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Supply–demand price decoupling in European-type day-ahead electricity markets

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ABSTRACT

In this paper, we consider the possibility of supply–demand price decoupling in European-type day-ahead electricity markets, considering also the possibility of the supply price exceeding the demand price for some periods. Using a simple market model and an illustrative example, we show that this approach can resolve the paradoxical rejection of block orders and thus potentially increase the total social welfare and surplus of bidders. However, it has additional implications, which must be considered in a potential application. The first is the non-uniqueness of the decoupled market-clearing prices, while the second is that price decoupling affects the relation between the sum of individual bid surpluses and the total social welfare, as these values may no longer be equal, and the approach may imply a nonzero income for the auctioneer. To tackle the issue of non-uniqueness of market-clearing prices, we propose an iterative three-step clearing method. In the second part of the paper, we consider realistic-sized examples, analyze how the proposed approach affects the market outcome. We show that the proposed method reduces the number of paradoxically rejected block bids by 34%–42% and slightly increases the total welfare. In addition, we define a measure (opportunity cost of paradox rejection) to characterize the level of paradox rejection in a clearing solution. We show that the proposed price decoupling-based clearing method may significantly (34%–44%) decrease the value of this measure compared to the conventional clearing approach. We also study the computational demand of the proposed method.

1. Introduction

Since the liberalization of electricity markets, different types of competitive environments (auctions) have emerged, which aim to match production with consumption. The characteristic difficulty of calculating the day-ahead electricity market dispatch is the non-convex nature of the related technical constraints and costs of generators, such as minimum output levels and start-up costs. Taking these properties into consideration appropriately within today's shifting electricity production landscape, affected by the renewable transition, market integration, and climate change, is a constant challenge for market/system operators and policymakers. As discussed by Contreras et al. [1], most day-ahead electricity markets (or power exchanges (PXs)) use single-round auctions to determine which of the participants' bids are fully or partially accepted and how the accepted bids will be paid off. Such day-ahead power exchanges contribute to the integration of renewable sources into the power mix by offering an appropriate trading platform for weather-dependent renewable generators, as the precise production prediction of such units is not possible or is challenging for periods longer than 24–48 h. Accordingly, improving the efficiency of such markets is the focus of research.

A key output of these multi-unit auctions are the market-clearing prices for the trading periods, which determine the payoff of participants with accepted bids. These clearing prices are made public after the evaluation of the auction and serve as an important indicator for traders and investors. As discussed e.g. in [2–6], in such non-convex markets, the existence of a market equilibrium supported by uniform prices is not guaranteed in general. To overcome this issue, practical applications compute near-equilibrium prices, often supplemented with side or 'uplift' payments to compensate generators whose income from the derived prices does not cover their production costs.

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The two dominant approaches used to solve the pricing problem are Incremental Price (IP) and Convex Hull pricing. While IP pricing determines the resulting prices based on the marginal cost of producing the next unit (increment) of electricity, Convex Hull pricing [7] calculates the smallest convex set (the 'convex hull') that encompasses the true cost structure. In contrast to IP pricing, which only reflects marginal generation costs and can result in scenarios where generators with non-convex costs are unable to fully recover their total production costs from the market, Convex Hull pricing results in prices that reflect the lowest possible cost of dispatch, but the derived prices are typically higher than the simple marginal cost, ensuring cost recovery for generators with non-convexities. The paper [6] summarizes the conceptual differences and computational implications of these two pricing methods well and argues that EU-like rules should be considered a computationally challenging variant of IP pricing (i.e., IP pricing without uplift payments). For a recent review of pricing approaches in coupled and non-convex markets, see [8].

1.1. Day-ahead electricity markets in the EU

In contrast to US day-ahead electricity markets, where supply-side participants can explicitly include the detailed technical constraints (e.g. minimal up-and-down times) and cost structure of the respective generating plants in the bidding process, the bidding products used in European markets have more limited potential to include such aspects.

As also emphasized by Chatzigiannis et al. [9], European day-ahead electricity markets (DAMs) may be classified as single-round, double-blind auctions, where the non-convexity of generation constraints and cost is essentially reflected in non-convex orders. Most such PXs allow the submission of 'fill-or-kill' type orders, potentially linking multiple periods. Such bids aim to allow participants to internalize non-convex production costs (like start-up costs) in the bidding process. The most common type of such orders is the block order, whose acceptance is described by binary variables.

The requirement for a single clearing price for each bidding zone and market period has been formulated by the European Network of Transmission System Operators for electricity [10]. However, this requirement implies that in general some of the submitted indivisible orders may be paradoxically accepted (even though they are 'out-of-the-money', i.e., imply losses for the participant who submitted the order), or paradoxically rejected (rejected, even though they are 'in-the-money') [11]. As discussed by Chatzigiannis et al. [9], *'even though both types of orders seem to be problematic, European PXs exclude only the paradoxically accepted orders from the attained market-clearing solution, since such transactions entail a negative welfare to the participants submitting them.'*. Accordingly, the general approach is to allow paradoxical rejection of block orders. As discussed in [12], there is a trade-off relationship between different objectives such as maximization of the total welfare, maximization of the volume to be traded, and minimization of the opportunity costs of paradoxically rejected block orders.

Regarding algorithmic approaches to handle block orders, while some of the proposed methods [9,13] use iterative methods in the solution algorithms to handle such orders, other models explicitly designate these orders in a single run [14].

1.2. Related literature

In this subsection, we review the literature related to pricing approaches in day-ahead market models that are based on the common principle of applying distinct supply and demand prices.

The idea of using separate prices for the supply and demand sides has been proposed earlier in various contexts, including several results corresponding to non-convex markets. The paper by Toczyłowski and Zoltowska [15] proposes the application of different price vectors for demand and supply in the case of a welfare-distribution problem. The method proposed in this paper uses a two-phase setup, where in the first step, a unit-commitment problem is solved to maximize the social welfare, while the welfare distribution is determined in the second step by calculating the buy and sell prices.

The idea of using decoupled supply and demand prices in the case of a European-style portfolio-bidding model with non-convex orders has been described in [16]. However, this paper illustrates the concept only through a simple example, without deeper exploration of the arising issues.

The paper by Ahunbay et al. [17] proposes a similar approach in the sense that it introduces a markup mechanism, according to which buyers pay an uplift to cover excess supply due to non-convexities, and analyzes the efficiency and runtimes of the introduced mechanism.

Regarding market models without non-convexities, the paper [18] proposes price decoupling in a pay-as-bid setting to increase the traded quantity in a single-period market, while the approach presented by Sleisz et al. [19] proposes different clearing prices for the demand and supply side to establish a novel cost allocation mechanism for local flexibility.

Most approaches presented in the literature for supply–demand price decoupling assume that demand prices must always be at least equal to supply prices (see, e.g., [15,17,19]). While this assumption seems plausible, as it guarantees profitability, it is not necessary for all periods in the case of a multiperiod market. The aim of the current paper is to propose and evaluate a market-clearing framework in which this assumption is relaxed, and in some periods demand prices may be lower than supply prices.

1.3. Contribution of the paper

Regarding potential perspectives that have been suggested for the innovative handling of block orders, we refer to Tanrisever et al. [20], who writes that *'Another innovative DAM design would consider a clearing mechanism with two separate prices for buying and selling electricity. In the absence of block orders, these two prices are equal to each other. However, in the presence of block orders, these prices may not converge to each other. Through this new definition of market-clearing prices, the market can avoid the notion of paradoxical orders, and the acceptance/rejection decisions can be determined based on the price corresponding to the order type (buy or sell). This new design may provide more total surplus in the market, and we believe the design of such markets is a promising research direction.'*

In this paper, we follow this idea and analyze what kind of clearing model may be constructed if we relax the single market-clearing price (MCP) requirement for each period, and instead, allow two potentially different prices for each trading period regarding demand and supply, which may diverge if it is necessary to resolve paradox rejection of block orders. The concept of supply–demand price decoupling has been recently proposed for pay-as-bid two-sided multi-unit auctions, without non-convex orders [18].

The MCP derived for individual periods is an important output of the current framework used in the clearing of European-type day-ahead electricity markets. While it is directly connected to the acceptance/rejection of convex bids and the payoff (or surplus) of bids, it also serves as an important signaling mechanism for investors and decision makers. While market-clearing and value-distribution mechanisms can be constructed without relying on the concept of the MCP, in practical applications, it is usually required to have such a central indicator. The approach proposed in this work retains this concept, as the potentially distinct clearing prices for buyer and seller-type market participants can further serve this purpose by providing price signals in a differentiated way.

As we will see, while the novel market-clearing paradigm based on the concept of supply–demand price decoupling has the potential to increase the total resulting welfare/surplus of the auction outcome via the possible resolution of paradoxically rejected block orders, the concept raises several questions and makes additional considerations necessary. On the one hand, these new considerations originate from the fact that, in the conventional model, both supply and demand are cleared at the same price. This pricing approach ensures an income-cost balance, as the total accepted quantities on both sides are equal. However, this balance must be reconsidered if we allow the decoupling of supply and demand prices.

The approach proposed in the current paper allows the divergence of supply and demand prices not only for periods including block orders but also for periods with only standard bids (allowing partial acceptance), to restore the income-cost balance.

In addition, the decoupling of prices raises issues related to the uniqueness of the market-clearing solution (or dispatch), which must also be addressed in a potential price decoupling-based market-clearing algorithm. Furthermore, price decoupling affects the relationship between total social welfare, which is conventionally used as the objective in the optimization problem, and the individual surplus of bids. While in the conventional clearing method, the sum of individual bid surpluses equals total welfare, this equivalence does not necessarily hold in a price-decoupled framework.

Taking the above considerations into account, in the current work, we propose a potential novel market-clearing algorithm of DAMs based on the decoupling of supply and demand clearing prices, demonstrate its implications through simple examples, and evaluate its performance on realistic bid data through numerical studies.

While the currently used clearing framework for European-type electricity markets has been successfully extended to handle multiple zones connected by capacity-constrained transmission lines simultaneously [21], and this is an important requirement for any future method potentially used in practice, in this article we restrict ourselves to a single-zone model. As we will see, the application of the price decoupling principle raises several non-trivial questions even in this context, which require careful consideration. A possible extension of the approach for a market-coupling framework is discussed in Section 6.1.

1.4. Outline

In Section 2, we define a basic (i.e., conventional) market model to be used as a reference and introduce the proposed concept of supply–demand price decoupling.

In Section 3, we show that this approach can resolve block orders, that are paradoxically rejected in the conventional (non-price-decoupled) clearing framework, thus increasing the total social welfare and the total surplus of bids. We also illustrate that the proposed concept raises issues regarding the uniqueness of clearing prices. Section 4 presents the results of the numerical tests conducted to evaluate the performance of the proposed clearing algorithm in the case of realistic data sets. Section 5 discusses the results, while Section 6 provides the conclusions.

The abbreviations and notations used throughout the manuscript are summarized in Table 12 of Appendix A.

2. Conceptualization

2.1. Conventional market model

As a reference, we consider a simplified multi-period market model that allows only two types of bids (simple bids and block orders). We use the simplest model possible, which can already describe the phenomenon of paradox rejection and can be easily modified to demonstrate the implications of the price decoupling approach.

Simple bids are characterized by three parameters: quantity (Q_i), price (P_i), and relevant time period (r_i). $Q_i > 0$ corresponds to demand bids, while $Q_i < 0$ corresponds to supply bids. The index sets of simple bids are denoted by SD and SS for demand and supply bids, respectively. Simple bids are divisible, i.e., they may be fully or partially accepted (or rejected).

Block orders (which are supply orders) are characterized by the following parameters: start period (s_j), end period ($e_j \geq s_j$), a quantity vector ($Q_{j,t}$ $s_j \leq t \leq e_j$) and a price (P_j).

The index set of block bids is denoted by BB. Block orders are indivisible, i.e., they must be either fully accepted for all included periods or fully rejected ('fill-or-kill' property).

We have three index sets in each time period t, SD_t and SS_t are the indices of simple demand and simple supply bids, respectively, for which $r_i = t$, while BB_t is the set of indices of block orders relevant for the period t, namely $BB_t = \{j \in BB : s_j \leq t \leq e_j\}$.

Let us denote the number of time periods by n_t, and the index set of time periods by T, i.e., $T = \{1, \dots, n_t\}$.

According to the usual assumption, the price of demand bids represents the utility of consumption for the bidder (per unit), while the price of supply bids reflects the cost of production (per unit). Given a set of simple bids and block orders (the latter may be empty), the market-clearing problem aims to determine a consistent set of market-clearing prices (MCPs) for each period, and the acceptance indicators of simple bids and block orders which maximize the resulting total social welfare (TSW), i.e., the total utility of consumption minus the total cost of production. Acceptance indicators are between zero and one (in the case of block orders, they are either 0 or 1). The acceptance of a simple bid is consistent with the MCP if it generates a nonnegative surplus for the bidder (e.g., in the case of demand bids, the corresponding MCP must not be higher than the bid price if a bid is accepted). The bid-acceptance conditions of block orders are similar, but the resulting total surplus is considered for the given block order. The detailed computational formulation of the problem is given in the next subsection.

2.1.1. Formulation of the market-clearing optimization problem

The reference clearing model can be viewed as a simplification of Chatzigiannis et al. [9]. Regarding the variables of the model, $x_i \in [0,1]$ for $i \in SD$ (or $i \in SS$) denotes the acceptance variable (or indicator) of the ith simple demand (supply) bid, while $y_j \in \{0,1\}$ for $j \in BB$ denotes the acceptance variable of the jth block order. The market-clearing price of period t is denoted by MCP_t. Problem 1 summarizes the formulas of the simple model of conventional market clearing.

Problem 1.

$$TSW^C = \max_{x,\ y,\ MCP} \sum_{t\in T} \left(\sum_{i\in SD_t \cup SS_t} x_i Q_i P_i + \sum_{j\in BB_t} y_j Q_{j,t} P_j \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in SD_t \cup SS_t} x_i Q_i + \sum_{j\in BB_t} y_j Q_{j,t} = 0 \qquad \forall t \in T \tag{2}$$

$$x_i > 0 \Rightarrow MCP_t \leq P_i, \quad x_i < 1 \Rightarrow P_i \leq MCP_t \qquad \forall t \in T,\ \forall i \in SD_t$$

$$x_i > 0 \Rightarrow P_i \leq MCP_t, \quad x_i < 1 \Rightarrow MCP_t \leq P_i \qquad \forall t \in T,\ \forall i \in SS_t \tag{3}$$

$$y_j = 1 \Rightarrow \sum_{s_j^B \leq t \leq e_j^B} Q_{j,t} P_j \geq \sum_{s_j^B \leq t \leq e_j^B} Q_{j,t} MCP_t \qquad \forall j \in BB. \tag{4}$$

Objective. The objective function is to maximize the total social welfare (TSW) – the 'total utility of consumption minus the total cost of production' – over all periods, as described by (1). We denote by TSW^C the optimum value, i.e., the maximum attainable TSW value in the conventional, non-decoupled price model.

Constraints. Two types of constraints are present in the proposed market-clearing problem: supply–demand constraints and bid-acceptance constraints. The supply–demand constraints given in Eq. (2) ensure the balance of accepted quantities on the demand and supply sides for each period. The bid-acceptance constraints, defined by Eqs. (3) and (4) describe how the MCP is related to the set of accepted and rejected bids.

Similar to the models proposed in [22–24], and the European market coupling tool Euphemia [25], the formulated model allows different quantities for each included period of the block order and the block

Table 1
Parameters of the simple bids in Example 1: bid ID (i), period (r_i), quantity (Q_i) [MWh] and price (P_i) [€/MWH].

i	r_i	Q_i [MWh]	P_i [€/MWh]	i	r_i	Q_i [MWh]	P_i [€/MWh]
1	1	3	5	3	1	−2	1
2	1	2	4	4	1	−4	3

order acceptance constraint (4) compares the bid price with the volume (quantity) weighted average market-clearing price across the respective periods as usually defined in the literature, see for example [26] (let us remember that quantities for supply orders are negative).

According to Eq. (3), the MCP_t values explicitly determine the full acceptance or rejection of all simple bids for the period t with $P_i^S \neq MCP_t$, and allow partial acceptance ($x_i^S \in [0,1]$) for bids with $P_i^S = MCP_t$. However, (4) indicates that we cannot require the acceptance of all block orders with an appropriate bid price. This means that the MCPs do not explicitly determine the set of accepted block orders, and some block orders may be paradoxically rejected, as illustrated later in Section 2.1.3. The implications described in these constraints may be implemented, for instance, by the so-called 'big-M' method within an optimization framework [27].

The surplus (SP) of individual simple bids can be calculated as described by Eq. (5), where MCP_t denotes the market-clearing price of the relevant period (note that $Q_i < 0$ in the case of supply bids):

$$SP_i = x_i Q_i (P_i - MCP_t) \qquad \forall\, t \in T,\ \forall\, i \in SD_t \cup SS_t. \tag{5}$$

For block orders, the surplus can be calculated using the formula described by Eq. (6):

$$SP_j = y_j \sum_{s_j^B \le t \le e_j^B} Q_{j,t}(P_j - MCP_t) \qquad \forall j \in BB. \tag{6}$$

The bid-acceptance constraints ensure nonnegative surplus values for accepted bids.

From the definition of the TSW described by Eq. (1), and the definition of the surplus values by Eqs. (5) and (6), it can be seen that the total surplus for all bids is equal to the TSW in the conventional model, as described by eq.

$$\sum_{i \in SD \cup SS} SP_i + \sum_{j \in BB} SP_j = TSW. \tag{7}$$

2.1.2. Aggregated curves

If only simple bids are present for a certain period, the solution to the market-clearing problem is straightforward and can be graphically expressed using the aggregated demand and supply curves. For example, let us consider the bids summarized in Table 1.

To obtain the aggregated demand curve of a given period, we consider the demand bids of the respective period ordered by descending price. The breakpoints of the aggregated demand curve are defined by the aggregate quantities; in this case, $Q^D = Q_1$, $Q_2^D = Q_1 + Q_2$. The aggregated supply curve is derived similarly, according to an increasing-price ordering. In this case, $Q_1^S = -Q_3$, $Q_2^S = -(Q_3 + Q_4)$. The aggregated demand and supply curves of Example 1 are depicted in Fig. 1.

In this case (when only simple bids are present), the total traded quantity and MCP of the corresponding period is determined by the intersection of the demand and supply curves. In this particular case, bids 1, 2, and 3 are fully accepted, while bid 4 is partially accepted ($x_1 = 1,\ x_2 = 1,\ x_3 = 1,\ x_4 = 0.75$) – thus its called the price-setter bid. The total traded quantity is equal to 5. The TSW can be calculated as $1 \cdot 3 \cdot 5 + 1 \cdot 2 \cdot 4 + 1 \cdot (-2) \cdot 1 + 0.75 \cdot (-4) \cdot 3 = 12$, while the surplus values of individual bids are as $SP_1 = 1 \cdot 3 \cdot (5-3) = 6$, $SP_2 = 1 \cdot 2 \cdot (4-3) = 2$, $SP_3 = 1 \cdot 2 \cdot (3-1) = 4$, and $SP_4 = 0$ (since the MCP is equal to the bid price).

The aggregated curves may also be used in cases where block orders are present in the respective period. In this case, only accepted block orders (more precisely, their component corresponding to the actual period) are included in the supply curve.

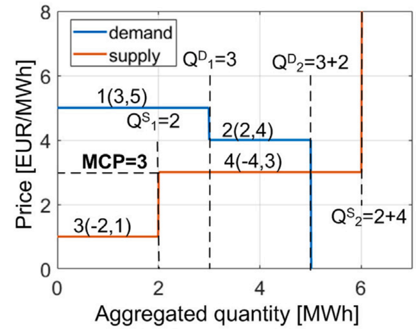

Fig. 1. The aggregated demand and supply curves for the bids summarized in Table 1, with bids annotated as $i(q_i, p_i)$.

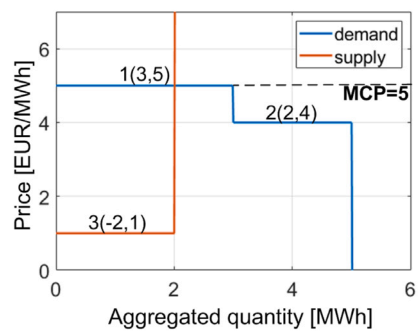

Fig. 2. Aggregated demand curves and resulting MCP in the case of the rejection of B1.

2.1.3. Paradox rejection of block orders

Let us now modify Example 1 by removing simple bid 4 and adding a block order B1 with the same quantity and price parameters. Accordingly, the partial acceptance of B1 is not allowed. Its full acceptance is also not feasible, since in this case, according to the acceptance constraint of the block order, the MCP must be at least 3, which implies the full acceptance of simple bid 3 as well, resulting in a total supply of 6 units that cannot be matched by demand.[1] Thus in this case, B1 will be rejected, and – as the total supply quantity will be 2 – simple bid 1 will be the price setter bid, as depicted in Fig. 2.

As the MCP will be equal to 5 in this case, B1 will be a paradoxically rejected block order, since the resulting MCP is higher than its price (3). The TSW is equal to 8 units, while the bid surpluses for the accepted bids are: $SP_1 = 0$, $SP_3 = 8$.

Potential (un)fairness implications of paradox rejection. The resulting surplus values of bids, defined by Eqs. (5) and (6) may be considered as the payoffs of participants in a value-distribution problem, where the total value to be distributed is the TSW (and each bid corresponds to a participant). Thus, the fairness of a surplus vector may be analyzed in the context of cooperative game theory [28,29]. The concept of the core [30] defines the set of stable payoff vectors according to the criterion that every coalition (i.e., subset of players) should receive at least the value achievable by the respective coalition without the cooperation of players outside the coalition. In our case, this value equals the TSW generated by a restricted clearing problem, considering only the bids included in the particular coalition. It may be shown that, if no block orders are present, the resulting surplus values constitute

[1] Another possibility would be to formulate the dual of Eq. (4) describing that if a block order is rejected, its surplus is not sufficient — this formulation would allow paradox acceptance. Both conditions cannot be used simultaneously, as in this case, there is no feasible MCP.

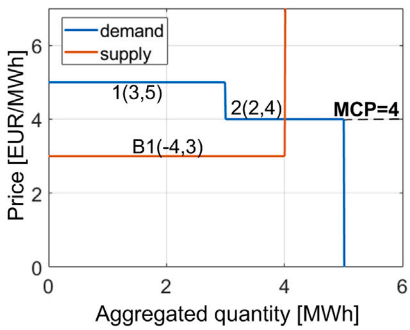

Fig. 3. Market clearing in the case of the coalition $\{1, 2, B1\}$.

stable payoff vectors. On the other hand, paradox rejection may induce unfairness as follows. If we consider the value achievable by the coalition $\{1, 2, B1\}$, we have to evaluate the TSW value in the market-clearing problem where we consider only simple bids 1, 2, and the block order B1. The clearing in this case is depicted in Fig. 3, resulting in a TSW of 7 units.

This means that the total payoff (the sum of bid surpluses) of the coalition $\{1, 2, B1\}$ should be at least equal to 7 units in the resulting dispatch to obtain a stable payoff. In contrast, as detailed before, only the surplus of bid 3 is nonzero in the case of the conventional market clearing, and thus the implied payoff vector is outside the core. The above example highlights that the phenomenon of paradox rejection of block orders is a potential source of unfairness in market clearing.

2.2. Price decoupled market model

As an alternative to the conventional market-clearing model described in Section 2.1, we now introduce the proposed price-decoupled (PD) market-clearing model. In this case, we assume that for each period t, there is a separate MCP for the demand and supply sides, denoted by MCP_t^D and MCP_t^S, respectively. The vectors of these values across market periods are denoted by MCP^D and MCP^S.

Consequently, the definitions of surplus (5) and (6), need to be changed to use different MCPs as (8), (9), (10) for simple demand bids, supply bids, and block orders, respectively:

$$SP_i = x_i Q_i (P_i - MCP_t^D) \qquad \forall t \in T,\ \forall i \in SD_t \tag{8}$$

$$SP_i = x_i Q_i (P_i - MCP_t^S) \qquad \forall t \in T,\ \forall i \in SS_t \tag{9}$$

$$SP_j = y_j \sum_{s_j^B \le t \le e_j^B} Q_{j,t} (P_j - MCP_t^S) \qquad \forall j \in BB \tag{10}$$

Therefore, the relation of the TSW and the total surplus value that has been described by Eq. (7) in the conventional model takes the more general form:

$$TSW = \sum_{i \in SD \cup SS} SP_i + \sum_{j \in BB} SP_j + \sum_{t \in T} \sum_{i \in SD_t} x_i Q_i MCP_t^D + \sum_{t \in T} \sum_{i \in SS_t} x_i Q_i MCP_t^S + \sum_{t \in T} \sum_{j \in BB_t} y_j Q_{j,t} MCP_t^S \tag{11}$$

Eq. (11) may be interpreted as follows. The payment for accepted supply bids may no longer equal the income from accepted demand bids. More precisely, the value generated by the clearing is the TSW, which is divided between the participants with accepted bids (in the form of bid surplus values) and the auctioneer (in the form of revenue resulting from the income-cost imbalance). Let us define the second part, which is the difference between the TSW and the total surplus, as the total revenue (TR), namely

$$TR = \sum_{t \in T} \left(\sum_{i \in SD_t} x_i Q_i MCP_t^D + \left(\sum_{i \in SS_t} x_i Q_i + \sum_{j \in BB_t} y_j Q_{j,t} \right) MCP_t^S \right). \tag{12}$$

Note that in the original model (Problem 1), that is, in the non-decoupled case ($MCP_t^D = MCP_t^S \quad \forall\, t$), TR is zero, so we get back the original definition of TSW (7).

The total revenue cannot be negative, since we should not require the auctioneer to pay. In other words, we require that the total income (over all periods) from accepted demand bids be at least equal to the total cost of accepted supply bids, see Eq. (17).

On the other hand, we do not want to allow TR to increase at the expense of SP, thus, we require the following inequality to be satisfied

$$TR \le TSW - TSW^C,$$

where TSW^C is the maximal total social welfare without price decoupling (see (1). Based on the definition of TR, this equivalently means that the total surplus should be at least TSW^C, see Eq. (18).

The implied PD market-clearing problem is described by Eqs. (13)–(17) of Problem 2.

Problem 2.

$$TSW^{PD} = \max_{x,\ y,\ MCP^D,\ MCP^S} \sum_{t \in T} \left(\sum_{i \in SD_t \cup SS_t} x_i Q_i P_i + \sum_{j \in BB_t} y_j Q_{j,t} P_j \right) \tag{13}$$

$$\text{s.t.} \quad \sum_{i \in SD_t \cup SS_t} x_i Q_i + \sum_{j \in BB_t} y_j Q_{j,t} = 0 \qquad \forall t \in T \tag{14}$$

$$x_i > 0 \Rightarrow MCP_t^D \le P_i, \quad x_i < 1 \Rightarrow P_i \le MCP_t^D \qquad \forall t \in T,\ \forall i \in SD_t$$

$$x_i > 0 \Rightarrow P_i \le MCP_t^S, \quad x_i < 1 \Rightarrow MCP_t^S \le P_i \qquad \forall t \in T,\ \forall i \in SS_t \tag{15}$$

$$y_j = 1 \Rightarrow \sum_{s_j^B \le t \le e_j^B} Q_{j,t} P_j \ge \sum_{s_j^B \le t \le e_j^B} Q_{j,t} MCP_t^S \qquad \forall j \in BB, \tag{16}$$

$$\sum_{t \in T} \sum_{i \in SD_t} x_i Q_i MCP_t^D \ge - \sum_{t \in T} \left(\sum_{i \in SS_t} x_i Q_i + \sum_{j \in BB_t} y_j Q_{j,t} \right) MCP_t^S \tag{17}$$

$$\sum_{t \in T} \sum_{i \in SD_t} x_i Q_i (P_i - MCP_t^D) + \sum_{t \in T} \sum_{i \in SS_t} x_i Q_i (P_i - MCP_t^S) + \sum_{t \in T} \sum_{j \in BB_t} y_j Q_{j,t} (P_j - MCP_t^S) \ge TSW^C \tag{18}$$

Assuming different clearing prices for demand and supply, the bid-acceptance constraints for simple bids are given by Eq. (15), while the bid-acceptance constraints for block orders are as described in (16). Note that the supply–demand balance constraint described by Eq. (14) and the objective function detailed in Eq. (13) are unaltered compared to (2) and (1) in Problem 1.

The constraint (17) contains product terms involving the variables x_i and y_i with MCP, thus making it a nonlinear constraint. In the rest of this section, we propose an equivalent model where the nonlinear constraints are replaced by linear ones. Thus, the solver can handle the new problem more efficiently.

Following the principles described by Sleisz and Raisz [31], we can replace the nonlinear terms with new variables M_i^D, M_i^S, M_j^B, as described in Eq. (19).

$$\sum_{t \in T} \sum_{i \in SD_t} M_i^D \ge - \left(\sum_{t \in T} \sum_{i \in SS_t} M_i^S + \sum_{j \in BB} M_j^B \right) \tag{19}$$

The new variables are defined by the following implications, which can be formulated as linear constraints through the use of binary variables (see Appendix B):

$$x_i > 0 \quad \Rightarrow \quad M_i^D = x_i Q_i P_i + Q_i MCP_t^D - Q_i P_i \qquad \forall\, t \in T,\ \forall i \in SD_t \tag{20}$$

$$x_i < 1 \quad \Rightarrow \quad M_i^D = x_i Q_i P_i \qquad \forall\, t \in T,\ \forall i \in SD_t \tag{21}$$

$$x_i > 0 \quad \Rightarrow \quad M_i^S = x_i Q_i P_i + Q_i MCP_t^S - Q_i P_i \qquad \forall\, t \in T,\ \forall i \in SS_t \tag{22}$$

$$x_i < 1 \quad \Rightarrow \quad M_i^S = x_i Q_i P_i \qquad \forall\, t \in T,\ \forall i \in SS_t \tag{23}$$

$$y_j = 0 \quad \Rightarrow \quad M_j^B = 0 \qquad \forall\, j \in BB \tag{24}$$

$$y_j = 1 \quad \Rightarrow \quad M_j^B = \sum_{s_j \le t \le e_j} MCP_t^S Q_{j,t} \qquad \forall\, j \in BB. \tag{25}$$

In (24) and (25), the variables y_j are binary, therefore we can reformulate these implications as linear constraints without introducing additional binary variables. However, for each constraint of type (20)–(23), we need to introduce a new binary variable. Replacing implications (20)–(23) with the following linear constraints reduces problem size and may decrease solution time.

$$M_i^D \le Q_i MCP_t^D \qquad \forall\, t \in T,\ \forall i \in SD_t \tag{26}$$

$$M_i^D \le x_i Q_i P_i \qquad \forall\, t \in T,\ \forall i \in SD_t \tag{27}$$

$$M_i^S \le x_i Q_i P_i + Q_i MCP_t^S - Q_i P_i \qquad \forall\, t \in T,\ \forall i \in SS_t \tag{28}$$

$$M_i^S \le 0 \qquad \forall\, t \in T,\ \forall i \in SS_t. \tag{29}$$

Similarly, the constraint (18) with the new variables is as folows:

$$\sum_{i \in SD} x_i Q_i P_i - M_i^D + \sum_{i \in SS} x_i Q_i P_i - M_i^S + \sum_{t \in T} \sum_{j \in BB_t} y_j Q_{j,t} P_j - M_j^B \ge TSW^C \tag{30}$$

Based on the above discussion, we solve the following equivalent problem instead of Problem 2:

Problem 2′.

max (13) $s.t.$ (14)–(16), (19), (24)–(30).

The equivalence of the two problems can be shown as follows. Given a feasible solution to Problem 2, we can construct a feasible solution for Problem 2′ with the same objective function value by setting:

- $M_i^D = x_i Q_i MCP_t^D$ for all $i \in SD$,
- $M_i^S = x_i Q_i MCP_t^S$ for all $i \in SS$, and
- $M_j^B = \sum_{s_j \le t \le e_j} y_j Q_{j,t} MCP_t^S$ for all $j \in BB$.

Note that M_i^D and M_i^S are well-defined, as simple bids are defined only for a single period.

Conversely, if we have a feasible solution for Problem 2′, we can show that constraint (17) is satisfied for the variables x, y, MCP^D, and MCP^S, which implies that these vectors form a feasible solution to Problem 2. The key observation is that the acceptance constraints (15) imply, through constraints (26)–(27) and (28)–(29), that $M_i^D \le x_i Q_i MCP_t^D$ for $i \in SD$ and $M_i^S \le x_i Q_i MCP_t^S$ for $i \in SS$.

To see how the first inequality is implied by the acceptance constraints, let us consider an index $i \in SD$. If $x_i = 0$, then $M_i^D \le 0$ from (27) and $x_i Q_i MCP_t^D = 0$. When $x_i = 1$, from (26) we have $M_i^D \le Q_i MCP_t^D$ and $x_i Q_i MCP_t^D = Q_i MCP_t^D$. If $0 < x_i < 1$, constraint (15) implies that $P_i = MCP_t^D$; thus, from (27) we have $M_i^D \le x_i Q_i MCP_t^D$. Therefore, $M_i^D \le x_i Q_i MCP_t^D$ is satisfied in all three cases. The inequality for the supply bids can be derived using a similar argument.

Furthermore, for all $j \in BB$, we have $M_j^B = \sum_{s_j \le t \le e_j} y_j Q_{j,t} MCP_t^S$ according to (24) and (25). Therefore, constraints (17) and (18) are also satisfied.

3. Results

3.1. The potential implications of supply–demand price decoupling

To demonstrate the potential benefits and implications of the price-decoupled market-clearing model proposed in Section 2.2, let us consider the bid set of Example 2, summarized in Tables 2 and 3.

Table 2
Parameters of the simple bids in Example 2: bid ID (i), period (r_i), quantity (Q_i) [MWh] and price (P_i) [€/MWh].

i	r_i	Q_i [MWh]	P_i [€/MWH]	i	r_i	Q_i [MWh]	P_i [€/MWH]
1	1	7	26	5	2	9	24
2	1	9	15	6	2	−3	12
3	1	−6	12	7	2	−3	15
4	1	−10	22				

Table 3
Parameters of the block order in Example 2: bid ID (j), start period (s_j), end period (e_j), quantities for periods 1 and 2 ($Q^B_{j,1}$, $Q^B_{j,2}$) [MWh] and price (P_j) [€/MWh].

j	s_i^B	e_i^B	$Q^B_{j,1}$ [MWh]	$Q^B_{j,2}$ [MWh]	P_i^B [€/MWh]
B1	1	2	−5	−5	16

Table 4
Acceptance indicators, TSW contribution, generated income/cost components (M) and surplus (SP) of individual bids under conventional clearing (superscript ^{C}), and in the case of price-decoupled clearing (superscript ^{PD}).

i	x^C/y^C	x^{PD}/y^{PD}	TSW^C	TSW^{PD}	M^C	M^{PD}	SP^C	SP^{PD}
1	1	1	182	182	154	105	28	77
2	0	0.444	0	60	0	60	0	0
3	1	1	−72	−72	−132	−132	60	60
4	0.1	0	−22	0	−22	0	0	0
5	0.667	1	144	216	144	216	0	0
6	1	1	−36	−36	−72	−45	36	9
7	1	0.333	−45	−15	−72	−15	27	0
B1	1	1	0	−160	0	−185	0	25
Total			151	175	0	4	151	171

3.1.1. Results of conventional clearing in the case of example 2

As reference, let us consider the results of the conventional clearing model in the case of Example 2.

Similar to Example 1, in the conventional clearing framework, it is not possible to accept block order B1, it will be a paradoxically rejected block order. According to the aggregate demand and supply curves depicted in Fig. 4, the MCPs are 22 and 24 for periods 1 and 2, respectively, while the total traded quantities (TTQs) are 7 and 6 for periods 1 and 2, respectively. The resulting TSW (see Table 4 later for details) is 151 units.

3.1.2. Results of PD clearing in the case of example 2

The PD clearing framework allows the acceptance of the block order (the paradox rejection is resolved) via the decoupling of supply and demand prices as $MCP_1^D = 15$, $MCP_1^S = 22$, $MCP_2^D = 24$, $MCP_2^S = 15$, as depicted in Fig. 5.

Let us note that the intersection of the supply and demand curves no longer explicitly determines the total traded quantity and the MCP. In period 1, the simple demand bids 1 and 2 are fully and partially accepted, respectively, while on the supply side, bid 3 and the block order are fully, while bid 4 is partially accepted, resulting in $TTQ_1 = 11$ units. In period 2, the simple demand bid 5 is fully accepted, as on the supply side the simple bid 6 and the block order are fully, while bid 7 is partially accepted, resulting in a $TTQ_2 = 9$ units.

Table 4 summarizes the acceptance indicators, the TSW contribution, the respective income/cost and the resulting surplus of individual bids in the case of conventional clearing and PD-based clearing.

The main implications of the PD clearing compared to the conventional model illustrated by example 2 may be summarized as follows.

- The PD-based clearing may yield higher TSW and total surplus values by resolving block orders that are paradoxically rejected in the output of the conventional clearing model.
- In contrast to the conventional market-clearing model, the TSW and the total surplus are not necessarily equal in the case of PD. The difference is equal to the resulting revenue of the auctioneer

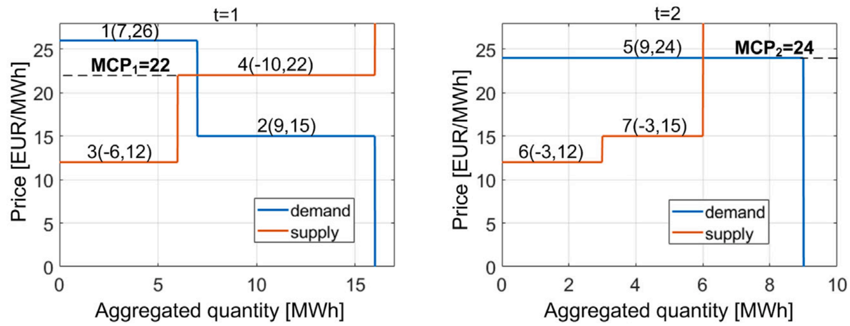

Fig. 4. Aggregated demand and supply curves of the bid set of Example 2 summarized in Tables 2 and 3 (the rejected block order is omitted in the supply curves), and the resulting MCPs in the case of conventional clearing in periods 1 and 2.

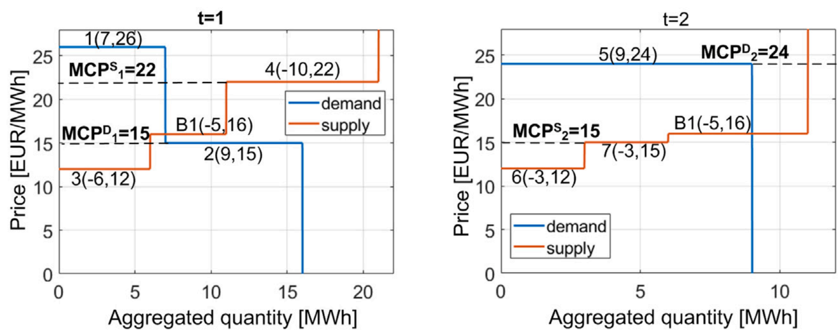

Fig. 5. Aggregated demand and supply curves of the bid set of Example 2 summarized in Tables 2. and 3 (the accepted block order is included in the supply curves), and the resulting MCPs for the demand and supply side in the case of PD clearing in periods 1 and 2.

(i.e., total income minus total cost). While in the result of the conventional clearing approach, the sum of the surplus values of individual bids equals the TSW for each period, this equality no longer holds with the proposed PD-based clearing algorithm.

- In addition, the demand and supply MCPs derived in the PD-based clearing in Problem 2 are not necessarily unique. While the MCP values $MCP_1^D = 15$, $MCP_1^S = 22$, $MCP_2^D = 24$, $MCP_2^S = 15$ and the x_i^{PD} bid-acceptance indicators summarized in Table 4 are a feasible, TSW-maximizing solution of Problem 2, other maximal solutions also exist. In the case of this particular example for any value of MCP_1^S, for which the inequalities $17 \leq MCP_1^S \leq 22$ hold, the solution (complemented with the unmodified other MCPs and acceptance indicators in the PD case) is still in the feasible set of Problem 2, resulting in the same TSW value of 175.

In the next subsection, we propose a three-step approach to address the general non-uniqueness of PD clearing prices.

3.2. The proposed algorithm for price-decoupled market clearing

Considering the implications discussed in Section 2.2, we propose an algorithm that aims to maximize the total social welfare and minimize the deviation of MCP^D and MCP^S from the MCP in conventional clearing. Let MCP^C denote the vector of MCP values from the solution to Problem 1 (the tth element of MCP^C is the resulting value of MCP_t in the solution of Problem 1), and let TSW^{PD} represent the TSW value from the solution to Problem 2. The proposed optimization problem is defined in Problem 3.

Problem 3.

$$\min_{x,\, y,\, MCP^D,\, MCP^S} \sum_{t\in T} \left((MCP_t^D - MCP_t^C)^2 + (MCP_t^S - MCP_t^C)^2 \right) \quad (31)$$

$$s.t. \quad \sum_{t\in T} \left(\sum_{i\in SD_t \cup SS_t} x_i Q_i P_i + \sum_{j\in BB_t} y_j Q_{j,t} P_j \right) = TSW^{PD} \quad (32)$$

$$(14)–(16), (19), (24)–(28). \quad (33)$$

Overall, the proposed method to derive the supply–demand price decoupled clearing dispatch for a market-clearing problem is as follows:

1. Solve Problem 1 to optimize the TSW without price decoupling and denote the resulting vector of MCPs by MCP^C.
2. Solve Problem 2 to optimize the TSW, allowing price decoupling, and denote the resulting TSW value by TSW^{PD}.
3. Solve Problem 3 to determine the final results of the clearing, taking into account MCP^C and TSW^{PD}.

The proposed three-step algorithm is visualized in Fig. 6.

4. Evaluation using realistic market data

4.1. Generating test problem instances

To evaluate the performance of the PD-based clearing method described in Section 3.2 under more realistic market configurations, appropriate bid sets are necessary; however, we consider it important to keep the statistical characteristics of real-life market configurations in the test sets as well. To achieve this goal, two functions have been developed and implemented in MATLAB, based on the principles described in [32]: the first calculates various statistical parameters from a given reference bid set, while the second generates bids based on the output of the first.

We used the data set from [33] to test the effectiveness of the price decoupling approach. The example assumes 14 bidding zones with interconnections and defines the connections and capacities between the bidding zones. These associations have been merged into a single

Table 5
Summary of bid dataset characteristics for 24_140_524 and 24_140_1048. $|D|$, $|S|$, and $|B|$ denote the number of simple demand, simple supply, and block bids, respectively. $\overline{Q}$ [100 MWh] and $\overline{P}$ [100 €/MWh] represent the average quantity and price for simple demand (subscript d), simple supply (s) bids, and block orders (b). $\overline{e_j - s_j}$ denotes the average number of periods for block orders.

Instance	$\|D\|$	$\|S\|$	$\|B\|$	$\overline{Q}_d$	$\overline{P}_d$	$\overline{Q}_s$	$\overline{P}_s$	$\overline{Q}_b$	$\overline{P}_b$	$\overline{e_j - s_j}$
24_140_524	3360	3360	524	3.83	1.60	2.94	0.90	1.26	0.61	11.61
24_140_1048	3360	3360	1048	3.76	1.51	2.76	0.91	1.21	0.57	11.08

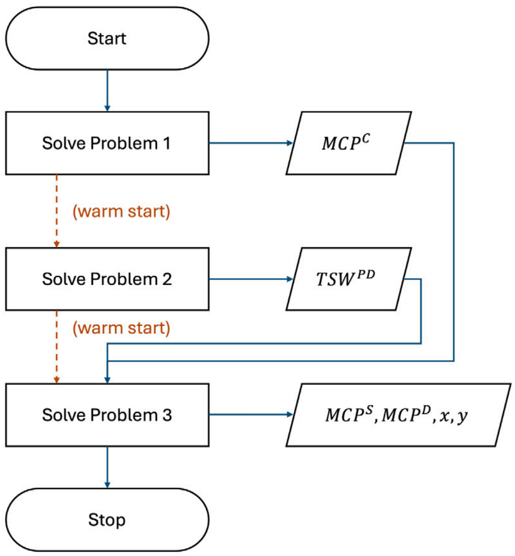

Fig. 6. The proposed three-step algorithm for the PD-based clearing.

zone in the case of our numerical tests, since we assumed a single-zone market in the proposed model.

The source data includes 936 supply and 336 demand offers, each with 10 different steps in terms of quantity and price; thus, we had 9360 supply and 3360 demand offers overall. We ignored the information declaring the bidding zone where the given bid is submitted. The data also includes 1048 block offers. Most of these offers are submitted for eight consecutive periods. For simplicity, we reduced the number of supply offers to match the number of supply and demand offers.

To create examples of different sizes, two setups were generated from the original data, each with a different number of block bids; however, the number of periods and submitted demand and supply offers per period remained the same. We configured market scenarios where market participants could submit offers for 24 periods, with each period containing 140 demand and 140 supply offers. The number of block orders in the scenarios are 524 and 1048, respectively. The two test sets are named as 24_140_524 and 24_140_1048, with each set containing 100 problem instances. Table 5 shows the main properties of the two bid datasets, such as the number of different bids and the average quantities, prices, and durations.

4.2. Implementation

We implemented the method described in Section 3.2 using FICO Xpress Optimizer v9.5.0 and FICO Xpress Mosel 64-bit v6.8.0 [34]. The numerical tests were conducted on a MacBook Pro with an Apple M3 CPU and 24 GB of unified memory.

We connected the three models using the mmjobs module of XPRESS. For Problem 2, we calculated an initial solution using the optimal solution of Problem 1, and similarly for Problem 3, we constructed an initial feasible solution from the optimal solution of Problem 2. This approach allowed us to warm start the solver in the second and third phase, significantly reducing the running time.

Table 6
Comparison of the number of paradoxically rejected block orders in the case of the conventional clearing (Par. BB conv) and the PD clearing (Par. BB PD) and opportunity cost of paradox rejections (OCPR) under the conventional and price decoupling models for problem sets 24_140_524 and 24_140_1048. Averages (AVG) and standard deviations (DEV) are reported for each case.

	24_140_524		24_140_1048	
	AVG	DEV	AVG	DEV
Par. BB conv.	29.77	4.86	47.34	7.52
Par. BB PD	17.30	6.24	31.40	10.55
Absolute difference	12.47		15.94	
Decrease (%)	42%		34%	
OCPR conv.	282.69	60.09	373.19	71.23
OCPR PD	158.19	65.95	247.23	87.86
Absolute difference	124.50		125.96	
Decrease (%)	44%		34%	

4.3. Numerical results

We analyzed and evaluated the performance of the PD-based clearing method compared to conventional clearing across multiple aspects.

The effect of price decoupling on the number paradox rejection of block orders is investigated in Table 6. In addition, we introduce an additional measure to characterize the rate of paradox rejection and its change due to the PD-based clearing. The *opportunity cost of paradox rejection (OCPR)* in the conventional case is defined as shown in Eq. (34):

$$OCPR_{\mathrm{C}} = -\sum_{t\in T}\sum_{j\in PBB_t} MCP_t Q_{j,t}, \tag{34}$$

where PBB_t represents the set of paradoxically rejected block offers defined in period t. Similarly, we can define this parameter in the case of price decoupling, as described by Eq. (35):

$$OCPR_{\mathrm{PD}} = -\sum_{t\in T}\sum_{j\in PBB_t} MCP_t^S Q_{j,t}. \tag{35}$$

OCPR measures the amount of revenue traders would receive if their offers were not paradoxically rejected. However, this definition requires caution, as accepting these block orders would make the remaining models infeasible; therefore, this profit cannot be realized in practice. Nevertheless, the percentage decrease in this parameter closely aligns with the decrease measured in the average number of paradoxically rejected block orders in all cases.

We also examined how price decoupling affects the total traded quantity (TTQ). In Example 2, the TTQ significantly increased as price decoupling could resolve the paradoxically rejected (single) block bid. However, in more realistic situations, the number of block bids in general, and the number of paradoxically rejected block bids that can be resolved by price decoupling account for only a small fraction of the total traded quantity. Therefore, as can be seen from Table 7, the average TTQ value increases by 0.03% and 0.02% for the two problem sets, respectively, and this is far from what we could see in Example 2.

We also examined how the PD approach affects the total social welfare in the case of realistic market scenarios. The average TSW values and deviations for the two test problem sets are shown in Table 8.

The results can also be evaluated based on the surplus values introduced in Section 2.1.1. In Table 9, we can see the effect of price

Table 7
Comparison of Total Traded Quantity (TTQ) under conventional and price decoupling models for the problem sets 24_140_524 and 24_140_1048. Averages (AVG) and standard deviations (DEV) are reported for each case.

	24_140_524		24_140_1048	
	AVG	DEV	AVG	DEV
TTQ conv.	10 851.48	72.88	11 118.57	58.67
TTQ PD	10 854.36	72.64	11 120.62	58.65
Absolute difference	2.88		2.05	
Increase (%)	0.03%		0.02%	

Table 8
Comparison of average TSW values (AVG) and their standard deviations (DEV) under the conventional and decoupled pricing models for the problem sets 24_140_524 and 24_140_1048.

	24_140_524		24_140_1048	
	AVG	DEV	AVG	DEV
TSW conv.	14 604.9333	69.8316	14 631.6879	72.1496
TSW PD	14 604.9385	69.8321	14 631.6909	72.1501
Absolute difference	0.0052		0.0029	
Increase (%)	$3.53 \cdot 10^{-5}$%		$2.01 \cdot 10^{-5}$%	

Table 9
Comparison of block bid (BB), simple demand (SD), and simple supply bid (SS) surpluses under conventional and price decoupling models for problem sets 24_140_524 and 24_140_1048. Averages (AVG) and standard deviations (DEV) are reported for each case.

	24_140_524		24_140_1048	
	AVG	DEV	AVG	DEV
BB surplus conv.	327.0903	27.4394	384.6646	47.2245
BB surplus PD	322.7674	26.6313	381.3453	46.5509
Absolute difference	4.3229		3.3193	
Decrease (%)	1.32%		0.86%	
SD surplus conv.	13 106.7582	80.6493	13 493.6424	76.8533
SD surplus PD	13 114.0404	80.9930	13 498.6005	76.4912
Absolute difference	7.2822		4.9581	
Increase (%)	0.06%		0.04%	
SS surplus conv.	1171.0848	33.8641	753.3809	27.3703
SS surplus PD	1168.1301	34.0367	751.7448	27.1334
Absolute difference	2.9548		1.6361	
Decrease (%)	0.25%		0.22%	

decoupling on the average and standard deviation of surplus values. The total surplus has been calculated for the three bid types: block bids (BB), simple supply (SS), and simple demand bids (SD).

The change in the overall TSW and surplus values is summarized in Table 10, while the average running times (in seconds) are shown in Table 11.

5. Discussion

5.1. Cross-period subsidization is necessary to increase TSW and total surplus via price decoupling

As illustrated by period 1 of Example 2 in Section 3.1.2, increasing the total traded quantity in some periods is necessary to resolve block orders originally paradoxically rejected. This is only possible via such a price decoupling, where $MCP_t^D < MCP_t^S$ that induces a revenue deficiency, as the total income from demand bids will be less than the total cost of supply bids for the particular period. According to inequality (17), this must be compensated by increasing the revenue in other periods — as in period 2 of Example 2. In other words, while in the conventional clearing, the TSW of each period is distributed among bids in that period (i.e., the total surplus of bids equals the TSW of the period for each period), in the case of PD, cross-period subsidization is necessary, if the TSW is increased via the resolution of block orders. This is an important aspect of the proposed approach, which, depending on the priorities of policymakers, may limit the practical application of the method.

Table 10
Comparison of total social welfare (TSW) and total surplus under conventional and price decoupling models for instances 24_140_524 and 24_140_1048.

	24_140_524	24_140_1048
TSW conv.	14 604.9333	14 631.6879
Total surplus conv.	14 604.9333	14 631.6879
TSW PD	14 604.9385	14 631.6909
Total surplus PD	14 604.9379	14 631.6906
Absolute difference	0.0006	0.0003

Table 11
Average runtimes (in seconds) for Problems 1–3 on problem sets 24_140_524 and 24_140_1048.

	24_140_524	24_140_1048
Problem 1	4.63	9.78
Problem 2	2.58	3.70
Problem 3	15.46	15.26

5.2. Evaluation of numerical results

As Table 6 shows, both the average number of paradoxically rejected block orders and the OCPR values are significantly decreased by the application of the PD-based clearing.

Regarding the results summarized in Tables 7, 8 and 10 of Section 4.3, it may be noted on the one hand that while the PD approach is capable to increase the TTQ, the TSW and the total surplus, the increase in the resulting TTQ and TSW values implied by the PD approach is limited in the case of the analyzed realistic market scenarios. This may be explained by multiple factors. In Example 2, described in 3.1, the resolution of the originally paradoxically rejected block order significantly affects the market outcome. While the total traded quantity (TTQ) increased from 13 to 20, i.e., by 53.85 percent, in contrast, in the case of the realistic examples, the average increase in the TTQ implied by the resolution of block orders, which were paradoxically rejected in the original setup is only 0.03 and 0.02% in average for the two problem sets, respectively, as most of the traded quantity originates from simple bids. This factor is determined by the test bid set, i.e., more precisely, the relation between the quantity parameters of block orders and simple (supply) bids.

Since the proposed approach may increase welfare only via the resolution of paradoxically rejected block orders, the potential of the PD method depends on the frequency of paradox rejection, which – due to the privacy of bid data – is hard to estimate in the case of real markets. The authors are aware of only one study, which publishes data about the rate of paradox rejection in the Belgian day-ahead market. Madani et al. [35] writes that *'Focusing only on block orders, in 2015, 1.3% or 357 of all the 26 425 offered supply and demand block orders were paradoxically rejected'*. While it is not possible to draw general conclusions about other markets based on this study, we may guess that while paradox rejection is definitely present in such markets, its significance may be limited.

On the other hand, constraint (18) also limits the TSW increase by excluding those dispatch configurations, where the TSW increase would imply a too high TR, thus decreasing the total surplus of participants.

In 51% of the examined cases, the total surplus deviates from the TSW by more than 1% of the TSW increase, i.e., the PD-based method does not necessarily imply the divergence of TSW and the total surplus.

Regarding the computational time of the proposed method in the case of larger problem instances, Table 11 shows that while the total computational time of the conventional method is about 5–10 s (Problem 1), the total running time of the proposed algorithm is about 22–27 s (the overall time of Problems 1, 2 and 3). This shows that, in the case of the test problem instances, the PD approach requires 2–3 times more computational effort compared to the conventional approach, while still providing a feasible numerical performance.

Let us note, however, that larger problem instances could present difficulties for standard MILP solvers, and the typical approach to efficiently solve these examples includes the use of state-of-the-art solvers like Euphemia [25] or other dedicated methods, using, e.g., Benders decomposition (see, e.g., [36]) In the case of the future application of the proposed method, the adjustment of these state-of-the-art approaches could be advisable according to the PD principle to efficiently deal with larger problem instances.

6. Conclusions and future work

In this work, we have illustrated that the PD-based market clearing can resolve paradoxically rejected block orders, thereby increasing the total social welfare (TSW) and the total surplus of bidders. While the proposed approach guarantees that the total surplus of participants in the case of price-decoupling is at least equal to the reference (non-price decoupled) case, it is possible that a part of the increment is realized as a nonzero revenue of the auctioneer. This auction revenue might be the subject of various policy decisions, i.e., it might be allocated to ensure sufficient supply of balancing and/or flexibility resources for the network as proposed by Sleisz et al. [19].

It has also been illustrated that the TSW-maximizing solution is not unique with respect to the decoupled clearing prices. Based on these considerations, we proposed a three-step framework, which calculates the reference MCPs in the first step, maximizes the welfare in the second step, and minimizes the deviation of MCPs from those of conventional clearing in the third step.

Numerical simulations on realistic data show that while the proposed method has limited potential to increase the overall TSW, the PD-based clearing reduces the number of paradoxically rejected block orders and the introduced OCPR measure by 32%–42%.

6.1. Future work

While the current work illustrates that the total value generated by the clearing (the TSW) may be increased by applying the PD approach, the proposed distribution of this value among participants is based on the outcome of the conventional model. Regarding step 3 of the proposed algorithm, forcing the resulting clearing prices – and thus the individual bid surpluses – to be as close as possible to the result of the traditional clearing is not the exclusive option to solve the price determination problem. As discussed in Section 2.1.3, the resulting bid surpluses in the result of the conventional clearing do not always align with fairness-related considerations. The non-uniqeness of MCPs in the PD model provides flexibility regarding the distribution of the value generated. While this value is maximized in step 2 of the proposed algorithm, step 3 may also be designed based on more complex considerations compared to the approach used in this paper. However, it is not trivial how to use the tools of cooperative game theory for examples with a large number of bids to achieve the most desired bid surplus values. The analysis of such approaches should be the subject of further studies. In addition, the aspect of incentive compatibility [37] also need to be considered in the design of such future pricing mechanisms.

As illustrated by Example 2 described in Section 3.1.2, in the case of the PD-based clearing, the total sum of bid surplus values is no longer necessarily equal to the total social welfare. Although the approach proposed in the current work, similarly to the traditional approach, aims to maximize the TSW in the conventional sense (as described by Eq. (13) in Problem 2), this approach may be reconsidered. A straightforward modification would be to consider the sum of individual bid surplus values as the objective function of the optimization. According to Eq. (11), the total surplus may be calculated as the difference between the TSW and the total revenue (total income minus the total cost); maximizing this difference, which may be easily obtained from the proposed formulation, would be a sufficient approach for this aim.

However, the above consideration will not necessarily hold in a multi-zone context. While according to the market-coupling model based on the traditional market clearing, prices of different zones will converge as long as the transmission constraints enable sufficient interzonal trading and generate congestion rent if a bottleneck appears, it is not clear what relations may be formulated in the relation of the total zonal incomes, costs, surpluses, TSW and the congestion rent in the case of a PD-based market coupling model. Although the flexibility offered by potentially distinct supply and demand prices may offer new possibilities to also address the market coupling problem, such models should be the subject of cautious future studies, considering all affected aspects and implications.

CRediT authorship contribution statement

Anita Varga: Writing – review & editing, Validation, Software, Methodology, Formal analysis, Data curation. **Botond Feczkó:** Visualization, Software, Methodology. **Marianna E.-Nagy:** Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Methodology, Formal analysis. **Dávid Csercsik:** Writing – review & editing, Writing – original draft, Visualization, Supervision, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: David Csercsik reports financial support was provided by National Research Development and Innovation Office. David Csercsik reports financial support was provided by Hungarian Academy of Sciences. Marianna E.-Nagy reports financial support was provided by Hungarian Academy of Sciences. The work of Anita Varga was partially supported by the Air Force Office of Scientific Research under award number FA9550-23-1-0370. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

See Table 12.

Appendix B

$$
\begin{aligned}
x_i > 0 \quad &\Rightarrow \quad M_i^D = x_i Q_i P_i \; + \; Q_i MCP_t^D - Q_i P_i \quad \leftrightarrow \\
x_i > 0 \quad &\Rightarrow \quad M_i^D \le x_i Q_i P_i \; + \; Q_i MCP_t^D - Q_i P_i \quad \& \\
x_i > 0 \quad &\Rightarrow \quad M_i^D \ge x_i Q_i P_i \; + \; Q_i MCP_t^D - Q_i P_i
\end{aligned}
$$

$$
\begin{aligned}
x_i - z_i &\le 0 \\
M_i^D - (1 - z_i)\overline{M}_i^D &\le x_i Q_i P_i \; + \; Q_i MCP_t^D - Q_i P_i \\
M_i^D + (1 - z_i)\overline{M}_i^D &\ge x_i Q_i P_i \; + \; Q_i MCP_t^D - Q_i P_i
\end{aligned}
$$

Table 12
List of abbreviations, parameters, and decision variables.

Abbreviations	
PX	Power exchange
IP	Incremental price
DAM	Day-ahead electricity market
MCP	Market-clearing price
PD	Price decoupling
TSW	Total social welfare
SP	Surplus
TR	Total revenue
OCPR	Opportunity cost of paradox rejection
Parameters	
SD	Index set of simple demand bids
SS	Index set of simple supply bids
BB	Index set of block bids
Q_i	Quantity of simple bid i
P_i	Price of bid i
r_i	Relevant period of simple bid i
$Q_{j,t}$	Quantity of block bid i in period t
MCP^C	Market-clearing prices obtained with conventional clearing
TSW^C	Total social welfare obtained by solving Problem 1
TSW^{PD}	Total social welfare obtained by solving Problem 2
SP_i	Surplus of bid i
$OCPR_C$	Opportunity cost of paradox rejection in the conventional case
$OCPR_{PD}$	Opportunity cost of paradox rejection with price decoupling
Decision variables	
$x_i \in [0,1]$	Acceptance variable of simple bid i
$y_i \in \{0,1\}$	Acceptance indicator of block bid i
MCP_t	Market-clearing price in period t (conventional model)
MCP_t^D	Market-clearing price in period t for the demand side
MCP_t^S	Market-clearing price in period t for the supply side
M_i^D, M_i^S, M_j^B	Auxiliary variables introduced to linearize (17)

Data availability

In silico data has been used for numerical tests, the generation method of these data is described in the article.

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