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Committee-Structured Games in a Cooperative Framework

Parishmita Boruah[1,2] · Surajit Borkotokey[1,3] · Rajnish Kumar[4]

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Abstract

We introduce the notion of committee-structured games (CSG) in a cooperative framework to analyze situations where committees affect players in generating their coalitional values. We represent the sets of committees by hypergraphs and obtain values as a function of both coalitions and hypergraphs. Based on how committee structures affect value generation, we present two models within the CSG framework. In the first model, the total value is generated by the committee set as a whole, while in the second model, it is obtained by aggregating contributions from all subsets of the set of committees. Accordingly, we define the Shapley value for the first model and the Aggregated Shapley value for the second, providing axiomatic characterizations for both. Further, we propose bidding mechanisms to show that the Shapley value and the Aggregated Shapley value for the class of CSG is a subgame perfect Nash equilibrium (SPNE) of the induced strategic game. Finally, we study an alternative Exchange Economy model by incorporating committees instead of commodities.

Keywords Cooperative game · Shapley value · Aggregated Shapley value · Hypergraph game · Committee structure · Exchange economy

✉ Surajit Borkotokey
sborkotokey@dibru.ac.in

Parishmita Boruah
pbaruahm@gmail.com

Rajnish Kumar
rajnish.kumar@qub.ac.uk

[1] Dibrugarh University, Dibrugarh, India

[2] Department of Mathematics, Rangapara College, Rangapara, India

[3] Corvinus Institute for Advanced Studies (CIAS), Corvinus University of Budapest, Budapest, Hungary

[4] Queen's University Belfast, Belfast, UK

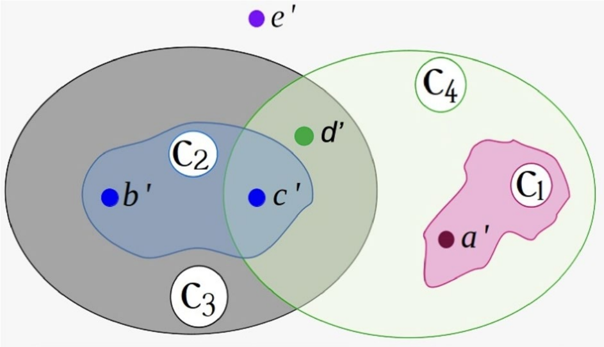

Fig. 1 Model hostel with a committee structure

1 Introduction

The purpose of this paper is to study situations where committees of various stakeholders of an organization are formed to smoothen the functioning of their coalitional activities. The employees of the organization are actively involved in diverse activities, leading to a situation where some employees become members of multiple committees, while others are not affiliated with any such committees. In the overall value generation process across different coalitions of employees, these committees play a crucial role by contributing additional benefits. To further motivate this idea, take the following example.

Example 1 Consider a model hostel with five boarders: a', b', c', d', and e'. The warden of the hostel constitutes four committees with these boarders, namely c_1 (Cultural), c_2 (Sports), c_3 (Mess Management), and c_4 (Discipline and Anti-ragging). The composition of these committees is as follows: $c_1 = \{a'\}, c_2 = \{b', c'\}, c_3 = \{b', c', d'\}, c_4 = \{a', c', d'\}$, as shown in Fig. 1. By identifying the values of the coalitions they form, cooperative game-theoretic techniques can be used to determine the overall contributions of each boarder. It represents the value generated by a coalition without distinguishing individual contributions to additional engagements within those committees. For instance, boarders b' and c' are members of both the sports and mess management committees, with different contributions in each. The cooperative game-theoretic model, however, fails to distinguish their contributions across these two committees. On the other hand, the network approach considers players' interactions pairwise, forming links between two players. For an overview of cooperative approaches to network games, see Borkotokey et al. (2019). While this captures some cooperative dynamics, it may overlook cases where work is carried out collectively by an entire coalition rather than individual interactions. For example, while the mess management committee collectively prepares the hostel menu, the specific inputs from each member may not be crucial, but the overall decision matters. Hypergraphs offer a way to represent committees, where each committee can be seen as a hyperlink, and all committees together form a hypergraph. However, relying solely on a hypergraph has its limitations. It fails to fully describe the impact of committees on each coalition or the surplus generated by committees for the coalitions. Additionally, players like e', in Fig. 1, who are not members of any committees, are not considered significant in the hypergraph game, potentially overlooking their contributions.

This motivates us to look for an alternative approach that emphasizes the effect of committees on players' cooperation in generating values. To address this, a new hybrid formalism of cooperative games and hypergraph games is proposed. This new game can be viewed as the cooperative game when there is no committee, and a hypergraph game in the presence of committees as hypergraphs. Cooperative games analyze how rational agents cooperate to achieve mutual gains, while hypergraph games extend this framework to represent scenarios where coalitions are formed based on hyperlinks, offering a powerful tool to study cooperation dynamics in complex systems. Myerson (1980) and van den Nouweland et al. (1992) independently introduced the notion of hypergraphs generalizing the graph structure. Each element of a hypergraph is called a hyperlink through which communication takes place. Jorzik (2023) extended the conference structure of hypergraph games into hypergraph network games. In our framework, committees are represented as hyperlinks to analyze their impact on cooperation. For this reason, we call them "committee-structured games" (CSGs). Other studies on hypergraph games can be found in Béal et al. (2021) and Moretti et al. (2022). Recent work in cooperative game theory also studies situations in which players may belong to multiple, possibly overlapping groups. Albizuri et al. (2006) analyze such settings through coalition configurations by extending the Owen value, and Algaba et al. (2025) consider more general cooperation structures using flow-based allocation methods. In these approaches, value is assigned through feasible coalitions or coalition profiles. In our framework, committees are modeled as hyperlinks, allowing committee effects, including contributions from non-members, to be captured directly in the allocation rules.

In the framework of CSGs, we discuss two models based on how the overall value or outcome is generated through the collaborations of committees. In the first model, the total value is generated by the committee set as a whole, without accounting for the contributions of its internal sub-committees (subsets of the committee set). In the second model, the total value is obtained by aggregating the contributions of the sub-committees, explicitly capturing the effects of its internal subsets.

A one-point solution concept (usually called value) or an allocation rule in a game theoretic setting assigns to each game a real vector, where every component of this vector gives the payoff allocation for each player in the game. From now on, unless stated otherwise, by a solution or an allocation rule we only mean a one-point solution concept. In this paper, we introduce the Shapley value and the Aggregated Shapley value for CSGs and provide their axiomatic characterizations. The Shapley value measures the productivity of a player through the so-called marginal contributions, see, Algaba et al. (2019) and Shapley (1953). We introduce the marginal contributions of the players in a committee setting and show that the allocation rule that divides this worth among the players is exactly the one that satisfies the properties of Linearity, Symmetry, Efficiency, and Non-Committee Member Contribution. In addition, we also characterize the Shapley value using Efficiency and the Fairness axiom. On the other hand, the Aggregated Shapley value measures a player 's productivity by considering marginal contributions across all subsets of the committee set. We characterize the Aggregated Shapley value by extending the axioms for the Shapley value in the CSG framework. Our axioms are specific to the CSGs and they differ from those in cooperative games in the following ways: In cooperative games, axioms like Linearity, Symmetry, and Efficiency determine players' payoffs based on the coalitions they form. However, in CSG, these axioms account for both coalitions and committees, reflecting the overlapping and interdependent nature of the committees the players form ex-ante. Consequently, a player's payoff depends not only on the coalitions she belongs to but also on her role within the committees to which she belongs. The axiom of Non-Committee Member Contribution is specifically designed for the CSG framework. It addresses situations where external contributions to committees influence the payoff distributions. The Fairness axiom, on the other hand, ensures that committees contribute in unison and therefore, members within the same committee are treated equitably.

Further, we present a bidding mechanism for implementing committee-based decision-making. Finally, we introduce an alternative Exchange Economy model by replacing goods and services with committees formed by players. This deviation from the traditional Exchange Economy model provides a different perspective and understanding of economic interactions within a committee-based framework. Some basic properties of ordinal and cardinal value allocation in an Exchange Economy are explored in Scafuri and Yannelis (1984) and Shafer (1980) by constructing examples. Their approach puts forward two interesting findings in our committee-based framework of the Exchange Economy:

R_1: Individuals not belonging to any committee can also avail the benefits of committees formed by other individuals.

R_2: Individuals sharing identical preferences may obtain different rewards depending upon their positions based on the organizational structure of the committees.

Guala et al. (2013) studied the impact of group or committee membership on individuals' cooperative behavior and contributions to shared resources, emphasizing the influence of group affiliation and knowledge symmetry. They observed that these factors could unintentionally lead to the exclusion of individuals not identified as in-group members, potentially decreasing overall cooperation. To address this concern, our Exchange Economy model considers how individuals outside the committee can still benefit from its activities, making cooperation more inclusive and reducing exclusions.The paper is organized as follows: Sect. 2 contains some basic concepts and Sect. 3 presents the mathematical formulation of the model. Section 4 discusses the Shapley value for CSG and its axiomatic characterization. Sect. 5 introduces the Aggregated Shapley value and presents its axiomatic characterizations. Sect. 6 provides bidding mechanisms. In Sect. 7, we explore the application of CSG in Exchange Economy and lastly in Sect. 8, we make the concluding remarks.

2 Preliminaries

In this section, we present some basic concepts and definitions of cooperative games and hypergraph games that are relevant for developing our model compiled from Shapley (1953), and van den Nouweland et al. (1992). Let $N = \{1, \cdots, n\}$ be a finite set of players. Each subset *S* of *N* is known as a coalition, and *N* is the grand coalition. The set of all coalitions of *N* is denoted by 2^N. We use |*S*| to denote the cardinality of *S*.

Definition 1 A cooperative game with transferable utility, or simply a TU game, is the pair (*N*, *v*) where *N* is the player set and a characteristic function $v : 2^N \to \mathbb{R}$ such that $v(\emptyset) = 0$.

When *N* is fixed, we denote a TU game just by its characteristic function *v*. For each $S \in 2^N$, *v*(*S*) is the value of the coalition *S*. Denote the set of all TU-games on a player set *N* by V^N.

Definition 2 A value for TU games is a function $\phi : V^N \to \mathbb{R}^{|N|}$, which assigns to each $v \in V^N$ an |*N*|-tuple of real numbers. Thus formally, for each $v \in V^N$, $\phi(v) = (\phi_1(v), \phi_2(v), \cdots, \phi_n(v))$ where $\phi_i(v) \in \mathbb{R}$ represents the payoff for player *i*.

Let us assume that the players in *N* make multilateral relationships among themselves. A hyperlink is a subset of *N* that is formed based on such multilateral relationships among its members. Thus, with an abuse of notations, we denote by 2^N the set of all hyperlinks on the player set *N*. A hypergraph is a pair (*N*, *E*), where *N* is the set of nodes (players) and $E = \{c_1, c_2, \cdots, c_k\}$ the set of hyperlinks, where $k \in \{1, 2, ..., (2^{|N|} - 1)\}$. If *N* is fixed, we represent a hypergraph only by *E*. Let $E_\emptyset$ denote the null hypergraph, the hypergraph with no hyperlink. Let H^N denote the set of all hypergraphs on the player set *N*. Two players *i* and *j* are connected directly in the hypergraph *E* if they belong to the same hyperlink in *E*. A hyperpath between two players *i* and *j* in *E* is a sequence of hyperlinks $c_1, c_2, \cdots, c_n$ in *E* such that $i \in c_1$ and $j \in c_n$ with the condition that there is at least one node common between c_k and c_{k+1}, $k \in \{1, 2, \cdots, n-1\}$. Players *i* and *j* are connected indirectly if they don't belong to the same hyperlink but are connected by a hyperpath. For each $S \subseteq N$, $E|_S$ denotes the set of hyperlinks of *E* formed by the players in *S*, i.e., $E|_S := \{c \in E | c \subseteq S\}$. It follows that $E|_S \subseteq 2^S$.

Definition 3 A hypergraph game is the pair (N, γ), where *N* is a finite set of players and $\gamma : H^N \to \mathbb{R}$ is a value function satisfying the condition $\gamma(E_\emptyset) = 0$.

For each $E \in H^N$, $\gamma(E)$ is the value obtained by hypergraph *E*. Denote the set of all value functions defined on H^N by G^N.

Remark 1 van den Nouweland et al. (1992) defined hypergraph games by extending communication situations where the value function is determined without considering whether players are connected in a hypergraph directly or indirectly. In our current study, we inherit the idea of network games given by Jackson and Wolinsky (1996) in defining the hypergraph games that distinguish the direct and indirect hyperlinks among the players. In our work, we include hyperlinks consisting of single players recognizing that a single player may suffice to execute a specific task. Recall for instance, in Example 1, the committee c_1 contains member a' only.

3 Model formulation

Assume that the players within *N* form distinct committees, labelled as c_1, c_2, and so forth, up to c_k. Let *E* be the set of all the committees. Essentially, *E* possesses the structure of a hypergraph, where the committees themselves act as hyperlinks. Denote *N*(*E*) the set of players that has at least one hyperlink in *E*.

A committee-coalition pair (co-co pair) is an element of $H^N \times 2^N$ of the form (*E*, *S*) where $E \in H^N$ and $S \in 2^N$. For two co-co pairs $(E, S), (F, T) \in H^N \times 2^N$, we define the notation $\sqsubseteq$ as follows.

$$(E, S) \sqsubseteq (F, T) \text{ if and only if } E \subseteq F \text{ and } S \subseteq T.$$

Definition 4 A CSG is a pair (*N*, *w*), where *N* is a finite set of players and *w* is a value function such that $w : H^N \times 2^N \to \mathbb{R}$ with $w(E_\emptyset, \emptyset) = 0$.

For each co-co pair $(E, S) \in H^N \times 2^N$, the quantity *w*(*E*, *S*) represents the value generated by the coalition *S* in the presence of the set *E* of committees. If no ambiguity arises on the player set *N*, we denote a CSG (*N*, *w*) simply by its value function *w*. Denote the set of all value functions on the player set *N* by W^N. W^N is a vector space with dimensions $2^{2^{|N|}} \times 2^{|N|} - 1$.

Remark 2 In a CSG, identical coalitions can yield different values depending on the hypergraph (committee) structure in which they are embedded. This extends the classical cooperative game framework, where coalition worth depends only on membership, by explicitly accounting for the organizational context and internal structure of committees.

For each non-empty co-co pair $(F, T) \in H^N \times 2^N$, we define the unanimity value function $u_{(F,T)}$ as follows.

$$u_{(F,T)}(E', S) = \begin{cases} 1 & \text{if } (F, T) \sqsubseteq (E', S) \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The set of unanimity value functions $(u_{(F,T)})_{(F,T) \neq (E_\emptyset, \emptyset) \in H^N \times 2^N}$ forms a basis for W^N. Each $w \in W^N$ can be written as a linear combination of $(u_{(F,T)})_{(F,T) \neq (E_\emptyset, \emptyset) \in H^N \times 2^N}$ as follows.

$$w = \sum_{(F,T)\neq(E_{\emptyset},\emptyset)\in H^N\times 2^N} \lambda_T^F(w)u_{(F,T)} \tag{2}$$

where, $\lambda_T^F(w) = \sum_{(F',T')\sqsubseteq(F,T)} (-1)^{|F|+|T|-|F'|-|T'|}w(F',T')$.

Remark 3 Given $v \in V^N$ and $\gamma \in G^N$, we can always define a CSG $w \in W^N$ as a function of *v* and γ. For example, the two games (i) $w = \gamma + v$, given by $w(E,S) = \gamma(E) + v(S)$ and (ii) $w = \gamma \cdot v$ given by $w(E,S) = \gamma(E) \cdot v(S)$ for $(E,S) \in H^N \times 2^N$ are CSG. It is worth noting that when a coalition of players does not form any committees among themselves, their overall value is equivalent to the value they generate as a whole, without any committees.

We now present two models that represent how value is generated through committees in the CSG framework. These are described below.

CV: For a given committee set $E \in H^N$, *w*(*E*, *N*) denotes the total value generated when all players in *N* operate within the structure defined by *E* in a CSG (*N*, *w*). This value is then distributed among the players according to a specified allocation rule. For example, in a group of firms collaborating on a joint project, *N* represents all participating firms, and *E* is the set of committees responsible for planning, oversight, and implementation. The value *w*(*E*, *N*) represents the total output the firms can collectively achieve under the guidance of these committees. In this model, the committee is treated as a single unit, which is why we call it the *Collective Value (CV)* model.

AV: The CV model works well when the influence of *E* depends only on the committee set as a whole. However, in many real-world situations, the contributions of subgroups within a committee set play an important role. For instance, in research collaborations or policy-making bodies, the effectiveness of the whole group depends on the performance of its subgroups. In such cases, each subset $E' \subseteq E$ has a distinct impact on the total value. The aggregated value is then defined as $\sum_{E'\subseteq E} w(E',N)$. This model generalizes CV by systematically incorporating the contributions of all subgroups, providing a richer and more accurate account of how total value is generated and distributed, especially when the internal structure of committees is important. That is why we call it the *Aggregated Value (AV)* model.

We next provide the formal definition of an allocation rule in the CSG framework, which determines how the value generated by the committees is distributed among the players.

Definition 5 An allocation rule for a CSG is a function $\phi : H^N \times W^N \to \mathbb{R}^{|N|}$ such that, for every $E \in H^N$ and $w \in W^N$, the vector $\phi(E,w) = (\phi_1(E,w), \phi_2(E,w), \ldots, \phi_n(E,w)) \in \mathbb{R}^{|N|}$ assigns a payoff to the game (*N*, *w*) associated with the committee structure *E*, where $\phi_i(E, w) \in \mathbb{R}$ denotes the payoff assigned to player $i \in N$.

As already mentioned in the Introduction, in the following, we study two allocation rules for the class of CSGs: (i) the Shapley value under the CV model and (ii) the Aggregated Shapley value under the AV model.

In Sect. 4, we present the Shapley value and its characterizations. The Myerson value (Myerson 1980) refines the Shapley value for TU games, where the coalition value depends on the underlying conference structure (hypergraph) without consdering how the players are connected in the conference. On the other hand, the Shapley value in a CSG accounts for the impact of the structure of the committees (hypergraph), i.e., it also considers how players are connected in the hypergraph. Thus, under our set-up, direct and indirect hyperlinks among players are assumed to have different contributions in the value generation process. In Sect. 5, we introduce the Aggregated Shapley value and provide its axiomatic characterizations. This allocation rule distributes the total aggregated value among players in a way that differs from the traditional approach of allocating a single generated value. It explicitly incorporates the contributions of sub-committees in the process of value generation.

From now on, wherever there is no ambiguity, we shall simply refer to the Shapley value and the Aggregated Shapley value within the CSG framework, without repeatedly specifying whether they correspond to the CV or AV model. Their precise meaning will be clear from the definitions provided in the latter sections.

4 The Shapley value in CSGs

We begin with a few definitions that are relevant for developing the Shapley value and its characterization.

Definition 6 For $E \in H^N$, the marginal contribution $m_i^w(E, S)$ of a player *i* to a co-co pair $(E, S) \sqsubseteq (E, N \setminus \{i\})$ in $w \in W^N$ is given by

$$m_i^w(E, S) = w(E|_{S\cup\{i\}}, S \cup \{i\}) - w(E|_S, S). \tag{3}$$

The marginal contributions in a CSG evaluate a player's contributions due to her inclusion or exclusion in a coalition and the committees together.

Definition 7 Let $E \in H^N$. Two players $i, j \in N$ are said to be symmetric in $w \in W^N$ with respect to *E*, if for all $S \subseteq N \setminus \{i, j\}$,

$$w(E|_{S\cup\{i\}}, S \cup \{i\}) = w(E|_{S\cup\{j\}}, S \cup \{j\}).$$

It follows from Definition 7 that two players *i* and *j* are symmetric in a CSG with respect to a committee *E* if their marginal contributions to every possible co-co pair are identical. This implies that *i* and *j* play identical roles in the game with respect to E. Here, symmetry should be attained on both coalitions and committees which is strong requirement unlike its counterpart in cooperative games.

Definition 8 A player $i \in N$ is called a non-committee member with respect to $E \in H^N$ if $i \notin N(E)$.

This implies that *i* is not a member of any committee in *E*. Consequently, $i \notin N(F)$ for all subsets $F \subseteq E$.

Definition 9 The Shapley value for the class W^N of CSGs is the function $\phi^{Sh} : H^N \times W^N \to \mathbb{R}^{|N|}$ given by

$$\phi_i^{Sh}(E, w) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \big(w(E|_{S \cup \{i\}}, S \cup \{i\}) - w(E|_S, S)\big) \quad (4)$$

for each $i \in N$, and $(E, w) \in H^N \times W^N$.
The Shapley value for a CSG assigns to each player $i \in N$ a fair share of the total value, based on their marginal contributions across various co-co pairs. It evaluates not only the individual contributions of players but also the structural influence of the committee, thereby extending the classical Shapley value to a setting where the outcome depends on both coalition formation and the committee structure.

Next, we introduce the axioms below in the framework of CSG.

Axiom1 *(Efficiency)For* $w \in W^N$ *andE* $\in H^N$*,wehave*$\sum_{i \in N} \phi_i(E, w) = w(E, N)$.

The Efficiency axiom states that the total value (which, in this model, corresponds to the collective value) *w*(*E*, *N*) generated by the grand coalition *N* in presence of the committee set *E* must be fully distributed among all players. That is, no value is lost or left undistributed.

Axiom 2 *(Linearity) For all* $v, w \in W^N$ *and scalars* $\alpha, \beta \in \mathbb{R}$*,*

$$\phi(E, \alpha v + \beta w) = \alpha \phi(E, v) + \beta \phi(E, w).$$

The Linearity axiom ensures consistency under aggregation or scaling of different games and similar to its counterpart in cooperative games.

Axiom 3 *(Symmetry) For two symmetric players i, j in* $w \in W^N$ *with respect to* $E \in H^N$*, we have* $\phi_i(E, w) = \phi_j(E, w)$*.*

The Symmetry axiom states that if two players are symmetric, then the allocation rule ϕ must assign them equal payoffs.

Axiom 4 *(Non-Committee Member Contribution) For a non-committee member $i \in N$ with respect to $E \in H^N$, we have $\phi_i(E, w) > 0$ if and only if there exists a coalition $S \subseteq N \backslash \{i\}$ such that*

$$w(E|_{S\cup\{i\}}, S \cup \{i\}) > w(E|_S, S),$$

and for all other coalitions $T \subseteq N \backslash \{i\}$,

$$w(E|_{T\cup\{i\}}, T \cup \{i\}) \geq w(E|_T, T),$$

with $w(E, N) > 0$.

The Non-Committee Member Contribution applies to players who are not part of any committee. It states that such a player receives a strictly positive payoff if and only if they increase the value of at least one coalition without decreasing the value of any other. This condition ensures that a player who contributes to the outcome, even without belonging to a committee, is not ignored in the allocation. The condition captures a fundamental fairness principle by valuing the impact of players outside the committee structure.

Remark 4 In classical cooperative games, players who do not contribute to the value of any coalition are assigned a payoff of zero. Such players are referred to as null players. Similarly, in network-based models, isolated players (those not connected to the network) are considered as non-productive and receive no share of the value. However, our proposed framework in CSG treats such players differently. A player who is not part of any committee, i.e., a non-committee member can still receive a positive payoff if she contributes to the overall value generation. Even without a formal involvement in the committee structure, her external contributions are also acknowledged. This form of fairness is unique to the committee setting and distinguishes CSG from traditional cooperative and network game models. It highlights the importance of recognizing valuable contributions that arise outside the formal decision-making bodies, reflecting a more inclusive and realistic view of cooperation. For example, external advisors on boards, non-core researchers in collaborations, and policy consultants can improve committee outcomes even without being formally involved as a committee member.

Axiom 5 *(Fairness) For all $E \in H^N$ and for any committee $c \in E$ and any two members $i, j \in c$,*

$$\phi_i(E, w) - \phi_i(E \backslash c, w) = \phi_j(E, w) - \phi_j(E \backslash c, w).$$

Under Fairness axiom, it is assumed that the committees contribute in a collective manner and individual contributions of the committee members are not identified. The individual contributions of the players are only counted only through the coalitions in the co-co pair. Thus, all members of a committee should receive an equal share of the value contributed by that committee. Alternatively, removing a committee *c* from the structure results in the same reduction in payoff for each of its members. This ensures fair treatment among committee members. The Fairness axiom in CSG framework is conceptually similar to Myerson (1980) Network Fairness and Balanced Contribution axioms, which ensure fair treatment and balanced marginal contributions among members of a cooperating group. The distinction is that Myerson's axioms operate at the individual and network-link level, whereas Fairness in a CSG applies at the committee (group) level, treating the committee's contribution as a unit and allocating it equally among its members.

In the following, we provide two characterizations of the Shapley value for the class of CSGs.

Theorem 1 *An allocation rule ϕ satisfies the Efficiency, Symmetry, Linearity, and Non-Committee Member Contribution if and ony if it is the Shapley value for CSG.*

Proof It is easy to show that the Shapley value for CSG satisfies the given axioms. We establish the uniqueness by the following procedure.

Let ϕ be an allocation rule satisfying the aforementioned axioms. Let $E \in H^N$, and for a fix $(F, T) \in H^N \times 2^N$, consider the unanimity value function $u_{(F,T)}$. Then the following cases arise.

Case 1: *Let* $i \notin N(E)$, *then* $i \notin N(F)$, *and also let* $i \notin T$. Here, we have $i \in N \setminus (T \cup N(F)$. Then the following sub-cases arise:

(a) Let $T \nsubseteq S$. It follows that $T \nsubseteq S \cup \{i\}$. Consequently, for any E, irrespective of the relation with F, we have $u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S\cup\{i\}}, S \cup \{i\}) = 0$.

(b) Let $T \subseteq S$. We have $T \subseteq S \cup \{i\}$. The following arise.

- If $F \subseteq E$ and $F \subseteq E|_S$, then $F \subseteq E|_{S\cup\{i\}}$ and we have

$$u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S\cup\{i\}}, S \cup \{i\}) = 1.$$

- If $F \subseteq E$ and $F \nsubseteq E|_S$, then $F \nsubseteq E|_{S\cup\{i\}}$ and we have

$$u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S\cup\{i\}}, S \cup \{i\}) = 0.$$

- If $F \nsubseteq E$, then $F \nsubseteq E|_S$, and $F \nsubseteq E|_{S\cup\{i\}}$ and we have

$$u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S\cup\{i\}}, S \cup \{i\}) = 0.$$

Thus, i does not increase the value of any co-co pair, i.e. all marginal contributions are zero. Since the choice of $i \in N \setminus (T \cup N(F))$ is arbitrary, and $u_{(F,T)}(E, N) > 0$, the Non-Committee Member Contribution axiom rules out a strictly positive payoff for them. Conversely, assigning a negative payoff would imply that i reduces the value of some co-co pair, which contradicts the structure of the CSG, as such players have no impact on $u_{(F,T)}(E, N)$. Furthermore, by Efficiency, $\sum_{j \in N} \phi_j(E, u_{(F,T)}) = u_{(F,T)}(E, N)$, so assigning a negative payoff to a non-contributor (players in $N \setminus (T \cup N(F))$ would force some other players to receive more than their actual marginal contributions in order to maintain the equality. This would undermine the consistency between Efficiency, the Non-Committee Member Contribution axiom, and the CSG value structure. Therefore, the only consistent assignment is $\phi_i(E, u_{(F,T)}) = 0 \quad$ for all $i \in N \setminus (T \cup N(F))$.

Case 2: *Let* $i \in N(E)$, *then* $i \notin N(F)$, *and let* $i \notin T$. Similar to Case 1. Next, by considering Cases 3, 4, 5, and 6, we show that the players in $T \cup N(F)$ are symmetric.

Case 3: *Let* $i, j \notin N(E)$, *then* $i, j \notin N(F)$, *and let* $i, j \in T$. Let for $i, j \in T$ but $i, j \notin S$, we have $T \not\subseteq S$. Therefore, $T \not\subseteq S \cup \{i\}$ and $T \not\subseteq S \cup \{j\}$. Then for any *E* irrespective of the relation with *F*, we have

$$u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S \cup \{i\}}, S \cup \{i\}) = u_{(F,T)}(E|_{S \cup \{j\}}, S \cup \{j\}) = 0.$$

Therefore, *i* and *j* are symmetric.

Case 4: *Let* $i, j \notin N(E)$, *then* $i, j \in N(F)$, *and let* $i, j \notin T$. Take $i, j \in N(F)$, $i, j \notin T$ and $i, j \notin S$.

(a) Let $T \not\subseteq S$, then $T \not\subseteq S \cup \{i\}$ and $T \not\subseteq S \cup \{j\}$. This implies,

$$u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S \cup \{i\}}, S \cup \{i\}) = u_{(F,T)}(E|_{S \cup \{j\}}, S \cup \{j\}) = 0.$$

(b) Let $T \subseteq S$, then $T \subseteq S \cup \{i\}$ and $T \subseteq S \cup \{j\}$. But for $F \subseteq E$ or $F \not\subseteq E$, we always have $F \not\subseteq E|_S, F \not\subseteq E|_{S \cup \{i\}}$ and $F \not\subseteq E|_{S \cup \{j\}}$ as $i, j \in N(F)$ and $i, j \notin S$. Therefore, $u_{(F,T)}(E|_S, S) = u_{(F,T)}(E|_{S \cup \{i\}}, S \cup \{i\}) = u_{(F,T)}(E|_{S \cup \{j\}}, S \cup \{j\}) = 0$. Thus, *i* and *j* are symmetric.

Case 5: *Let* $i, j \in N(E)$, $i, j \in N(F)$, *and* $i, j \in T$. Similar to Cases 3 and 4, we can conclude that players in $T \cup N(F)$ are symmetric.

Case 6: *Let* $i, j \in N(E)$, $i, j \notin N(F)$, *but* $i, j \in T$. In this case, we can likewise conclude that players in *T* are symmetric.

Thus, by Symmetry, $\phi_i(E, u_{(F,T)}) = \phi_j(E, u_{(F,T)})$ for all $i, j \in T \cup N(F)$. Therefore, for any $i \in T \cup N(F)$, we have

$$\sum_{i \in T \cup N(F)} \phi_i(E, u_{(F,T)}) = |T \cup N(F)| \phi_i(E, u_{(F,T)}).$$

Now, by Efficiency,

$$\sum_{i \in N} \phi_i(E, u_{(F,T)}) = u_{(F,T)}(E, N) = 1,$$

It follows that

$$\begin{aligned} &\sum_{i \in N \setminus (T \cup N(F))} \phi_i(E, u_{(F,T)}) + \sum_{i \in T \cup N(F)} \phi_i(E, u_{(F,T)}) = 1 \\ &\implies 0 + |T \cup N(F)| \phi_i(E, u_{(F,T)}) = 1 \\ &\implies \phi_i(E, u_{(F,T)}) = \frac{1}{|T \cup N(F)|}. \end{aligned} \tag{5}$$

Thus,

$$\phi_i(E, u_{(F,T)}) = \begin{cases} \dfrac{1}{|T \cup N(F)|} & \text{if } i \in T \cup N(F) \\ 0 & \text{otherwise.} \end{cases}$$

Recall that W^N is a Euclidean space of dimension $2^{2^{|N|}} \times 2^{|N|} - 1$. Since for each non-empty co-co pair $(F, T) \in H^N \times 2^N$, we have a value function $u_{(F,T)}$. Therefore, there are $2^{2^{|N|}} \times 2^{|N|} - 1$ value functions of the type $u_{(T_r, F_{r'})}$, where $r' \in \{1, \cdots, 2^{2^{|N|}}\}$ and $r \in \{1, \cdots, 2^{|N|}\}$. The matrix formed by these value functions is triangular and hence invertible. Therefore, $u_{(T_r, F_{r'})}$'s are linearly independent. This completes the proof. □

Theorem 2 *The Shapley value for the class of CSG is the unique allocation rule that satisfies Efficiency and Fairness.*

Proof It directly follows from the definition that the Shapley value for CSG satisfies the given axioms. We establish the uniqueness by the following procedure.

Let $\phi^1 : H^N \times W^N \to \mathbb{R}^{|N|}$ and $\phi^2 : H^N \times W^N \to \mathbb{R}^{|N|}$ be two different allocation rules that satisfy the Efficiency and Fairness axiom. Let *E* be a committee set with minimum number of committees such that $\phi^1(E, w) \neq \phi^2(E, w)$. By the minimality of *E*, if *c* is any committee of *E*, then $\phi^1(E \setminus c, w) = \phi^2(E \setminus c, w)$. Therefore, by Fairness for any $i, j \in c$,

$$\phi_i^1(E, w) - \phi_j^1(E, w) = \phi_i^1(E \backslash c, w) - \phi_j^1(E \backslash c, w) = \phi_i^2(E \backslash c, w) - \phi_j^2(E \backslash c, w) = \phi_i^2(E, w) - \phi_j^2(E, w).$$

This yields $\phi_i^1(E, w) - \phi_i^2(E, w) = \phi_j^1(E, w) - \phi_j^2(E, w)$, whenever *i* and *j* are in the same committee *c* and in the induced committee set $E|_S$ by coalition *S* such that $c \subseteq S$. Thus, we may write $\phi_i^1(E, w) - \phi_i^2(E, w) = f_S(E)$, where $f_S(E)$ depends on *S* and *E* only not on *i*. But by Efficiency,

$$
\begin{aligned}
&\sum_{i\in N}\phi_i^1(E,w)=\sum_{i\in N}\phi_i^2(E,w)\\
&\implies \sum_{i\in N}(\phi_i^1(E,w)-\phi_i^2(E,w))=0\\
&\implies |N|f_S(E)=0\\
&\implies f_S(E)=0.
\end{aligned}
$$

Hence, $\phi^1(E,w)=\phi^2(E,w)$, a contradiction. Therefore, there is only one allocation rule that satisfy Efficiency and Fairness. □

Remark 5 The logical independence of each set of axioms in Theorem 1 is shown below.

(i) Consider the allocation rule defined by

$$\phi_i(E,w)=\frac{1}{2}\cdot\phi_i^{Sh}(E,w),\quad \text{for all } i\in N,$$

where $\phi^{Sh}(E,w)$ denotes the Shapley value for the CSG (*N*, *w*). For any game with $w(E,N)>0$, this rule satisfies Linearity, Symmetry, and the Non-Committee Member Contribution, but violates Efficiency, since only half of the total value *w*(*E*, *N*) is distributed among the players.

(ii) Define the allocation rule as

$$\phi_i(E,w)=\alpha_i\cdot w(E,N),\quad \text{for all } E\in H^N,\ i\in N,\ \text{and } w\in W^N,$$

where the weights $\alpha_i\in\mathbb{R}^+$ satisfy $\sum_{i\in N}\alpha_i=1$ and $w(E,N)>0$. This rule satisfies Linearity, Efficiency, and the Non-Committee Member Contribution, but it fails to satisfy Symmetry unless all α_i's are equal.

(iii) Let $k\in\mathbb{R}_+$ be a fixed threshold. Define the allocation rule by

$$\phi_i(E,w)=\begin{cases}\dfrac{w(E,N)}{|N|} & \text{if } w(E,N)\le k\\ \phi_i^{Sh}(E,w) & \text{if } w(E,N)>k\end{cases}\quad \text{for all } i\in N.$$

This rule satisfies Efficiency, Symmetry, and the Non-Committee Member Contribution. However, it fails Linearity due to the piecewise definition.

(iv) Consider the allocation rule defined by

$$\phi_i(E,w)=\begin{cases}\dfrac{w(E,N)}{|N(E)|} & \text{if } i\in N(E)\\ 0 & \text{if } i\notin N(E)\end{cases}$$

This allocation rule satisfies Linearity, Symmetry, and Efficiency. However, it fails the Non-Committee Member Contribution, as non-members receive zero payoff regardless of any external contribution.

Remark 6 The logical independence of each set of axioms in Theorem 2 is given below.

(i) Fix a player $i^* \in N$. Define the allocation rule by

$$\phi_i(E, w) = \begin{cases} w(E, N) & \text{if } i = i^* \\ 0, & \text{otherwise} \end{cases} \quad \text{for all } E \in H^N,\ w \in W^N.$$

This rule satisfies Efficiency, as the entire value *w*(*E*, *N*) is allocated. However, it violates Fairness, since other members of any committee receive no share, even though they jointly contributed to the value.

(ii) Define the allocation rule by

$$\phi_i(E, w) = \begin{cases} \dfrac{1}{|N(E)|} \cdot \dfrac{1}{2} w(E, N) & \text{if } i \in N(E) \\ 0 & \text{otherwise,} \end{cases} \quad \text{for all } E \in H^N,\ w \in W^N,$$

This rule satisfies Fairness. However, it violates Efficiency because only half of the total value *w*(*E*, *N*) is distributed.

5 The Aggregated Shapley value in CSGs

Observe that the Shapley value discussed in Sect. 4 depends only on the hypergraph *E*, and on the sub-hypergraphs of *E* that are induced by coalition *S*. Now, given a pair (*E*, *w*), if we define a cooperative game (N, v_E) such that for each $S \subseteq N$, $v_E(S) = w(E|_S, S)$, we can easily obtain that $\phi_i^{Sh}(E, w) = \phi_i^{Sh}(N, v)$, where $\phi_i^{Sh}(N, v)$ is the Shapley value for TU games. It follows that, the characterizations given by Theorem 1 and Theorem 2 become alternative characterizations of the Shapley value for TU games. Thus, we can conclude that in this way, the CV model is merely an extension of Myerson (1980) even though, we obtain axioms specific to our CSG model in the form of Non-Committee Member Contribution and Fairness. Therefore, we look for an alternative value in CSGs and in this section, we present the Aggregated Shapley value for the class of CSGs under the AV model and provide its axiomatic characterizations. However, before formally defining the Aggregated Shapley value, we provide further motivation for the AV model to explain why this study is important.

As mentioned already, in particular, there are situations where treating a committee set as a single unit fails to capture its total generated value, because the contributions of its subsets significantly affect the outcome. To illustrate this idea, we present the following example.

Example 2 Consider a research collaboration where N is the set of scientists, and $E = \{c_1, c_2, c_3\}$ represents three research groups (committees) formed from these scientists. Group c_1 (theoretical research) develops models and abstract frameworks, group c_2 (experimental research) provides validation through experiments, and group c_3 (applied research) focuses on applying results to real-world problems. Since some scientists may belong to more than one group, their expertise and effort can impact multiple activities simultaneously. Each subgroup $E' \subseteq \{c_1, c_2, c_3\}$ generates a specific contribution in the presence of the full scientist set N: individually, $\{c_1\}$ may produce theoretical insights, $\{c_2\}$ may supply experimental data, and $\{c_3\}$ may deliver prototypes or applications; pairwise, $\{c_1, c_2\}$ may yield validated theoretical models, $\{c_1, c_3\}$ may transform abstract ideas into usable designs, and $\{c_2, c_3\}$ may develop empirically tested technologies; finally, $\{c_1, c_2, c_3\}$ working jointly may achieve ground breaking results that none of them could accomplish alone. Let $w(E', N)$ denote the value or outcome generated by a subgroup E' in the presence of the entire research team N. The total outcome of the collaboration is then defined as the sum of all these contributions $\sum_{E' \subseteq E} w(E', N)$. This aggregated value represents the overall output of the research project, which arises from the independent and joint contributions of all possible subgroups E' of E. This formulation highlights that the total impact of the research collaboration does not only come from the grand collaboration $\{c_1, c_2, c_3\}$, but also from the partial and overlapping contributions of smaller groups. For example, even if the full collaboration across all three groups does not succeed, valuable outcomes may still comes from $\{c_1, c_2\}$ or from $\{c_3\}$ working independently. In collaborative situations such as research, the true impact arises from a complex web of subgroup interactions. By aggregating the contributions of every $E' \subseteq E$, this framework provides a complete and accurate measure of total outcome, ensuring that both independent achievements and synergies are reflected in group evaluation, allocation, and negotiation.

More such situations arise in policy-making committees, cross-functional teams, and other collaborative environments. To allocate the aggregated value generated in these scenarios among the players in a CSG, we introduce the Aggregated Shapley value, which accounts for the contributions of each subset $E' \subseteq E$. The formula is given below.

Definition 10 The Aggregated Shapley value for the class W^N of CSGs is the function $\phi^{ASh} : H^N \times W^N \to \mathbb{R}^{|N|}$ defined by

$$\phi_i^{ASh}(E, w) = \sum_{S \subseteq N \setminus \{i\},\, E' \subseteq E} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \Big(w(E'|_{S \cup \{i\}}, S \cup \{i\}) - w(E'|_S, S) \Big) \quad (6)$$

for each $i \in N$, and $(E, w) \in H^N \times W^N$.

The Aggregated Shapley value assigns to each player $i \in N$ a fair share of the aggregated value, based on their marginal contributions across different co-co pairs. It accounts not only for the individual roles of players but also for the structural influence of committee subsets, thereby extending the Shapley value for CSGs to settings where outcomes depend jointly on coalition formation and internal committee structure. Unlike the Myerson value, which considers only individual connectivity in a network, the Aggregated Shapley value incorporates the contributions of all subgroups within committees, capturing the impact of internal committee structure on total outcomes.

To characterize the Aggregated Shapley value, we extend only those definitions and axioms from Sect. 4 that require modification. The remaining definitions and axioms are adopted from the earlier section and are not restated here to avoid repetition.

Definition 11 For $E \in H^N$, the aggregated marginal contribution $m_i^w(E', S)$ of a player *i* to a co-co pair $(E', S) \sqsubseteq (E, N \setminus \{i\})$ in $w \in W^N$ is given by

$$m_i^w(E', S) = w(E'|_{S\cup\{i\}}, S \cup \{i\}) - w(E'|_S, S). \tag{7}$$

The aggregated marginal contributions in a CSG evaluate a player's contributions due to her inclusion or exclusion in a coalition and the subsets of committees together.

Definition 12 Let $E \in H^N$. Two players $i, j \in N$ are said to be aggregated-symmetric in $w \in W^N$ with respect to *E*, if for all $E' \subseteq E$ and $S \subseteq N \setminus \{i, j\}$,

$$w(E'|_{S\cup\{i\}}, S \cup \{i\}) = w(E'|_{S\cup\{j\}}, S \cup \{j\}).$$

It follows from Definition 12 that two players *i* and *j* are aggregated-symmetric in a CSG with respect to a committee set *E* if their marginal contributions to every possible co-co pair are identical. This means that *i* and *j* play equivalent roles across all subsets $E' \subseteq E$ and all coalitions $S \subseteq N \setminus \{i, j\}$.

Axiom 6 *For $w \in W^N$ and $E \in H^N$, we have $\sum_{i\in N} \phi_i(E, w) = \sum_{E'\subseteq E} w(E', N)$.*

The Aggregated Efficiency axiom ensures that the total aggregated value from all committee subsets is fully distributed among the players.

Axiom 7 *(Aggregated Symmetry) For two aggregated-symmetric players i, j in $w \in W^N$ with respect to $E \in H^N$, we have $\phi_i(E, w) = \phi_j(E, w)$.*

The Aggregated Symmetry axiom implies that aggregated-symmetric players receive equal payoffs, reflecting their identical contributions across all subsets of committees.

Recall the definition of a non-committee member from Sect. 4. The following axiom extends the non-committee member principle to the aggregated setting, taking into account contributions across all subsets of committees.

Axiom 8 *(Aggregated Non-Committee Member Contribution) For a non-committee member $i \in N$ with respect to $E \in H^N$, we have $\phi_i(E, w) > 0$ if and only if there exists at least one co-co pair $(F, S) \sqsubseteq (E, N \setminus \{i\})$ such that*

$$w(F|_{S\cup\{i\}}, S \cup \{i\}) > w(F|_S, S),$$

and for all other co-co pair $(E', T) \sqsubseteq (E, N \setminus \{i\})$,

$$w(E'|_{T\cup\{i\}}, T \cup \{i\}) \geq w(E'|_T, T),$$

with $\sum_{E'\subseteq E} w(E', N) > 0$, then $\phi_i(E, w) > 0$.

The Linearity and Fairness axiom remains the same as in Sect. 4. Note that the Aggregated Shapley value does not satisfy Fairness in general, as committee contributions are calculated through player-specific marginal contributions across all subcommittees. The following theorem provides a characterization of the Aggregated Shapley value for the class of CSGs. The proof of Theorem 3 follows similar steps as Theorem 1, with the extension to all $E' \subseteq E$. As the proof closely follow the arguments of the previous theorem with straightforward extensions, it is omitted here.

Theorem 3 *An allocation rule ϕ satisfies Aggregated Efficiency, Aggregated Symmetry, Linearity, and Aggregated Non-Committee Member Contribution if and only if it coincides with the Aggregated Shapley value for CSGs.*

Remark 7 The set of axioms in Theorems 3 are logically independent. The independence argument can be easily extended from Remark 5.

6 A bidding mechanism

In this section, we adopt a strategic approach to characterize the Shapley value for CSG, closely following the bidding mechanism developed by Pérez-Castrillo and Wettstein (2001) for classical cooperative games. We adapt their mechanism step by step to our CSG framework, where the presence of a committee structure introduces an additional layer of coordination. In particular, when a player leaves the game, any committee involving that player dissolves, and the remaining players enter a bidding process over the remaining committees.

This approach fits within the broader literature known as the Nash program, which seeks to provide non-cooperative foundations for cooperative solution concepts by showing that such outcomes arise as equilibrium behavior in strategic settings. In our case, we show that the Shapley value for CSG emerges as a subgame perfect Nash equilibrium (SPNE) of the induced strategic game.

Unlike other mechanisms within the Nash program that recover the Shapley value only in expectation, the Pérez-Castrillo and Wettstein (2001) mechanism yields it exactly. This feature makes it particularly suitable for application to the committee setting. Furthermore, the mechanism is both simple and intuitive, which signifies its importance and practical relevance.

To illustrate the mechanism in a practical context, consider a group of researchers forming committees to collaborate on academic projects. Each researcher submits bids to join committees by proposing how the total project value should be divided among participants. The bidding proceeds sequentially: one researcher proposes an allocation of the project value, and others choose whether to accept or reject the offer. If the proposal is accepted, the committee is formed and receives the corresponding value. Otherwise, the next researcher makes a new proposal. Importantly, if a researcher decides to leave the game, any committee involving that researcher dissolves, and the remaining participants continue the bidding process over the remaining committees. This rule reflects the dependency of committee stability on its members.

Despite its simplicity, this mechanism captures the strategic dynamics of coalition and committee formation and reproduces the Shapley value as the SPNE of the induced strategic game. A step by step description of the mechanism's implementation in the CSG setting follows.

Definition 13 For a given $E \in H^N$, a value function *w* is zero monotonic if and only if

$$w(E|_S, S) + \sum_{i \in T \setminus S} w(E|_{\{i\}}, \{i\}) \leq w(E|_T, T), \quad \text{for all } S \subseteq T \subseteq N.$$

Zero monotonicity implies that under a committee structure, the value generated by a large co-co pair is at least as much as the value of a subgroup combined with the individual contributions of the remaining players acting alone. It reflects that cooperation within the committee is more effective than acting alone.

Axiom 9 *(Balanced Contribution Property) For a given $E \in H^N$, an allocation rule ϕ is said to satisfy the Balanced Contribution Property, if for all $w \in W^N$, and $i, j \in N$, we have*

$$\phi_j(E|_N, w) - \phi_j(E|_{N \setminus \{i\}}, w) = \phi_i(E|_N, w) - \phi_i(E|_{N \setminus \{j\}}, w).$$

It signifies that an allocation rule is balanced if, the difference in contributions of any two players, say *i* and *j* to *N* under ϕ reflects the same difference when considering their absence from *N*. This ensures a symmetrical and equitable distribution of contributions that when people join or leave a group, the changes they make to the group should be consistent.

6.1 Bidding Process

Let us consider a scenario where there is only one player, say $N = \{i\}$ with $E \in H^N$. In this case, the player obtains a payoff of $w(E|_{\{i\}}, \{i\})$.

Suppose we are already acquainted with the rules governing the bidding mechanism in scenarios involving a maximum of $(|N| - 1)$ players. The bidding game for a group of players denoted as $N = \{1, \cdots, n\}$, and operating with the set of committees $E \in H^N$, adheres to the subsequent procedure:

$t = 1$ *(Bid Submission):* Each player *i* in the set *N* submits bids $b_j^i \in \mathbb{R}$ for every $j \neq i$. This stage involves players adopting individual strategies $(b_j^i)_{j\neq i} \in \mathbb{R}^{|N|-1}$. For every player $i \in N$, we calculate $B^i = \sum_{j\neq i} b_j^i - \sum_{j\neq i} b_i^j$. Let α be the player that maximizes B^i. If multiple players yield the maximum, α is selected arbitrarily from among them. Once determined, player α becomes the proposer and makes payments of b_j^α to each player $j \neq \alpha$.

$t = 2$ *(Proposal Presentation):* Player α puts forth an offer y_j^α in $\mathbb{R}$ to each player $j \neq \alpha$. Consequently, the strategy for player α at this stage is denoted by a vector $(y_j^\alpha)_{j\neq\alpha}$ in $\mathbb{R}^{|N|-1}$ if player α assumes the role of the proposer.

$t = 3$ *(Offer Acceptance/Rejection):* The remaining players, excluding α, sequentially either accept or reject the offer presented. In case of a rejection, the offer is considered declined. Otherwise, it is accepted. In the event of a rejection, all players except α then participate in the bidding mechanism, where the player set becomes $N \setminus \{\alpha\}$, and the committee set is $E|_{N\setminus\{\alpha\}}$. Player α receives a payment of $w(E|_{\{\alpha\}}, \{\alpha\})$. In contrast, if accepted, each player $j \neq \alpha$ receives y_j^α, and player α obtains the sum of the values generated by the grand coalition in the presence of *E* minus the payments $\sum_{j\neq\alpha} y_j^\alpha$.

Given the value function *w*, in the case of proposal rejection, player α receives a final payment of $w(E|_{\{\alpha\}}, \{\alpha\})$. The final payments for other players are determined by adding the bid b_j^α received to the outcome of the mechanism involving players $N \setminus \{\alpha\}$. When the proposal is accepted, the final payment for any player *j* other than α is given by $y_j^\alpha + b_j^\alpha$, while player α receives $w(E, N) - \sum_{j\neq\alpha} y_j^\alpha - \sum_{j\neq\alpha} b_j^\alpha$. In order to analyze the outcome of the bidding mechanism, we consider the following recursive definition of the Shapley value.

$$\phi_i^{Sh}(E, w) = \frac{1}{|N|}\left(w(E, N) - w(E|_{N\setminus\{i\}}, N \setminus \{i\})\right) + \frac{1}{|N|}\sum_{j\neq i} \phi_i^{Sh}(E|_{N\setminus\{j\}}, w).$$

For a given $E \in H^N$, by defining $\phi_i^{Sh}(E|_{\{i\}}, w) = w(E|_{\{i\}}, \{i\})$ for every player $i \in N$, the above equation characterizes the Shapley value for any CSG (*N*, *w*).

Theorem 4 *For $E \in H^N$, the bidding mechanism implements the Shapley value for (N, w) in SPNE, where $w \in W^N$ is zero-monotonic.*

Proof The proof employs mathematical induction on the number of players in *N*, with a constraint on the set of committees $E \in H^N$. The theorem is initially established for the case where $k = 1$, i.e., in a one-player game. In this scenario, the Shapley value of the player is equivalent to the value of her standalone coalition in *E*.

Next, we assume that the theorem holds for $k = |N| - 1$ and proceed to establish its validity for $k = |N|$. Let $N = \{1, 2, ..., n\}$. We begin by establishing that the Shapley value's payoff serves as an equilibrium outcome. To do so, we construct an SPNE that results in the Shapley value as an SPNE outcome. This is achieved through the following strategies:

- At $t = 1$: Each player $i \in N$ announces a bid of $b_j^i = \phi_j^{Sh}(E|_N, w) - \phi_j^{Sh}(E|_{N\setminus\{i\}}, w)$ for every $j \neq i$.
- At $t = 2$: If player $i \in N$ is the proposer, she offers $y_j^i = \phi_j^{Sh}(E|_{N\setminus\{i\}}, w)$ to every $j \neq i$.
- At $t = 3$: If player $i \in N$ is not the proposer, she accepts any offer greater than or equal to $\phi_i^{Sh}(E|_{N\setminus\{j\}}, w)$ made by player $j \neq i$, and rejects any offer strictly smaller than $\phi_i^{Sh}(E|_{N\setminus\{j\}}, w)$.

The strategies outlined above yield the Shapley value for players who are not the proposers, since $x_i^\alpha = b_i^\alpha + y_i^\alpha = \phi_i^{Sh}(E|_N, w)$, for $i \neq \alpha$. Here, x_i^α represents the payment or payoff received by player *i* when α becomes the proposer. Additionally, following these strategies, since the grand coalition is formed, the proposer also attains her Shapley value. To further establish the effectiveness of the strategies, we demonstrate that all net bids B^i amount to zero. Under the above-mentioned strategies,

$$B^i = \sum_{j\neq i} b_j^i - \sum_{j\neq i} b_i^j = \sum_{j\neq i} \left(\phi_j^{Sh}(E|_N, w) - \phi_j^{Sh}(E|_{N\setminus\{i\}}, w)\right) - \sum_{j\neq i} \left(\phi_i^{Sh}(E|_N, w) - \phi_i^{Sh}(E|_{N\setminus\{j\}}, w)\right).$$

By Balanced Contribution Property, we deduce that

$$\phi_j^{Sh}(E|_N, w) - \phi_j^{Sh}(E|_{N\setminus\{i\}}, w) = \phi_i^{Sh}(E|_N, w) - \phi_i^{Sh}(E|_{N\setminus\{j\}}, w),$$

leading to the conclusion that $B^i = 0$.

The validity of the preceding strategies as an SPNE is based on the condition that the strategies at $t = 2$ and $t = 3$ constitute best responses, assuming that:

$$w(E|_N, N) - w(E|_{\{i\}}, \{i\}) \geq \sum_{j\neq i} \phi_j^{Sh}(E|_{N\setminus\{i\}}, w) = w(E|_{N\setminus\{i\}}, w).$$

This holds as follows: In the event of rejection, when $j \neq i$, proposer *i* receives $w(E|_{\{i\}}, \{i\})$, while players $j \neq i$ engage in the bidding mechanism involving the set of players $N \setminus \{i\}$ with $E|_{N\setminus\{i\}}$. By the induction argument, the outcome of this mechanism is the Shapley vector $\left(\phi_j^{Sh}(E|_{N\setminus\{i\}}, w)\right)_{j\neq i}$.

Turning to the strategies at $t = 1$, it becomes apparent that if player *i* increases her total bid $\sum_{j\neq i} b_j^i$, this guarantees that she will be the proposer, but her payoff for $j \neq i$ will decrease. Conversely, if player *i* decreases her total bid, another player becomes the proposer, and player *i* still receives her Shapley value. Any alteration in bids that maintains a constant total bid affects only the proposer's identity, leaving player i's payoff unchanged.

Subsequently, we establish that any SPNE necessarily results in the Shapley value. This argument proceeds through a sequence of claims:

Claim (a): Within any SPNE, at stage $t=3$, *all players apart from the proposer* α *accept the offer if* $y_i^\alpha > \phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ *for all players* $i \neq \alpha$. *Conversely, if* $y_i^\alpha < \phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ *for at least some* $i \neq \alpha$, *the offer is rejected.*

Note that in the event of a rejection, the induction argument indicates that a player $i \neq \alpha$ receives a payoff of $\phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$. We label the last player who must decide to accept or reject the offer at $t=3$ as β. When the decision-making reaches player β, signifying no prior rejections, their optimal strategy dictates accepting any offer exceeding $\phi_\beta^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ and declining any offer below $\phi_\beta^{Sh}(E|_{N\setminus\{\alpha\}}, w)$. The second-to-last player (referred to as $\beta-1$) anticipates the response of player β. Consequently, if $y_{\beta-1}^\alpha > \phi_{\beta-1}^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ and $y_\beta^\alpha > \phi_\beta^{Sh}(E|_{N\setminus\{\alpha\}}, w)$, $\beta-1$ accepts the offer when the game reaches her. If $y_{\beta-1}^\alpha < \phi_{\beta-1}^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ and $y_\beta^\alpha > \phi_\beta^{Sh}(E|_{N\setminus\{\alpha\}}, w)$, $\beta-1$ rejects the offer. When $y_\beta^\alpha < \phi_\beta^{Sh}(E|_{N\setminus\{\alpha\}}, w)$, $\beta-1$ becomes indifferent between accepting or rejecting an offer $y_{\beta-1}^\alpha$, recognizing that player β is bound to reject any offer if it reaches her. In all scenarios, the offer is rejected. Employing the same reasoning, the argument can be applied iteratively to prove claim (a) in reverse order.

Claim (b): If $w(E|_N, N) > w(E|_{N\setminus\{i\}}, N\setminus\{i\}) + w(E|_{\{i\}}, \{i\})$, *the only SPNE of the game that starts at* $t=2$ *is the following: At* $t=2$, *player* α *proposes an offer of* $y_i^\alpha = \phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ *to all* $i \neq \alpha$; *at* $t=3$, *every player* $i \neq \alpha$ *accepts any offer* $y_i^\alpha \geq \phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ *and rejects the offers below this threshold.*

If $w(E|_N, N) = w(E|_{N\setminus\{i\}}, N\setminus\{i\}) + w(E|_{\{i\}}, \{i\})$, additional SPNE can coexist alongside the one previously mentioned. In fact, any set of strategies where, at $t=2$, the proposer offers $y_j^\alpha \leq \phi_j^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ to a specific player $j \neq \alpha$, and at $t=3$, player *j* rejects any offer $y_j^\alpha \leq \phi_j^{Sh}(E|_{N\setminus\{\alpha\}}, w)$, also serves as an SPNE.

In all the SPNE, the final payoffs to players α and $i \neq \alpha$ are, respectively given by,

$$w(E|_N, N) - w(E|_{N\setminus\{\alpha\}}, N\setminus\{\alpha\}) - \sum_{j\neq\alpha} b_j^\alpha$$

and

$$\phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w) + b_i^\alpha.$$

It is evident that the proposed strategies constitute an SPNE. Now, suppose that

$$w(E, N) > w(E|_{N\setminus\{\alpha\}}, N\setminus\{\alpha\}) + w(E|_{\{\alpha\}}, \{\alpha\}).$$

In this scenario, rejecting the offers made by player α cannot be part of an SPNE. Player α would receive $w(E|_{\{\alpha\}}, \{\alpha\})$, and she can improve her payoff by offering $\phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w) + \dfrac{\epsilon}{|N|-1}$ to every $i \neq \alpha$. Here, ϵ is chosen such that $\epsilon < w(E|_N, N) - w(E|_{N\setminus\{\alpha\}}, N \setminus \{\alpha\}) - w(E|_{\{\alpha\}}, \{\alpha\})$ and $\epsilon > 0$, ensuring that her offers are accepted (as per claim (a)). Therefore, in an SPNE, the proposal must be accepted. This implies $y_i^\alpha \geq \phi_i(E|_{N\setminus\{\alpha\}}, w)$ for all $i \neq \alpha$. However, an offer such that $y_j^\alpha \geq \phi_j^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ for some $j \neq \alpha$ cannot be part of an SPNE, since α could still offer $\phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w) + \dfrac{\epsilon}{|N|-1}$ to every $i \neq \alpha$, with $\epsilon < y_j^\alpha - \phi_j^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ and $\epsilon > 0$. These offers would be accepted, and α's payoff would increase.

In any SPNE, we have that for all players *i* except α, their respective payoff y_i^α is determined by the function $\phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$. Finally, acceptance of the proposals implies that, at $t = 3$, every agent $i \neq \alpha$ accepts an offer if $y_i^\alpha \geq \phi_i^{Sh}(E|_{N\setminus\{\alpha\}}, w)$.

If the condition $w(E|_N, N) = w(E|_{N\setminus\{\alpha\}}, N \setminus \{\alpha\}) + w(E|_{\{\alpha\}}, \{\alpha\})$ is satisfied, then the proposer needs to offer at least $\sum_{j\neq\alpha} \phi_j^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ for the offer to be accepted by every other player.

Using a similar argument as in the previous scenario, in any equilibrium where the offer is accepted, it must involve a proposal of exactly $\phi_j^{Sh}(E|_{N\setminus\{\alpha\}}, w)$ for every player *j* other than α. In case of rejection, the proposer would receive payoffs equivalent to $w(E|_{\{\alpha\}}, \{\alpha\})$. Consequently, any offer that results in rejection can also be seen as an SPNE.

It is important to note that following the first set of strategies leads to offer acceptance and the formation of the grand coalition and committee set E, whereas following the second set of strategies leaves the proposer on her own. The latter strategies only constitute an SPNE when the condition $w(E, N) = w(E|_{N\setminus\{\alpha\}}, N \setminus \{\alpha\}) + w(E|_{\{\alpha\}}, \{\alpha\})$ holds. This condition ensures that the final payoffs align with those stated in the claim.

Claim (c): In any SPNE, we have $B^i = B^j$ for all players i and j, implying $B^i = 0$ for all $i \in N$.

Let us examine this claim step by step: Denote the set $\Omega = \{i \in N | B^i = \max_j(B^j)\}$. If $\Omega = N$, the claim is met, as the sum of all payments is $\sum_{i\in N} B^i = 0$. Otherwise, we can show that any player *i* within Ω can adjust their bids to reduce the sum of payments in case they win. Importantly, these adjustments don't alter the set Ω, so the player maintains the same chance of winning while gaining a higher expected payoff.

Let us consider a player *j* who is not in Ω. Let player $i \in \Omega$ change her strategy by announcing: $b_k'^i = b_k^i + \delta$ for all $k \in \Omega$ and $k \neq i$; $b_j'^i = b_j^i - |\Omega|\delta$; and $b_l'^i = b_l^i$ for all $l \notin \Omega$ and $l \neq j$. The new bids are: $B'^i = B^i - \delta$; $B'^k = B^k - \delta$ for all $k \in \Omega$ and $k \neq i$; $B'^j = B^j + |\Omega|\delta$ and $B'^l = B^l$ for all $l \notin \Omega$ and $l \neq j$. By selecting δ to be sufficiently small such that $B^i + |\Omega|\delta < B^i - \delta$ (noting that $B^j < B^i$), we have $B'^l < B'^i = B'^k$ for all *l* not in Ω, including *j*, and for all $k \in \Omega$. Consequently, the set Ω remains unchanged. However, It is evident that $\sum_{h\neq i} b_h^i - \delta < \sum_{h\neq i} b_h^i$.

This process establishes that in any situation where Ω isn't the entire set *N*, a player in Ω can indeed adjust their strategy to reduce the sum of payments while maintaining the same set Ω.

Claim (d): In any SPNE, every player receives the same payoff regardless of who is selected as the proposer.

We have already established that all the bids B^i are equal across players. If a player, say *i*, were to have a strict preference to be the proposer, she could improve her payoff by making a slight increase to one of her bids b^i_j. Similarly, if player *i* would prefer another player, say *j*, to be the proposer, she could improve her payoff by decreasing b^i_j.

However, the fact that player *i* does not take such actions in the equilibrium indicates that she has no incentives to deviate from the current strategy. In other words, she is content with either role and her expected payoff remains the same regardless of the proposer's identity.

Claim (e): In any SPNE, the final payment received by each player corresponds to their Shapley value.

Note first that, if player *i* is the proposer, her final payoff is given by:

$$x^i_i = w(E|_N, N) - w(E|_{N\setminus\{i\}}, N \setminus \{i\}) - \sum_{j\neq i} b^i_j.$$

Now, if player $j \neq i$ is the proposer, player *i*'s final payoff is given by:

$$x^j_i = \phi_i(E|_{N\setminus\{j\}}, w) + b^j_i.$$

Therefore, the sum of payoffs to player *i* over all possible choices of the proposer is given by:

$$\begin{aligned}\sum_j x^j_i &= \left[w(E|_N, N) - w(E|_{N\setminus\{i\}}, N \setminus \{i\}) - \sum_{j\neq i} b^i_j\right] + \left[\phi^{Sh}_i(E|_{N\setminus\{j\}}, w) + b^j_i\right] \\ &= |N|\phi^{Sh}_i(E|_N, w).\end{aligned}$$

Moreover, since player *i* is indifferent to all possible choices of the proposer, we have $x^j_i = x^k_i$ for all *j*, *k*. Therefore, $x^j_i = \phi^{Sh}_i(E|_N, w) = \phi^{Sh}_i(E, w)$ for all $j \in N$. □

Remark 8 The bidding mechanism described above for the Shapley value naturally extends to the Aggregated Shapley value. In the aggregated framework (AV model), each player's bids and proposals consider their marginal contributions across all subsets $E' \subseteq E$, ensuring that the resulting SPNE implements the Aggregated Shapley value for any CSG (*N*, *w*). The mechanism reduces to the Shapley value bidding mechanism for CV model when considering a single subset $E' = E$.

7 Application of CSG in an Exchange Economy

By employing the framework of CSG in an Exchange Economy due to Aumann (1975), Scafuri and Yannelis (1984), Shafer (1980), and Shapley (1969), we expect to obtain the results R_1 and R_2 mentioned in Sect. 1.

Consider an Exchange Economy where goods and services are replaced by committees formed by players. Instead of exchanging commodities, players are engaged in forming and participating in committees based on shared interests, skills, or goals. These committees act as the pivotal elements for economic interactions, representing clusters of people with specific expertise or resources. The players select committees strategically, considering factors such as the committee's specialization, negotiation power, and the potential for collaboration. The utility (or value) in this economy lies in the collective efforts within these committees, redefining the notion of economic transactions. The concept of value allocation, introduced by Shapley (1969) for economies without transferable utility, has provided a way to characterize some allocations in an Exchange Economy that reflect the bargaining strength of the players, like the Shapley value in TU games.[1] A value allocation may be either in the context of cardinal utility, as described by Shapley (1969), or in a purely ordinal framework as formulated by Aumann (1975).

Let $E = \{c_i\}_i \in H^N$ be the set of committees formed by the players set $N = \{1, 2, \ldots, n\}$. Let $\succsim$ be the preference order of the committees. The term preference ordering is referred to a complete preorder $\succsim$ of $\mathbb{R}_+^{|E|}$, where $\mathbb{R}_+^{|E|}$ denotes the non-negative $|E|$-Euclidean space of committees. Denote by $\mathcal{P}^*$, the space of all such preference orderings. For a given $\succsim \in \mathcal{P}^*$, $c_i \succsim c_j$ implies that "c_i is at least as good as c_j", and $c_i \succ c_j$ is defined to mean $c_i \succsim c_j$ and not $c_j \succsim c_i$, i.e., "c_i is better than c_j".

A finite Exchange Economy for $E \in H^N$ is a sequence of triplets $\mathcal{E} = \{(E, u_i, \alpha_i)\}_{i \in N}$, where N is the finite set of players, and (E, u_i, α_i) are the characteristics of player i. Here, $\alpha_i \in \mathbb{R}_+^{|E|}$ is i's initial endowment and $u_i : E \to \mathbb{R}$ is her value function.

An allocation for $\mathcal{E}$ is $\{x_i\}_{i \in N}$ such that $x_i \in E$ for each i and $\sum_{i \in N}(x_i - \alpha_i) = 0$. To each finite Exchange Economy $\mathcal{E}$ with $E \in H^N$, we may associate a TU game (N, v_u) as follows:

$$v_u(S) = \max \left\{ \sum_{i \in S} u_i(x_i) \,\middle|\, \sum_{i \in S}(x_i - \alpha_i) = 0 \right\}. \tag{8}$$

In TU games, the Shapley value inherently accounts for a player' s contributions across all possible coalitions. The distinction between the Shapley value and the Aggregated Shapley value arises only in CSGs, where the internal effects of the sub-committees

[1] The fundamental idea, which allows the application of solution concepts designed for games with transferable utility to those without transferable utility, is termed "the principle of irrelevant alternatives" by Shapley (1969).

create additional aggregation considerations. Once the exchange economy model is mapped to a TU game (N, v_u) by (8), the coalition values $v_u(S)$ already capture the total contributions of players across all subsets $S \subseteq N$. Consequently, in this TU setting, the Aggregated Shapley value coincides with the Shapley value. This explanation ensures a smooth transition from the committee-structured framework to the TU formulation, highlighting that aggregation is automatically embedded in the Shapley value for TU games. Therefore, we refer only to the Shapley value here, as both coincide in this setting.

Moreover, the purpose of this discussion is to illustrate the application of the CSG framework in an exchange economy, rather than to analyze specific allocation rules. One may explore different allocation rules and their consequences within this framework, which could provide directions for future research.

An “ordinal" value allocation for $\mathcal{E}$ is defined as $\{x_i\}_{i\in N}$ if and only if $\sum_{i\in N}(x_i - \alpha_i) = 0$ and there exists value functions $u = (u_i)_{i\in N}$ for each $\succsim_i$ such that $(u_i(x_i))_{i\in N}$ is the Shapley value of the game (N, v_u). On the other hand, $\{x_i\}_{i\in N}$ is a “cardinal" value allocation for $\mathcal{E}$ if and only if there exists non negative numbers $(\lambda_i)_{i\in N}$, not all zero, such that $(\lambda_i u_i(x_i))_{i\in N}$ is the Shapley value of $(N, v_{\lambda u})$, where $\lambda u = (\lambda_i u_i)_{i\in N}$.

Players with the same characteristics are termed identical players. Further, a cardinal value allocation is said to be symmetric if identical players are assigned with the same value.

The determination of a value allocation depends on two factors: players' preferences and their initial endowments. Example 2 from Shafer (1980), presented below, illustrates the concept of ordinal value allocation and points out that players with no initial endowment can receive more of all commodities than they are initially endowed with.

Example 3 Let there be three players $N = \{1, 2, 3\}$ and two commodities x and y. Value functions and endowments are

$$u_1(x_1, y_1) = (\frac{1}{2}x_1^\beta + \frac{1}{2}y_1^\beta)^{\frac{1}{2}}, \quad \alpha_1 = (0, 0),$$

$$u_2(x_2, y_2) = (\frac{1}{2}x_2^p + \frac{1}{2}y_2^p)^{\frac{1}{2}}, \quad \alpha_2 = (1, 0),$$

$$u_3(x_3, y_3) = (\frac{1}{2}x_3^p + \frac{1}{2}y_3^p)^{\frac{1}{2}}, \quad \alpha_3 = (0, 1)$$

for $0 \le p < \beta \le 1$. Calculating v_u by using Eq. (8) and noting that for all x, y, $(\frac{1}{2}x_1^\beta + \frac{1}{2}y_1^\beta)^{\frac{1}{2}} \ge (\frac{1}{2}x_2^p + \frac{1}{2}y_2^p)^{\frac{1}{2}}$ with equality only if $x = y$, we get the Shapley values for players 1, 2 and 3 respectively as follows:

$s_1 = \frac{1}{3}[(\frac{1}{2})^{\frac{1}{\beta}} - (\frac{1}{2})^{\frac{1}{p}}], s_2 = \frac{1}{2} - \frac{1}{6}[(\frac{1}{2})^{\frac{1}{\beta}} - (\frac{1}{2})^{\frac{1}{p}}]$ and $s_3 = \frac{1}{2} - \frac{1}{6}[(\frac{1}{2})^{\frac{1}{\beta}} - (\frac{1}{2})^{\frac{1}{p}}]$.

The value allocation is given by $(x_i, y_i) = (s_i, s_i)$ for $i = 1, 2, 3$. This shows that the value allocation assigns positive consumption to a player without any endowment in the initial stage.

This example can also be seen as a cardinal allocation rule with $\lambda_1 = \lambda_2 = \lambda_3 = 1$.

By substituting the commodities set $\{x, y\}$ with the committee set $E = \{c_1 = \{2\}, c_2 = \{3\}\}$, and adopting an approach similar to Example 3 as discussed above, we will find that player 1 who do not belong to any committees gains the benefit from both committee c_1 and c_2. We can extend this to a large number of players with more committees due to Shafer (1980). Thus, it establishes the result R_1.

Adding an additional player (player 4) with characteristics identical to player 1 and assigning weights $\lambda_4 = 0$ and $\lambda_1 = \lambda_2 = \lambda_3 = 1$, Shafer (1980) constructed another example by extending the concept of Example 3 above, where the Shapley values of players 1, 2, and 3 remain the same but that of player 4 is 0. Hence, the value allocation is $(x_i, y_i) = (s_i, s_i)$ for $i = 1, 2, 3, 4$ and players 1 and 4 are not treated symmetrically.

This example is further extended by Scafuri and Yannelis (1984) giving positive weights $\lambda_i > 0$ to all the players $i \in N$ and shows that players with identical preferences and identical endowments can be treated very differently at a cardinal value allocation, i.e., cardinal value allocations may assign different values to identical players when all players are given strictly positive weights. If we interpret the weights as job levels, for instance, individuals with the same preferences may obtain varying compensations, highlighting the impact of organizational structures on reward distribution. This establishes the result R_2.

8 Concluding remarks

This study explores a new approach to decision-making methods that builds on the complex interactions between committees and players. We examine how committee structures influence cooperation and generate value among players. This is illustrated through the two models: CV and AV. The Shapley value and the Aggregated Shapley value, along with their axiomatic characterizations, provide a strong theoretical foundation. The Aggregated Shapley value, in particular, captures the contributions of internal committee structures. Further, the proposed bidding mechanism offers practical strategies for implementing decision-making processes within committees. Moreover, the incorporation of committees in the Exchange Economy builds a new economic interaction, where individuals join committees or groups based on their interests, skills, or goals.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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References

Algaba E, Fragnelli V, Sánchez-Soriano J (Eds.) (2019) Handbook of the Shapley value. CRC Press
Algaba E, Rémila E, Solal P (2025) Flow methods for cooperative games with generalized coalition configuration: E. algaba et al. J Optim Theory Appl 207(2):22
Albizuri MJ, Aurrecoechea J, Zarzuelo JM (2006) Configuration values: extensions of the coalitional Owen value. Games Econ Behav 57(1):1–17
Aumann RJ (1975) Values of markets with a continuum of traders. Econometrica. https://doi.org/10.2307/1913073
Béal S, Rémila E, Solal P (2021) Coalitional games with collaboration hypergraphs. Group Decis Negot 30(6):1357–1385
Borkotokey S, Gogoi L, Kumar R (2019) Network games: the cooperative approach. In: Network Theory and Agent-Based Modeling in Economics and Finance. Singapore, Springer Singapore, pp 429–458
Chun Y (1989) A new axiomatization of the Shapley value. Games Econ Behav 1(2):119–130
Guala F, Mittone L, Ploner M (2013) Group membership, team preferences, and expectations. J Econ Behav Organ 86:183–190
Jackson MO, Wolinsky A (1996) A strategic model of social and economic networks. J Econ Theory 71(1):44–74
Jorzik N (2023) Axiomatization and core stability of allocation rules for hypergraph situations. Int Game Theory Rev 25(02):2350007
Moretti S, Ozturk M, Tsoukiás A (2022) Negotiation frameworks for cooperative games with partial cooperation. Group Decis Negot 31(2):289–317
Myerson RB (1980) Conference structures and fair allocation rules. Internat J Game Theory 9:169–182
Pérez-Castrillo D, Wettstein D (2001) Bidding for the surplus: a non-cooperative approach to the Shapley value. J Econ Theory 100(2):274–294
Scafuri AJ, Yannelis NC (1984) Non-symmetric cardinal value allocations. Econometrica. https://doi.org/10.2307/1913510
Shafer WJ (1980) On the existence and interpretation of value allocation. Econometrica. https://doi.org/10.2307/1911109
Shapley L (1953) A value for n-person Games. Ann. Math. Study28, Contributions to the Theory of Games, Kuhn HW, Tucker AW (ed), pp 307-317
Shapley LS (1969) Utility comparison and the theory of games. The Shapley Value. Essays in Honor of Lloyd S. Shapley, pp 307-319
van den Nouweland A, Borm P, Tijs S (1992) Allocation rules for hypergraph communication situations. Internat J Game Theory 20:255–268

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