The results from our model reveal that when the standard deviation of an asset increases, the expected return decreases if the investor's aspiration level is higher than the expected payoff of the asset. Conversely, the expected return increases if the aspiration level is lower than the expected payoff. By accounting for the aspiration level, our model provides a plausible explanation for the negative risk-return relationship. It offers a straightforward way of understanding this phenomenon, highlighting how investors' desires for certain payoffs can impact their risk-taking behavior and, consequently, the expected returns they seek.

In conclusion, our model, based on expected utility theory and incorporating the concept of aspiration levels, offers a comprehensive explanation for the negative risk-return relationship observed in financial markets. By providing insights into how investors' aspirations may affect their risk preferences, the model enhances our understanding of decision-making processes in the context of investment and portfolio management. Understanding how investors' aspirations influence their risk-taking behavior can have practical implications too. Portfolio managers can consider investors' different aspiration levels when constructing investment strategies, tailoring risk profiles to align with clients' individual preferences and goals.

In summary, our findings challenge the idea that the negative risk-return relationship is solely due to irrational behavior. Our results suggest sophisticated investors can exhibit this negative relation for specific stocks, aligning with empirical evidence. Additionally, our approach provides a straightforward framework that may explain phenomena in asset pricing, including the equity premium puzzle, extreme volatility puzzle, and the preference for lottery stocks. This contributes to a more nuanced understanding of financial markets and investor behavior.

CRediT authorship contribution statement

Barna Bakó: Conceptualization, Formal analysis, Methodology, Writing – original draft, Writing – review & editing. **Gábor Neszveda:** Conceptualization, Formal analysis, Methodology, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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