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Violin Virtuosi

Do their Performances Fade over Time?

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Abstract *In many professional activities humans are getting better generation by generation. This is supposed to be the case, for instance, in sports and in science. Is it true in the arts? In this paper, we consider violinists from the time period in which audio and video recordings became possible. Based on the number of YouTube views, and by employing different aggregation methods, we find that listening to violinists from the mid of the previous century does not seem to be significantly less attractive to audiences than listening to contemporary violinists. Methodologically, our analysis contributes to the growing literature on the aggregation of incomplete lists. In particular, we introduce a generalization of the Nash collective utility function for incomplete lists.*

Keywords: Group decisions and negotiations, multi-criteria decision making, aggregation of incomplete lists, Nash collective utility function, top violinists.

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1 Introduction

The original motivation for this paper stems from a communication between an anonymous mathematician and an anonymous violinist in which one of the authors served as an intermediary. The statement by the mathematician was that contemporary violinists must be better than older violinists – as it supposedly the case for tennis players, chess players, and many others, – because of the development of practicing and preparation methods, the availability of better material as well as technical and non-technical progress. The professional violinist strongly disagreed and claimed that the great virtuosos' performances of the last century were still the yard stick by which every contemporary artist would have to be judged, and that only little improvement, if at all, can be observed in the performing arts. In this paper, we try to resolve these conflicting standpoints from the viewpoint of the audience by comparing the number of views of different violinists on YouTube. On deliberation, this turns out to be a non-trivial task. In particular, since the character, popularity and difficulty of music pieces differ greatly, merely considering the total number of views of violinists is not an appropriate approach. Instead, we investigate 46 different music pieces of central importance to the classical repertoire and consider the number of views of altogether 128 violinists piece by piece.

This approach still poses the methodological problem of how to aggregate the findings for each piece. Employing notions from social choice theory, one can view the 128 violinists as the alternatives or candidates to be ranked, and the 46 pieces as the criteria or judges or voters. The 'voters' can assign either cardinal values to the candidates (e.g. the number of views) or ordinal ranks (obtained from the ordering of candidates by their popularity). We apply techniques from preference and utility aggregation (Blackorby et al., 2002), as well as pairwise comparison matrices (Saaty, 1980) in order to obtain a ranking of violinists. One fundamental difficulty is that in our context not all candidates are ranked by all voters, i.e. not all violinists have recorded all pieces. Thus, we have to aggregate based on *incomplete* lists.

Our proposal of how to deal with this missing information contributes to the recent theoretical literature, mainly developed in computer science, on the aggregation of incomplete lists with a wide range of applications such as search engines and spam filters, see, e.g. Dwork et al. (2002). In social choice theory, the problem of aggregating incomplete orders has been addressed by a great number of scholars, see e.g. Pini et al. (2009) for a comprehensive treatment of three central results (Arrow's theorem, the Gibbard-Satterthwaite theorem and the Muller-Satterthwaite theorem) with incomplete preferences. Our goal is to arrive at a ranking of classical violinists in terms of their popularity. For this purpose, the theorem by Arrow provides the relevant background.[1] Pini et al. (2009) prove various generalizations of Arrow's

[1] The issue of possible individual manipulation of aggregation procedures as considered by the Gibbard-Satterthwaite theorem may clearly also play a role in classical music competitions, see for instance Kontek and Kenner (2023). This issue is not addressed here.

impossibility theorem if individual and/or collective rankings are incomplete. In this paper, we want to derive a *complete* ranking of candidates (violinists) based on a particular kind of incomplete rankings, which we call 'lists'. A list is a strict partial order that is complete on a subset of alternatives. We note here that a variant of Arrow's theorem in this setting follows from the results of Pini et al. (2009).

The generalization of Arrow's impossibility theorem to the present context implies that there is no 'ideal' method of ranking alternatives based on incomplete lists. Our solution to this general problem is to consider various reasonable (but necessarily 'non-ideal') aggregation methods and to compare the respective results with each other. We thus examine adaptations to the present framework of a number of different ordinal ranking methods that have been proposed in the literature in the light of Arrow's impossibility theorem. Specifically, we consider appropriate adaptations of the Borda count and of the Copeland method. Moreover, we look at the network-based extensions through Markov chains of the Borda count and the Copeland method as proposed by Dwork et al. (2002).

The data in our application in fact provide more than just ordinal information. Indeed, the number of views of a piece performed by a violinist is a cardinal value. Therefore, identifying these views with cardinal 'utilities' we can also employ the theory of the aggregation of individual utility functions into a social welfare ordering, or a collective utility function; see d'Aspremont and Gevers (2002) for a survey of this theory. Specifically, we adapt and generalize the Nash collective utility function, which multiplies (positive) individual utilities, to our framework with incomplete lists. On the set of complete lists, the Nash collective utility function is the only collective utility function that satisfies a natural condition of scale independence. We prove that on the set of all possibly incomplete lists there does not exist a continuous collective utility function satisfying scale independence, and we argue that our extension of the Nash collective utility function thus seems to be the best compromise if one wants to keep the scale independence property at least on the subdomain of all complete lists. Alternative approaches involve the utilitarian and relative utilitarian collective utility functions, respectively, and we consider both of them as well.

Saaty (1980) developed a general method for multi-criteria decision making which can be used for ranking alternatives. Central to his method are pairwise comparison matrices (PCM). In our context, PCMs can be regarded as a refinement of the pairwise comparisons carried out by the Copeland method, or as a coarsening of the available cardinal values (i.e. views) used by the collective utility approach. More specifically, we employ the so-called eigenvector method (EM) here to arrive at a ranking of the alternatives.[2]

Our two main results are: (i) the ranking of violinists is quite robust against the particular

[2]There are other methods known in the literature, but Bozóki, Csató and Temesi (2016) found only minor differences in the results of the most common alternative methods to EM.

ranking method employed, with Hilary Hahn followed by Itzhak Perlman as the two most popular classical violinists, and (ii) among the top violinists there is a significant number of older artists from the previous century; for instance, Jascha Heifetz and David Oistrakh, two of the greatest violinists of all times, appear safely in the top 10 no matter which specific ranking method is employed. Therefore, we arrive at the conclusion that violinists from earlier decades are almost as attractive to audiences based on YouTube views as today's active violinists. This is in stark contrast to popular music, where no music video prior to the launch of YouTube in 2005 appears in the list of 30 most watched videos (see `https://en.wikipedia.org/wiki/List_of_most-viewed_YouTube_videos`).

Further Related Literature

Applications of social choice and multi-criteria decision theory abound. Many problems require the aggregation of rankings of objects based on inputs from multiple sources like in automated decision making, machine learning (see, e.g. Volkovs and Zemel, 2014), database middleware (see, e.g. Masthoff, 2004), or in the determination of the results in sport competitions (see, e.g. Csató, 2023, and Ausloos, 2024). The problem also arises in coding theory since the alternatives can be regarded as letters and the rankings as strings (see, e.g. Bortolussi et al., 2012). The issue of aggregating rankings also emerges in the link analysis in networks and web search algorithms (see, e.g. Borodin et al., 2005). Applications dealing with the aggregation of incomplete preferences range from student paper competitions (Hochbaum and Moreno-Centeno, 2021), the ranking of cities as destination for tourists (Dopazo and Martnez-Céspedes, 2015) to the ranking of teams in sports competition (Ausloos, 2024). For the ranking of individual tennis players of different decades, see e.g. Bozóki et al. (2016) and Temesi et al., 2024).

There is a large literature on the assessment of great artists in music. For instance, Campbell (2011) discusses great violinists from the early stages on from the point of view of a musician in an informal way, and an in-depth analysis of the art of Jascha Heifetz is carried out by Sarlo (2010).

The structure of the paper is as follows. In the next section (Section 2) we describe the collection of data. Section 3 provides the theoretical background of our inquiry. Specifically, we review results on the aggregation of ordinal and cardinal preference information and provide generalizations of these results to the aggregation of incomplete lists. Our main theoretical contribution is the generalization of the well-known Nash collective utility functions to the case of incomplete lists (see Section 3.2.2). Section 4 describes the (ordinal and cardinal) methods used to arrive at the different rankings of violinists. Section 5 contains the results and the statistical analysis, and Section 6 concludes. An appendix demonstrates the robustness of our main result with respect to the specific choice of parameters.

2 Collection of Data

First, we had to select the set of violinists for comparison and the set of pieces, which we employ as judges. We chose 46 violin pieces 29 of which are violin concertos, 9 are pieces originally composed for violin and either orchestra or piano, and 8 are among the most difficult solo violin pieces (cf. Tables 1-3). The list comes very close to the list of graded violin pieces by Chen (2023). Clearly, one could have added further pieces, but we believe that the most significant ones are included in our list and are sufficiently representative in order to address our initial question and to rank violinists based on the respective views.

There are many sources containing the list of greatest classical violinist of all time. We started with those listed as the top 25 violinist of all time on Classic FM (2022) and are sufficiently viewable on YouTube. Namely, Joshua Bell, Nicola Benedetti, Midori Goto, Hilary Hahn, Jascha Heifetz, Janine Jansen, Fritz Kreisler, Gidon Kremer, Yehudi Menuhin, Viktoria Mullova, Anne-Sophie Mutter, Ginette Neveu, David Oistrakh, Itzhak Perlman, Gil Saham, Isaac Stern and Maxim Vengerov in alphabetical ordering. We added those not appearing in the previous list of violinists, but appearing on Nicolas' (2024) list of 20 all time greatest violinists, who are James Ehnes, Kyung-Wa Chung, Nathan Milstein and Ruggiero Ricci. There is also a page on which visitors can vote on the greatest violinist of all time (ranker.com, 2024) from which we added those not mentioned so far, but appearing in the latter top 30. These are Leonidas Kavakos, Pinchas Zuckerman, Julia Fischer, Arthur Grumiaux, Henryk Szeryng, Leonid Kogan, Ray Chen and Michael Rabin. We added to the list Renaud Capucon, Sarah Chang, Mischa Elman, Christian Ferras, Zino Francescati, Ivry Gitlis, Ida Haendel, Nigel Kennedy and Shlomo Mintz. So far we arrived at a list of 38 violinists. Clearly, nobody would object that we have listed already great violinists, but any listing is arbitrary. Since we wanted to keep our list open, we added every violinist who is among the six most viewed ones for at least one of the 46 pieces and achieves at least 50 thousand views in that piece, either in April 2024 or October 2024. In this way we arrived to 128 violinists. Their names can be found in Tables 1-3.

For obvious reasons we had to restrict the list of investigated violinists to those ones being active in the time period when recordings were possible. Even though recordings are available already for all-time greats like Joseph Joachim,[3] Eugene Ysaÿe[4] or Pablo de Sarasate[5] they are of limited number, low-quality and short length. Therefore, it would not be appropriate to include them into our analysis.

Tables 1-3 contain the six most viewed violinists for each of the 46 pieces ordered decreas-

[3] https://adp.library.ucsb.edu/index.php/mastertalent/detail/103104/Joachim_Joseph
[4] https://adp.library.ucsb.edu/index.php/mastertalent/detail/102401/Ysae_Eugne
[5] https://adp.library.ucsb.edu/index.php/mastertalent/detail/102893/Sarasate_Pablo_de?Matrix_page=6

	1st	2nd	3rd	4th	5th	6th
Bach No. 1. 1041	Verhey	Bell	Fischer	Hahn	Grumiaux	Oistrakh
	29623756	2689471	2446233	1476595	1277655	982881
Bach No. 2, 1042	Verhey	Hahn	Bell	Sato	Siwoo Kim	Jansen
	29623756	4152924	1471997	1165480	421555	358166
Bach Double Concerto	Oistrakh	Menuhin	Sato	Hahn	Yang	Chen
	5143111	5143111	3507137	2125309	2087080	2087080
Barber	Shaham	Hadelich	Stern	Meyers	Hahn	Schmidt
	345117	229488	227390	220978	216199	209855
Bartok no. 2	Stern	Hadelich	Zimmermann	Chung	Mullova	Markovici
	257049	218792	196014	158662	114447	56045
Beethoven	Hahn	Perlman	Vengerov	Steinbacher	Lozakovich	Kang
	11860196	9781215	3732355	2471720	2464426	1921866
Brahms	Hahn	Perlman	Oistrakh	Chung	Bomsori Kim	Fischer
	8854891	2671456	1062274	993047	820617	547803
Brahms Double Concerto	Mutter	Oistrakh	Stern	Fischer	Frang	Perlman
	1917173	489198	297386	175909	132152	129209
Bruch No. 1	Cointet	Jansen	Bell	Duenas	Himari	Chang
	3016491	1653826	1617465	1430086	1410426	899718
Dvorak A min	Fischer	Hahn	Chung	Oistrakh	He	Bell
	741393	302525	277355	252785	223828	183361
Glazunov A min	Hahn	Markovici	Gluzman	Shumsky	Fischer	Benedetti
	434398	97122	85484	83559	63408	60235
Korngold D maj	Hahn	Hagen	Perlman	Heifetz	Benedetti	Chen
	659308	148297	135870	110545	101240	81938
Lalo	Duenas	Meyers	Hadelich	Pavalec	Repin	Markovici
	822370	396502	357225	306439	279802	271881
Mendelssohn No. 2	Chen	Hahn	Jansen	Perlman	Chang	Protsenko
	5932856	5186249	3967639	3760314	3456272	2767365
Mozart No. 3	Suk	Hahn	Oistrakh	Studer	Baráti	Yoon
	4035501	2597865	2100391	1528766	731720	651568
Mozart No. 4	Suk	Oistrakh	Hahn	Chua	Ko	Baráti
	4035501	2100391	1832827	1351666	1155688	731720
Mozart No. 5	Suk	Bomsori Kim	Hahn	Oistrakh	Baráti	Porter
	4035501	3126151	2702941	2100391	731720	466510
Paganini No. 1	Lee	Himari	Hahn	Duenas	Chang	Kang
	15574190	5334114	3404328	2684316	1920252	1745144
Paganini No. 2	Mae	Quint	Takamatsu	Garrett	Milenkovich	Kang
	23815446	5056407	3756791	3434971	3315824	2635165
Prokofiev No. 1	Hahn	Kavakos	Fischer	Perlman	Batiashvili	Oistrakh
	1216499	259189	218170	204665	135288	126914
Prokofiev No. 2	Shoji	Kavakos	Jansen	Josefowicz	Fischer	Oistrakh
	245189	240618	204681	190623	182527	141387
Saint-Saens No. 3	Bell	Kang	In Mo Yang	Vengerov	Milstein	Fischer
	496491	420777	385269	207551	180709	165060
Shoshtakovich No. 1	Hahn	Bomsori Kim	Kogan	Khachatryan	Vengerov	Oistrakh
	785937	218696	159979	157646	126596	125791
Sibelius	Hahn	Vengerov	Chang	Chen	Bell	Oistrakh
	8053011	6487626	6101546	2345744	997800	759590
Tchaikovsky	Baeva	Bell	Fischer	Midori	Heifetz	Shoji
	6462644	5568233	4459984	4423596	4301034	3598208
Vieuxtemps No. 5	Chang	Ko	Josefowicz	Hsu	Donghyun Kim	Heifetz
	395343	218416	178623	89670	89451	86854
Vivaldi 4 seasons	Banfalvi	Agostini	Freivogel	Samuelson	Mae	Perlman
	262357520	70908089	62001257	41595528	30498054	22612327
Wieniawski No. 1	Himari	Chen	Yoon	Perlman	Widjaja	Midori
	1318996	853260	352442	138636	126918	94500
Wieniawski No. 2	Bomsori Kim	Mintz	Rabin	Tchumburidze	Shaham	Perlman
	2122611	1263588	242969	233549	186669	143004

Table 1: Top 6 viewed violin concertos by violinists most viewed video

	1st	2nd	3rd	4th	5th	6th
Beethoven: Kreutzer Sonata	Shinohara	Mutter	Oistrakh	Kopatchinskaya	Bell	Han
	7951670	5677568	1609857	689927	551830	315123
Elgar: Salut d'amour	Chang	Abrami	Petryshak	Okumura	Hope	Midori
	4349678	1260226	1145412	1106779	609798	402875
Kreisler: Liebesleid	Kreisler	Meyers	Kang	Perlman	Ko-woon Yang	Mae
	1537109	1005724	577023	458317	441943	440297
Massenet: Thais Meditation	Panfili	Vengerov	Kang	Mutter	Milstein	Perlman
	5297104	4242233	1931684	1405857	872557	801985
Ravel: Tzygane	Fesneau	Kopatchinskaya	Midori	Oistrakh	Perlman	Szeryng
	946759	758394	662734	443908	331678	264694
Saint-Saens: Introduction &	Shinohara	Koelman	Bomsori Kim	Shin	Chen	Perlman
Rondo Capriccioso	6401985	4376795	2850293	1668463	1362397	1182580
Sarasate: Zigeunerweisen	Himari	Ko	Perlman	Panfili	Han	Takamatsu
	8984959	8520908	5732722	3332187	2263194	1682064
Vaughn: The lark ascending	Nolan	Hahn	Benedetti	Park	Hwang	Liebeck
	8340138	1655061	846592	332157	300426	299948
Vitali Chaconne	Chang	Grytsay	Heifetz	Francescati	Chen	Ko
	1901251	1696677	1416701	1301300	907145	828832

Table 2: Top 6 viewed other violin with accompaniment pieces by violinists most viewed video

	1st	2nd	3rd	4th	5th	6th
Bach Chaconne	Perlman	Hahn	Chung	Menuhin	Shoji	Grumiaux
	2735409	2623754	2342688	1690601	1300495	1084813
Ernst: Grand Caprice	Hahn	Leong	Barti	Feng	Frang	Kang
	2174049	323522	250785	225312	118878	71098
Ernst: Last rose of summer	Midori	Hahn	Kang	Vengerov	Ricci	Boulier
	1205043	520561	298272	168450	89792	74681
Locateli: Harmonic Labyrinth	Chua	Oistrakh	Gringolts	Szeryng		
	305115	143822	56480	56084		
Paganini: Caprices	Markov	Studer	Garrett	Heifetz	Krylov	Shin
	12648045	11946608	8276323	7850046	6069111	4701666
Paganini: God save the king	Roman Kim	Kavakos	Feng	Gibboni	Zimmermann	
	1049012	240458	109899	95983	68279	
Paganini: Nel cor piu non mi sento	Kogan	Kavakos	He	Accardo		
	468768	314634	123078	80632		
Ysaye: Sonatas	Vengerov	Chua	Tompkins	Hahn	Chen	Hadelich
	998656	914411	482298	300207	272507	219952

Table 3: Top 6 viewed difficult violin solos by violinists most viewed video

ingly by views from left to right. Under the names we put the respective number of views of their performance of that piece. Frequently, artists have recorded several performances of the same piece; in most cases, looking at the most viewed performance contains sufficient information since usually their second most viewed performance of the same piece received far less views. However, there are some exceptions; therefore, we also gathered the three most viewed performances of each piece and each violinist, and carried out all calculations based on the sum of these views. The latter results can be found in the Appendix.

We also need to make explicit *which* uploaded videos we took into account since in many cases performances are uploaded by movements or sometimes even by parts. In brief, we took into account the most viewed movement of a performance. The reason for this is simply that we cannot tell for how long one video was watched. So even if the full performance is gathered in one video it is not clear how many movements have been watched by a viewer. In a few cases, recordings of more than one piece are included in a video. If the whole recording just contains recordings by the same performer we attribute the recording to each piece of that recording. We emphasize that we had to apply this rule in a very few cases.

For further consideration in the derivation of our rankings below, we require that a violinist has to pass the 50 thousand threshold of views in at least 10 of the 46 pieces. We impose this restriction in order to avoid outlier effects. Altogether 32 among the 128 violinists occurring in Tables 1-3 pass this minimum requirement.[6]

Our central question is now how we can rank violinists based on the data summarized in Tables 1-3. Before we describe the concrete methods employed, we need to provide some theoretical background from multi-criteria decision-making. This is dome in the next section.

[6]Evidently, the number 10 is somewhat arbitrary. From a technical point of view at least a minimum of 8 pieces are required if we employ pairwise comparison matrices and do not want to deal with incomplete such matrices.

3 Background: Aggregation of Incomplete Lists

3.1 Aggregating Strict Partial Orders into a Complete Social Ranking: Arrow's Theorem Generalized

Let $A = \{a_1, \ldots, a_m\}$ be the set of alternatives (violinists) and $N = \{1, \ldots, n\}$ the set of voters (pieces). Denote by $\mathcal{I}$ the set of all profiles of strict partial orders (asymmetric and transitive binary relations on A), i.e. the set of possibly incomplete individual preferences, and by $\mathcal{P} \subseteq \mathcal{I}$ the subset of all linear orders (i.e. complete partial orders).

In our context, the partial orders to be aggregated have a particular structure. While they may be incomplete (because not all violinists have recorded all pieces), they form complete orders on a *subset* (because the number of views allows, for each piece, the comparison of all violinists that have recorded that piece). We refer to such partial orders as 'lists.'

Definition 1. A *list (on A)* is a strict partial order $\succ \subseteq A \times A$ such that $\succ$ is complete on some subset $B \subseteq A$ with $\#B \geq 2$.

We denote by $\mathcal{L}$ the set of all (possibly incomplete) lists on A, and by $\mathcal{L}^n$ the domain of all profiles of individual lists on A.

A social welfare function defined on the subdomain $\mathcal{L}^n \subseteq \mathcal{I}^n$ assigns to each profile of (possibly incomplete) lists a linear order, i.e. a complete social ranking of all alternatives in A. Formally, we have the following definition.

Definition 2. A mapping $F : \mathcal{L}^n \to \mathcal{P}$ is called a *social welfare function*, henceforth, SWF on the domain $\mathcal{L}^n$.

In this way, we have extended the usual notion of a SWF as mapping from $\mathcal{P}^n$ to $\mathcal{P}$ to the domain of all profiles of lists.

We turn to the appropriate generalizations of the well-known notions of Pareto property, independence of irrelevant alternatives and dictatorship in our present context.

The first is the weak Pareto property stating that if all voters rank a above b, then so must the social ranking.

Definition 3. A SWF F satisfies the *weak Pareto property* (or is *weakly Paretian)* (WP) if, for all profiles $\Pi = (\succ_1, \ldots, \succ_n) \in \mathcal{L}^n$ and $a, b \in A$ we have

$$(\forall i \in N : a \succ_i b) \implies a \succ b,$$

where $\succ = F(\succ_1, \ldots, \succ_n)$.

Let us consider next the extension of the well-known IIA condition, which requires that, for any pair of distinct alternatives, if in two profiles these two alternatives are ranked in the same way voter by voter, then the SWF must rank these two alternatives in both profiles in the same way. The following condition formalizes this general principle in our context of possibly incomplete orders; it is exactly the version used in Pini ez al. (2009).

Definition 4. The SWF F satisfies *independence of irrelevant alternatives (IIA)* if, for all distinct $a, b \in A$, and all profiles $\Pi = (\succ_1, \ldots, \succ_n), \Pi' = (\succ'_1, \ldots, \succ'_n) \in \mathcal{L}^n$ we have

$$\big(\forall i \in N : a \succ_i b \Leftrightarrow a \succ'_i b \text{ and } b \succ_i a \Leftrightarrow b \succ'_i a\big) \implies \big(a \succ b \Leftrightarrow a \succ' b\big), \tag{3.1}$$

where $\succ = F(\succ_1, \ldots, \succ_n)$ and $\succ' = F(\succ'_1, \ldots, \succ'_n)$.

In our context, the natural notion of dictatorship is as follows.

Definition 5. A SWF F is *dictatorial* (or a *dictatorship*) if there exists a voter h such that, for all $(\succ_1, \ldots, \succ_n) \in \mathcal{L}^n$ and all $a, b \in A$ we have

$$a \succ_h b \implies a \succ b,$$

where $\succ = F(\succ_1, \ldots, \succ_n)$.

Observe that a dictator thus imposes the ordering of those alternatives on which she has an opinion. Clearly, there can be at most one such voter. Also note that in our context it is not possible to define a dictator as a voter who imposes *exactly* her (incomplete) preference as the social ranking since we assume the social ranking always to be complete.[7]

The following version Arrow's impossibility theorem follows from Pini et al. (2009, Th. 7).

Theorem 1. *Suppose that $m \geq 3$; then every SWF $F : \mathcal{L}^n \to \mathcal{P}$ that satisfies the weak Pareto property (WP) and independence of irrelevant alternatives (IIA) is dictatorial.*

Remark 1. One can define lexicographic dictatorships that satisfy all conditions of Theorem 1 as follows. First, if on some profile voter 1 has incomplete preferences one may let that voter decide the ranking of those alternatives on which she has a opinion; then, voter 2 may decide on the remaining alternatives on which voter 2 has an opinion, and so on. Evidently, while formally possible such lexicographic dictatorships are not particularly attractive as aggregation methods.

Remark 2. One may wonder if a result akin to Theorem 1 is true on all subdomains $\mathfrak{D}$ of profiles with $\mathcal{P}^n \subseteq \mathfrak{D} \subseteq \mathcal{I}^n$. Remarkably, the answer is, no. There are such subdomains $\mathfrak{D}$ on which there exist non-dictatorial SWFs satisfying IIA and WP; however, these SWFs are not particularly attractive and violate a very mild monotonicity condition. A systematic and detailed analysis of this issue is provided in Puppe and Tasnádi (2024).

[7] This is in contrast to Pini et al. (2009) who introduce different types of dictators, the 'strong' and 'weak' dictators.

3.2 Incorporating Cardinal Information: Social Welfare Orderings and Collective Utility Functions

It is well-known that Arrow's impossibility can be overcome by giving up the IIA condition and allowing for interpersonal (i.e. inter-criteria) comparisons. This approach lends itself naturally to our context since we can use even cardinal information, namely the number of views per piece, for such inter-criteria comparison.

In this section, we explore this route. The number of views of a piece can take on only non-negative values, thus there is a common minimum value, the zero which is naturally identified with the lack of any information stemming from that particular voter (i.e. piece). Although, strictly speaking, the values are integers, it is natural to embed them into the non-negative reals. Specifically, let $\mathbb{R}_+ = [0, \infty)$ and $\mathbb{R}_{++} = (0, \infty)$; by $\mathcal{U} = \mathbb{R}^n_+$ we denote the set of all *utility profiles*, where we identify the number of views on piece i with the 'utility' that a candidate (i.e. violinist) receives from that piece. Let $\mathcal{D}(U)$ stand for the set of pieces for which a violinist has a strictly positive number of views given the utility profile $U \in \mathcal{U}$. Summarizing, a utility profile contains the number of views for each piece and a given violinist. Violinists are then compared based on their respective utility profiles, i.e. a matrix $V \in \mathcal{U}^A$ contains all data necessary in order carry out these comparisons (in the tables above, pieces are represented as rows of this matrix and violinists as columns).

Next, we review and adapt some well-known concepts that have been developed for 'complete' utility profiles, i.e. for utility profiles $U \in \mathbb{R}^n_{++}$ for which $\mathcal{D}(U) = N$. A social welfare ordering compares violinists in terms of their associated utility profiles; formally, we have the following definition.

Definition 6. A *social welfare ordering (SWO)* $\succeq$ on $\mathcal{U}$ is a preference ordering (complete and transitive binary relation) on the set of all utility profiles.

The interpretation is that a utility profile is preferred by the social welfare ordering if and if it corresponds to a higher aggregate value ('social welfare'). In what follows, we assume that SWOs satisfy:

- *Anonymity:* Only the voters' utilities matter not their identities, and
- *Monotonicity:* A unilateral increase in one voter's utility increases social welfare.

In our context, the anonymity condition says that all pieces contribute equally to the value of a violinist, or in other words, that the identity of pieces does not matter. One might question this assumption by arguing that some pieces are more central to the repertoire than others. This can be addressed by weighing pieces differently. While this is possible in principle, it would require a systematic musical inquiry to determine the weights that is beyond the scope of the present paper. Moreover, we have tried to partly solve this problem by identifying the core of those pieces that are commonly held by music experts to be central to the violin repertoire. The monotonicity condition seems uncontroversial.

A collective utility function assigns a numerical value to any utility profile.

Definition 7. A *collective utility function (CUF)* assigns to each utility profile $U \in \mathcal{U}$ a real value. A CUF W *represents* the SWO $\succeq$ if $(U \succeq U' \Leftrightarrow W(U) \geq W(U'))$.

CUFs are sometimes considered to be simpler mathematical objects than SWOs, but note that some important SWOs cannot be represented by a CUF, e.g. the lexicographic SWOs. On the other hand, we know from Debreu's famous representation theorem (Debreu, 1959) that every *continuous* SWO can be represented by a continuous CUF.

3.2.1 The Utilitarian and Relative Utilitarian Collective Utility Functions

An obvious candidate in order to account for cardinal information is the well-known utilitarian CUF which takes the arithmetic mean as a measure of social welfare (see, e.g., Moulin, 1988). In our context, the utilitarian CUF simply maximizes the average number of views that a violinist receives. We can obtain this solution by employing the following minimization of squared distance approach. Specifically, consider for any profile $U \in \mathcal{U}$, the problem

$$\arg\min_{z \in \mathbb{R}_+} \sum_{i \in \mathcal{D}(U)} (u_i - z)^2 . \tag{3.2}$$

This has the solution

$$u^* = \frac{1}{k} \sum_{i \in \mathcal{D}(U)} u_i,$$

where $k = \#\mathcal{D}(U)$ is the number of pieces for which the candidate (violinist) at hand receives a positive number of views. Let us denote the utilitarian CUF by

$$W_{util}(U) \;:=\; \frac{1}{k} \sum_{i \in \mathcal{D}(U)} u_i.$$

The utilitarian CUF treats pieces differently in the sense that pieces with more total views have a larger weight in the comparisons between violinists. One could address this by normalizing the number of views also from above and measure utility in terms of the *fraction* of views that a violinist receives from a given piece. This approach gives rise to the so-called 'relative utilitarian' CUF first axiomatized by Dhillon and Mertens (1999), see also Sprumont (2019) and Peitler and Schlag (2024) for recent contributions. Formally, for each piece i, let $\bar{u}_i$ denote the maximal number of views that any violinist achieves in that piece given the data summarized in the matrix $V \in \mathcal{U}^A$. Then define

$$W_{rel-util}(U) \ := \ \frac{1}{k} \sum_{i \in \mathcal{D}(U)} u_i / \bar{u}_i.$$

3.2.2 Scale Independence and the Extended Nash Collective Utility Function

Both the utilitarian and the relative utilitarian CUFs involve a particular comparison between the value of one view of any given piece and that of one view of another piece; indeed, in case of the utilitarian CUF each single view has exactly the same value, and in the case of the relative utilitarian CUF their normalized value is the same. But one can question if the values of views across pieces are in fact commensurable. This is analogous to the case of utility theory in which many researchers have rejected the idea that individual utilities are interpersonally comparable. The following property of scale independence expresses exactly this idea that utilities (views) are incomparable across voters (pieces).

Definition 8. A SWO $\succeq$ is *scale independent (SI)* if

$$\forall U, U' \in \mathcal{U} \, \forall Z \in \mathbb{R}^n_{++} : U \succeq U' \iff U \bullet Z \succeq U' \bullet Z, \tag{3.3}$$

where $U \bullet Z = (u_1 z_1, \ldots, u_n z_n)$.

In the case of complete utility profiles (i.e. with strictly positive utilities) the scale independence condition characterizes the so-called Nash CUF.

Definition 9. $W_N(U) = \prod_i u_i$ is the *Nash CUF.*

The following result is Theorem 2.3 in Moulin (1988), see also d'Aspremont and Gevers (1977). The set of all complete utility profiles with $\mathcal{D}(U) = N$ is denoted by $\mathcal{U}_c = \mathbb{R}^n_{++}$.

Theorem 2 (D'Aspremont and Gevers, 1977)**.** *The CUF W_N satisfies SI on $\mathcal{U}_c$. Conversely, any continuous SWO on $\mathbb{R}_{++}$ that satisfies SI is represented by W_N.*

The next result shows that the characterization of the Nash CUF does not carry over to the case of incomplete utility profiles.

Theorem 3. *There does not exist a continuous SWO that satisfies SI on $\mathcal{U}$.*

Proof. By contradiction, suppose that there exists a continuous and SWO $\succeq$ that satisfies SI on $\mathcal{U}$. Then, by continuity it can be represented by a continuous CUF W, which on $\mathcal{U}_c$ equals W_N by Theorem 2. Take a proper incomplete utility profile U and a complete utility profile U' such that $U \succeq U'$ and $W(U') > 0$. Note that one can choose such utility profiles U and U' because the value of W_N is arbitrarily close to zero, and by monotonicity there exists a U

such that $W(U)$ is positive. Then pick a sufficiently small $\varepsilon > 0$ and a sufficiently large $\lambda > 0$ such that for $U'' \in \mathbb{R}^N_{++}$ defined by $u''_i = \varepsilon$ if $i \in \mathcal{D}(U)$, and $u''_i = \lambda$ if $i \notin \mathcal{D}(U)$ we have $W(U' \bullet U'') > W(U \bullet U'')$, a contradiction. □

By Theorem 3, there does not exist a CUF that satisfies SI on the larger domain of incomplete utility profiles. Nevertheless, we can try to extend the Nash CUF to the larger domain so as to satisfy SI at least on $\mathcal{U}_c$. As a starting point we take the logarithm of W_N in order to transform the product into a sum; we then minimize for any profile $U \in \mathcal{U}$ the sum of the squared difference in views piece by piece. In other words, we solve the problem

$$\arg\min_{z \in \mathbb{R}_+} \sum_{i \in \mathcal{D}(U)} (\log u_i - \log z)^2 . \tag{3.4}$$

Let $k = \#\mathcal{D}(U)$. Since the quadratic deviations are minimized by the arithmetic mean, we obtain for the solution u^* of (3.4),

$$\log u^* = \frac{1}{k} \sum_{i \in \mathcal{D}(U)} \log u_i$$

from which we get

$$u^* = \sqrt[k]{\prod_{i \in \mathcal{D}(U)} u_i}.$$

In particular, by minimizing the sum of the differences of the logarithms of the available (and thus positive) views, we have to take the product of views and thereafter the k-th root of the obtained product, where k is the number of pieces with positive views. We call this CUF on $\mathcal{U}$ the *extended Nash CUF*. (Note that the extended Nash CUF as defined is indeed a monotone transform of W_N on $\mathcal{U}_c$.) The extended Nash CUF represents an arguably optimal compromise in view of the impossibilities uncovered by Theorems 1 and 3: it does not satisfy IIA but it uses cardinal information in a way that does not involve inter-criteria comparisons on the class of complete utility profiles.

Evidently, one can also here normalize the utilities and express them in terms of fractional numbers of views, i.e. consider the following variant of the extended Nash CUF. For all $U \in \mathcal{U}$,

$$W_{ext-N}(U) \;=\; \sqrt[k]{\prod_{i \in \mathcal{D}(U)} (u_i/\bar{u}_i)}.$$

This is the form in which we will use the extended Nash CUF in our application below.

Remark 3. On complete utility profiles on the entire set of real numbers (including negative reals) the utilitarian CUF can be characterized by a condition of 'zero independence' (see d'Aspremont and Gevers, 1977) in a way similar to the characterization of the Nash CUF in terms of SI on the positive reals. As in the case of SI one can show that no continuous SWO can satisfy zero independence on the class of *incomplete* utility profiles (appropriately defined).

4 Application to the Ranking of Violinists

What ranking of violinists do our data on the number of views on YouTube suggest? Theorem 1 above shows that any ranking will violate some desirable property. Therefore, our approach to answer the question is to look at different ranking methods and to investigate the robustness of the results with respect to the specific method employed.

A naïve first approach is to look at the violinst who receives the most first places. The clear winner on this 'plurality count' criterion is Hilary Hahn who is the most viewed violinist in 7 of the 46 pieces, followed by Sarah Chang who is the most viewed violinist in 3 pieces. But in view of the available information, looking only at the number of first places is evidently not appropriate. Instead, we will use both ordinal and cardinal ranking methods that make use of the available information. We describe these next.

Given a list $\succ \in \mathcal{L}$, we denote the domain of $\succ$, i.e. the set of comparable alternatives, by $\mathcal{D}(\succ)$, and for all $B \subseteq A$, by $\succ_{|B}$ the restriction of $\succ$ to $B \cap \mathcal{D}(\succ)$, i.e. the preference relation that is defined on $B \cap \mathcal{D}(\succ)$ and maintains the ordering of these alternatives as in $\succ$. In addition, we denote the set of complete linear orders on B by $\mathcal{P}_B$. The set of all incomplete lists is thus given by $\mathcal{L} = \cup_{\emptyset \neq B \subseteq A} \mathcal{P}_B$.

We derive a profile $\Pi = (\succ_1, ..., \succ_n) \in \mathcal{L}^n$ of incomplete lists from the data on views V by

$$a_j \succ_i a_k :\Longleftrightarrow\ V(i, a_j) > V(i, a_k) > 0$$

for any $i \in N$ and any $a_j, a_k \in A$.[8]

4.1 Ordinal Methods

The list of possible SWFs on $\mathcal{P}^n$ is long and some of them have nontrivial and multiple extensions to $\mathcal{L}^n$. Therefore, in this paper we restrict ourselves to the most basic ones: the Borda count, the Copeland method and the MedRank rule. To illustrate these SWFs in our context we use the profile given in Table 4. In all what follows, we use the fixed tie-breaking rule $a \tau b \tau c \tau d \tau e$ to resolve ties in order to arrive to the linear ordering.

1. The **Borda count**, henceforth denoted by BC, orders the alternatives based on the sum of their ranks. In particular, an alternative with a lower sum of ranks is preferred over

[8] It is worth mentioning that we simplify our analysis by considering only strict preferences without loss of generality since the possibility of identical number of views by different violinists for a given piece is negligible.

Rank	$\succ_1$	$\succ_2$	$\succ_3$	$\succ_4$	$\succ_5$	$\succ_6$	$\succ_7$
1	e	d	a	e	d	c	c
2	a	a	d	b	a	e	e
3	d	b	c	d	e		d
4	b		b		b		a
5	c				c		

Table 4: Illustrative incomplete profile

an alternative with a higher sum of ranks. As for any SWF, one possibility would to put all unranked alternatives at the lowest rank, which is against our interpretation of incomparable alternatives as missing information. We therefore consider the so-called 'modified' Borda count which better accounts for the missing information. Translating ranks into scores, if $\succ$ compares $k \leq n$ alternatives, then the highest ranked alternative gets k points, the second highest ranked alternative $k-1$, and so forth.

In case of the profile in Table 4 the sum of modified Borda scores of alternatives a, b, c, d and e are 15, 8, 10, 17 and 15, respectively. Therefore, the social ordering determined by BC is $d \succ a \succ e \succ c \succ b$. Formally, the SWF BC_τ is the *modified Borda count* if for all $(\succ_i)_{i=1}^n \in \mathcal{L}^n$ and all pairs of distinct alternatives a and b we have

$$aBC_\tau\left((\succ_i)_{i=1}^n\right)b \quad :\Longleftrightarrow \quad \sum_{a\in\mathcal{D}(\succ_i)}^{n} |\mathcal{D}(\succ_i)| - rk[a,\succ_i] > \sum_{b\in\mathcal{D}(\succ_i)} |\mathcal{D}(\succ_i)| - rk[b,\succ_i] \text{ or}$$
$$\sum_{a\in\mathcal{D}(\succ_i)} |\mathcal{D}(\succ_i)| - rk[a,\succ_i] = \sum_{b\in\mathcal{D}(\succ_i)} |\mathcal{D}(\succ_i)| - rk[b,\succ_i] \text{ and } a\tau b,$$

where $rk[a,\succ_i]$ stands for the position of a in $\succ_i$.

2. The **Copeland method** is based on pairwise comparisons of alternatives. An alternative beats another if it is ranked higher by more voters than vice versa; in this case, the former alternative wins while the other looses. This procedure is carried out for any pair of distinct alternatives. The Copeland method ranks alternatives based on the numbers of their pairwise wins. This, in fact, is the usual way how round-robin tournaments are organized. Of course, possible ties have to be broken by a tie-breaking rule. Again we consider the profile given in Table 4 and employ the same tie-breaking rule as in case of the previously introduced SWFs. We can see that a beats alternatives b and c, b beats alternative c, c does not beat any other alternative, d beats alternatives a, b and c, and finally e beats a, b and d. Therefore, the Copeland method arrives to the linear ordering $d \succ e \succ a \succ b \succ c$. We shall denote by CM the Copeland method, which we define now formally. For a given profile $(\succ_i)_{i=1}^n \in \mathcal{L}$ we say that alternative $a \in A$ *beats*

alternative $x \in A$ if $\#\{i \in N \mid a \succ_i x\} > \#\{i \in N \mid x \succ_i a\}$, i.e. a wins over x by pairwise comparison. We shall denote by $l[a, (\succ_i)_{i=1}^n]$ the number of alternatives beaten by alternative $a \in A$ for a given profile $(\succ_i)_{i=1}^n$. Then, the SWF CM_τ is the *Copeland method* if for all $(\succ_i)_{i=1}^n \in \mathcal{L}$ and all pairs of distinct alternatives a and b we have

$$a CM_\tau \left((\succ_i)_{i=1}^n\right) b \quad :\Longleftrightarrow \quad \begin{array}{l} l[a, (\succ_i)_{i=1}^n] > l[b, (\succ_i)_{i=1}^n] \text{ or} \\ l[a, (\succ_i)_{i=1}^n] = l[b, (\succ_i)_{i=1}^n] \text{ and } a\tau b. \end{array}$$

3. The **MedRank rule** determines for each alternative $a \in A$ the highest rank h_a such that a appears more than $\#\{i \in N \mid a \in \mathcal{D}(\succ_i)\}/2$ times in a given profile among the alternatives that ranked at h_a or higher; alternatives are then ranked according to their h_a value in descending order. Looking at Table 4, we see that no alternative receives a majority (of 4 votes) when counting only the numbers of top ranked alternatives. Now taking also the second ranked alternatives into consideration we see that both a and e appear four times, hence $h_a = h_e = 2$ with the tie-breaking rule τ giving priority to a. Admitting also all third ranked alternatives, d appears 6 times in the first three rows hence $h_d = 3$. If we also take the fourth ranked alternatives into account c and b appear 6 and 4 times, respectively, hence $h_b = h_c = 4$. Thus, employing the tie breaking rule τ, the MedRank rule gives the ranking $a \succ e \succ d \succ b \succ c$.

   In general, for each alternative a the rank h_a is the median rank in all rankings of voters who rank alternative a. We shall denote the *MedRank rule* by MR, and by MR_τ the variant employing the tie-breaking rule τ, i.e.,

$$a MR_\tau \left((\succ_i)_{i=1}^n\right) b :\Longleftrightarrow h_a < h_b \text{ or } \left(h_a = h_b \text{ and } a\tau b\right).$$

4. Dwork et al. (2002) proposed the **Markov chain extension of the Borda count** for incomplete lists as follows. The alternatives are taken as the states of the Markov chain. For a given preference profile Π and a given alternative $a \in A$ pick each preference relation in which a is ranked with equal probability. Then, for the selected preference relation $\succ_i$ choose each ranked alternative in $\mathcal{D}(\succ_i)$ with equal probability. If the selected alternative b is ranked higher than a, that is $b \succ_i a$, then move to state b; otherwise stay in state a. For the profile in Table 4 the derived transition matrix is show in Table 5. The stationary point of this Markov-process equals $[0.21239061, 0.04694411, 0.16922061, 0.25486873, 0.31657595]$, and ranking the alternatives according to these probabilities we arrive at the social ranking $e \succ d \succ a \succ c \succ b$. In general, the *Markov chain extension of the Borda count* ranks the alternatives according to the probabilities of the stationary point of the Markov-process defined above; below,

	a	b	c	d	e
a	$\frac{1}{5}(\frac{4}{5}+\frac{2}{3}+1+\frac{4}{5}+\frac{1}{4}) = \frac{211}{300}$	$0 = 0$	$\frac{1}{5}\frac{1}{4} = \frac{1}{20}$	$\frac{1}{5}(\frac{1}{3}+\frac{1}{5}+\frac{1}{4}) = \frac{47}{300}$	$\frac{1}{5}(\frac{1}{5}+\frac{1}{4}) = \frac{9}{100}$
b	$\frac{1}{5}(\frac{1}{5}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}) = \frac{59}{300}$	$\frac{1}{5}(\frac{2}{5}+\frac{1}{3}+\frac{1}{4}+\frac{2}{3}+\frac{2}{5}) = \frac{123}{300}$	$\frac{1}{5}\frac{1}{4} = \frac{1}{20}$	$\frac{1}{5}(\frac{1}{5}+\frac{1}{3}+\frac{1}{4}\frac{1}{5}) = \frac{59}{300}$	$\frac{1}{5}(\frac{1}{5}+\frac{1}{3}+\frac{1}{5}) = \frac{11}{75}$
c	$\frac{1}{5}(\frac{1}{5}+\frac{1}{4}+\frac{1}{5}) = \frac{13}{100}$	$\frac{1}{5}(\frac{1}{5}+\frac{1}{5}) = \frac{2}{25}$	$\frac{1}{5}(\frac{1}{5}+\frac{2}{4}+\frac{1}{5}+1+1) = \frac{58}{100}$	$\frac{1}{5}(\frac{1}{5}+\frac{1}{4}+\frac{1}{5})$ $\frac{13}{100}$	$\frac{1}{5}(\frac{1}{5}+\frac{1}{5}) = \frac{2}{25}$
d	$\frac{1}{6}(\frac{1}{5}+\frac{1}{4}) = \frac{9}{120}$	$\frac{1}{6}(\frac{1}{3}) = \frac{1}{18}$	$\frac{1}{6}(\frac{1}{4}) = \frac{1}{24}$	$\frac{1}{6}(\frac{3}{5}+1+\frac{3}{4}+\frac{1}{3}+1+\frac{2}{4}) = \frac{251}{360}$	$\frac{1}{6}(\frac{1}{5}+\frac{1}{3}+\frac{1}{4}) = \frac{47}{360}$
e	$\frac{1}{5}\frac{1}{5} = \frac{1}{25}$	$0 = 0$	$\frac{1}{5}(\frac{1}{2}+\frac{1}{4}) = \frac{3}{20}$	$\frac{1}{5}\frac{1}{5} = \frac{1}{25}$	$\frac{1}{5}(1+1+\frac{3}{5}+\frac{1}{2}+\frac{3}{4}) = \frac{77}{100}$

Table 5: Borda transition matrix

we denote the induced SWF by 'BordaMC.'

5. Dwork et al. (2002) also proposed the **Markov chain extension of the Copeland Method** for incomplete lists. For a given preference profile Π the set of alternatives ranked by any voter are the states of the Markov chain. Pick any alternative $a \in A$ with equal probability. Then for any other alternative b move to state b if and only if b is preferred to a by the majority of voters who rank both a and b. For the profile in Table 4 the derived transition matrix is shown in Table 6. The stationary point of this Markov-

	a	b	c	d	e
a	$\frac{1}{5}(1+0+0+1+1)$	0	0	$\frac{1}{5}$	$\frac{1}{5}$
b	$\frac{1}{5}$	$\frac{1}{5}(0+1+1+0+0)$	0	$\frac{1}{5}$	$\frac{1}{5}$
c	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}(0+0+1+0+1)$	$\frac{1}{5}$	0
d	0	0	0	$\frac{1}{5}(1+1+1+1+0)$	$\frac{1}{5}$
e	0	0	0	0	$\frac{1}{5}(1+1+1+1+1)$

Table 6: Copeland transition matrix

process equals $[0, 0, 0, 0, 1]$; ranking the alternatives according to these probabilities and employing the tie-breaking rule ,we arrive to the social ranking $e \succ a \succ b \succ c \succ d$. (The result is non-surprising since e is the Condorcet-winner.) In general, the *Markov chain extension of the Copeland method* ranks the alternatives according to the probabilities of the stationary point of the Markov-process so defined; below, we denote the induced SWF by 'CopMC.'

6. We also employ a method based on pairwise comparison matrices introduced by Saaty (1980). The (i, j)th entry of the pairwise comparison matrix (PCM) contains the ratio of the preferences in a profile in which alternative a_i beats alternative a_j. For the profile in Table 4 the PCM is shown in Table 7. In determining the ranking for the PCM in

	a	b	c	d	e
a	1	4/0	3/1	2/3	1/2
b	0/4	1	2/1	1/4	0/3
c	1/3	1/2	1	1/3	2/2
d	3/2	4/1	3/1	1	1/3
e	2/1	3/0	2/2	3/1	1

Table 7: Pairwise comparison matrix

Table 7, we replace $x/0$ by 10 and $0/x$ by 0.1, for simplicity. The weights for the ranking are then determined by the eigenvector associated with the dominant real eigenvalue of this modified PCM. The non-normalized weights are 0.6510, 0.1841, 0.7285, 0.5029, 1, resulting in the ranking of alternatives $e \succ c \succ a \succ d \succ b$. The SWF induced by this method is referred to as 'Saaty' below.

4.2 Cardinal Methods

Among the collective utility functions, we employ the utilitarian ('Util'), the relative utilitarian ('RUtil'), the Nash ('Nash') and the extended normalized Nash ('RNash') CUF, respectively. Note that we only consider 'utilities' (i.e. views) that pass the 50 thousand threshold; therefore we do not run into problems with zero values in case of the Nash and relative Nash CUF (see Table 8 for the results).

5 Results and statistical analysis

Our main goal is to confirm or refute the hypothesis that the most famous violinists from the early period of recordings are as attractive to today's viewers as the prominent contemporary violinists. To test this statistically, we produced different rankings of violinists based on the methods described above, and grouped the artists into active and non-active violinist.

Table 8 contains the rankings according to the ten methods defined above. From the 128 violinists under consideration only 32 violinists had 10 uploaded videos passing the 50 thousand view threshold.[9] In each column we can see the rank positions of the violinists by the respective method.[10]

The names of the violinists are ordered based on their positions in the last column. Indeed, we believe that the Relative Nash CUF is the most sound ranking because it satisfies the scale independence property at least on all full utility profiles, and because it gives each piece the same weight. At first sight we can see that the positions of most of the violinists are 'similar' for all ten employed methods. We obtained the most deviations for Saaty's eigenvalue method. The rank correlation matrix in Table 9 shows the correlation between any pair of rankings. Most pairs of rankings have a very high rank correlation (values larger than 0.7), or high rank correlation (between 0.5 and 0.7). There is a medium level of correlation only between the Nash CUF and Saaty's method.

We can also observe that Hilary Hahn is ranked first based on eight out of ten methods, while Itzhak Perlman is the most frequent second ranked violinist. Maxim Vengerov is also

[9] We have taken into account only videos uploaded until the 31th of December 2023; the data were collected during October 2024 with the data for each piece collected on the same day.

[10] We have also compiled rankings based on requiring just 8 or 6 pieces passing the threshold, and alternatively considered a minimum threshold of only 25000 views. These rankings are available on request from the authors. They all support our null hypotheses as well.

ranked in the top 10 by all methods. Joshua Bell, Sarah Chang, Julia Fischer, Jascha Heifetz and David Oistrakh are ranked by at least eight methods in the top 10.

	Cop	MBorda	MedRank	BordaMC	CopMC	Saaty	Util	RUtil	Nash	RNash
Hilary Hahn	1	2	1	1	1	1	1	1	2	1
Sarah Chang	3	10	4	4	3	3	6	2	19	2
Maxim Vengerov	8	5	4	7	6	5	4	3	5	3
David Oistrakh	6	3	2	3	4	18	5	4	3	4
Itzhak Perlman	2	1	3	2	2	7	2	7	1	5
Ray Chen	11	11	9	12	7	10	11	6	13	6
SoHyuon Ko	16	23	12	21	19	9	14	5	28	7
Jascha Heifetz	8	7	4	9	8	12	9	11	9	8
Julia Fischer	5	8	4	6	11	19	10	8	8	9
Augustin Hadelich	20	19	19	13	24	20	22	10	12	10
Joshua Bell	8	5	4	5	9	8	7	12	6	11
Janine Jansen	3	4	11	8	5	4	3	13	10	12
Clara-Jumi Kang	12	9	9	10	16	14	13	21	7	13
Leonidas Kavakos	31	26	19	24	27	30	27	14	18	14
Anne-Sophie Mutter	16	12	18	15	13	24	12	16	14	15
David Garrett	12	16	26	16	17	13	8	22	24	16
Sayaka Shoji	7	17	12	14	10	2	17	9	22	17
Gil Shaham	27	24	24	19	25	22	28	18	20	18
Kyung Wha Chung	15	18	26	22	12	15	20	17	16	19
Ai Takamatsu	12	13	15	17	15	6	15	28	11	20
Isaac Stern	23	13	12	11	18	25	23	19	4	21
Nicola Benedetti	32	31	19	31	31	28	30	29	31	22
Maria Duenas	16	22	19	23	21	17	21	15	29	23
Frank P. Zimmermann	24	32	26	32	30	29	32	24	32	24
Leonid Kogan	29	25	31	25	28	31	26	23	17	25
Soojin Han	19	15	19	18	14	23	19	30	23	26
Nathan Milstein	26	30	29	28	32	27	29	27	30	27
Yehudi Menuhin	21	20	15	20	23	16	16	20	15	28
Zia Hyunsu Shin	27	21	32	26	29	21	18	26	26	29
Shlomo Mintz	21	28	24	27	20	11	25	25	27	30
Pinchas Zuckerman	29	29	15	29	22	26	31	31	25	31
Daniel Lozakovich	24	25	29	30	26	32	24	32	21	32
Runs test (Z_r)	0.567	1.636	0.567	1.636	1.101	0.567	1.636	0.567	-1.569	0.567
Wilcoxon rank-sum test (Z-value)	-0.661	-0.114	-0.410	-0.114	-0.433	-1.299	-0.570	-0.251	0.980	-0.752

Table 8: Rankings and Z-values

After these simple observations we turn to the test of our main hypothesis. Since we have no information about the distribution of views we carry out two well-known non-parametric tests. The standardized Z-values for the runs test and the Wilcoxon rank sum test can be found in the last two rows of Table 8 for each method.[11] For both tests we have to form two groups. Out of the 32 violinists appearing in Table 8 Kyung Wha Chung, Jascha Heifetz, Leonid Kogan, Yehudi Menuhin, Nathan Milstein, David Oistrakh and Isaac Stern are inactive, while the other 25 violinists are all active.

To illustrate the tests we take the last column of Table 8. Following the ranking by the (extended) relative Nash CUF we obtain the sequence A, A, A, I, A, A, A, I, A, A, A, A, A, A, A, A, A, A, I, A, I, A, A, A, I, A, I, I, A, A, A, A, where A and I stands for active and

[11] For a detailed description ot these two tests we refer to Walpole et al. (2016).

	Cop	MBorda	MedRank	BordaMC	CopMC	Saaty	Util	RUtil	Nash	RNash
Cop	1.0000	0.8796	0.7703	0.8672	0.9283	0.8369	0.9130	0.7231	0.6284	0.7561
MBorda	0.8796	1.0000	0.7972	0.9531	0.9064	0.6688	0.9285	0.6648	0.8668	0.7559
MedRank	0.7703	0.7972	1.0000	0.8491	0.8197	0.6422	0.7389	0.7393	0.7109	0.7909
BordaMC	0.8672	0.9531	0.8491	1.0000	0.8827	0.6807	0.8794	0.7771	0.8464	0.8405
CopMC	0.9283	0.9064	0.8197	0.8827	1.0000	0.7841	0.8845	0.7188	0.7060	0.7577
Saaty	0.8369	0.6688	0.6422	0.6807	0.7841	1.0000	0.7720	0.6342	0.4175	0.6213
Util	0.9130	0.9285	0.7389	0.8794	0.8845	0.7720	1.0000	0.7053	0.6946	0.7698
RUtil	0.7231	0.6648	0.7393	0.7771	0.7188	0.6342	0.7053	1.0000	0.5576	0.9106
Nash	0.6284	0.8668	0.7109	0.8464	0.7060	0.4175	0.6946	0.5576	1.0000	0.6393
RNash	0.7561	0.7559	0.7909	0.8405	0.7577	0.6213	0.7698	0.9106	0.6393	1.0000

Table 9: Rank correlation matrix

inactive, respectively. We can see 13 runs, where a run is a maximal consecutive subsequence of 'A's or 'I's. Intuitively, if the this number is relatively large we cannot separate the two groups and our hypothesis cannot be refuted. Carrying out a respective one-tailed test, the respective Z values have to be at least -1.65 at a significance level of 5 percent. We can see that this is satisfied by all Z values, and therefore we can confirm our hypothesis.

Turning to the Wilcoxon rank-sum test, we still have to separate our 32 violinists into two groups and determine the sum of the ranks for each group. The Wilcoxon test checks whether the distributions for the two groups are sufficiently similar or not. Assuming that the active violinists still do not get less attention on YouTube, we carry out a one-sided test. At a significance level of 5 percent the Z values shown in the last line of Table 8 have to be larger than -1.65. We can see that this is the case for all ten methods. Thus, based on the Wilcoxon test we can also affirm our hypothesis. In fact, the respective Z values a far larger than -1.65 for both tests.

6 Concluding Remark

One may question if counting the number of YouTube views is in fact an appropriate basis for judging our hypothesis. But note that our approach arguably even favors the younger generation, for instance because many of the younger artists maintain their own YouTube channels. Moreover, even though theoretically neutral with respect to violinists, the YouTube search and recommendation algorithm naturally favors newly uploaded content. These considerations thus appear to even strengthen our main result that older violinists are as attractive to contemporary audiences as contemporary artists. In addition, there is good reason to believe that our rankings and results are robust with respect to the precise source of data; indeed, similar results can be expected if data were collected from other platforms such as Spotify, Apple Music Classical or IDAGIO.

Appendix

In this appendix, for each violinist and piece, the three most viewed items with at least 25.000 views are added. It can be the case that none, one, two or three videos satisfy this criterion. In addition, we require that the at most top 3 viewed items get altogether at least 50.000 views, and that the ranked violinist in Table 10 have 10 such pieces. In Table 10 we can see that Hilary Hahn is ranked first by 8 out of 10 methods. Itzhak Perlman is ranked second most frequently and always ranked in the top 10. Among the legendary artists of the previous century, David Oistrakh and Jascha Heifetz are ranked in the top 10 by all employed methods. For all ten methods the Wilcoxon-sum test accepts our null hypotheses that the inactive violinists are as attractive to the viewers as contemporary active violinist (at a 5% significance level). Concerning the runs test only the Nash CUF misses slightly the 5% significance level, for all other methods the runs test accepts our null hypotheses safely. Out of the 34 violinists appearing in Table 10 Kyung Wha Chung, Jascha Heifetz, Leonid Kogan, Yehudi Menuhin, Nathan Milstein, David Oistrakh and Isaac Stern are inactive, while the other 27 violinists are all active.

The rank correlation matrix in Table 11 shows between most of the rankings very high rank correlation (values larger than 0.7) or high rank correlation (between 0.5 and 0.7). There is only a medium level of correlation between the Nash CUF and Saaty's method.

	Cop	MBorda	MedRank	BordaMC	CopMC	Saaty	Util	RUtil	Nash	RNash
Hilary Hahn	1	2	1	1	1	1	1	1	2	1
Bomsori Kim	6	19	2	8	5	3	14	3	30	2
Itzhak Perlman	2	1	2	2	2	10	2	5	1	3
Maxim Vengerov	10	5	7	6	12	5	5	4	7	4
David Oistrakh	5	3	2	3	4	7	7	6	3	5
Sarah Chang	3	10	5	5	3	2	6	2	18	6
Jascha Heifetz	4	4	7	4	6	9	8	7	4	7
Ray Chen	13	12	7	12	10	14	12	10	14	8
Julia Fischer	7	8	6	7	9	21	9	8	9	9
SoHyuon Ko	20	26	11	24	19	20	16	14	28	10
Augustin Hadelich	25	21	27	22	27	18	24	11	13	11
Joshua Bell	14	6	10	9	11	11	10	13	6	12
Janine Jansen	9	7	11	10	8	13	4	12	10	13
Anne-Sophie Mutter	17	11	17	14	17	25	11	17	11	14
David Garrett	11	12	22	13	14	6	3	21	21	15
Clara-Jumi Kang	11	9	11	11	16	16	13	23	8	16
Sayaka Shoji	7	15	11	15	7	4	15	9	22	17
Leonidas Kavakos	31	25	22	25	25	31	27	18	15	18
Gil Shaham	28	22	29	18	26	22	29	19	20	19
Kyung Wha Chung	18	18	26	21	21	23	21	16	17	20
Isaac Stern	25	16	17	17	20	29	23	20	5	21
Ai Takamatsu	16	14	16	16	15	8	18	29	12	22
Nicola Benedetti	33	32	11	27	30	27	30	27	32	23
Maria Duenas	15	23	22	20	13	12	22	15	29	24
Soojin Han	19	17	29	23	18	26	19	28	24	25
Leonid Kogan	29	26	33	26	31	32	28	24	19	26
Frank P. Zimmermann	21	34	28	33	34	24	34	25	34	27
Nathan Milstein	24	28	17	28	22	28	31	30	31	28
Yehudi Menuhin	22	20	17	19	24	17	17	22	16	29
Shlomo Mintz	27	29	21	29	23	15	25	26	27	30
Pinchas Zuckerman	33	31	22	31	29	33	32	32	25	31
Zia Hyunsu Shin	23	24	31	30	32	30	20	31	26	32
Daniel Lozakovich	31	29	31	32	28	34	26	33	23	33
Antal Zalai	29	33	34	34	33	19	33	34	33	34
Runs test (Z_r)	0.478	0.478	-0.605	0.478	-0.605	0.478	0.478	0.478	-1.689	-0.606
Wilcoxon rank-sum test (Z-value)	-0.234	0.298	-0.128	0.192	-0.234	-0.958	-0.532	-0.106	1.171	-0.575

Table 10: Rankings and Z-values based on the top 3 viewed videos

	Cop	MBorda	MedRank	BordaMC	CopMC	Saaty	Util	RUtil	Nash	RNash
Cop	1.0000	0.8559	0.7662	0.8915	0.9261	0.8276	0.8859	0.7992	0.5478	0.7961
MBorda	0.8559	1.0000	0.7104	0.9486	0.8620	0.6593	0.9222	0.7376	0.8663	0.7929
MedRank	0.7662	0.7104	1.0000	0.8275	0.8483	0.6750	0.7358	0.7639	0.5306	0.8136
BordaMC	0.8915	0.9486	0.8275	1.0000	0.9199	0.7574	0.8995	0.8533	0.7717	0.8894
CopMC	0.9261	0.8620	0.8483	0.9199	1.0000	0.8011	0.8726	0.8225	0.5963	0.8252
Saaty	0.8276	0.6593	0.6750	0.7574	0.8011	1.0000	0.7299	0.7137	0.3561	0.6895
Util	0.8859	0.9222	0.7358	0.8995	0.8726	0.7299	1.0000	0.7455	0.6920	0.7953
RUtil	0.7992	0.7376	0.7639	0.8533	0.8225	0.7137	0.7455	1.0000	0.5707	0.9306
Nash	0.5478	0.8663	0.5306	0.7717	0.5963	0.3561	0.6920	0.5707	1.0000	0.6275
RNash	0.7961	0.7929	0.8136	0.8894	0.8252	0.6895	0.7953	0.9306	0.6275	1.0000

Table 11: Rank correlation matrix for the top 3 viewed videos

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