Heliyon 11 (2025) e43537

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Fuzzy model-based analysis of user feedback for product development insights

Anna Sudár [a,b,c,*], Borbála Berki [a,b,c], Ildikó Horváth [a,b,c]

[a] Corvinus Institute for Advanced Studies, Corvinus University of Budapest, Fővám tér 8, Budapest, 1093, Hungary
[b] Institute of Data Analytics and Information Systems, Corvinus University of Budapest, Fővám tér 8, Budapest, 1093, Hungary
[c] Hungarian Research Network, Piarista u. 4, Budapest, 1052, Hungary

ARTICLE INFO

Keywords:
Fuzzy modeling
Sparse datasets
User feedback
Interpretability
Product development

ABSTRACT

The topic of this paper belongs to the modeling of human-software experience to support product development. The primary objective of this paper is to present a structured modeling framework, incorporating user feedback to support software product development. The proposed modeling framework constructs a model that outlines the relationship between measurable parameters of objective and subjective user feedback and the evaluation metrics of software and user capability, motivated by the typical disparity between feedback parameters and capability metrics. A key benefit of the framework is its visually enhanced interpretability power. The resulting model is based on a manageable set of linguistic fuzzy rules, designed to remain at a level comprehensible to human intuition, thereby enabling developers to understand the primary causal relationships. A further benefit of the framework is its adaptability across the entire application lifecycle; it can initiate with minimal user feedback and is computationally equipped to handle the gradually increasing volume of feedback over time. Thus, this approach serves two critical purposes: first, to guide the development team in identifying features that require enhancement to better align with user needs; and second, to optimize the alignment between the application's functionalities and the diverse capabilities of its user base. Beyond this proposed framework, the primary novelty in the field of modeling lies in the special modification of the TS fuzzy model transformation method that enables the selection of principal components within the fuzzy rules and facilitates the use of interpretable fuzzy sets, along with a pseudo-inverse-based model adaptation that mitigates the computational load as the number of users grows. The effectiveness of the proposed framework is demonstrated through four use case examples. The first use case is in a hypothetical example for illustrative purposes to intuitively clarify the framework's functionality. The second models user demographics, digital content engagement, and VR platform adaptation preferences. The third addresses the subjective and objective challenges of users when performing tasks in 2D versus 3D environments. The fourth examines the experience of spatial presence in 3D environments.

1. Introduction

The primary objective of this paper is to propose a modeling framework that utilizes user feedback to better understand the users of a software product (app) to enhance further development. The paper demonstrates and evaluates this framework in various use-case

* Corresponding author at: Corvinus Institute for Advanced Studies, Corvinus University of Budapest, Fővám tér 8, Budapest, 1093, Hungary.
E-mail addresses: anna.sudar@uni-corvinus.hu (A. Sudár), borbala.berki@uni-corvinus.hu (B. Berki), ildiko.horvath@uni-corvinus.hu (I. Horváth).

https://doi.org/10.1016/j.heliyon.2025.e43537
Received 22 August 2024; Received in revised form 18 June 2025; Accepted 18 June 2025

Available online 27 June 2025
2405-8440/

scenarios. The proposed framework offers two key benefits. First, it produces a model with high visual and linguistic interpretability, making it accessible to the app development team, including members without a deeper mathematical background. Second, it adapts seamlessly throughout the app's entire lifecycle —from the initial stages, when only sparse user feedback is available, to later phases when abundant feedback supports its ongoing development and adjustment.

1.1. Goals and motivation of the paper

This paper proposes a modeling framework that describes the relationship between objective and subjective user feedback and various feature metrics of the app. This framework is motivated by the recognition that the information gathered from or provided by users differs significantly from the evaluation metrics and feature metrics used by the development team.

The primary goal of the modeling framework is to provide the development team with interpretable insights, rather than to derive a closed-form formula or a high-accuracy approximation from a trained neural network that functions as a black-box model, lacking interpretability.

- Thus, the goal of the proposed modeling framework produce a TS fuzzy model in which the antecedent fuzzy sets offer a visual interpretation of input sensitivity and meaningful categories, while the fuzzy rules explain the linguistic IF-THEN causal relationships within the model.
- The framework has a goal to offer a trade-off between model accuracy, in terms of approximation, and the number of antecedents and fuzzy rules, as an excessive number of these components becomes challenging for human comprehension.
- The modeling framework also aims to be equipped with mathematical solutions that allow it to adapt across the entire lifecycle of the app—from the early stages, when only sparse user feedback is available, to the later success phase, when large volumes of feedback enable further model refinement.

1.2. State of the art and literature overview

Product development often struggles with limited data, particularly in prototyping, where incomplete measurements hinder robust analysis and decision-making [1,2]. Strategies like targeted sampling [3] and data imputation [4] offer partial solutions but fail to adapt to real-time feedback shaped by socio-demographic and cognitive factors.

Product adoption depends heavily on user contexts, cognitive styles, and personalized experiences, which significantly complicates user modeling [5–7]. As the dimensions of user characteristics grow, capturing all potential combinations becomes increasingly challenging, making effective user modeling essential for guiding software design, optimizing adoption strategies, and addressing these complexities. The increasing complexity in user modeling and the demand for adaptive methodologies underscore the relevance of fuzzy-based multicriteria decision-making (FMCDM) approaches, which offer robust solutions to the challenges of uncertainty and sparse data in product design and decision-making contexts. Shahmohammad et al. [8] emphasize the need for interpretable, efficient FMCDM models for robust decision-making in dynamic settings. Parallel advances in AI-driven methods, like deep learning and neural networks, optimize product design decisions under data scarcity. Li et al. [9] and Zou et al. [10] showcase the power of deep residual networks and data-driven models in evaluating complex design criteria early in development. AI techniques, such as multidimensional neural networks [11] and collaborative deep learning [12,13], enhance FMCDM by enabling personalized recommendations and addressing user data inconsistencies. Applications like Wang and Slowik's [14] study of AI in consumer behavior illustrate FMCDM's relevance in consumer-focused domains. Adaptive algorithms further align FMCDM with dynamic user data, as seen in Nilashi et al.'s [15] use of neuro-fuzzy systems for accurate recommendations despite data challenges. The integration of AI with FMCDM promises advanced, adaptable tools for early-stage decision-making in new product development, excelling in environments with limited or evolving data.

The proposed method in this paper is compared to existing solutions across three hierarchical levels, focusing on interpretability for product development. Instead of assessing accuracy, this framework emphasizes relevant literature at each level to position our approach and clarify where our model provides unique contributions, particularly in handling sparse data challenges.

1.2.1. First level

The top-level summarizes findings that provide high-level guidance for product development stages, particularly when data is limited or complex to manage. Rather than directly comparing our proposed method, this level highlights foundational strategies from the literature on data management and adaptability in product development. The focus is on general recommendations, such as adaptable methods and the importance of interpretability, which serve to inform practical, accessible decision-making. Lee and Ahmed-Kristensen [16] identify four patterns of data-driven design activities that enhance flexibility in new product development, offering insights into adaptable approaches that are beneficial when data availability is constrained. Similarly, a recent paper examines the integration of product development data for further ontological analysis, underscoring structured data handling as a foundation for effective decision-making [17]. Wilberg et al. [18] address the need for comprehensive data strategies to improve interpretability and support variable data contexts, even during early development. Another study explores the use of product data in designing materials supply systems, providing adaptable strategies that help mitigate limitations posed by inconsistent data availability [19]. Together, these studies underscore the importance of flexibility and interpretability in data handling to guide practical, accessible decision-making in product development.

1.2.2. Second level

The second level focuses on user feedback-driven and cognitive modeling techniques, highlighting the benefits of these approaches for product development. We explore studies addressing the cognitive aspects of feedback-based modeling and compare these to traditional feedback and statistical methods, which often lack flexibility in low-data environments. In a recent study, researchers examine the cognitive aspects of feedback-based modeling, demonstrating how cognitive analysis can extract key concepts to enhance product design by aligning closely with user perceptions [20]. Kolbachev et al. [21] explore human cognitive methods in product design, underscoring the role of human factors in creating systems that are more responsive to user needs. Another study further integrates consumer preferences with external data through advanced modeling techniques, showcasing the value of incorporating both subjective feedback and broader contextual data for a refined design process [22]. Traditional statistical methods face limitations not only in product development but also in cases involving sparse or noisy data. Challenges like misinterpreting non-significant p-values [23,24], reliance on strict significance thresholds (e.g., $p < 0.05$) [25], sensitivity of p-values to sample size [25], and limitations in sample size recommendations—such as the commonly cited “magic number” of five participants in usability studies, which may miss critical insights [26]—can lead to misleading conclusions. These constraints highlight the need for models that can interpret variable data more flexibly, as standard statistical approaches often fall short in developmental contexts where data may be inconclusive and sample sizes limited. Such methods struggle to account for users’ nuanced, evolving requirements due to their rigid structure. Our fuzzy modeling framework approach addresses these limitations, adapting dynamically to user feedback to meet the interpretive needs of early development with greater effectiveness.

1.2.3. Third level

The third level closely aligns with our proposed TS fuzzy model, allowing for a comparison with similar approximation methods cited in the literature. In mathematical modeling, numerous approximation techniques can capture input-output relationships, yet many lack the interpretability required for practical applications. For our model, the development team prioritized features such as visually interpretable antecedent fuzzy sets, clear causality expressed through IF-THEN fuzzy rules, and a focused set of primary rules via principal component analysis to maintain clarity and usability. One category of techniques yields a closed-form formula; however, this approach does not provide the intuitive understanding necessary for those without a mathematical background. Similarly, artificial neural network-based methods, while powerful, operate as black-box models that fail to offer the transparent insights needed by the team. In contrast, our approach is grounded in the interpretability strengths of fuzzy logic, aligning with findings by Hsiao and Su [27], who demonstrated how fuzzy systems enhance decision-making by capturing nuanced relationships in a format accessible to non-experts. Additionally, another research highlights the value of multi-layer fuzzy inference systems in creating structured, interpretable models suitable for complex decision-making environments [28]. By drawing from these studies, our model is able to provide a mathematically robust yet transparent framework. This balance between rigor and interpretability ensures that the model remains accessible to the development team, supporting informed decision-making while maintaining analytical depth.

1.3. Novel contributions of the paper

The novel contribution of this paper from a modeling standpoint is twofold. First, the proposed modeling framework is built on the TS fuzzy model transformation where we extend the TS fuzzy model transformation [29–31] in more than one way. On the one hand, we replace the first step of the TS fuzzy model transformation (the rectangular gridding-based sampling step) with an interval gridding technique for projecting random located sparse data onto a rectangular grid whose density can be adaptively increased as more data is collected in subsequent stages. On the other hand, the higher-order singular value decomposition (HOSVD) that is carried out as a core step of the TS fuzzy model transformation is generally applied to the task of finding good approximation tradeoffs; however, in our case, we apply this step to finding useful tradeoffs between the precision and human interpretability of the resulting fuzzy rule set. Further, when determining the shape of the antecedent fuzzy sets – unlike in the existing literature on the TS fuzzy model transformation – is not to manipulate the convex hull, but rather to generate antecedent fuzzy sets that are as easily interpretable by the human eye as possible, in that they will preferably comprise “dominant” and “less dominant” components for any given input value combination.

From a practical standpoint, the novelty of this paper lies in its ability to provide development teams with a clear understanding of user needs and feedback. The model offers high visual and linguistic interpretability. This ensures that the insights derived from both objective and subjective feedback are easy to understand and apply in decision-making.

Another key strength of the model is its adaptability throughout the app’s lifecycle. In the early stages, when user feedback is sparse, the model effectively interprets limited data to guide initial development decisions. When critical information is missing, experts’ contribution plays a crucial role in filling these gaps, drawing on their experience to provide necessary insights. As the app evolves and user feedback becomes more abundant, the model seamlessly adjusts, ensuring its relevance and supporting ongoing sophistication and optimization.

Practical applications include recommendation systems for newly launched services with few initial users. Additionally, the method is suitable for other scenarios where inferences must be made based on sparse or noisy data, offering an adaptable tool for tackling challenges across different stages of product development. By bridging the gap between feedback data and capability metrics, the model empowers teams to make informed decisions, enhancing both the development process and the end-user experience.

Within the fuzzy model, fuzzy input sets are used to represent the relative dominance or subordinate of different dimension values in contributing to a general outcome. By integrating this model into a recommendation system, a flexible and adaptive tool is developed that can provide meaningful interpretations and actionable insights even with minimal data. In this context, the inhibitive

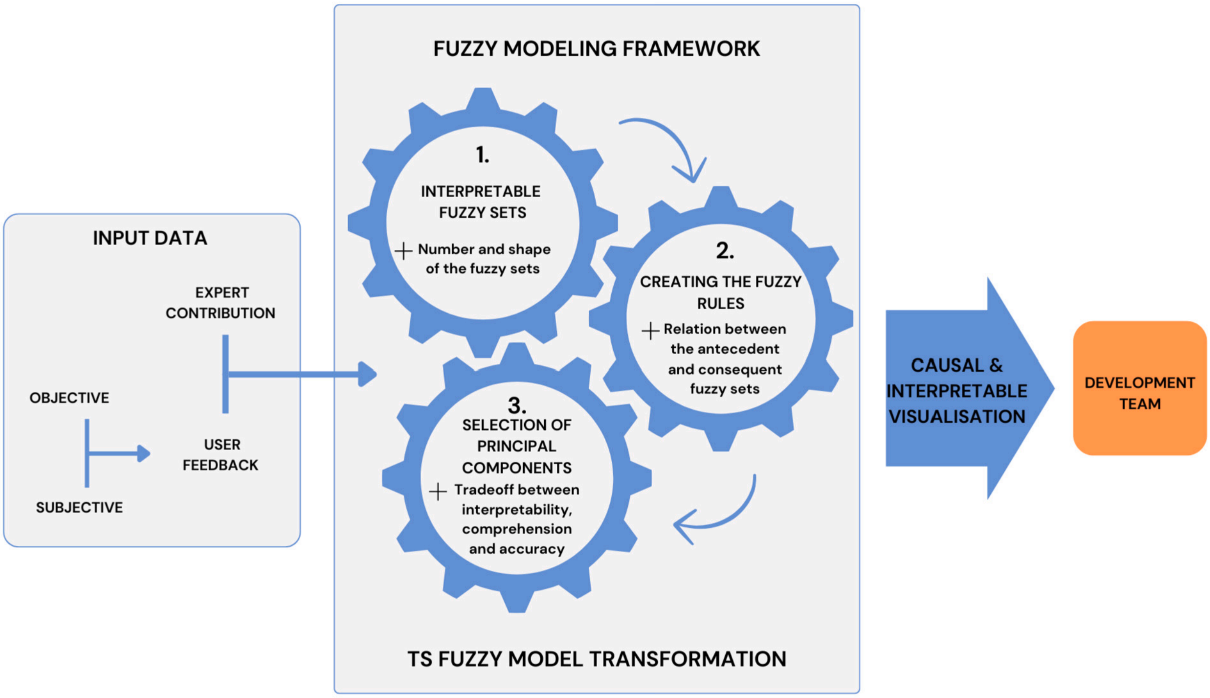

Fig. 1. Framework for Fuzzy Model-Based Analysis of User Feedback.

costs associated with user testing are highly relevant – initially, the number of users involved in testing is often limited to around 80-100 users per application. As the user base expands into the 1000s during a general launch, the model can be fine-tuned to ensure reliable performance. The mathematical foundations presented in this paper ensure the model's adaptability and robustness, even with a large and diverse user base.

Fig. 1 presents the proposed framework for fuzzy-model-based analysis of user feedback, integrating both objective and subjective data and expert contribution to enhance understanding of user needs. The process involves defining fuzzy sets, creating fuzzy rules and selecting principal components to balance interpretability, comprehension and accuracy. The outcomes is a causal and interpretable visualisation that supports decision-making and improvements throughout the app's lifecycle.

Conventional methods typically demand well-structured datasets to yield accurate predictions, yet these methods often break down in the face of the noise and uncertainty typical of early-stage product data. In contrast, the fuzzy model's unique ability to handle uncertainty and adapt to dynamic data environments makes it an ideal choice for early product development stages as well.

In the broadest sense, by presenting this fuzzy modeling framework, the paper aims to provide a valuable tool for enhancing the decision-making capabilities of researchers and developers working with sparse datasets.

1.4. Structure of the paper

The paper is structured as follows. In Section 2, we summarize the notations and concepts used in the paper. This section also introduces a set of symbols with specific meanings that are systematically used throughout the paper. These symbols are defined at the outset and consistently understood within this framework, ensuring clarity and uniformity in the mathematical exposition. This section also reviews essential concepts, such as the definition of the TS fuzzy model and its transfer function, which will be used later. Section 3 introduces the proposed framework, describing its specific objectives, such as achieving interpretable and adaptive user feedback models. This section details the framework's ability to evolve with increasing data availability, offering a novel variant of the TS fuzzy model transformation that handles sparse datasets and adapts to a large amount of feedback when it increases. Section 4 provides practical applications, starting with an illustrative example designed to intuitively clarify the workflow of the proposed framework. This example uses a hypothetical scenario to demonstrate how fuzzy membership functions can be applied to categorize user behaviors, offering a visual and conceptual introduction to the methodology. Following this, three real-world use cases demonstrate the framework's versatility in virtual reality (VR) development. These use cases explore key aspects such as demographics and digital content preferences, task complexities in 2D vs. 3D environments, and spatial presence in VR settings. Section 5 presents a comparative analysis, including Table 1, which highlights the differences and advantages of our proposed fuzzy modeling framework compared to traditional approaches in product development. Collectively, these sections demonstrate the framework's potential as a valuable decision-support tool.

2. Notations and basic concepts

In this section, we briefly summarize the mathematical notations and the key concepts used for the derivations in this paper.

2.1. Notations

- Lower-case letters of the English alphabet, such as $i, j, m, n, g \dots$ will be used as indices with lower bound 1 and an upper bound denoted by the corresponding upper-case character, e.g. $I, J, M, N, G \dots$. In case we have a number of dimensions to index, these can be differentiated between using a further lower index; e.g. $i_1, i_2, \dots i_n$ can take values between 1 and $I_1, I_2, \dots I_N$, respectively.
- The notations $a \in \mathbb{R}$, $\mathbf{a} \in \mathbb{R}^I$, $\mathbf{A} \in \mathbb{R}^{I_1 \times I_2}$, $\mathcal{A} \in \mathbb{R}^{I^N}$ are used to denote scalars, vectors, matrices and tensors, respectively, where the notation $\mathbb{R}^{I^N}$ is equivalent to $\mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$.
- $\mathbf{1}$ denotes a vector whose elements are all 1.
- $rank(\mathbf{A})$ denotes the rank of matrix $\mathbf{A}$.
- $rank_n(\mathcal{A})$ denotes the n-mode rank of tensor $\mathcal{A}$, see [32].
- $[\cdot]_{index}$ is used to refer to elements of a vector, matrix or tensor, as in e.g. $[\mathcal{A}]_{i_1,i_2,\dots i_N} = a_{i_1,i_2,\dots i_N}$ of $\mathcal{A}$.
- $\{\mathcal{A}\}_{(i)}$ denotes the i-mode unfolding of $\mathcal{A}$, see [32]. $\{\mathcal{A}\}_{(i)}$ is a matrix whose columns are the vectors from dimension i of $\mathcal{A}$. Thus the size of $\{\mathcal{A}\}_{(i)}$ is $I_i \times \prod_{n=1,n\neq i}^{N} I_n$.
- $\mathcal{A} \in co\{\forall n : \mathcal{B}_n\}$ denotes that $\mathcal{A}$ is within the convex hull defined by the vertices $\mathcal{B}_n$.
- $\mathbf{U}$ denotes an orthonormed matrix;
- $\mathbf{W}$ denotes a matrix that satisfies the properties:

$$\mathbf{1} = \mathbf{W}\mathbf{1} \quad \text{and} \quad \forall i, j : 0 \leq [\mathbf{W}]_{i,j}.$$

- $\mathbf{U} \rightarrow_{ctr} \mathbf{W}$ denotes the Convex Transformations that transform the orthonormed matrix $\mathbf{U}$ to $\mathbf{W}$. Such transformations in the TS Fuzzy model transformation related literature include the Sum Normalisation – Non Negativeness (SNNN), Normalized (NO), Close to Normalized (CNO), Relaxed Normalized (RNO), Inverse Normalized (INO), and the Inverse Relaxed Normalized (IRNO) transformations [29,30,33]. These transformations guarantee further characteristics of $\mathbf{W}$ that are advantageous in TS Fuzzy model based design.
- The notation $f(\mathbf{p})$ is used to represent a bounded continuous function $f(\mathbf{p}) \in \mathbb{R}^J$, where $\mathbf{p} \in \Omega \subset \mathbb{R}^N$. Here, Ω is defined by the intervals $\omega_1 \times \omega_2 \times \dots \times \omega_N$, where $p_n \in \omega_n$, and ω_n denotes the interval $[\omega_n^{min}, \omega_n^{max}]$.

2.2. Basic concepts

This section reviews two fundamental concepts from the fuzzy literature to be utilized in the proposed framework.

Definition 1. TS Fuzzy model

We recall that the widely used form of the fuzzy rules in the TS fuzzy model is as follows [34,35,29–31]:

$$\text{IF } A_{1,i_1} \text{ AND } A_{2,i_2} \dots \text{AND } A_{N,i_N} \text{ THEN } B_{i_1,i_2,\dots i_N},$$

where N denotes the number of inputs of the TS fuzzy model and the antecedent fuzzy sets are denoted by A_{n,i_n}. The consequents are denoted by $B_{i_1,i_2,\dots i_N}$. The antecedent fuzzy set A_{n,i_n} is defined by the membership function $w_{n,i_n}(p_n)$. The antecedent fuzzy sets are given in a Rusini-partition, which means that:

$$\forall n, p : \mathbf{w}_n(p)\mathbf{1} = 1; \quad \forall n, i, p : 0 \leq [\mathbf{w}_n(p)]_i,$$

where $\mathbf{w}_n(p) = \begin{bmatrix} w_{n,1}(p) & w_{n,2}(p) & \dots & w_{n,I}(p) \end{bmatrix}$ is a vector of the membership functions. The consequents $B_{i_1,i_2,\dots i_N}$ can represent scalar or vector (in case of multi-output) denoted as $\mathbf{s}_{i_1,i_2,\dots i_N} \in \mathbb{R}^J$ and structured in a tensor $\mathcal{S} \in \mathbb{R}^{I^N \times J}$. Further, let the observation fuzzy sets be singleton sets over the input value p_n, as is also widely adopted in the case of TS fuzzy models.

Definition 2. Transfer function of a TS Fuzzy model

When product-sum-gravity inference operator is used then the transfer function of the TS Fuzzy model defined in Definition 1 is [34,35,29–31]:

$$f(\mathbf{p}) = \sum_{i_1}^{I_1} \sum_{i_2}^{I_2} \dots \sum_{i_N}^{I_N} \prod_{n=1}^{N} w_{n,i_n}(p_n)[\mathcal{S}]_{i_1,i_2,\dots i_N}.$$

This transfer function can also be expressed using the tensor product operation as [29–31]:

$$f(\mathbf{p}) = \mathcal{S} \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{w}_n(p_n). \tag{1}$$

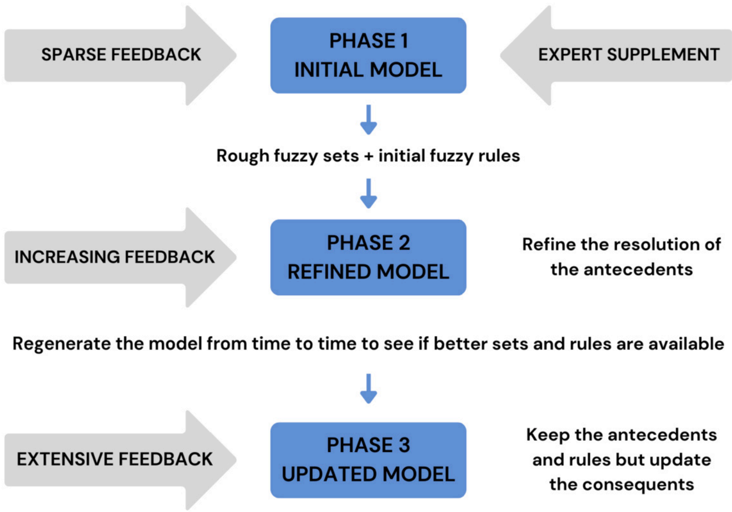

Fig. 2. Phases of the modeling framework.

The tensor product operation utilized in Eq. (1) is a concept that was introduced in tensor algebra in [32]. Since then, Eq. (1) has been widely employed in the literature on TP model and TS Fuzzy model transformations [29–31].

3. The proposed modeling framework

First of all, let us outline the main properties that the proposed method should address.

- **Visualising**
  One of the most important properties is that the resulting model should describe the input in a very informative and visualizable way:
  - **Visualising of the input categories:** It should classify the input into categories.
  - **Determining the membership values of input elements in different categories:** Then it should determine and visualise the most dominant elements of these categories and the overlapping of these categories. These are actually forming the antecedent fuzzy sets of the resulting model.
  - **Balancing between accuracy - number of categories - human comprehension:** The method should aim to minimise the number of categories, as a high number of categories can hinder human comprehension. Therefore, the model must balance reducing the number of principal categories with maintaining accuracy. The focus is on enabling an intuitive understanding of input membership within these categories, even if this requires a slight trade-off in precision, given that the output represents a distribution rather than an exact value.
  - **Noise filtering:** The function to be modeled exhibits a significant degree of variability, as user feedback may differ for the same case, even from a single user. This means that noise-like variations between input and output must be filtered out. This requirement aligns with the need for a trade-off between filtering out noise and details with small relevance while retaining the essential, principal elements of the information. This is closely related to the previous point; however, the focus here is not on identifying the main components but on filtering out the minor ones.
  - **Identifying causality:** To enhance human understanding of the process between input and output, we need to identify linguistically explainable IF-THEN rules, such as ‘IF input, THEN output.’ This approach allows the development team to readily grasp how inputs influence outputs. The goal here is to retain only the principal rules, avoiding an overwhelming number that could obscure clarity in unnecessary details.
  - **Refining and updating with stable computational load:** The model should be able to operate effectively with both small and large datasets, ensuring that computational load remains manageable even as the dataset grows significantly. Furthermore, the model should allow revisiting the visualization of input categories and causal rules, as these features in the initial small dataset may differ significantly from those in the subsequently larger dataset.

In the following let us discuss the three main phases in the lifecycle of the software product. Fig. 2 depicts the main phases.

- **Phase 1: Launch preparation**
  In this phase, the app is not released. It is under user testing. Since performing measurements based on a large user pool can be costly, the number of users involved in this phase of data collection is generally limited to about 80-100 users. Thus the initial TS fuzzy model is created on sparse data set. Those areas of the input space (e.g. border of the input space) that are not covered by user feedback are supplemented with expert opinion.

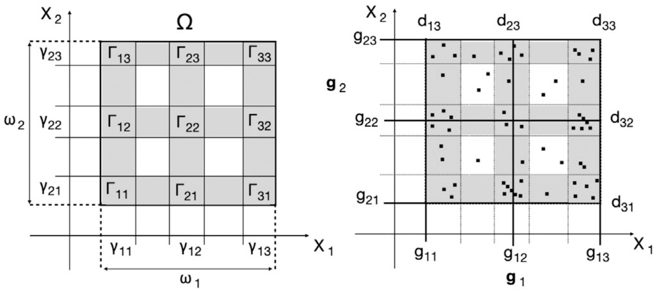

Fig. 3. Illustration of a 2-dimensional interval-grid and one way in which this interval grid could be transformed into a grid.

- **Phase 2: Refining the TS fuzzy model**
  In this phase, the app is released and the number of users can be expected to increase into the thousands. The goal is to update the initial model, while keeping the number of antecedent fuzzy sets low and only the resolution of the antecedent fuzzy sets is improved.
- **Phase 3: Updating the TS fuzzy model**
  In this phase, the number of users is around a million – hence, the number of data points obtained in real time is very high. This is especially true when modeling an increasing number of apps is done in parallel. As a result, the feature space for product assessment is well covered by the data points, therefore automatic updates and refinements can be made to the TS fuzzy model on a regular basis.

Before introducing the key steps of the proposed method let us define the following concepts.

Definition 3. Interval grid

Let us consider an N-dimensional hypercube denoted by $\Omega = \omega_1 \times \omega_2 \ldots \times \omega_N$. Assume that for each dimension, n, we have G_n disjoint (non-overlapping) interval- grid $\gamma_{n,g_n} = [\gamma^{min}_{n,g_n}, \gamma^{max}_{n,g_n}] \subset \omega_n$, where $\forall n, g : \gamma^{max}_{n,g} < \gamma^{min}_{n,g+1}$. Based on the above, the corresponding interval-grid is defined by grid cells:

$$\Gamma_{g_1,g_2,\ldots g_N} = \gamma_{1,g_1} \times \gamma_{2,g_2} \times \ldots \times \gamma_{N,g_N}.$$

The interval-grid covers Ω:

$$\forall n : \omega^{min}_n = \gamma^{min}_{n,1}; \text{ and } \omega^{max}_n = \gamma^{max}_{n,G_n}.$$

The left-hand side in Fig. 3 shows an illustration of Ω defined by intervals $\omega_1 \times \omega_2$ and the grid intervals $\gamma_{1,g}$ and $\gamma_{2,g}$, which in turn determine grid cells Γ_{g_1,g_2} for an $N = 2$ dimensional case. For the sake of generality, we can see that there is a gap between the grid intervals in the figure. In practical scenarios, however, the union of intervals would cover the whole space Ω.

Method 1. Transforming the interval-grid to grid tensor

Let each grid interval $\gamma_{n,g}$ be represented by its center (or any otherwise dominant point). Let these points be stored in vector $\mathbf{g}_n \in \mathbb{R}^{G_n}$), i.e.:

$$g_{n,g} = \frac{\gamma^{min}_{n,g} + \gamma^{max}_{n,g}}{2}$$

except at the border of hypercube edge ω_n, where

$$g_{n,1} = \omega^{min}_n \quad \text{and} \quad g_{n,G} = \omega^{max}_n.$$

These points of the grid intervals define the coordinates of the grid cell $\Gamma_{g_1,g_2,\ldots g_N}$ as:

$$\mathbf{d}_{g_1,g_2,\ldots g_N} = \begin{bmatrix} g_{1,g_1} & g_{2,g_2} & \ldots & g_{N,g_N} \end{bmatrix} \in \mathbb{R}^N.$$

These coordinate vectors form the grid tensor $\mathcal{D} \in \mathbb{R}^{G^{N \times N}}$.

The right-hand side of Fig. 3 illustrates how the interval grid is replaced by a grid. It shows that each interval $\gamma_{n,g}$ is replaced by grid $g_{n,g}$ on dimension n. It also illustrates that g_{1,g_1} and g_{2,g_2} define the coordinate vectors $d_{g_1,g_2} = \begin{bmatrix} g_{1,g_2} & g_{2,g_2} \end{bmatrix}$. Note that in this case, what we have done is to transform the interval-grid to a grid that is defined by a grid tensor (the coordinates of the tensor correspond to grid intervals that are not necessarily equidistant and may span differently sized ranges).

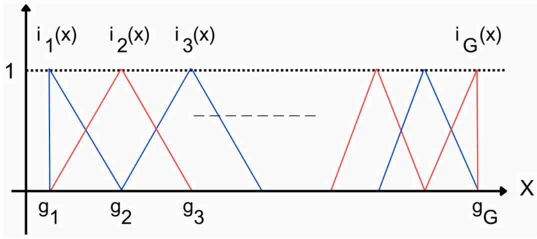

Fig. 4. Illustration of the zero order membership functions $i_g(x)$.

Method 2. Determination of the representative tensor with increasing resolution

Consider a function $y = f(\mathbf{p})$ whose closed formula is unknown, but is sampled over a point-cloud consisting of K different arbitrary points $\mathbf{p}_k = \begin{bmatrix} p_{k,1} & p_{k,2} \end{bmatrix} \in \Omega$ as $y_k = f(\mathbf{p}_k)$ as depicted by points on the right image of Fig. 3. We use the term "sampled" to signify that the function f unknown in analytical terms. When the interval-grid is constructed, a key requirement is to ensure that each grid cell $\Gamma_{g_1,g_2,\dots g_N}$ includes at least t^{min} different points $\{\mathbf{p}_k\}_{g_1,g_2,\dots g_N}$. Now, let us select y_t the closets to the grid point $\mathbf{d}_{g_1,g_2,\dots g_N}$, and define

$$f_{g_1,g_2,\dots g_N} = y_t.$$

If we have Z different y_z over the same point $\mathbf{p}_t$ then we use the average as

$$f_{g_1,g_2,\dots g_N} = \frac{\sum_{t=z}^{Z} y_z}{Z}.$$

More generally, $f_{g_1,g_2,\dots g_N}$ can be defined as any aggregate of y_t or any 'representative' value of the grid cell $\Gamma_{g_1,g_2,\dots g_N}$. These values $f_{g_1,g_2,\dots g_N}$ can be used to construct the representative tensor:

$$[\mathcal{F}]_{g_1,g_2,\dots g_N} = f_{g_1,g_2,\dots g_N}.$$

Thus, the representative tensor $\mathcal{F}$ and the grid tensor $\mathcal{D}$ is assigned to each other element-wise, and hence, the $f_{g_1,g_2,\dots g_N}$ can then be used as a rough estimate of the function over the grid as:

$$f([\mathcal{D}]_{g_1,g_2,\dots g_N}) \approx \hat{f}([\mathcal{D}]_{g_1,g_2,\dots g_N}) = [\mathcal{F}]_{g_1,g_2,\dots g_N}, \tag{2}$$

where the hat over $\mathcal{D}$ denotes "approximation". When K is increasing we split the cells (keeping T^{min} in each new cell). As a consequence the density of the grid defined by $\mathcal{D}$ and the size of the representative grid tensor will in turn increase. This leads to a better resolution of the approximation in Eq. (2). As a consequence the representative grid tensor will converge to a sampled tensor over a dense grid defined by $\mathcal{D}$ with increasing K.

The right-hand side of Fig. 3 shows a case where each grid cell has at least $t^{min} = 1$ sampled data points (dots in the figure).

Definition 4. Zero order membership function system

Let us consider a vector $\mathbf{i}_n(x_n) = \begin{bmatrix} i_{n,1}(x_n) & i_{n,2}(x_n) & \dots i_{n,G_n}(x_n) \end{bmatrix}$ defined over a given grid as an interpolation between grid points:

$$\mathbf{i}_n(x_n) = \lambda_n[\mathbf{I}]_g + (1-\lambda_n)[\mathbf{I}]_{g+1}, \quad \text{where} \quad \lambda_n = \frac{g_{n,g+1} - x_n}{g_{n,g+1} - g_{n,g}},$$

and $[\mathbf{g}]_g \le x_n \le [\mathbf{g}]_{g+1}$, and $[\mathbf{I}]_g$ represents the g-th row of the identity matrix $\mathbf{I}$. Fig. 4 illustrates the functions $i_n(x_n)$ for $n = 1$.

3.1. Determination of the TS fuzzy model

Based on the above, let us consider a function $f(\mathbf{x})$ whose closed formula is unknown, but its output is known over K different samples $\mathbf{p}_k$ as $y_k = f(\mathbf{p}_k)$. The proposed modeling framework has the following steps:

- **Step 1: Determination of the representative tensor**

Method 3. Method of Step 1

Let us define the hypercube interval grid γ_{n,g_n} in such a way that each cell $\Gamma_{n_1,n_2,\dots n_G}$ contains, at a minimum, t^{min} different points. Further, let us assume that the resolution of the grid tensor $\mathcal{F}$ is acceptable in the given phase of the app development. In this case, a multi-linear approximation can be determined as:

$$\hat{f}(\mathbf{p}) = \mathcal{F} \boxtimes_{n=1}^{N} \mathbf{i}_n(p_n), \tag{3}$$

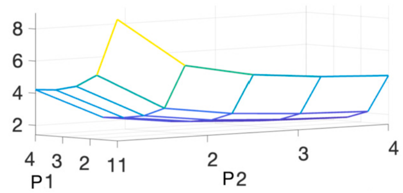

Fig. 5. Illustration of the multi-linear approximation based on the representative matrix.

based on grid tensor $\mathcal{F}$ and grid $\mathcal{D}$.

As an illustrative example, consider the hypercube $\Omega = \begin{bmatrix}1 & 4\end{bmatrix} \times \begin{bmatrix}1 & 4\end{bmatrix}$. Further, assume that 5×5 interval grid is defined. Further, consider the function:

$$f(\mathbf{p}) = \left(1 + p_1^{-2} + p_2^{-1.5}\right)^2, \quad \text{where} \quad \mathbf{p} = \begin{bmatrix}p_1 & p_2\end{bmatrix}. \tag{4}$$

Fig. 5 illustrates a 2-dimensional case based on the multi-linear approximation of the function given in Eq. (4) in which the representative tensor is a matrix and has the following values:

$$\mathcal{F} = \begin{bmatrix} 8.9761 & 5.7940 & 4.8896 & 4.5693 & 4.4568 \\ 5.2776 & 3.1042 & 2.4846 & 2.2444 & 2.0902 \\ 4.5207 & 2.5321 & 2.0040 & 1.7714 & 1.6558 \\ 4.2885 & 2.3401 & 1.8141 & 1.6011 & 1.4887 \\ 4.1596 & 2.2390 & 1.7364 & 1.5207 & 1.4102 \end{bmatrix}, \tag{5}$$

and in which case the grid vectors are $\mathbf{g}_1 = \mathbf{g}_2 = \begin{bmatrix}1 & 2 & 3 & 4 & 5\end{bmatrix}$ in dimensions p_1 and p_2 respectively. Note that this representation arises as an interpolation between neighboring values in a matrix (tensor), whose coordinates are interpreted over a grid tensor.

- **Step 2: Select the main components to enhance interpretability for human comprehension.**

Method 4. Method of Step 2
Let us execute the compact HOSVD (CHOSVD) on tensor $\mathcal{F}$, thus obtaining:

$$\mathcal{F} \overset{chosvd}{=\!=} \mathcal{D} \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{U}_n. \tag{6}$$

Here, the core tensor $\mathcal{D}$ has the size of $R_1 \times R_2 \times \ldots \times R_N$ and orthonormed matrices $\mathbf{U}_n$ have the size $G_n \times R_n$, where $R_n = rank_n(\mathcal{F})$. Each row of matrices $\mathbf{U}_n$ defines the weights of the elements (subtensors) of the core tensor to be combined to obtain each element of $\mathcal{F}$, such as

$$[\mathcal{F}]_{g_1, g_2, \ldots g_N} = \mathcal{D} \underset{n=1}{\overset{N}{\boxtimes}} [\mathbf{U}_n]_{g_n},$$

where $[\mathbf{U}_n]_g$ is the g-th row of matrix $\mathbf{U}_n$. One can create the multi-linear approximation based on the TP structure. Based on Eqs. (3) and (6), we have that:

$$\hat{f}(\mathbf{x}) = \left(\mathcal{D} \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{U}_n\right) \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{i}_n(x_n) = \mathcal{D} \underset{n=1}{\overset{N}{\boxtimes}} \overline{\mathbf{u}}_n(x_n), \quad \text{where} \quad \overline{\mathbf{u}}_n(x_n) = \mathbf{i}_n(x_n)\mathbf{U}_n. \tag{7}$$

The CHOSVD guarantees that the number of the piece-wise linear weighting functions is the absolute minimum. If one would like to further decrease the number of weighting functions $\mathbf{u}_n(x_n)$ by dimensions (for example, with the goal of increasing the human interpretability of the resulting fuzzy rule set), then non-zero singular values and the corresponding columns of $\mathbf{U}_n$ can be discarded. As a trade-off, this will result in further approximation errors over the grid, which are bounded by the sum of the discarded singular values. For further details on the rank and approximation trade-off of the TP model transformation is given in [36–39].
Thus, assume that we finally keep I_n singular values by dimension, which will lead to I_n number of antecedent fuzzy sets by dimensions and I^N number of fuzzy rules in the TS fuzzy model at the end. Therefore the size of $\mathcal{D}$ will be $I_1 \times I_2 \times \ldots \times I_N \times J$ and the size of $\mathbf{U}_n$ will be $G_n \times I_n$.

Fig. 6 illustrates a case where the weighting functions $\overline{\mathbf{u}}_n(x_n) = \begin{bmatrix}u_{n,1}(x_n) & u_{n,2}(x_n) & u_{n,3}(x_n)\end{bmatrix}$ are obtained from Eq. (5). Since $\mathcal{F}$ in this case is a matrix, SVD (instead of HOSVD) is applied, resulting in:

$$\mathcal{F} = \mathbf{U}_1 \mathbf{D} \mathbf{U}_2^T.$$

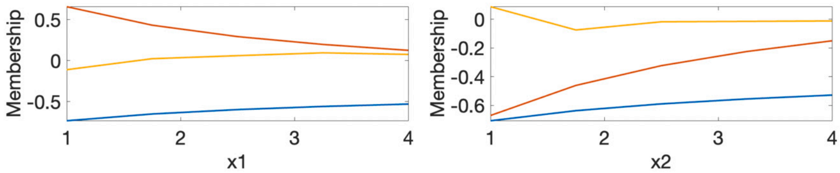

Fig. 6. Illustration of the weighting functions whose values may not be between 0 and 1, and, hence, may not interpretable as fuzzy sets.

Since the range of $\mathcal{F}$ is $R_1 = R_2 = 3$ in this case, this will result in only 3 antecedent fuzzy sets per dimension (9 fuzzy rules in total), which can be readily understood by humans. Therefore, we keep all non-zero singular values. As a result we have:

$$\mathbf{U}_1 = \begin{bmatrix} -0.7280 & 0.6720 & -0.0997 \\ -0.6481 & 0.4347 & 0.0964 \\ -0.5960 & 0.2925 & 0.1275 \\ -0.5589 & 0.1934 & 0.1338 \\ -0.5307 & 0.1231 & 0.1026 \end{bmatrix} \quad \text{and} \quad \mathbf{U}_2 = \begin{bmatrix} -0.6988 & -0.6924 & 0.1332 \\ -0.6331 & -0.4583 & -0.0993 \\ -0.5872 & -0.3208 & -0.1092 \\ -0.5538 & -0.2202 & -0.1061 \\ -0.5276 & -0.1477 & -0.0796 \end{bmatrix}$$

The functions $\mathbf{i}_n(x_n)\mathbf{U}_n$ are shown in Fig. 6. Note that these functions might be interpretable as Fuzzy membership functions representing the degree to which different subtensors should be taken into account, but for the fact that they are not scaled between 0 and 1 (as is required of membership functions).

- **Step 3: Transforming to Ruspini-partitioned CNO type antecedent fuzzy sets**
Since matrices $\mathbf{U}_n$ are real-valued and can contain even negative elements, the resulting piece-wise linear weighting functions in $\mathbf{u}_n(x_n)$ can also have negative values as illustrated above. In order to obtain a TS fuzzy model with Ruspini-partitioned fuzzy sets, the piece-wise weighting functions are further transformed by Convex Transformations. In order to obtain fuzzy sets whose dominant elements are emphasised via higher membership values, leading to a representation that is closer to human intuition, the Close-to-Normal (CNO) type Convex Transformation is applied here [29,30,33]. The CNO type Convex Transformation increases the membership values of the functions to 1 at least over one element. If this is not possible, then it gets as close to one as possible. This means that the dominant elements of each antecedent fuzzy set will be emphasized by higher membership values and over those elements the other fuzzy sets will have lower or even zero membership values. This helps to extract classifications of the elements, expressed through more understandable and visualizable membership functions, clarifying their role in the TS fuzzy model. Thus the following method is proposed:

Method 5. Method of Step 3
We execute convex transformation here as

$$\mathbf{U}_n \underset{ctr}{\rightarrow} \mathbf{W}_n,$$

Here, $\mathbf{W}_n$ still satisfy Eq. (7), and at least one element of each column will equal 1 or will get as close as possible to 1. Note that the Convex Transformation may increase the number of columns by one, therefore, the size of $\mathbf{W}_n$ is $G_n \times T_n$, where $I_n \leq T_n \leq I_n + 1$. The core tensor is calculated as

$$\mathcal{S} = \mathcal{D} \underset{n=1}{\overset{N}{\boxtimes}} \left(\mathbf{W}_n\right)^+, \quad \text{therefore} \quad \mathcal{F} = \mathcal{S} \underset{n=1}{\overset{N}{\boxtimes}} \mathbf{W}_n. \tag{8}$$

Based on Eqs. (3) and (8), this leads to

$$\hat{f}(\mathbf{x}) = \mathcal{S} \underset{n=1}{\overset{N}{\boxtimes}} \overline{\mathbf{w}}_n(x_n), \quad \text{where} \quad \mathbf{w}_n(x_n) = \mathbf{i}_n(x_n)\mathbf{W}_n. \tag{9}$$

Continuing our example, Fig. 7 illustrates the antecedent fuzzy sets $\overline{\mathbf{w}}_n(x_n) = \begin{bmatrix} w_{n,1}(x_n) & w_{n,2}(x_n) & w_{n,3}(x_n) \end{bmatrix}$. In the current very simple case, we arrived at the same model as Eq. (3). The resulting matrices $\mathbf{W}_n$ are:

$$\mathbf{W}_1 = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 \\ 0.8368 & 0.0779 & 0.0853 \\ 0.7075 & 0.1977 & 0.0948 \\ 0.6167 & 0.2802 & 0.1030 \\ 0.5485 & 0.3420 & 0.1096 \end{bmatrix} \quad \text{and} \quad \mathbf{W}_2 = \begin{bmatrix} -0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.1839 & 0.8161 \\ 0.0119 & 0.2947 & 0.6934 \\ 0.0223 & 0.3736 & 0.6041 \\ 0.0664 & 0.3966 & 0.5371 \end{bmatrix}$$

The figures demonstrate that the weighting functions have no negative values and have a dominant maximum or minimum value. This example demonstrates that the inputs can be effectively classified into at least 3 antecedents. It further shows that one antecedent strongly dominates at an input value of 1, while the other two categories exhibit low dominance across the entire input range.

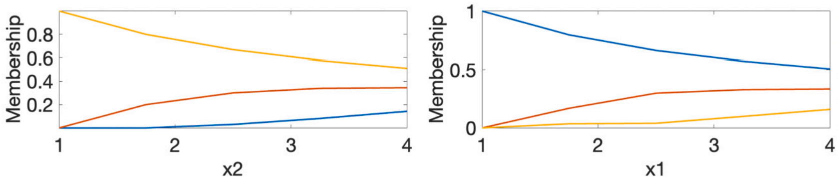

Fig. 7. Illustration of the CNO type membership functions.

- **Step 4: Refining the TS Fuzzy model in Phase 2**
  As the number of points for K increases, we can split the grid intervals, leading to a larger grid tensor and representative tensor. This results in antecedent fuzzy sets defined over a denser grid, making the piecewise linearly interpolated membership functions of the antecedents smoother.

  Method 6. Method of Step 4
  Assume that we introduce a new grid on dimension z. Let us denote the location of this extra grid by $g_{z,e}$. The values of the membership functions on dimension z - that is the new row $[\mathbf{W}_z]_e$ to be inserted into $\mathbf{W}_z$, which can then be determined over $g_{1,e}$. Since

$$\left(\mathcal{S} \underset{n=1,n\neq z}{\overset{N}{\boxtimes}} \mathbf{W}_n\right) \times_z [\mathbf{W}]_{z,e} = \mathcal{F}', \quad \text{that is} \quad \{\mathcal{F}'\}_{(z)} = [\mathbf{W}]_{z,e} \left\{\mathcal{S} \underset{n=1,n\neq z}{\overset{N}{\boxtimes}} \mathbf{W}_n\right\}_{(z)}.$$

  Here, $\mathcal{F}'$ is the new slice to be inserted in the z-th dimension, positioned between the $(e-1)$-th and $(e+1)$-th elements of $\mathcal{F}$. Its size is $G_1 \times G_2 \times \ldots \times G_{z-1} \times 1 \times G_{z+1} \times \ldots \times G_N$. Therefore

$$[\mathbf{W}]_{z,e} = \{\mathcal{F}'\}_{(z)} \left(\left\{\mathcal{S} \underset{n=1,n\neq z}{\overset{N}{\boxtimes}} \mathbf{W}_{n_{(z)}}\right\}\right)^{+}.$$

- **Step 5: Updating the TS fuzzy model in Phase 3**
  In the first two phases of the app development, we have approximately a thousand data points, and the size of the corresponding representation tensor results in an acceptable computational load for executing HOSVD. This means that the HOSVD can be periodically executed to identify better or additional antecedents, if they exist. However, when the number of points is very high, resulting in a dense grid and high-resolution representation tensor, executing the HOSVD becomes computationally challenging. Since the causal relationship between the input and output of the TS fuzzy model is well defined by the fuzzy rules and the shape of the antecedents at the end of Phase 2, we may retain the existing rules and antecedents, focusing only on updating the TS fuzzy model to improve its accuracy in Phase 3. Therefore the following method is proposed here:

  Method 7. Method of Step 5
  We use the pseudo TP model transformation to recalculate the core tensor $\mathcal{S}$ based on the updated representative tensor $\mathcal{F}$ as follows:

$$\mathcal{S}' = \mathcal{F}' \overset{N}{\underset{n=1}{\boxtimes}} \mathbf{W}^{+}.$$

  This is a simple tensor product computation that does not require executing HOSVD.

4. Illustrative showcase of the workflow

The dataset analysed in this study has also been used in prior works [40,41]. However, this study presents a new analysis of previously collected data, which serves as the basis for the novel approach and interpretation introduced in this manuscript.

In this section, we present four examples that demonstrate the practical application of the framework discussed in Section 3. The first example offering a real-life scenario to clarify the workflow outlined in our framework, though it is not based on specific measurement data. This example aims to provide a clear and intuitive understanding of the framework's functionality.

Following this, three additional examples apply the fuzzy modeling framework to real measurement-based use cases, each highlighting different facets of virtual reality development. These case studies explore key questions relevant to VR developers and sales professionals: Who are the ideal potential customers? Which VR work environments should be recommended for each target group? How do task complexity and workflows impact VR usage? And what approaches can help identify the best-suited target audience to optimize sales? The topic of each section will be as follows:

Demographics, Digital Content, and VR Adoption Section 4.2 explores how initially sparse data on user demographics and digital content usage can still be used to understand VR platform adoption. The system provides early insights into user acceptance, highlighting key factors influencing VR adoption across different age groups and content usage patterns.

Task Difficulty in 2D vs. 3D Section 4.3 examines the system's ability to assess the subjective and objective difficulties of tasks performed in 2D versus 3D environments. By analyzing user interactions and performance in different virtual settings, the recommendation system offers valuable feedback on the optimal environment for various educational and work-related tasks.

Spatial Presence Analysis Section 4.4 focuses on analyzing spatial presence scores based on users' navigational and previous VR experiences. This use case illustrates how the system can adapt to users' abilities and continuously refine recommendations, ensuring an optimized and effective VR experience.

Collectively, these use cases demonstrate the flexibility and adaptability of the recommendation system. By integrating diverse data sources and evolving through continuous updates, the system offers actionable insights that can significantly enhance decision-making processes. Experts can apply these recommendations to make informed decisions about deployment and optimization, ensuring an efficient and user-centric approach to product adoption.

4.1. Illustrative showcase I.: Age-based fuzzy membership function for health recommendations

Imagine an app that offers health recommendations based on user behavior. The age of users might be a significant input, as different age groups have varying needs and behaviors. In this case, we might define fuzzy membership functions for three age categories: "young," "middle-aged," and "elderly."

- Young (0–30 years): This membership function peaks at a lower age and gradually decreases toward middle age. Young users may prioritize fitness and social features, so this input could influence the app to highlight those features in recommendations.
- Middle-aged (30–60 years): This membership function reaches its peak around middle age and gradually decreases toward young and elderly categories. Middle-aged users may have different preferences, such as balancing health recommendations for managing stress or work-life balance. Here, the input would guide the app to provide content relevant to this group.
- Elderly (60+ years): The membership function peaks for ages 60 and above, capturing needs that may differ significantly from younger groups. Elderly users might prioritize health monitoring or features to support physical wellness. For these users, the input influences the app to emphasize features aligned with their specific requirements.

By visualizing these membership functions, we gain a clearer picture of how age influences the model. If we see a strong overlap between "middle-aged" and "elderly" categories, this suggests the app could adapt features to reflect the shared priorities of these groups. Alternatively, if the overlap is minimal, the model recognizes age as a distinct factor in user needs, signaling the development team to design specific features for each age group. This visualized distribution ultimately explains the role and importance of age in influencing the app's behavior and recommendations.

There are two main reasons for using principal component analysis to extract only the most important IF-THEN rules. First, a high number of antecedents for each input reduces human comprehension; too many antecedents are challenging for humans to interpret. Second, the number of fuzzy rules increases exponentially with the number of antecedent fuzzy sets, making the task overly complex and hindering intuitive understanding.

Another important objective is to filter out noise-like information resulting from distributional variance. Therefore, a balance must be struck between retaining principal components and removing noise-like elements, all while ensuring that the resulting antecedents and fuzzy rules remain comprehensible for human interpretation. Continuing from the health recommendations example, the "young," "middle-aged," and "elderly" membership functions may have overlapping ranges, potentially introducing noise-like information that could the interpretability of our rules. Principal component analysis allows us to retain only the principal components, focusing the model on clear distinctions or shared characteristics relevant to each age group, thus preserving interpretive accuracy. In cases where user feedback is limited before the app's full release—given the cost and time required for extensive human testing—the model is built on sparse data. In this case, rules are constructed directly from the available feedback. A key issue is that the boundaries of the input space are often not covered by user feedback, meaning the model cannot interpolate between rules where no boundary information exists. In such situations, we consult product experts to supplement missing information based on their experience and understanding of user psychology. Additionally, at this stage, the initial model is carefully evaluated to ensure it is both informative and sufficiently accurate. If it meets these criteria, the model, including its antecedents and fuzzy rules, will be retained and refined when thousands of additional feedback points become available. This ensures that the model remains robust and insightful as data grows, aligning recommendations with nuanced user needs across the defined age categories.

4.2. Use case example I.: Demographics, digital content, and VR adoption

Despite the impressive information-sharing capabilities of 3D virtual reality and the rapid advancements behind the technology, its adoption in office and workplace environments remains limited. High initial costs and user acceptance are major limitations. Understanding the cognitive processes that form Attitude Toward Technology (ATT) is crucial, as it significantly influences the adoption of new technologies [42,43]. This example aims to explore the factors that drive or hinder the acceptance of VR in virtual environments, particularly focusing on perceived utility as a decisive factor in user acceptance.

4.2.1. Measurement description

The study was carried out using an online survey in April 2024. The survey was distributed through social media platforms and participation was voluntary, with no compensation offered to respondents. A total of 132 (anonymous) individuals completed the survey. The survey collected demographic and educational background information, as well as objective indicators such as VR training, frequency of VR use, time spent actively using VR, number of digital contents used concurrently in a single project or workflow, and the number of collaborative partners.

Subjective measures were also collected through the survey, focusing on users' acceptance of VR. A 5-point Likert scale, ranging from 'strongly disagree' to 'strongly agree', was employed to gauge user perceptions.

The first step was to aggregate the questionnaire. The objective was to define two input parameters and one output parameter to validate the new methods. In this sample, the input parameters included users' age and the number of digital contents used in a single project, while the output parameter was the level of VR acceptance. This approach aimed to enhance the decision-making process in sales by considering users' digital content handling habits. Users were categorized into age groups as follows:

- Gen Z college students aged 18-21 (1. age group)
- Gen Z employees aged 22-28 (2. age group)
- Gen Y users aged 29-45 (3. age group)
- Gen X members aged 46 and above -no upper age limit specified (4. age group)

The categorization of Generation Z users into two distinct groups was based on expert opinions indicating that, after entering the workforce, these users engage in significantly more live projects and manage more digital content simultaneously within a single workflow.

In terms of the number of digital content used, the users were divided into four groups:

- Users working with 1-10 digital contents
- Users working with 11-20 digital contents
- Users working with 21-30 digital contents
- Users working with over 30 digital contents

The ranges for categories of digital content quantity were determined with input from experts in the field of Cognitive Infocommunications [44–46] and Cognitive Aspects of Virtual Reality (cVR) [41]. Experts agreed that defining additional categories beyond 30 digital contents is unnecessary, as managing such a large number requires complex information processing and advanced cognitive skills, with only a few users falling into higher categories.

In addition, all of the data obtained was based on the users' self-assessment in relation to a 'typical project'; hence, the numbers obtained were expected to be inherently noisy to some degree.

4.2.2. *Creation of fuzzy model and analysis thereof*

Clustering the data points into a single matrix, we obtained the following matrix (in which the rows correspond to content quantity – from low to high – and in which the columns correspond to age group – from youngest to oldest):

$$\mathcal{F} = \begin{bmatrix} 3.28 & 3.37 & 3.1 & 3.33 \\ 4.33 & 4.16 & 4 & 3.5 \\ 3 & 2.83 & 4.5 & 3.33 \\ 4 & 2.71 & 3.33 & 4 \end{bmatrix}$$

Using SVD, while keeping only one singular value, we obtain a mean squared approximation error of only 0.44, which, on a scale of $1-5$ is acceptable. Hence, based on the derivations leading to Eq. (9), we obtain the following core tensor:

$$\mathcal{S} = \begin{bmatrix} 4.12 & 3.8 \\ 3.47 & 2.93 \end{bmatrix}$$

along with the interpolated weighting functions shown on Fig. 8, based on which we obtain the following reconstruction:

$$\tilde{\mathcal{F}} = \begin{bmatrix} 3.4 & 2.93 & 3.47 & 3.22 \\ 4.08 & 3.8 & 4.12 & 3.97 \\ 3.56 & 3.13 & 3.62 & 3.4 \\ 3.64 & 3.24 & 3.7 & 3.49 \end{bmatrix}$$

The fuzzy sets, as shown in Fig. 8 represent user groups based on digital content usage and age, providing interpretable categories for VR acceptance analysis:

Based on Fig. 8, we can identify linguistically explainable IF-THEN rules that are straightforward for the development team to understand, such as "IF input 1 and input 2, THEN output." The authors aimed to provide examples that illustrate the interpretability of the visual representation rather than presenting the complete set of rules. This approach focuses on clarity by highlighting key rules.

1. **IF** the user belongs to age group 1 (18-21 years) **AND** works with fewer than 10 digital content items in a project, **THEN** VR acceptance is moderate/neutral.
2. **IF** the user belongs to age group 1 **AND** and uses 10-20 digital content items **THEN** VR acceptance significantly increases.

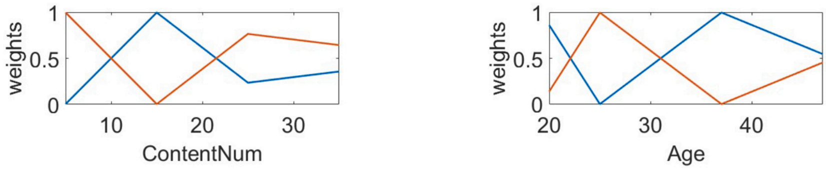

Fig. 8. Weighting functions for content quantity and age group.

3. **IF** the user belongs to age group 1 **AND** works with more than 21 digital content items in a project, **THEN** then VR acceptance decreases again.
4. **IF** the user belongs to age group 2 **AND** works with fewer than 10 digital content items in a project **THEN** VR acceptance is moderate/neutral.
5. **IF** the user belongs to age group 2 **AND** uses 11-20 different digital contents for their work **THEN** VR acceptance is particularly high.
6. **IF** the user belongs to age group 2 **AND** uses more than 21 digital content items in a project **THEN** VR acceptance drastically decreases.
7. **IF** the user belongs to age group 3 **AND** uses 21-30 digital content items **THEN** VR acceptance is optimal for them.
8. **IF** the user belongs to age group 4 **AND** works with any number of digital content items **THEN** VR acceptance is neutral.

Based on this example, the fuzzy sets of the subsequent use cases can also be described as linguistic rules.
Detailed Mathematical Perspective:

- By referring to the reconstructed values (i.e., a matrix which has the same dimensions as $\mathcal{F}$), we can see that when the content number is close to 15 (blue weighting function for ContentNum on Fig. 8) and the age of the user is close to 25 (red weighting function for Age)—which corresponds to the second item in the second row of matrix $\mathcal{F}$ (4.16)—we need to take into consideration the first row (blue ContentNum) and second column (red age group) of the core tensor $\mathcal{S}$, obtaining an approximation of 3.8.
- Conversely, when the content number is close to 0 (red weighting function for content num) and the age group is close to 20 (mostly blue weighting function for age group), in which case we need to consider the second row and (mostly) the first column of the core tensor, obtaining an approximation to the 3.28 that is somewhat less than 3.47, but still closer to 3.47 than 2.93.

Thus, we see that in the case of the weighting functions for both content number and age group (on Fig. 8), blue corresponds to the first row and first column of the core tensor, respectively.

This structured inquiry not only allows us to interpret the acceptance score of VR technology depending on age group and number of contents generally used, but also enables us to make qualitative judgments based on these independent variables. For example, we see that the blue colored weighting function for ContentNum (hence, the first row of the core tensor) really starts to play a role for users who prefer to use at least about 10 different content elements. Therefore, we can say that the first row of the core tensor, with the relatively higher values of 4.12 and 3.8, seems to have increasing weight as the number of content elements grows—at least up to a point, after which less benefits may accrue.

Practical Insights and Simplified Explanation:
To make the results more intuitive: In Fig. 8, the weighting functions illustrate VR acceptance based on two primary factors: age groups and the number of digital contents managed by users.

- **Content Quantity and VR Acceptance:** The model shows that users who handle 11–20 digital content are more likely to accept VR. This range seems to balance the cognitive load and VR capabilities, making the experience comfortable and efficient.
- **Age Group Impact:** Younger users, particularly those aged 22–28, are more open to VR adoption. This demographic benefits the most from VR due to their technical adaptability and preferences for engaging digital content.

Key Insight: The optimal user profile for VR adoption includes individuals aged 22–28 who handle 11–20 digital contents. This combination maximizes VR acceptance and provides actionable insights for marketing or development efforts targeting similar user groups.

4.3. Use case example II.: Task difficulty evaluation in 2D vs. 3D

A highly relevant theory in educational psychology, Cognitive Load Theory (CLT), seeks to describe and demonstrate how the human cognitive system processes information and what this means for instructional design. The goal of cognitive load theory in this context is to clarify how increased information processing demands during learning tasks can impact students' capacity for taking in and storing new information in long-term memory [47–50].

4.3.1. Measurement description

In the study associated with this dataset, we aimed to assess differences in the subjective and objective difficulty of questionnaires filled out by test subjects based on content that was shared on a 2D web-based interface as well as two different 3D desktop VR spaces (for more information on this study, see [40] and [51]).

In the 2D case, participants used a classical Google Sites page accessed via Chrome browser. The learning materials and questionnaires were serially embedded in the site, requiring users to scroll through the documents and then complete the associated questionnaires. Documents and questionnaires were presented in a top-to-bottom linear order.

For two separate groups of users, the same documents and questionnaires were laid out in two different 3D desktop VR spaces using the MaxWhere[1] platform.

In the dataset examined here, we consider only the age groups of users, the experimental condition (whether 2D, the first 3D space or the second 3D space), and the perceived difficulty reported by the users.

We distinguished between the following age groups:

- Ages 18-23
- Ages 24-29
- Ages 30-34
- Ages 35-55

Perceived level of difficulty was obtained from users on a Likert scale of 1-7, with 7 corresponding to the highest perceived difficulty.

4.3.2. Creation of fuzzy model and analysis thereof

Based on the above, the clustered matrix contained 4 rows (one per age group) and 3 columns (one per experimental condition):

$$\mathcal{F} = \begin{bmatrix} 3.25 & 2.75 & 1.75 \\ 4.25 & 7 & 1.5 \\ 4 & 3.75 & 3.25 \\ 4.5 & 3 & 5.5 \end{bmatrix}$$

Using SVD, while keeping only one singular value, we obtain a mean squared approximation error of 1.04 for the whole dataset, which, on a scale of $1-7$ is higher than desired but still acceptable for our purposes. Note also that keeping more singular values, as well as differentiating among different age groups (more fine-grained or separated at different ages) did not have a significant impact on the error with respect to the whole dataset.

For the above reasons, we decided to keep the model with only one singular value. Based on the derivations leading to Eq. (9), we obtain the following core tensor:

$$\mathcal{S} = \begin{bmatrix} 5.14 & 3.42 \\ 2.78 & 2.42 \end{bmatrix}$$

and the weighting functions shown on Fig. 9, based on which we obtain the following reconstruction:

$$\tilde{\mathcal{F}} = \begin{bmatrix} 2.7 & 2.78 & 2.42 \\ 4.78 & 5.14 & 3.42 \\ 3.85 & 4.08 & 2.97 \\ 4.4 & 4.7 & 3.24 \end{bmatrix}$$

Detailed Mathematical Perspective:

By analyzing the weighting functions and core tensor in Fig. 9, we can observe the following:

- The 2D scenario is closely related to the first VR space ("Spaceship") in that the model considers the first column of the core tensor, interpolating—based on age—between 5.14 and 2.78;
- The second VR space ("Sublimus") is unique in that in this condition, the model instead considers the second column of the core tensor, interpolating between the generally lower values of 3.42 and 2.42 (such that for younger age groups, it prefers the second difficulty value of these two.

Despite some uncertainty in the model—likely due to the limited number of data points—the results indicate that users found the "Sublimus" VR space less challenging overall. This observation is further supported by external evidence, such as the pupillometry data in [40] which suggests a lower cognitive load for tasks completed in "Sublimus".

Practical Insights and Simplified Explanation:

To better understand the results, Fig. 9 illustrates how task difficulty varies across 2D and 3D environments based on user feedback.

[1] https://maxwhere.com/.

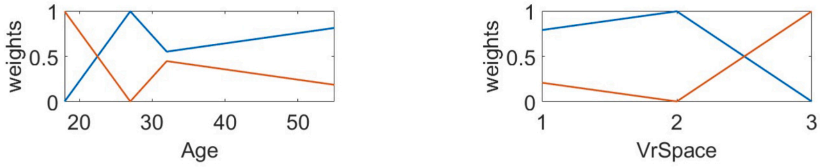

Fig. 9. Weighting functions for age group and experimental condition (2D scenario corresponding to 1, first VR space, i.e. "Spaceship" corresponding to 2, and second VR space, i.e. "Sublimus" corresponding to 3.

- Task Complexity and Environment: The results indicate that tasks with moderate complexity receive higher user preference scores in both 2D and 3D environments. This outcome suggests that tasks should maintain an optimal level of complexity—avoiding extremes of simplicity or difficulty—to support engagement and cognitive performance.
- The weighting functions and core tensor derived in this study further highlight the impact of environmental familiarity on task difficulty perceptions. Users tend to perform better in environments they frequently use. The "Sublimus" VR space, with its cleaner design and reduced navigational requirements, demonstrates lower difficulty ratings compared to the more complex "Spaceship" VR environment. This indicates that designing environments with user familiarity and simplicity in mind can significantly reduce perceived task difficulty.

Key Insight: The optimal task performance in both 2D and 3D environments occurs when tasks are designed with moderate complexity and set in familiar environments. This combination reduces cognitive load and maximizes user engagement, making it particularly effective for educational and professional VR applications. Designers can use these insights to create intuitive, user-friendly experiences that balance challenge and usability, ensuring sustained performance and satisfaction.

4.4. Use case example III.: Spatial presence analysis

This section demonstrates the application of the fuzzy modeling framework for analyzing spatial presence scores and interpreting the underlying fuzzy sets and rules derived from small and scarce data sets. The objective is to describe spatial presence scores based on users' navigational experience and prior VR usage. This recommendation system is particularly relevant because a high degree of presence contributes to good situational awareness [52] and can enhance engagement and enjoyment of mediated content [53,54].

4.4.1. Measurement description

A total of 112 individuals were involved in this use case, with 75 male and 37 female participants. The average age of the participants was 23.9 years. All individuals volunteered and gave their written consent to participate.

The participants were selected from two distinct studies [55,40], each with unique tasks in various virtual environments. Despite the differences in tasks and environments, all virtual reality experiences were administered using desktop virtual reality through the MaxWhere software platform. This ensured uniformity in navigation and interaction methods for all scenarios.

Participants were asked to complete a questionnaire with identical questions after their virtual reality sessions to be able to thoroughly compare their experiences across diverse virtual settings.

Participants were divided into 4 groups based on their prior experience with utilizing desktop VR software, including whether they were first-time users or had used VR for varying durations such as between 30-60 minutes, between 1-4 hours or more than 4 hours.

At the end of the VR session, participants rated their experience in 3D desktop VR navigation using a 10-point Likert scale based on five statements detailed in [56]. Participants were then grouped into six clusters based on their scores:

- Users with navigation score ≤ 0.4
- Users such that $0.4 <$ navigation score ≤ 0.6
- Users such that $0.6 <$ navigation score ≤ 0.7
- Users such that $0.7 <$ navigation score ≤ 0.8
- Users such that $0.8 <$ navigation score ≤ 9
- Users such that $0.9 <$ navigation score ≤ 1

To measure spatial presence, we used the Igroup Presence Questionnaire (IPQ), which assesses the sense of presence using 14 items rated on a 7-point scale from 0 to 6 [57,58]. Lower scores indicate disagreement. The questionnaire includes three subscales: spatial presence, involvement, and experienced realism. This study focuses on spatial presence, which evaluates the connection between the virtual environment and users' bodies, reflecting the feeling of physical presence in the simulated environment.

4.4.2. Creation of fuzzy model and analysis thereof

Based on the above, the clustered matrix contained 4 rows (one per VR experience group) and 5 columns (one per navigation score group), with values between 0 and 6 corresponding to spatial presence assessments:

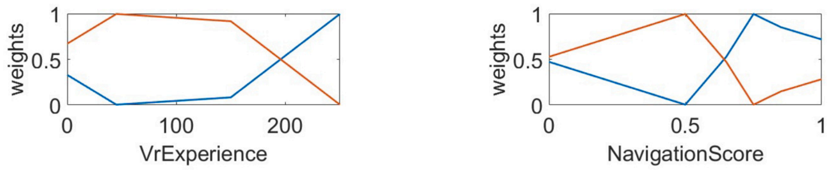

Fig. 10. Weighting functions for VR experience (in minutes) and navigation score (on a scale of 0 to 1).

$$\mathcal{F} = \begin{bmatrix} 2.4 & 0 & 2.8 & 4.4 & 2.6 & 3.4 \\ 0 & 0.6 & 3.4 & 4.4 & 4 & 1.2 \\ 3 & 1.4 & 2.8 & 2.4 & 3.2 & 3.4 \\ 3.2 & 0.2 & 1.2 & 5.2 & 4.6 & 4.2 \end{bmatrix}$$

Using SVD, while keeping only one singular value, we obtain a mean squared approximation error of 0.97 for the whole dataset, which, on a scale of $0-6$ is reasonable for our purposes. At the same time, keeping more singular values, as well as differentiating among different (more fine-grained or separated at different ages) did not have a significant impact on the error with respect to the whole dataset.

Based on the derivations leading to Eq. (9), we obtain the following core tensor:

$$\mathcal{S} = \begin{bmatrix} 5.2 & 0.01 \\ 3.5 & 0.85 \end{bmatrix}$$

and the weighting functions shown on Fig. 10, based on which we obtain the following reconstruction:

$$\tilde{\mathcal{F}} = \begin{bmatrix} 2.22 & 0.57 & 2.41 & 4.06 & 3.55 & 3.08 \\ 2.1 & 0.85 & 2.25 & 3.5 & 3.12 & 2.76 \\ 2.13 & 0.78 & 2.29 & 3.64 & 3.23 & 2.84 \\ 2.46 & 0.01 & 2.75 & 5.2 & 4.44 & 3.74 \end{bmatrix}$$

Detailed Mathematical Perspective:
By analyzing the weighting functions and core tensor in Fig. 10, we can observe the following:

- Sensitivity to Navigation at Higher Experience Levels: For users with more than 200 minutes of VR experience, their sense of presence becomes increasingly dependent on their navigation skills. The model assigns greater weight to the higher core tensor values (up to 5.2) for users with high navigation scores, amplifying the immersive effect for these skilled individuals.
- Stable Presence at Low Experience Levels: For users with less than 200 minutes of VR experience, presence scores remain relatively stable, ranging between 0.85 and 3.5, regardless of navigation skill. This stability suggests the influence of a "novelty factor", where the novelty of VR itself maintains a consistent sense of presence for beginners, independent of their navigation ability.

This structured mathematical inquiry highlights how prior VR experience and navigation skills interact to influence users' sense of presence, with the model interpolating between the respective values of the core tensor.

Practical Insights and Simplified Explanation:
To make the findings more intuitive, Fig. 10 evaluates the relationship between users' prior VR experience and navigation skills, providing actionable insights for adapting VR applications:

- Beginner Users: Users with limited VR experience (less than 200 minutes) exhibit stable presence scores, largely unaffected by navigation ability. This suggests that beginners are primarily influenced by the novelty of VR. Developers could focus on simple, engaging interfaces, such as tutorials or orientation sessions, to maximize this effect while gradually introducing core navigation concepts.
- Experienced Users: For users with more than 200 minutes of VR exposure, navigation skills play a critical role in enhancing their sense of presence. Advanced navigation options, such as customizable control schemes or more sophisticated interactions, could significantly enhance immersion for this group.

The above seems to suggest that a kind of "novelty factor" could be at play in creating a sense of presence for users. At the same time, we would expect that the collection of more data could help us create a more precise model with a more nuanced interpretation. In Fig. 10, the model uses weighting functions to evaluate the relationship between users' prior VR experience (in minutes) and navigation skill (scaled from 0 to 1), showing how these factors influence users' sense of presence in VR.

- Sensitivity to Navigation at Higher Experience Levels: The model indicates that once users surpass approximately 200 minutes of VR experience, their sense of presence becomes increasingly sensitive to their navigation skills. Specifically, higher navigation scores correlate with stronger presence experiences, as the model assigns greater weight to high core tensor values (up to 5.2), amplifying the immersive effect for these skilled users.

Table 1
Comparison of the proposed fuzzy modeling framework and traditional approaches in product development.

Feature	Fuzzy modeling framework	Other methods
Interpretability	High interpretability through visually interpretable antecedent fuzzy sets and IF-THEN rules, designed for intuitive understanding by development teams, including those without a deep mathematical background	Many traditional models (e.g., statistical or neural networks) lack transparency, often functioning as “black-box” models that are difficult for non-experts to interpret.
Adaptability to sparse data	Designed to handle sparse or noisy data, making it suitable for product development and situations with limited data.	Standard statistical methods and some machine learning models require complete, well-structured datasets and struggle in low-data scenarios.
Mathematical structure	Applies the Takagi-Sugeno (TS) fuzzy model transformation, which offers a balance between complexity and clarity through principal component analysis and minimal fuzzy rules.	Traditional methods, like closed-form solutions or neural network-based models, lack flexibility for trade-offs between simplicity and human comprehension.
Flexibility and refinement over time	Gradually adapt as more data becomes available, with a pseudo-inverse-based model adaptation that scales with increasing user feedback.	Many traditional approaches lack flexibility and cannot dynamically adjust to new data without significant re-training.
Causal clarity	Provides linguistically structured IF-THEN rules that clarify causal relationships between inputs and outputs, aiding decision-making processes and help to form recommendations.	Statistical models like correlation analyses (e.g., Spearman’s rank) only indicate associations, often failing to reveal causal insights.
Noise handling	Designed to filter out minor, noise-like variations while retaining core data elements, ensuring that essential information remains highlighted.	Conventional statistical techniques may struggle to differentiate noise from signal in sparse, noisy data environments.

- Stable Presence in Low Experience Levels: For users with limited VR experience, presence scores remain consistent (between 0.85 and 3.5) across various navigation skills. This consistency suggests that beginners may experience a “novelty factor”, where the initial VR experience provides a stable sense of presence regardless of navigation ability, potentially due to the novelty and initial excitement of VR.

Recommendation for VR developers: Fig. 10 suggests that VR applications could be optimized by adapting user experiences based on VR familiarity and navigation ability:

- Beginner Users: For newcomers with low VR experience, focus on providing an engaging yet simple navigation interface. As presence scores are stable at this stage, beginners might benefit from straightforward tutorials or orientation sessions that maintain the novelty effect while introducing them to core navigation skills.
- Experienced Users: For users with over 200 minutes of VR exposure, developers should enhance the depth and complexity of navigational features. Since experienced users’ sense of presence is sensitive to their navigation skills, offering advanced navigation options, such as customizable control schemes or more sophisticated in-world interactions, could significantly boost immersion.

Key Insight: Fig. 10 highlights the importance of user-specific adjustments. For less experienced users, VR should focus on maximizing the novelty effect with simple, intuitive controls, while more experienced users would benefit from advanced navigational features that enrich their immersive experience. Modifying VR experiences in this way could improve user engagement, skill progression, and overall satisfaction across varied user levels.

5. Comparison

The comparison in Table 1 highlights the strengths of our proposed fuzzy modeling framework in product development contexts, especially when working with sparse or noisy data.

6. Conclusions

This paper proposes a fuzzy modeling framework to help app development teams better understand and evaluate the nature of user feedback in relation to app feature metrics. The framework is built upon a modified version of the TS fuzzy model transformation developed in this study. The framework results in a highly visualizable categorization of inputs that highlights category dominance through fuzzy set membership functions. Furthermore, it provides a causal flow between input and output in the form of IF-THEN linguistic rules. The framework is equipped with principal component analysis to retain the most important rules and filter out less significant, noise-like rules. This feature helps the development team maintain a clear focus, providing a comprehensible and manageable set of rules for improved understanding and overview. The framework is designed to start with sparse information and progressively improve as the volume of user feedback grows and keeps the computational load on a lower level. The paper includes use cases that demonstrate how the proposed method effectively visualizes and provides causal descriptions, offering highly informative insights. Overall, the fuzzy modeling framework introduced in this paper provides a valuable tool for developers, researchers, and marketers, enabling them to navigate the uncertainties of product development with increased quality. As the model continues to adapt and refine itself with the accumulation of new data, it holds significant potential to mitigate risks and support the success of new product launches.

CRediT authorship contribution statement

Anna Sudár: Writing – review & editing, Writing – original draft, Validation, Software, Methodology. **Borbála Berki:** Writing – review & editing, Writing – original draft, Software, Methodology. **Ildikó Horváth:** Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Conceptualization.

Consent to participate

Each participant read and accepted the declaration of consent before the measurements.

Consent for publication

Each participant read and accepted the declaration of consent before the measurements and accepted that the data provided can be used and published in scientific form

Code availability

Not applicable.

Ethics approval

Ethics statement: Ethical approval for the research involved in this study was granted by Széchenyi István University on 10 October 2022 (ref no.: 14/2022).

In the research of the previously cited papers ([40,41]), electronic informed consent was obtained from all the participants. The questionnaires were anonymized, and participants were free to opt out of participation in the study whenever they were uncomfortable.

Funding

This research received no external funding.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The research presented in this paper was supported by the Hungarian Research Network (HUN-REN), and was carried out within the HUN-REN Cognitive Mapping of Decision Support Systems research group.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.heliyon.2025.e43537.

Data availability

Data will be made available on request.

References

[1] Nouhaila Idrissi, Ahmed Zellou, A systematic literature review of sparsity issues in recommender systems, Soc. Netw. Anal. Min. 10 (1) (2020) 15.
[2] Mehrbakhsh Nilashi, Rabab Ali Abumalloh, Mesfer Alrizq, Ahmed Almulihi, O.A. Alghamdi, Murtaza Farooque, Sarminah Samad, Saidatulakmal Mohd, Hossein Ahmadi, A hybrid method to solve data sparsity in travel recommendation agents using fuzzy logic approach, Math. Probl. Eng. 2022 (1) (2022) 7372849.
[3] David Guest, Employee engagement: a sceptical analysis, J. Org. Effect. People Perform. 1 (2) (2014) 141–156.
[4] Yaqiu Li, Qijie Zhou, Ye Fan, Guangze Pan, Zongbei Dai, Baimao Lei, A novel machine learning-based imputation strategy for missing data in step-stress accelerated degradation test, Heliyon 10 (4) (2024).
[5] Arun Vishwanath, Gerald M. Goldhaber, An examination of the factors contributing to adoption decisions among late-diffused technology products, New Media Soc. 5 (4) (2003) 547–572.

[6] Indranil Chakraborty, Paul Jen-Hwa Hu, Dai Cui, Examining the effects of cognitive style in individuals' technology use decision making, Decis. Support Syst. 45 (2) (2008) 228–241.
[7] Ildikó Horváth, Personalized learning opportunity in 3d vr, in: 2020 11th IEEE International Conference on Cognitive Infocommunications (CogInfoCom), 2020, pp. 000425–000430.
[8] Farshad Nezhad Shahmohammad, Yashar Pourrahimian, Naeimeh Akbari-Gharalari, Synthesizing complexity: trends, challenges, and future directions in fuzzy-based multicriteria decision-making (fmcdm) methods, Appl. Soft Comput. (2024) 112362.
[9] Baodong Li, Jiafu Su, Boqiao Yuan, Lvcheng Li, Yihuan Zhao, Zhidan Qin, Li Qian, A supplier selection decision-making approach for complex product development based on hesitant fuzzy information, Axioms 12 (11) (2023) 1006.
[10] Zou Lai, Siqin Fu, Hang Yu, Shulin Lan, Chen Yang, A data-driven decision-making approach for complex product design based on deep learning, in: 2021 IEEE 24th International Conference on Computer Supported Cooperative Work in Design (CSCWD), IEEE, 2021, pp. 238–243.
[11] Marie Saade, Maroun Jneid, Imad Saleh, Enhancing decision-making in new product development: forecasting technologies revenues using a multidimensional neural network, in: Information Systems: 17th European, Mediterranean, and Middle Eastern Conference, EMCIS 2020, Dubai, United Arab Emirates, November 25–26, 2020, Proceedings 17, Springer, 2020, pp. 715–729.
[12] Hao Wang, Naiyan Wang, Dit-Yan Yeung, Collaborative deep learning for recommender systems, in: Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2015, pp. 1235–1244.
[13] Xiangnan He, Lizi Liao, Hanwang Zhang, Liqiang Nie, Xia Hu, Tat-Seng Chua, Neural collaborative filtering, in: Proceedings of the 26th International Conference on World Wide Web, 2017, pp. 173–182.
[14] Fenglei Wang, Adam Slowik, Ai-empowered consumer behavior analysis for trustworthy track recommendation over musical dance electronic products, Heliyon 10 (18) (2024).
[15] Mehrbakhsh Nilashi, Othman bin Ibrahim, Norafida Ithnin, Nor Haniza Sarmin, A multi-criteria collaborative filtering recommender system for the tourism domain using expectation maximization (em) and pca–anfis, Electron. Commer. Res. Appl. 14 (6) (2015) 542–562.
[16] Boyeun Lee, Saeema Ahmed-Kristensen, Four patterns of data-driven design activities in new product development, Proc. Des. Soc. 3 (2023) 1925–1934.
[17] Jessica Pickel, Sebastian Bickel, Stefan Goetz, Sandro Wartzack, Integration of product development data for further ontological utilization, Proc. Des. Soc. 4 (2024) 463–472.
[18] Julian Wilberg, Isabell Triep, Christoph Hollauer, Mayada Omer, Big data in product development: need for a data strategy, in: 2017 Portland International Conference on Management of Engineering and Technology (PICMET), IEEE, 2017, pp. 1–10.
[19] Eva Johansson, Lars Medbo, On the use of product data in the design of the materials supply system, Int. J. Manuf. Technol. Manag. 15 (7) (2004) 641–650.
[20] Mingdong Li, Shanhe Lou, Hao Zheng, Yixiong Feng, Yicong Gao, Siyuan Zeng, Jianrong Tan, A cognitive analysis-based key concepts derivation approach for product design, Expert Syst. Appl. 236 (2024) 121289.
[21] Evgeny Kolbachev, Elena Sidorova, Polina Vaneeva, Human factor and cognitive methods in the design of products and production systems of mechanical engineering in the framework of nbic convergence, in: Intelligent Human Systems Integration 2020: Proceedings of the 3rd International Conference on Intelligent Human Systems Integration (IHSI 2020): Integrating People and Intelligent Systems, February 19-21, 2020, Modena, Italy, Springer, 2020, pp. 288–293.
[22] Li Hui, Jian Ni, Fangzhu Yang, Product design using generative adversarial network: incorporating consumer preference and external data, preprint, arXiv:2405.15929, 2024.
[23] Balazs Aczel, Bence Palfi, Aba Szollosi, Marton Kovacs, Barnabas Szaszi, Peter Szecsi, Mark Zrubka, Quentin F. Gronau, Don van den Bergh, Eric-Jan Wagenmakers, Quantifying support for the null hypothesis in psychology: an empirical investigation, Adv. Methods Pract. Psychol. Sci. 1 (3) (July 2018) 357–366.
[24] Peter Adriaan Edelsbrunner, Christian Thurn, Improving the utility of non-significant results for educational research: a review and recommendations, Educ. Res. Rev. (October 2023) 100590.
[25] Willem M. Otte, Christiaan H. Vinkers, Philippe C. Habets, David G.P. van IJzendoorn, Joeri K. Tijdink, Analysis of 567,758 randomized controlled trials published over 30 years reveals trends in phrases used to discuss results that do not reach statistical significance, PLoS Biol. 20 (2) (February 2022) e3001562.
[26] Roobaea Alroobaea, Pam J. Mayhew, How many participants are really enough for usability studies?, in: 2014 Science and Information Conference, IEEE, August 2014, pp. 48–56.
[27] Shih-Wen Hsiao, Kuan-Yun Su, Application of fuzzy logic in decision-making for product concept design, in: 2024 IEEE 7th Eurasian Conference on Educational Innovation (ECEI), IEEE, 2024, pp. 81–86.
[28] Weijun Liu, Jianming Qi, Yu Jin, Zhiyong Zhou, Xu Zhang, Application of fuzzy analytic hierarchy process–multi-layer fuzzy inference system in product design evaluation, J. Intell. Fuzzy Syst. (2023) 1–24, Preprint.
[29] P. Baranyi, Y. Yam, P. Várlaki, Tensor product model transformation in polytpic model based control, in: Automation and Control Engineering, CRC Press, Taylor & Frances Group, March 2017.
[30] P. Baranyi, TP-model transformation based control design frameworks, in: Control Engineering, Springer, July 2016.
[31] P. Baranyi, Dual-control-design TP and TS fuzzy model transformation based control optimisation and design, in: Topics in Intelligent Engineering and Informatics (TIEI, vol. 17), Springer, December 2023.
[32] L. De Lathauwer, B. De Moor, J. Vandewalle, A multilinear singular value decomposition, SIAM J. Matrix Anal. Appl. 21 (4) (2000) 1253–1278.
[33] Péter Várkonyi, Domonkos Tikk, Péter Korondi, Péter Baranyi, A new algorithm for rno-ino type tensor product model representation, in: 2005 IEEE International Conference on Intelligent Engineering Systems, INES, vol. 5, 2005, pp. 263–266.
[34] Y. Yam, Fuzzy approximation via grid point sampling and singular value decomposition, IEEE Trans. Syst. Man Cybern. 27 (6) (December 1997) 933–951.
[35] Y. Yam, P. Baranyi, C.T. Yang, Reduction of fuzzy rule base via singular value decomposition, IEEE Trans. Fuzzy Syst. 7 (2) (April 1999) 120–132.
[36] P. Baranyi, Y. Yam, D. Tikk, R.J. Patton, Trade-off between approximation accuracy and complexity: TS controller design via HOSVD based complexity minimization, in: J. Casillas, O. Cordón, F. Herrera, L. Magdalena (Eds.), Interpretability Issues in Fuzzy Modeling, in: Studies in Fuzziness and Soft Computing, vol. 128, Springer-Verlag, 2003, pp. 249–277.
[37] A.R. Varkonyi-Koczy, A. Ruano, P. Baranyi, O. Takacs, Anytime information processing based on fuzzy and neural network models, in: 18th IEEE Instrumentation and Measurement Technology Conference, 2001, pp. 1247–1252.
[38] P. Baranyi, P. Korondi, R.J. Patton, H. Hashimoto, Trade-off between approximation accuracy and complexity for TS fuzzy models, Asian J. Control 6 (1) (2004) 21–33.
[39] P. Baranyi, Y. Yam, Complexity reduction of a generalised rational form, in: D. Ruan, E.E. Kerre (Eds.), Fuzzy IF-THEN Rules in Computational Intelligence: Theory and Applications, Kluwer, 2000, pp. 135–159, chapter 7.
[40] Anna Sudár, Ádám B. Csapó, Comparing desktop 3D virtual reality with web 2.0 interfaces: identifying key factors behind enhanced user capabilities, Heliyon 10 (11) (June 2024) e31717.
[41] Ildikó Horváth, Ádám B. Csapó, Borbála Berki, Anna Sudár, Péter Baranyi, Definition, background and research perspectives behind 'Cognitive Aspects of Virtual Reality' (cVR), Infocommun. J. 15 (Special Issue) (2023) 9–14.
[42] Marc Fetscherin, Christoph Lattemann, User acceptance of virtual worlds, J. Electron. Commer. Res. 9 (3) (2008) 231.
[43] Mark L. Gillenson, Daniel L. Sherrell, et al., Enticing online consumers: an extended technology acceptance perspective, Inf. Manag. 39 (8) (2002) 705–719.
[44] Péter Baranyi, Ádám Csapó, Definition and synergies of cognitive infocommunications, Acta Polytech. Hung. 9 (1) (2012) 67–83.
[45] Péter Baranyi, Adam Csapo, Gyula Sallai, Cognitive Infocommunications (CogInfoCom), Springer International Publishing, 2015.
[46] Péter Baranyi, Borbála Berki, Ádám B. Csapó, Concepts of cognitive infocommunications, Infocommun. J. 16 (Special Issue) (2024) 37–48.

[47] John Sweller, Jeroen J.G. van Merriënboer, Fred Paas, Cognitive architecture and instructional design: 20 years later, Educ. Psychol. Rev. 31 (2019) 261–292.
[48] John Sweller, Jeroen J.G. Van Merrienboer, Fred G.W.C. Paas, Cognitive architecture and instructional design, Educ. Psychol. Rev. (1998) 251–296.
[49] John Sweller, Chapter Two - Cognitive Load Theory, Psychology of Learning and Motivation, vol. 55, Academic Press, 2011, pp. 37–76.
[50] Jan L. Plass, Roxana Moreno, Roland Brünken, Cognitive Load Theory, Cambridge University Press, 2010.
[51] Anna Sudár, Ildikó Horváth, Design suggestions for digital workflow oriented desktop vr spaces, Infocommun. J. 16 (Spec i) (2024) 119–127.
[52] Jamie I. Cross, Christine C. Boag-Hodgson, Tim J. Mavin, Measuring presence and situational awareness in a virtual reality flight simulator, Aviat. Psychol. Appl. Human Fact. 13 (2) (September 2023) 83–94.
[53] Daniel M. Shafer, Corey P. Carbonara, Lucy Popova, Spatial presence and perceived reality as predictors of motion-based video game enjoyment, Presence, Teleoper. Virtual Environ. 20 (6) (December 2011) 591–619.
[54] Paul Skalski, Ron Tamborini, Ashleigh Shelton, Michael Buncher, Pete Lindmark, Mapping the road to fun: natural video game controllers, presence, and game enjoyment, New Media Soc. 13 (2) (2011) 224–242.
[55] Borbála Berki, Experiencing the sense of presence within an educational desktop virtual reality, Acta Polytech. Hung. 17 (2) (2020) 255–265.
[56] Borbála Berki, Does effective use of MaxWhere VR relate to the individual spatial memory and mental rotation skills?, Acta Polytech. Hung. 16 (6) (aug 2019) 41–53.
[57] Thomas Schubert, Frank Friedmann, Holger Regenbrecht, Embodied presence in virtual environments, in: Visual Representations and Interpretations, Springer, 1999, pp. 269–278.
[58] Thomas Schubert, Frank Friedmann, Holger Regenbrecht, The experience of presence: factor analytic insights, Presence, Teleoper. Virtual Environ. 10 (3) (2001) 266–281.