# From Planned to Dynamic Obsolescence

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**Abstract**

We consider a two-period model where a durable-goods monopolist might engage in *"dynamic obsolescence"* by *changing* the durability of the good in period 2 from what was planned in period 1 (e.g. through software updates) *after* consumers have bought it. We show that given the opportunity to do so, in the subgame perfect Nash equilibrium of the game the monopolist chooses to reduce the durability in the second period. Moreover, we show that committing to a durability level leads to higher profits than the profits under dynamic durability adjustment. Our findings indicate that welfare can be enhanced by regulatory support for credible commitment mechanisms, such as warranties and buybacks.

**Keywords:** Durable Goods; Monopoly; Planned Obsolescence; Dynamic Obsolescence; Time Inconsistency; Welfare

**JEL classification:** D42, D60, L12, L50, L63, L68, O30

## 1 Introduction

It is well established that a monopolist producer of a durable good can practice planned obsolescence, i.e. they can choose the durability of the good below the socially optimal level to ensure repeat purchases in the future. Manufacturers can plan this in advance of production (e.g. car production, see Swan (1972)) or in some cases they can make the product obsolete by the introduction of a new product (e.g. textbooks, cf. Waldman (1993)).

Advances in new technologies, however, provide the manufacturers the opportunity to *adjust* (increase or decrease) the durability of certain products via software updates after the products have

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been bought and are actively used by consumers. That is, this can be done without choosing the specific product durability for the entirety of the product's lifespan when the good is introduced or without launching a new version in the future. For example, in 2017 Apple was fined 25 million euros by France's competition and fraud watchdog DGCCRF for deliberately slowing down older iPhone models without revealing it to the consumers.[1,2] Moreover, in 2020 Apple reached a settlement agreement in a US class action lawsuit that accused them of "secretly throttling" some iPhone models and agreed to pay 500 million USD.[3,4] Likewise, video game developers have been known to suddenly discontinue games, so that even customers who bought a game are no longer able to play it. This has spurred initiatives to address this on an EU-level.[5] Other industries might also face the same problem, which will only exacerbate in the future. For instance, in the automobile industry new cars typically have built-in technologies which rely on regular software updates.[6]

The possibility of dynamic adjustment of product durability presents an additional commitment problem to the manufacturer. To contrast it with the established concept of *planned obsolescence* we introduce the term *dynamic obsolescence* to refer to this phenomenon. In the case of planned obsolescence, the product durability is chosen in advance. In contrast, if a firm has the option to alter the durability at a later date, it might be tempted to do so due to its misaligned incentives across time, even if the adjustment is costly. In particular, it might benefit from reducing the durability because doing so increases repeat-purchase demand and hence profit. Such a firm might use a pricing scheme different from the one under planned obsolescence, possibly generating significant welfare implications. From a welfare perspective, planned obsolescence itself has been shown to have an adverse effect on consumers.[7] Planned obsolescence is also criticized because of its environmental effects as less durable products lead to greater waste (e.g. in the electronics industry). Thus, it is detrimental to sustainability and hurts the society as a whole (Rivera and Lallmahomed, 2016; Satyro et al., 2018). Intuitively, dynamic obsolescence could be even worse for consumers and the environment as manufacturers could repeatedly engage in durability downgrades. Dynamic obsolescence allows them a higher degree of flexibility in terms of timing and actions. Hence, the producer has more ways to extract surplus and exploit consumers at the expense of generating a greater negative externality.

We analyze how the lack of commitment to a predetermined durability level (i.e. the possibility

[1] DGCCRF stands for Direction générale de la concurrence, de la consommation et de la répression des fraudes (Directorate general for competition policy, consumer affairs and fraud control). See `https://www.bbc.com/news/technology-51413724`.

[2] The fact that Apple has confirmed that they implemented the same practice in later models establishes dynamic durability adjustment as a real phenomenon.

[3] In a similar case against Apple in the United Kingdom, the plaintiffs seek 1.6 billion GBP in compensation, see `https://www.bbc.com/news/technology-67911517`.

[4] Samsung has also been fined for the same reason, see `https://www.theguardian.com/technology/2018/oct/24/apple-samsung-fined-for-slowing-down-phones`.

[5] E.g. "Stop Destroying Videogames." `https://citizens-initiative.europa.eu/initiatives/details/2024/000007_en`.

[6] In fact, car manufacturers have a track record of deactivating built-in features of their cars. For instance, in 2023 Mazda demoted the embedded remote start feature to a subscription feature in cars sold in 2020, see `https://www.motor1.com/news/729233/mazda-connected-services-remote-start-subscription/`.

[7] See e.g. Bulow (1986); Rust (1986); Waldman (1993).

of dynamic obsolescence), affects the pricing of the good, the consumers' welfare, and the firm's profits. Because more consumer surplus is extracted as market power increases, we focus on dynamic obsolescence in a monopolistic market. We analyze this setup in a simple two-period model. The monopolist first selects a specific durability level through the choice of product's hardware components in period 1 to start sales.[8] In the second period he can choose to adjust the durability of all goods that are already in use by consumers. The monopolist does this *without* introducing a new product, e.g. by issuing a software update for the existing products. Consumers are aware that the adjustments that happen in the second period could affect the purchased product's durability. They take this into account when making their purchasing decisions in both periods. However, they do not know that the change will happen for sure. For instance, if caught, the monopolist might decide to reverse their decision to save face when under pressure from the public opinion.[9,10]

We show that in equilibrium the monopolist always chooses to *lower the durability* in the second period (Theorem 1).[11] While our finding is in line with the existing literature on time-inconsistent behavior of durable-goods monopolists (Waldman, 1996; Nahm, 2004; Utaka, 2006), to our knowledge this is the first paper that establishes that a *dynamic* and *partial* (rather than complete) reduction in durability is an equilibrium strategy for producers. This explains the behavior of companies like Apple, who have the opportunity to engage in a *deliberate and gradual* reduction of the durability of their products to induce repeat purchases in the future. This approach from companies can be much more subtle than simply discontinuing/obsoleting all existing products, and hence its effects might be easier to overlook. However, given the omnipresence of digital technologies, one can guess that dynamic obsolescence will become more and more prevalent as a company strategy.

After we establish the legitimacy of dynamic obsolescence as an equilibrium phenomenon, we contrast its welfare effects to the ones of planned obsolescence. In our model dynamic and planned obsolescence lead to the same *expected* consumer welfare.[12] Furthermore, we show that dynamic obsolescence can harm the firm (Theorem 2).[13] Therefore, if the monopolist could commit to the initial durability level, then he would do so. In the past, traditional product design used to tie the hands of manufacturers because, once the producer sold the product, it generally had no access to it. In contrast, the latest technological advancements naturally erode the firm's commitment power that is inherent in planned obsolescence as the product design allows (and in some cases requires)

[8] For example, the product can be made less durable through a choice of components with mismatched life cycles.

[9] Cf. the case of Apple phones becoming unusable after consumers used third party repair shops created many disgruntled consumers and resulted in Apple changing its decision and issuing a software update to fix this (`https://www.wired.com/2016/02/apple-shouldnt-get-to-brick-your-iphone-because-you-fixed-it-yourself/`).

[10] See also the case of Revolv in which consumers were offered full refunds when their lifetime subscription was cancelled after public backlash (`https://www.slashgear.com/nest-is-now-offering-full-refunds-to-revolve-hub-owners-14436384/`).

[11] It is theoretically possible that the monopolist also *gains from increasing* the durability of the good. For example, in the gaming industry the durability of a game can be measured by the time it remains appealing to the players, which can be increased through frequent updates and new downloadable content (DLCs). See `https://gameworldobserver.com/2022/11/30/todd-howard-elder-scrolls-6-games-playable-for-20-years`.

[12] Note that the consumers are worse-off after a successful durability adjustment (i.e. ex-post), as in the case of Apple slowing down their phones.

[13] In van den Berg et al. (2012) commitment might be welfare improving in a different setup with an oligopolistic market.

regular software updates. For example, nowadays, besides typical electronics, even big household appliances like washing machines, dishwashers and fridges require an internet connection to retain all of their functionality.[14]

Our findings lead to several relevant policy implications. Since *not engaging* in dynamic obsolescence is better for the companies, even though dynamic obsolescence is equilibrium behavior when they cannot commit to a durability level, firms have incentives to introduce commitment devices such as offering extended warranties and buybacks. Because firms can alter the Terms and Conditions and implement them through software updates, devising an effective commitment device might require external enforcement, e.g. via the regulators.[15]

### 1.1 Related Literature

The idea that a durable-goods monopolist would choose an inefficiently short lifespan for its product has been explored extensively (Martin, 1962; Kleiman and Ophir, 1966; Levhari and Srinivasan, 1969; Swan, 1970; Schmalensee, 1970; Barro, 1972; Bulow, 1986; Rust, 1986; Muller and Peles, 1988; Fethke and Jagannathan, 2002). Most of these studies focus on monopolists that directly choose the durability of the product when it is produced, rather than indirectly reducing its durability by introducing new versions of the product, like in Waldman (1993), Choi (1994), Utaka (2006), Iizuka (2007) and Utaka (2022); or by improving the quality of the product in future sales, like in Waldman (1996), Lee and Lee (1998), and Nahm (2004).

The majority of the aforementioned studies argue that monopolists end up with a socially inefficient level of durability. In contrast, Strausz (2009) shows that planned obsolescence can also benefit the consumers, as it increases the frequency of repurchases, giving the consumers more opportunities to punish the producers for low quality products, which consequently increases the quality of the offered products. Likewise, when the producer is not a monopolist but operates in a competitive market, planned obsolescence might be necessary for survival (Grout and Park, 2005) and the firm might even produce a good that is too durable (Fishman, Gandal, and Shy, 1993).[16]

We also take the approach where a monopolist chooses how much durability will be built into the product, but allow for a durability adjustment after consumers make a purchase. This is reflective of the possibility for dynamic adjustment through software updates and differentiates our study from most of the planned obsolescence literature. So far the literature has not directly addressed the problem of dynamic obsolescence as this issue typically applies to markets with relatively recent technological developments.

A similar setup is considered only in Shankar (2024). There are, however, several important differences between the models. Shankar (2024) works with a continuum of consumer types and

[14] See, for example LG ThinQ Washer or Whirlpool Smart Appliances.

[15] Deep Cycle Systems, a manufacturer of lithium batteries, allegedly reneged on their warranty guarantees by secretly amending conditions. See https://www.carexpert.com.au/opinion/dcs-batteries-suing-youtuber-for-honest-review-sets-scary-precedent for details.

[16] Kinokuni, Ohori, and Tomoda (2019) explore the effect of disposal fees of solid wastes on products produced by a monopolist and show that the monopolist might *increase* the durability of the product in equilibrium.

intrinsic value of product durability whereas we consider two consumer types and no intrinsic value of durability. Moreover, we assume that adjusting product durability is costly and does not happen with certainty as the monopolist might be forced to reverse the changes and stick to the initial durability.[17] These differences lead to significantly different conclusions. Despite allowing the possibility of partial obsolescence as we do (i.e. in the second period the monopolist makes obsolete a fraction of the goods which have been sold on the market), Shankar (2024) finds that it is never done in equilibrium. More precisely, in his framework this is always dominated by either full obsolescence or no obsolescence. Therefore, the author concludes that the observed behavior by companies like Apple is not due to pursuing optimality, but due to technological constraints (i.e. new software inevitably slowing down the old hardware it works on).

Contrary to the findings of Shankar (2024), in this article we show that partial obsolescence is typically the equilibrium behavior of the monopolist producer. More importantly, the durability is *dynamically* adjusted when the monopolist cannot commit to following planned obsolescence. This is the most profitable strategy given that there is uncertainty that the durability adjustment will go through. Nevertheless, we establish the existence of cases in which dynamic durability adjustment occurs in equilibrium even when there is no uncertainty (Example 2).

The empirical studies that have looked into the conduct of companies like Apple have been divided in identifying the root causes for it. On the one hand, Sun, Kong, Khan, and Pecht (2019) argue that some software updates necessarily slow down phones to preserve battery life and are an ad hoc response not driven by profit concerns. Similarly, Makov and Fitzpatrick (2021) state that software updates and repairability do not have a serious effect on most iPhone models, with some exceptions. On the other hand, Barros and Dimla (2021) describe a transition away from planned obsolescence in the smartphone industry (through the aesthetic features of the product) to technology-driven obsolescence, i.e. dynamic obsolescence. Although Shankar (2024) provides theoretical analysis in line with this transition, its findings predict full destruction of the product, which is typically not an observed practice.

The rest of the article is organized as follows. Section 2 formally introduces the model. Section 3 provides the main results of the article and Section 4 concludes.

## 2 Model Setup

We consider a two-period model, in which a monopolist produces a durable good at a marginal cost of production $m > 0$. There are two types of consumers, a "high" type ($H$) and a "low" type ($L$). The mass of each type of consumers is 1. Consumers of different types differ in their valuations of the object, but all consumers of the same type have identical valuations. The *per-period* valuation of a type $t \in \{L, H\}$ consumer is $v_t$, where $v_H \geq v_L > m$.[18] We assume that $2(v_L - m) \geq v_H - m$ (which reduces to $2v_L \geq v_H + m$), so that the per-period surplus of providing the good to both

[17] Such uncertainty can arise for a number of reasons. If the adjustment gets detected, this can thwart the monopolist's attempt due to the public outcry or legal pressure from regulators.

[18] Note that this case also incorporates the case of a single type, i.e. $v_H = v_L$.

types is greater than the one from providing the good to only one type of consumers; i.e. the monopolist serves both types of consumers in at least one of the periods.[19,20]

The product durability is the probability that the product *sold in period 1* still functions in period 2. We let $\delta_1 \in [0, 1]$ denote the durability rate chosen by the firm in period 1. This durability is governed by the intrinsic product characteristics, e.g. hardware. Because we have a continuum of identical consumers of each type, the durability rates will also be the *fraction* of the goods sold in period 1 that are still usable in period 2. The cost of building infrastructure capable of producing a good of durability level $\delta_1$ in period 1 is $F(\delta_1)$. We assume that $F(\cdot)$ is continuously differentiable and that $F'(\delta_1) \geq 0$ for each $\delta_1 \in [0, 1]$ and $F(0) = F'(0) = 0$.[21] That is, if the monopolist wants to produce a more durable good, he incurs a (weakly) higher fixed cost.

Unlike standard models, we assume that in the beginning of *period 2* the seller can attempt to alter the durability of the products sold *in period 1*. Let $\delta_2 \in [0, 1]$ be the durability rate *targeted* by the monopolist in period 2. This durability is governed by software updates. Note that when $\delta_2 = \delta_1$ we have planned obsolescence and when $\delta_2 \neq \delta_1$ we have dynamic obsolescence. The durability adjustment in period 2 is successful with probability $\alpha$.[22] To make the problem non-trivial, we assume that $\alpha > 0$, as otherwise we have planned obsolescence by default. Moreover, in all formal statements in the paper we assume $\alpha < 1$. Nevertheless, in Example 2 we illustrate that dynamic obsolescence can also occur in equilibrium when $\alpha = 1$. Therefore, the product durability in period 2 is a random variable which takes two values: $\delta_1$ with probability $1 - \alpha$ and $\delta_2$ with probability $\alpha$. When the intervention is successful, the fraction of the goods that are sold in period 1 and become obsolete (i.e. unusable) in period 2 is $1 - \delta_2$. Similarly, when it is unsuccessful it is $1 - \delta_1$.

The probability $\alpha$ is common knowledge to all market participants.[23] Rational consumers can perfectly deduce the *optimal* values of $\delta_1$ and $\delta_2$ chosen by the profit-maximizing monopolist. Nevertheless, at the time of the purchase decisions in period 1, the period 2 durability is *uncertain* as this is based on the adjustment probability $\alpha$.

Durability adjustment is a *costly* intervention for the monopolist (Banker, Datar, Kemerer, and Zweig, 1993; Jansen and Brinkkemper, 2006; Berhe, Maynard, and Khomh, 2023). To capture this effect, we let $C(\delta_1, \delta_2)$ denote the *adjustment cost* of the durability level from $\delta_1$ to $\delta_2$. The monopolist pays $C(\delta_1, \delta_2)$ irrespective of whether the adjustment is successful or not. The cost

[19] Although this assumption is not without loss of generality, it is not essential for the qualitative results about dynamic obsolescence summarized in Theorem 1. The formal analysis when this assumption is relaxed is available upon request.

[20] It is worth noting that Waldman (1996) makes the opposite assumption to remove the time inconsistency problem of the monopolist in order to focus on the effects of introducing a higher quality product in the second period. On the other hand, our results have time inconsistency at their core.

[21] Our results would hold with a positive but sufficiently small $F(0)$.

[22] The probability $1-\alpha$ can alternatively be interpreted as the probability that the durability adjustment is detected by a competition authority or by the consumers themselves, which leads to the producer being forced to fulfill the promise to deliver durability $\delta_1$ product to consumers.

[23] Kinokuni, Ohkawa, and Okamura (2010) analyze a related setup with two types of consumers. However, in their model it is the consumers *and not the monopolist* who can affect the durability in period 2 through product maintenance. Hence, in their paper the monopolist's commitment problem is not present.

function $C(\cdot)$ is differentiable almost everywhere in both arguments. In the rest of the article, whenever it is differentiable we denote its derivative with respect to the $i^{th}$ argument (where $i \in \{1,2\}$) by $C_i$. For example, $C_2(\delta_1, \delta_2) = \partial C(\delta_1, \delta_2)/\partial \delta_2$.

We impose the following assumptions on the adjustment cost function.

**Assumption 1.** *For all $\delta$ it holds that $C(\delta, \delta) = 0$.*

Assumption 1 implies that maintaining the same durability level in two periods comes at no additional cost.

**Assumption 2.** *For every $\delta' > \delta$, (i) $C_1(\delta', \delta) \geq 0 \geq C_2(\delta', \delta)$ and (ii) $C_1(\delta, \delta') \leq 0 \leq C_2(\delta, \delta')$. Moreover, at least one inequality holds strictly for some $\delta, \delta'$.*

Assumption 2 implies that a greater durability adjustment (i.e. making the difference between $\delta'$ and $\delta$ higher) comes at a (weakly) higher cost. In the extreme case, making a perfectly durable good perfectly perishable (adjusting from $\delta_1 = 1$ to $\delta_2 = 0$) is costly.

The monopolist and the consumers do not discount the future.[24] Each period is divided into *two stages* and the timeline of the game is as follows. In stage 1 of period 1, the monopolist chooses $\delta_1$. In stage 2, the monopolist chooses the period 1 price $p_1$ for the product and consumers make their purchase decisions. In stage 1 of period 2, the monopolist chooses $\delta_2$ and nature decides *if the adjustment is successful or not.* In stage 2, the monopolist chooses the period 2 price $p_2$ and consumers make their purchase decisions, i.e. they either make a first-time purchase in case they bought nothing in period 1 or a repurchase in case their product is unusable (this implies that the monopolist has to produce more goods in period 2). The timeline of the model is summarized in Figure 1 below.

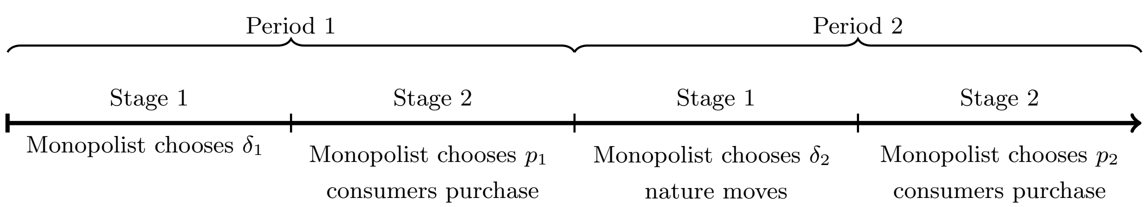


Figure 1: Timeline of the model.

## 3 Analysis

The solution concept we employ is *subgame perfect Nash equilibrium* (SPNE). Hence, the game is solved using backward induction. In period 2, the monopolist chooses the targeted durability level $\delta_2$ and the price $p_2$ depending on the durability realization to maximize the expected profit from purchases in period 2 only. In contrast, in period 1, the monopolist maximizes the expected profit from period 1 and period 2. Furthermore, the monopolist takes the optimal price and

[24] All results survive if the monopolist and the consumers have a sufficiently high common discount factor.

target durability in period 2 into consideration when choosing the initial durability level $\delta_1$ and the corresponding price $p_1$.

The expected profit across the two periods is given by

$$\mathbb{E}\pi_1 = (p_1 - m)D_1(p_1, \delta_1) - F(\delta_1) + \mathbb{E}\pi_2(p_2, \delta_2), \tag{1}$$

where $D_1(p_1, \delta_1)$ is the demand in period 1 and $\mathbb{E}\pi_2(p_2, \delta_2)$ is the expected profit from period 2 sales, where $\delta_2$ is the targeted durability by the monopolist. Given the optimal price $p_2^*$, the monopolist chooses $\delta_2$ to maximize $\mathbb{E}\pi_2(p_2^*, \delta_2)$. Similarly, $\delta_1$ and $p_1$ are chosen to maximize $\mathbb{E}\pi_1$ given optimal $p_2^*$ and $\delta_2^*$. The exact forms of the expected profits and the demand function are relegated to the Appendix.

Finally, notice that since consumers make decisions to maximize their surplus, in equilibrium all consumers of the same type make identical purchasing decisions.[25]

We begin with the familiar planned obsolescence setup as a benchmark. This is done in two instances depending on whether the monopolist has commitment power (Subsection 3.1) or not (Subsection 3.2). Next, we establish dynamic obsolescence as an equilibrium phenomenon (Subsection 3.3) which is then juxtaposed with the benchmark with respect to the profitability (Subsection 3.4), social welfare (Subsection 3.5), and consumer welfare in case of quadratic costs (Subsection 3.6). This section concludes with two specific examples of dynamic obsolescence (Subsection 3.7).

### 3.1 Benchmark: Commitment Power and Planned Obsolescence

We start the analysis by considering a monopolist who has the **commitment power** to keep the durability level unchanged in period 2, i.e. he engages in planned obsolescence. Because in this case $\delta_2 = \delta_1$, the adjustment cost is $C(\delta_1, \delta_2) = 0$ and the adjustment probability is as if $\alpha = 0$.

**Proposition 1.** *When the monopolist has commitment power, the profit-maximizing durability is $\delta_1 > 0$, the optimal prices are $p_1 = v_L + \delta_1 v_L$ and $p_2 = v_L$, and the monopolist sells to both types of consumers in period 1.*

All proofs are relegated to the Appendix. The specific proof of Proposition 1 is postponed to after Theorem 1 as it readily follows from it.

The intuition for Proposition 1 is as follows. Because $2v_L \geq v_H + m$, in period 2 the monopolist charges $p_2 = v_L$ which is the highest acceptable price for low-type consumers and thus extracts all surplus from them.[26] The same condition implies that for any durability level chosen by the monopolist selling the good to both types in period 1 generates a higher (lifetime) profit. The price chosen in period 1 is equal to the expected value of purchasing the good in period 1 by a low-type consumer, i.e. $p_1 = v_L + \delta_1 v_L$.

[25] Otherwise the monopolist can always lure the consumers who do not purchase anything by slightly reducing the price.

[26] Because $v_L > m$, in any subgame perfect equilibrium, all consumers of low type buy the product when $p_2 = v_L$.

Given the optimal prices, from the perspective of period 1, the profit is given by

$$\hat{\pi}(\delta_1) = 4(v_L - m) + 2\delta_1 m - F(\delta_1). \tag{2}$$

The monopolist never chooses to make the good fully perishable because he faces the following trade-off. Making the good slightly durable leads to a reduction in the variable costs of producing the replacement goods in period 2 (captured by $2m$) but increases the fixed costs (the increase is captured by $F'(0)$). Because $F'(0) = 0$, making the good slightly durable comes at (virtually) no cost. Therefore, the benefit from the reduction in variable cost from replacing broken products outweighs the increased fixed cost.[27]

### 3.2 No Commitment Power and Planned Obsolescence

Because a product sale generates a positive surplus, a monopolist who **cannot commit to keeping a certain durability** level has an incentive to reduce the durability of products which were already sold. This happens because lower durability leads to higher demand in period 2 since consumers whose products break down are willing to repurchase it under certain prices. Therefore, it is natural to check whether the presence of misaligned intertemporal incentives precludes planned obsolescence from being an optimal strategy for the monopolist. This is addressed in the following lemma.

**Lemma 1.** *When the monopolist has no commitment power, there exists an adjustment cost function $C(\cdot)$ for which planned obsolescence is chosen by the monopolist in equilibrium.*

As the proof readily follows, we omit it and provide the intuitive explanation instead. For planned obsolescence to be an equilibrium, a deviation to dynamic obsolesce should not be profitable. In the presence of adjustment costs $C(\cdot)$, a downward adjustment of durability has two effects. First, lower durability implies that in period 2 there will be more repurchases because all consumers with a broken product will buy the product again in period 2. Because $v_L > m$, these repurchases increase the profits for the monopolist. In fact, the expected *marginal* benefit from such repurchases is at most $2\alpha(v_L - m)$.[28] Second, this adjustment is costly for the monopolist and the marginal adjustment cost is $-C_2(\delta_1, \delta_2)$. Hence, it is easy to see that planned obsolescence can be optimal if the second effect outweighs the first. Similarly, an upward adjustment of durability implies that in period 2 there will be fewer repurchases and adjustment costs, both of which have a negative effect on profitability. Thus, such an adjustment is not profitable.

An example of a cost function for which planned obsolescence is an equilibrium strategy is $C(\delta_1, \delta_2) = q\sqrt{|\delta_1 - \delta_2|}$, where $q > 0$. Observe that when $\delta_1 > \delta_2$

$$C_2(\delta_1, \delta_2) = -\frac{q}{2\sqrt{\delta_1 - \delta_2}}. \tag{3}$$

[27] Note that if $F'(1) \leq 2m$ then $\delta_1 = 1$ is profit-maximizing.

[28] In case only high types purchase the product in period 1, this expected marginal benefit is $\alpha(v_L - m)$.

As $\delta_2$ approaches $\delta_1$, $C_2$ tends to $-\infty$. That is, the marginal cost of decreasing the durability (given by $-C_2(\delta_1, \delta_2)$) is very high. The example showcases the fact that planned obsolescence can be optimal only in the cases in which the marginal adjustment costs are substantial.[29]

### 3.3 Dynamic Obsolescence

Contrary to the conclusion of Lemma 1, the examples of Apple and Samsung clearly indicate that in reality firms do engage in *dynamic obsolescence*. A possible explanation for this observation is that environments in which durability adjustment is done through software updates are unlikely to exhibit such high marginal adjustment costs. In order to analyze this possibility we impose the following assumption.

**Assumption 3.** *(i)* $\lim_{\delta_2 \to \delta_1^-} C_2(\delta_1, \delta_2) = 0$, *(ii)* $\lim_{\delta_1 \to 0^+} C_1(\delta_1, 0) = 0$.

Assumption 3 implies that (infinitesimally) small adjustments to the product durability come at virtually no cost. Although this is an extreme case which ensures that there is a limited negative effect of the durability adjustment, it simplifies our analysis without introducing substantial qualitative limitations.

The optimal equilibrium behavior in period 2 depends on the monopolist's (pricing) decision in period 1. The two possibilities in period 1 are either (i) the monopolist chooses the price $p_1$ that is acceptable to both types of consumers, or (ii) he chooses $p_1$ that is acceptable only to high-type consumers. We refer to the first case as **full market coverage** and to the second case as **partial market coverage**. Although each case depends on the parameters of the model, in both cases we have the same qualitative result which is summarized in the theorem below.[30]

**Theorem 1** (Equilibrium Dynamic Obsolescence)**.** *When the monopolist has no commitment power, the profit-maximizing durability levels are* $\delta_1 > \delta_2 \geq 0$. *That is, in equilibrium the monopolist practices dynamic obsolescence.*

The intuition behind this result is as follows. Because $2v_L \geq v_H + m$, the monopolist finds it optimal to sell to both types of consumers in period 2. Therefore $p_2 = v_L$, the highest acceptable price to low-type consumers.

Under full market coverage, in period 2 the product is demanded by consumers whose products (purchased in period 1) broke down. Given a period 2 realization of durability $\delta$ (which is either $\delta_1$ or $\delta_2$), the demand is $2(1-\delta)$. Thus, the expected profit in period 2 from choosing the targeted durability $\delta_2$ is:

$$\mathbb{E}\pi_2^f(p_2 = v_L, \delta_2) = 2\alpha(v_L - m)(1-\delta_2) + 2(1-\alpha)(v_L - m)(1-\delta_1) - C(\delta_1, \delta_2), \quad (4)$$

[29] Another function which leads to planned obsolescence being optimal is $C(\delta_1, \delta_2) = b\,|\delta_1 - \delta_2|$ for a sufficiently high $b > 0$.

[30] In the case of quadratic costs which is explicitly analyzed in Section 3.6, full market coverage is optimal for the monopolist.

where superscript $f$ denotes full coverage.

Likewise, under partial market coverage, in period 2 the product is demanded by both high-type consumers whose products (purchased in period 1) broke down and by low-type consumers who buy the product for the first time. Thus, the demand is $2-\delta$ and the expected profit in period 2 is:

$$\mathbb{E}\pi_2^p(p_2 = v_L, \delta_2) = \alpha(v_L - m)(2-\delta_2) + (1-\alpha)(v_L - m)(2-\delta_1) - C(\delta_1, \delta_2), \quad (5)$$

where superscript $p$ denotes partial coverage.

As can be seen from Equations (4) and (5) choosing $\delta_2 < \delta_1$ has two effects. First, a reduction in $\delta_2$ leads to positive profits from higher demand in period 2. Second, it negatively affects profits due to a higher adjustment cost. Under Assumption 3, the first effect dominates the second and thus, a downward adjustment is profitable.

Theorem 1 also shows that $\delta_1$ is positive in equilibrium. The intuition is as follows. Given optimal prices and targeted durability $\delta_2$, choosing a higher durability in period 1 has two effects. On the one hand, it allows the monopolist to avoid (re)production costs in period 2 as fewer objects need to be replaced in period 2 when the product is more durable. On the other hand, higher durability in period 1 implies higher fixed cost $F(\cdot)$ and a non-negative adjustment cost $C(\cdot)$ (as $\delta_2 < \delta_1$ whenever $\delta_1 > 0$). However, when $\delta_1$ is sufficiently close to 0, $F'(0) = 0$ and Assumption 3 together imply that the negative effect of higher fixed costs is smaller than the positive effect from reducing (re)production costs. Thus, choosing a positive durability level is more profitable than making the product perishable from the start. Hence, $\delta_1 > 0$.

Although the monopolist decreases the durability in both cases of market coverage, the prices charged in period 1 are different. Under full market coverage, low type consumers are purchasing the product in period 1 and have an expected value of $v_L + \alpha\delta_2^f v_L + (1-\alpha)\delta_1^f v_L$, where $\delta_1^f$ and $\delta_2^f$ are the optimal durability levels under *full* market coverage. Because $p_2 = v_L$, low-type consumers get *zero surplus* from purchase in period 2, and hence the price in period 1 is the expected value, $p_1 = v_L + \alpha\delta_2^f v_L + (1-\alpha)\delta_1^f v_L$.

Under partial coverage, in period 1 only high-type consumers purchase the product. Therefore, the monopolist sets a price to make them indifferent between buying in period 1 and in period 2. High-type consumers have a surplus of $v_H - v_L > 0$ from purchase in period 2. In addition, the expected surplus from a purchase in period 1 is the expected value of the product plus the surplus from repurchasing the product in period 2 in case it breaks down. Therefore, $p_1 = v_H + \alpha\delta_2^p v_L + (1-\alpha)\delta_1^p v_L$, where $\delta_1^p$ and $\delta_2^p$ are the optimal durability levels under *partial* market coverage.

### 3.4 Profitability: Planned vs. Dynamic Obsolescence

We compare the monopolist's profits under planned and dynamic obsolescence to determine if commitment power benefits him. The following proposition summarizes the results.

**Theorem 2** (Profitability Comparison)**.** *Under Assumptions 1-3, the monopolist generates greater profits under planned obsolescence than under dynamic obsolescence.*

The intuition is as follows. Under planned obsolescence, the monopolist always has an option to choose the same durability level in period 1 as the one chosen under dynamic obsolescence. Moreover, the monopolist can choose the same level of market coverage (full or partial). If the monopolist chooses $\delta_1$ in period 1 under dynamic obsolescence, the period 1 price is $p_1 = v_t + \alpha\delta_2 v_L + (1-\alpha)\delta_1 v_L$ where $t \in \{L, H\}$ is the type of consumer targeted in period 1. In contrast, the price charged under planned obsolescence under the same $\delta_1$ is $p_1 = v_t + \delta_1 v_L$. By Theorem 1, $\delta_2 < \delta_1$ under dynamic obsolescence. Thus, even when the monopolist chooses the same durability level $\delta_1$, he charges a higher price to consumers under planned obsolescence. This higher price is sufficient to compensate for lower profits from period 2 sales even when the commitment is to suboptimal durability level $\delta_1$. At the same time, planned obsolescence makes the monopolist avoid adjustment costs in period 2. The two effects are positive and hence, planned obsolescence benefits the monopolist.

Theorem 2 also suggests that the monopolist has an ex ante incentive to find a commitment device to maintain a specific durability level across periods. Examples of such commitment devices are extended warranties and buybacks. The conditions outlined by the warranty policies could specify a wider range of conditions under which the monopolist has to offer a free replacement of a faulty product. This effectively increases the costs of decreasing durability. However, nowadays many companies provide the terms and conditions for product use in a digital form on their websites, thus making them easily amendable and subject to similar dynamics as a durability adjustment.[31] The adherence to the initially specified terms can therefore only be enforced by a third-party market participant, i.e. a regulator.

### 3.5 Socially Optimal Durability Levels

In this section we determine the durability levels that are socially optimal, i.e. the levels that maximize total welfare. Because $v_L > m$, the total welfare is maximized when all consumers get the product in period 1 and when all broken products are replaced in period 2. Given this, when the durability levels are $\delta_1$ and $\delta_2$, the total welfare is

$$\begin{aligned} TW(\delta_1, \delta_2) &= v_H + v_L - 2m + \underbrace{(\alpha\delta_2 + (1-\alpha)\delta_1)(v_H + v_L)}_{\text{surplus from goods still in use in period 2}} \\ &\quad + \underbrace{(\alpha(1-\delta_2) + (1-\alpha)(1-\delta_1))(v_H + v_L - 2m)}_{\text{surplus from resupplied goods}} - F(\delta_1) - C(\delta_1, \delta_2) \qquad (6) \\ &= 2(v_L + v_H - 2m) + 2\alpha\delta_2 m + 2(1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, \delta_2). \end{aligned}$$

Increasing the durability has two effects on the total welfare. On the one hand, more durable products are less likely to break, therefore leading to lower production costs to replace the broken items. On the other hand, the fixed cost of producing more durable products is higher. Nonetheless, under the imposed assumptions on $F(\cdot)$ it is socially optimal to choose $\delta_1 > 0$. At the same

[31] The case of Deep Cycle Systems, mentioned in Footnote 15, is an example of such conduct.

time, because $F'(\delta) \geq 0$ and $C_2(\delta, \delta) = 0$, it is always (weakly) less costly to start with lower durability and increase it in period 2, rather than starting with a higher durability without making adjustments. That is, under Assumptions 1-3 for $\delta_2 > \delta_1, F(\delta_2) \geq F(\delta_1) + C(\delta_1, \delta_2)$. For this reason, it is socially optimal to either produce a perfectly durable product (i.e. $\delta_1 = \delta_2 = 1$) or to dynamically increase the durability. This is summarized in the following proposition.

**Proposition 2.** *Total welfare is maximized either when the product is perfectly durable, or when the durability is positive in period 1 and is dynamically increased.*

The proposition implies that the equilibrium under dynamic obsolescence is never socially optimal because the direction of adjustment to achieve a social optimum is the opposite to the one taken by a profit-maximizing monopolist.

### 3.6 Consumer Welfare with Quadratic Costs

Although we can make conclusions about the profitability of dynamic obsolescence and the well-being of society as a whole given any abstract cost functions (i.e. the adjustment cost functions $F(\cdot)$ and $C(\cdot, \cdot)$), making inferences about consumer welfare might not be possible when the costs are not specific. This stems from the fact that the period 1 price (and hence consumer surplus) depends on whether the monopolist decides to fully cover the market or not. It is difficult to determine if full market coverage is optimal for general cost functions. For this reason, we provide the consumer welfare analysis in the case of quadratic costs. In particular, let the cost functions be given by

$$F(\delta_1) = a(\delta_1)^2, \quad C(\delta_1, \delta_2) = b(\delta_1 - \delta_2)^2, \tag{7}$$

where $a > 0$ and $b > 0$. It is easy to verify that the two functions satisfy Assumptions 1-3. Moreover, for technical reasons, we assume that $m < a$. This condition allows us to avoid a corner solution where the monopolist sets $\delta_1 = 1$, but it also has economic meaning. Intuitively, $2m$ is the monopolist's benefit from avoiding the cost of supplying a replacement for the obsolete good to both types of consumers (when the good is made more durable). In contrast, $2a$ is the marginal cost from making the good infinitely durable ($F'(1) = 2a$). When $m < a$ this cost outweighs the benefit from avoiding the production costs of supplying the replacement for the obsolete goods. Hence, making the good infinitely durable is not in the monopolist's best interest.[32]

The following lemma describes the equilibrium market coverage decision.

**Lemma 2.** *Let $m < a$. It is optimal for the monopolist to sell the durable good to both types of consumers in period 1.*

Lemma 2 implies that the equilibrium prices are $p_1 = v_L + \alpha\delta_2 v_L + (1 - \alpha)\delta_1 v_L$ and $p_2 = v_L$.

[32] This argument also applies to the case of planned obsolescence. When $m < a$, the monopolist will not make the good infinitely durable when he can commit to keeping the durability the same across the two periods.

The monopolist extracts all surplus from the low type. The surplus of high-type consumers is

$$\begin{aligned} v_H - p_1 + \alpha\delta_2 v_H + (1-\alpha)\delta_1 v_H + \alpha(1-\delta_2)(v_H - p_2) + (1-\alpha)(1-\delta_1)(v_H - p_2) = \\ = 2(v_H - v_L). \end{aligned} \tag{8}$$

Hence, the consumer surplus is independent of the durability choices of the monopolist.

Under planned obsolescence the monopolist will fully cover the market (Proposition 1). Thus, the equilibrium prices are $p_1 = v_L + \delta_1 v_L$ and $p_2 = v_L$. The prices are set such that the monopolist extracts all surplus from the low-type consumers. The surplus of high-type consumers is

$$v_H - p_1 + \delta_1 v_H + (1-\delta_1)(v_H - p_2) = 2(v_H - v_L). \tag{9}$$

Since Equations (8) and (9) are identical, the consumer surplus under planned and dynamic obsolescence is the same, which is formalized below.

**Proposition 3.** *Consumers are equally well-off under planned and dynamic obsolescence.*

Proposition 3 and Theorem 2 imply that planned obsolescence is socially more desirable than dynamic obsolescence. Society as a whole benefits if the monopolist finds ways to commit to maintaining the same durability level over time.

It should be emphasized that the result in Proposition 3 makes an observation from an *ex ante* perspective. Conditional on a successful durability adjustment, i.e. from an *ex post* perspective, consumers' welfare is *lower* compared to the case of planned obsolescence. This observation is in line with our motivating example of Apple where consumers experienced a welfare loss as a result of the reduced durability of their phones.

### 3.7 Illustrative Examples

We conclude the analysis with two examples which provide a simple illustration of the phenomena we have discussed so far. Example 1 showcases a situation in which dynamic obsolescence is the equilibrium strategy for any $\alpha \in (0,1)$. One might naturally wonder if the uncertainty of the durability adjustment is the root cause of this. In Example 2, we show that even for the extreme case of $\alpha = 1$, dynamic obsolescence emerges in equilibrium.

**Example 1** (Dynamic Obsolescence)**.** Let $v_H = 7, v_L = 5$. Take a monopolist with cost functions given by (7) where $a = b = 4$ and $m = 3$. The optimal choice of $\delta_1$ and $\delta_2$ depends on $\alpha$. In particular, there exists a cutoff value $\bar{\alpha}$ which satisfies the conditions of inequality (A.27) in the Appendix so that when $\alpha \leq \bar{\alpha}$, $\delta_1 = \frac{m}{a}$ and $\delta_2 = \frac{3-2\alpha}{4}$. When $\alpha \geq \bar{\alpha}$, the optimal $\delta_1 = \frac{(1-\alpha)m}{a+b}$ and $\delta_2 = 0$. Figure 2 illustrates the equilibrium values of $\delta_1, \delta_2$ for a range of $\alpha$'s. The equilibrium values of $\delta_2$ are given by Equation (A.19). With this choice of parameter values, for any $\alpha \in (0,1)$, dynamic obsolescence is an equilibrium strategy.

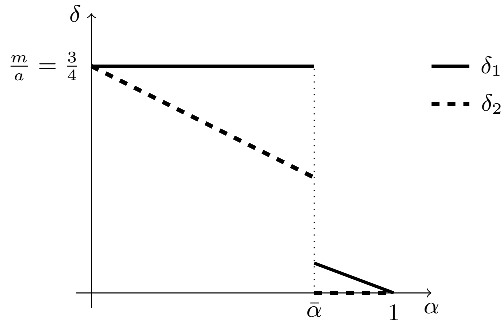


Figure 2: Equilibrium values of $\delta_1$ and $\delta_2$ as a function of the durability adjustment probability $\alpha$ when $v_H = 7, v_L = 5, m = 3$ and with quadratic costs. In this case $\bar{\alpha} \approx 0.74$.

The price in period 2 is $p_2 = v_L = 5$ and in period 1 it is

$$p_1 = v_L + \alpha\delta_2 v_L + (1-\alpha)\delta_1 v_L = \begin{cases} 8.75 - \frac{5\alpha^2}{2} & \text{if } \alpha \leq \bar{\alpha}, \\ 5 + \frac{15(1-\alpha)^2}{8} & \text{if } \alpha > \bar{\alpha}. \end{cases} \tag{10}$$

Observe that $p_1$ is decreasing in $\alpha$. △

**Example 2** (Dynamic Obsolescence when $\alpha = 1$)**.** Let $v_H = 5, v_L = 4$. Take a monopolist with cost functions given by (7) where $a = b = 4$ and $m = 3$. In this case, the right-hand side of Inequality (A.27) is greater than 1, implying that the optimal $\delta_1 = \frac{m}{a} > 0$ even when $\alpha = 1$. The target durability is $\delta_2 = \frac{3-\alpha}{4} > 0$. That is, the monopolist engages in dynamic obsolescence even when $\alpha = 1$, i.e. when the adjustment goes through with certainty. The prices are $p_2 = v_L = 4$ and $p_1 = v_L + \alpha\delta_2 v_L + (1-\alpha)\delta_1 v_L = 7 - \alpha^2$. △

# 4 Discussion and Conclusion

In this article we show that when a monopolist producer of a durable good does not have commitment power to fix the durability ex ante, they would choose different durability levels in different periods, i.e. the durability will be altered once the product has been purchased by the consumers. Moreover, this behavior is not going to be beneficial for the monopolist.

A monopolist might seek ways to establish commitment power through buybacks and extended warranties so that he can engage in planned obsolescence. However, technological advancements allow (and sometimes compel) the producer to have access to the product that is in the hands of the consumers, as such products require up-to-date software. This perpetual access erodes the producer's commitment power, making the phenomenon of dynamic obsolescence more prevalent in different industries. Finally, although the welfare maximizing durability adjustment is dynamic, the monopolist's adjustment is in the opposite direction to the socially optimal one and therefore harms welfare.

There are several possible directions for generalizing the result to richer environments. For instance, the choice of dynamic obsolescence as optimal strategy for a producer may be affected by the level of competition in the market. It stands to reason that with more competition, reduction in durability in period 2 might lead to reduction in sales, thus making the dynamic durability adjustment less beneficial. Additionally, if the producer can introduce a product of different quality on the market, this will change the producer's incentive structure and may lead to different conclusions. Addressing these problems is left for future research.

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# Appendix A Proofs

## A.1 Proof of Theorem 1

As explained in the main text, there are two possibilities in period 1: either (i) the monopolist chooses the price $p_1$ that is acceptable to both types of consumers, or (ii) he chooses $p_1$ that is acceptable only to high-type consumers. The first case is referred to as **full market coverage** and to the second case as **partial market coverage**.

### Case 1: Full market coverage in period 1

***Period 2 pricing.*** Let $\delta$ be the realization of period 2 durability. Then $\delta = \delta_2$ with probability $\alpha$ and $\delta = \delta_1$ with probability $1 - \alpha$. When all consumers made purchases in period 1, $1 - \delta$ measure of high-type consumers and $1 - \delta$ measure of low-type consumers are without the object in period 2. Because the monopolist has market power, he will choose the price $p_2$ that extracts all surplus from at least one type of consumers. Thus, the monopolist charges either $p_2 = v_L$ or $p_2 = v_H$.

When $p_2 = v_L$, the consumers of each type will make the purchase if they do not have the good. Thus, the demand in period 2 is $2(1 - \delta)$ and the monopolist's profit (excluding the sunk costs $F(\cdot)$ and $C(\cdot)$) is $2(1 - \delta)(v_L - m)$. When $p_2 = v_H$, only consumers of high type make a purchase. Thus, the demand in period 2 is $1-\delta$ and the monopolist's profit is $(1-\delta)(v_H-m)$. Charging $p_2 = v_L$ is optimal if $2(v_L-m) \geq v_H - m$, or if $2v_L \geq v_H + m$. Thus, the monopolist chooses $p_2 = v_L$ if the market is fully covered in period 1.

***Durability choice in period 2.*** Let $\delta_1$ be the durability chosen by the monopolist in period 1. Let $\pi_2^f(\delta_2)$ be the monopolist's expected profit of period 2 from adjusting the durability to $\delta_2$, conditional on the profit-maximizing choice of $p_2$, i.e. $\pi_2^f(\delta_2) = \mathbb{E}\pi_2^f(p_2 = v_L, \delta_2)$. Then,

$$\pi_2^f(\delta_2) = 2\alpha(1-\delta_2)(v_L-m) + 2(1-\alpha)(1-\delta_1)(v_L-m) - C(\delta_1,\delta_2). \tag{A.1}$$

Let $\delta_2^f(\delta_1)$ be the optimal durability level. It is the durability that maximizes the above profit. To find the optimal durability level, we consider the derivative with respect to $\delta_2$. Observe that for $\delta_2 > \delta_1$

$$\frac{\partial \pi_2^f(\delta_2)}{\partial \delta_2} = -2\alpha(v_L-m) - C_2(\delta_1,\delta_2) < 0, \tag{A.2}$$

where the inequality follows from Assumption 2, $\alpha > 0$ and $v_L > m$. Moreover, Assumption 3 implies that $\delta_2^f(\delta_1) < \delta_1$ whenever $\delta_1 > 0$. In addition, there exists $\bar{\delta}^f(\alpha, v_L, m) > 0$ sufficiently close to 0 such that $\delta_2^f(\delta_1) = 0$ for each $\delta_1 < \bar{\delta}^f(\alpha, v_L, m)$.

***Period 1 pricing.*** When choosing the price in period 1, to fully cover the market, the monopolist has to make it acceptable to low-type consumers. The monopolist chooses $p_1$ that makes low-type consumers indifferent between purchasing the product in period 1 and delaying the purchase until period 2. Because $p_2 = v_L$ a purchase delay leads to a surplus of zero to low-type consumers. In contrast, purchasing the good in period 1 generates a surplus of

$$v_L - p_1 + \alpha\delta_2^f(\delta_1)v_L + (1-\alpha)\delta_1 v_L + \underbrace{\alpha(1-\delta_2^f(\delta_1))\cdot 0 + (1-\alpha)(1-\delta_1)\cdot 0}_{\text{period 2 payoff when the good breaks down}}. \tag{A.3}$$

The price in period 1 then is $p_1 = v_L + \alpha\delta_2^f(\delta_1)v_L + (1-\alpha)\delta_1 v_L$.

***Durability choice in period 1.*** Let $\pi_1^f(\delta_1)$ be the monopolist's expected profit across two periods when the monopolist fully covers the market and when he chooses $\delta_1$, i.e. $\mathbb{E}\pi_1^f$. Then, the above computations imply

$$\begin{aligned}\pi_1^f(\delta_1) &= 2(p_1-m) - F(\delta_1) + \overbrace{2[\alpha(1-\delta_2^f(\delta_1)) + (1-\alpha)(1-\delta_1)](p_2-m) - C(\delta_1,\delta_2^f(\delta_1))}^{\pi_2^f(\delta_2^f(\delta_1))} \\ &= 4(v_L-m) + 2\alpha\delta_2^f(\delta_1)m + 2(1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1,\delta_2^f(\delta_1))\end{aligned} \tag{A.4}$$

The cost functions $F(\cdot)$ and $C(\cdot)$ are continuous. Moreover, $\delta_2^f(\delta_1)$ is a continuous function of $\delta_1$. So, $\pi_1^f(\delta_1)$ is also continuous in $\delta_1$ and the profit-maximizing durability level exists.

Earlier, we have seen that $\delta_2^f(\delta_1) = \delta_1$ only if $\delta_1 = 0$. Thus, to prove dynamic obsolescence in this case, it suffices to show that $\delta_1 = 0$ does not maximize the profit. Recall that for any $\delta_1 < \bar{\delta}^f(\alpha, v_L, m)$, $\delta_2^f(\delta_1) = 0$. Thus, for any such $\delta_1$ (including $\delta_1 = 0$), the profit is $\pi_1^f(\delta_1) =$

$4(v_L - m) + 2(1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, 0)$. The derivative with respect to $\delta_1$ is $\partial \pi_1^f(\delta_1)/\partial \delta_1 = 2(1-\alpha)m - F'(\delta_1) - C_1(\delta_1, 0)$. Continuous differentiability of $F(\cdot)$, $F'(0) = 0$ and Assumption 3 together imply that $\partial \pi_1^f(\delta_1)/\partial \delta_1 > 0$ for any $\delta_1 > 0$ sufficiently close to 0. Thus, an increase in $\delta_1$ leads to a higher profit. The profit is maximized at $\delta_1 > 0$. This, in turn, implies that $\delta_2^f(\delta_1) < \delta_1$. The monopolist chooses to dynamically adjust the durability if he chooses to fully cover the market in period 1.

**Case 2: Partial market coverage in period 1**

***Period 2 pricing.*** Like in the previous case, let $\delta$ be the realization of period 2 durability. Then $\delta = \delta_2$ with probability $\alpha$ and $\delta = \delta_1$ with probability $1-\alpha$. When only high-type consumers made purchases in period 1, $1-\delta$ measure of high type consumers and measure 1 of low-type consumers are without the object in period 2. Because the monopolist has market power, he will choose the price $p_2$ that extracts all surplus from at least one type of consumers. Thus, the monopolist charges either $p_2 = v_L$ or $p_2 = v_H$.

When $p_2 = v_L$, the consumers of each type will make the purchase if they do not have the good. Thus, the demand in period 2 is $2-\delta$ and the monopolist's profit (excluding the sunk costs $F(\cdot)$ and $C(\cdot)$) is $(2-\delta)(v_L - m)$. When $p_2 = v_H$, only consumers of high type make a purchase. Thus, the demand in period 2 is $1-\delta$ and the monopolist's profit is $(1-\delta)(v_H - m)$. Charging $p_2 = v_L$ is optimal if

$$(2-\delta)(v_L - m) \geq (1-\delta)(v_H - m) \iff \delta(v_H - v_L) \geq v_H + m - 2v_L. \tag{A.5}$$

Since $2v_L \geq v_H + m$, the right-hand side of the last inequality is non-positive. Moreover, $v_H \geq v_L$. Thus, the above inequality is satisfied for any $\delta \in [0,1]$ and thus $p_2 = v_L$.

***Durability choice in period 2.*** Let $\delta_1$ be the durability chosen by the monopolist in period 1. Let $\pi_2^p(\delta_2)$ be the monopolist's expected profit of period 2 from adjusting the durability to $\delta_2$, conditional on the profit-maximizing choice of $p_2$, i.e. $\pi_2^p(\delta_2) = \mathbb{E}\pi_2^p(p_2 = v_L, \delta_2)$. Then,

$$\pi_2^p(\delta_2) = \alpha(2-\delta_2)(v_L - m) + (1-\alpha)(2-\delta_1)(v_L - m) - C(\delta_1, \delta_2). \tag{A.6}$$

Let $\delta_2^p(\delta_1)$ be the optimal durability level. Observe that for $\delta_2 > \delta_1$

$$\frac{\partial \pi_2^p(\delta_2)}{\partial \delta_2} = -\alpha(v_L - m) - C_2(\delta_1, \delta_2) < 0, \tag{A.7}$$

where the inequality follows from Assumption 2, $\alpha > 0$ and $v_L > m$. Moreover, Assumption 3 implies that $\delta_2^p(\delta_1) < \delta_1$ whenever $\delta_1 > 0$. In addition, there exists $\bar{\delta}^p(\alpha, v_L, m) > 0$ sufficiently close to 0 such that $\delta_2^p(\delta_1) = 0$ for each $\delta_1 < \bar{\delta}^p(\alpha, v_L, m)$. Observe that $\bar{\delta}^f(\alpha, v_L, m) > \bar{\delta}^p(\alpha, v_L, m)$ for any $\alpha \in (0,1)$ and $v_L > m$.

***Period 1 pricing.*** When choosing the price in period 1, the monopolist chooses the highest price $p_1$ that is acceptable to high-type consumers. Whenever $v_H > v_L$ such a price will not be acceptable to low-type consumers and thus they will refrain from purchasing the good in period 1. The monopolist chooses $p_1$ at which high-type consumers are indifferent between purchasing the product in period 1 and delaying the purchase until period 2. Because $p_2 = v_L$ a purchase delay leads to a surplus of $v_H - v_L$ to high-type consumers. In contrast, purchasing the good in period 1 generates a surplus of

$$\begin{aligned} v_H - p_1 + \alpha\delta_2^p(\delta_1)v_H + &(1-\alpha)\delta_1 v_H \\ &+ \underbrace{\alpha(1-\delta_2^p(\delta_1))(v_H - v_L) + (1-\alpha)(1-\delta_1)(v_H - v_L)}_{\text{period 2 payoff when the good breaks down}} \\ &= v_H - p_1 + (v_H - v_L) + \alpha\delta_2^p(\delta_1)v_L + (1-\alpha)\delta_1 v_L. \end{aligned} \tag{A.8}$$

The period 1 price is the one that equates the above surplus to $v_H - v_L$, i.e. $p_1 = v_H + \alpha\delta_2^p(\delta_1)v_L + (1-\alpha)\delta_1 v_L$.

***Durability choice in period 1.*** Let $\pi_1^p(\delta_1)$ be the monopolist's expected profit across two periods when the monopolist partially covers the market and when he chooses $\delta_1$, i.e. $\mathbb{E}\pi_1^p$. Then, the above computations imply

$$\begin{aligned} \pi_1^p(\delta_1) &= (p_1 - m) - F(\delta_1) + \overbrace{[\alpha(2-\delta_2^p(\delta_1)) + (1-\alpha)(2-\delta_1)](p_2 - m) - C(\delta_1, \delta_2^p(\delta_1))}^{\pi_2^p(\delta_2^p(\delta_1))} \\ &= 2(v_L - m) + (v_H - m) + \alpha\delta_2^f(\delta_1)m + (1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, \delta_2^p(\delta_1)). \end{aligned} \tag{A.9}$$

The cost functions $F(\cdot)$ and $C(\cdot)$ are continuous. Moreover, $\delta_2^p(\delta_1)$ is a continuous function of $\delta_1$. So, $\pi_1^p(\delta_1)$ is also continuous in $\delta_1$ and the profit-maximizing durability level exists.

Earlier, we have seen that $\delta_2^p(\delta_1) = \delta_1$ only if $\delta_1 = 0$. Thus, to prove the statement, it suffices to show that $\delta_1 = 0$ does not maximize the profit. Recall that for any $\delta_1 < \bar{\delta}^p(\alpha, v_L, m)$, $\delta_2^p(\delta_1) = 0$. Thus, for any such $\delta_1$ (including $\delta_1 = 0$), the profit is $\pi_1^p(\delta_1) = 2(v_L - m) + (v_H - m) + (1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, 0)$. The derivative with respect to $\delta_1$ is $\partial\pi_1^p(\delta_1)/\partial\delta_1 = (1-\alpha)m - F'(\delta_1) - C_1(\delta_1, 0)$. For the same reasons as in case 1, $\partial\pi_1^p(\delta_1)/\partial\delta_1 > 0$ for any $\delta_1 > 0$ sufficiently close to 0. The profit is maximized at $\delta_1 > 0$, which, in turn, implies that $\delta_2^p(\delta_1) < \delta_1$. The monopolist chooses to dynamically adjust the durability if he chooses to partially cover the market in period 1. ■

## A.2 Proof of Proposition 1

As the main text indicates, the case of planned obsolescence is equivalent to $\alpha = 0$, $\delta_1 = \delta_2$, and $C(\delta_1, \delta_2) = 0$. All intermediate results in this case follow from the analysis of Theorem 1. That is, for both full and partial market coverage, the price that maximizes profit in period 2 is $p_2 = v_L$. When the monopolist fully covers the market in period 1, the price is $p_1 = v_L + \delta_1 v_L$, for partial coverage it is $p_1 = v_H + \delta_1 v_L$.

The expected profit under full coverage is given by $\hat{\pi}^f(\delta_1) = 4(v_L - m) + 2\delta_1 m - F(\delta_1)$. Let $\delta_1^*$ be the profit-maximizing durability when the market is *partially covered*. Then, the profit under partial market coverage is $\hat{\pi}^p = 2(v_L - m) + (v_H - m) + \delta_1^* m - F(\delta_1^*)$. When fully covering the market, the monopolist can also choose $\delta_1 = \delta_1^*$. Therefore, the difference in profits must satisfy

$$\hat{\pi}^f(\delta_1^*) - \hat{\pi}^p \geq 2(v_L - m) - (v_H - m) + \delta_1^* m \geq \delta_1^* m, \tag{A.10}$$

where the inequality follows from $2v_L \geq v_H + m$. Moreover, observe that $d\,\hat{\pi}^p/d\,\delta_1 = m - F'(\delta_1)$. Because $F'(0) = 0$ and $m > 0$, $\delta_1^* > 0$ must hold. As a result, $\hat{\pi}^f - \hat{\pi}^p \geq \delta_1^* m > 0$. The monopolist gets a greater profit when he fully covers the market in period 1 when engaging in planned obsolescence. ■

### A.3 Proof of Theorem 2

Let $\pi_1$ be the maximum (lifetime) expected profit from dynamic obsolescence, and $\delta_1$ and $\delta_2$ be the optimal durability levels under *dynamic* obsolescence chosen in periods 1 and 2 respectively. Similarly, let $\hat{\pi}_1$ be the maximum (lifetime) expected profit from planned obsolescence, and $\hat{\delta}_1$ be the optimal durability chosen under planned obsolescence.

Suppose that under dynamic obsolescence the monopolist decides to fully cover the market in period 1. Then, the profit from the dynamic obsolescence is

$$\pi_1 = 4(v_L - m) + 2\alpha\delta_2 m + 2(1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, \delta_2). \tag{A.11}$$

When the monopolist commits to the durability level in period 1, by Proposition 1 he will fully cover the market. Therefore, $\hat{\pi}_1 = 4(v_L - m) + 2\hat{\delta}_1 m - F(\hat{\delta}_1)$. Moreover, the monopolist has the option to choose $\delta_1$, the durability chosen under dynamic obsolescence. If $\hat{\delta}_1 \neq \delta_1$ the difference between the two profits is

$$\begin{aligned} \hat{\pi}_1(\delta_1) - \pi_1 &\geq 2\delta_1 m - 2\alpha\delta_2 m - 2(1-\alpha)\delta_1 m + C(\delta_1, \delta_2) \\ &> 2\delta_1 m - 2\alpha\delta_1 m - 2(1-\alpha)\delta_1 m + C(\delta_1, \delta_2) = C(\delta_1, \delta_2) \geq 0, \end{aligned} \tag{A.12}$$

where the strict inequality follows from $\delta_2 < \delta_1$ and the last inequality follows from Assumptions 1 and 2. Therefore, $\hat{\pi}_1 > \pi_1$ when the monopolist chooses to fully cover the market in period 1 under dynamic obsolescence.

Now, suppose that under dynamic obsolescence the monopolist decides to partially cover the market in period 1. Then, the profit from the dynamic obsolescence is

$$\pi_1 = 2(v_L - m) + (v_H - m) + \alpha\delta_2 m + (1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, \delta_2). \tag{A.13}$$

Under planned obsolescence, the profit from full market coverage is higher than the profit from partial market coverage (Proposition 1). Thus, $\hat{\pi}_1 \geq 2(v_L - m) + (v_H - m) + \tilde{\delta}_1 m - F(\tilde{\delta}_1)$ for any $\tilde{\delta}_1 \in [0, 1]$. In particular, this holds for $\tilde{\delta}_1 = \delta_1$, the optimal period-1 durability under dynamic obsolescence.

The difference between the two profits is thus

$$\begin{aligned}\hat{\pi}_1 - \pi_1 &\geq \delta_1 m - \alpha\delta_2 m - (1-\alpha)\delta_1 m + C(\delta_1, \delta_2) \\ &> \delta_1 m - \alpha\delta_1 m - (1-\alpha)\delta_1 m + C(\delta_1, \delta_2) = C(\delta_1, \delta_2) \geq 0,\end{aligned} \tag{A.14}$$

where the strict inequality follows from $\delta_2 < \delta_1$ and the last inequality follows from Assumptions 1 and 2. Therefore, $\hat{\pi}_1 > \pi_1$ when the monopolist chooses to partially cover the market in period 1 under dynamic obsolescence.

Combining the two results, we conclude that the monopolist gets more profit if he can commit to a durability level in period 1. ■

## A.4 Proof of Proposition 2

The total welfare is

$$TW(\delta_1, \delta_2) = 2(v_L + v_H - 2m) + 2\alpha\delta_2 m + 2(1-\alpha)\delta_1 m - F(\delta_1) - C(\delta_1, \delta_2). \tag{A.15}$$

First, observe that for any $\delta_2 < \delta_1$

$$\frac{\partial TW(\delta_1, \delta_2)}{\partial \delta_2} = 2\alpha m - C_2(\delta_1, \delta_2) \geq 2\alpha m > 0, \tag{A.16}$$

where the first inequality follows from Assumption 2 and the last equality follows from $\alpha > 0$ and $m > 0$. Thus, the welfare-maximizing $\delta_2$ must be such that $\delta_2 \geq \delta_1$. This immediately implies that $\delta_2 = 1$ whenever $\delta_1 = 1$.

Suppose that $\delta_1 < 1$. By Assumption 3, for each $\delta_1 < 1$, there exists $\varepsilon(\delta_1) > 0$ such that $\partial TW(\delta_1, \delta_2)/\partial\delta_2 > 0$ for any $\delta_2 \in (\delta_1, \delta_1 + \varepsilon(\delta_1))$. Thus, the welfare maximizing $\delta_2$ is such that $\delta_2 > \delta_1$ unless $\delta_1 = 1$.

Last, observe that

$$\frac{\partial TW(\delta_1, \delta_2)}{\partial \delta_1} = 2(1-\alpha)m - F'(\delta_1) - C_1(\delta_1, \delta_2) > 0 \tag{A.17}$$

for any $\delta_1 > 0$ sufficiently close to 0, which follows from continuous differentiability of $F(\cdot)$, $F'(0) = 0$ and Assumption 3. Thus, $\delta_1 > 0$ and $\delta_2 \geq \delta_1$. ■

## A.5 Proof of Lemma 2

We first find the optimal durability levels under full and partial market coverage.

**Full market coverage**

Suppose that the monopolist fully covers the market in period 1. Plugging the cost functions given in equation (7) into the period 2 profit function under full coverage given in equation (A.1), we get

$$\frac{\partial \pi_2^f(\delta_2)}{\partial \delta_2} = -2\alpha(v_L - m) - C_2(\delta_1, \delta_2) = -2\alpha(v_L - m) + 2b(\delta_1 - \delta_2). \tag{A.18}$$

Recall that $\delta_2^f(\delta_1)$ is the profit-maximizing durability in period 2 under full coverage. Using the above derivative, the first-order condition implies

$$\delta_2^f(\delta_1) = \begin{cases} 0 & \text{if } \delta_1 \le \frac{\alpha(v_L - m)}{b}, \\ \delta_1 - \frac{\alpha(v_L - m)}{b} & \text{otherwise.} \end{cases} \tag{A.19}$$

Using the functional forms for the cost functions in equation (7) and the expression for $\delta_2^f(\delta_1)$ in the monopolist's total expected profits given by equation (A.4), we get

$$\pi_1^f(\delta_1) = \begin{cases} 4(v_L - m) + 2(1-\alpha)\delta_1 m - (a+b)(\delta_1)^2 & \text{if } \delta_1 \le \frac{\alpha(v_L - m)}{b}, \\ 4(v_L - m) - \frac{\alpha^2(v_L^2 - m^2)}{b} + 2\delta_1 m - a(\delta_1)^2 & \text{otherwise.} \end{cases} \tag{A.20}$$

The derivative with respect to $\delta_1$ is

$$\frac{\partial \pi_1^f(\delta_1)}{\partial \delta_1} = \begin{cases} 2(1-\alpha)m - 2(a+b)\delta_1 & \text{if } \delta_1 < \frac{\alpha(v_L - m)}{b}, \\ 2m - 2a\delta_1 & \text{if } \delta_1 > \frac{\alpha(v_L - m)}{b}. \end{cases} \tag{A.21}$$

The second derivative is negative and so the profit function is concave. Observe that $\delta_1 = (1-\alpha)m/(a+b)$ is a local maximizer if

$$\frac{(1-\alpha)m}{a+b} < \frac{\alpha(v_L - m)}{b} \iff \alpha > \frac{bm}{bm + (a+b)(v_L - m)}. \tag{A.22}$$

Similarly, $\delta_1 = m/a$ is a local maximizer if

$$\frac{m}{a} > \frac{\alpha(v_L - m)}{b} \iff \alpha < \frac{bm}{a(v_L - m)}. \tag{A.23}$$

When $bm/(a(v_L - m)) \ge 1$, $\delta_1 = m/a$ is a local maximizer for any $\alpha < 1$. In addition, it is easy to verify that for any $\alpha \in (0,1)$, $a > 0$, $b > 0$ and $v_L > m$,

$$\frac{bm}{bm + (a+b)(v_L - m)} < \frac{bm}{a(v_L - m)}. \tag{A.24}$$

Therefore, when $\alpha \le bm/(bm + (a+b)(v_L - m))$, the profit-maximizing durability level is $\delta_1 = m/a$. Similarly, when $\alpha \ge bm/(a(v_L - m))$ (if such $\alpha < 1$ exists), the profit-maximizing durability is

$\delta_1 = (1-\alpha)m/(a+b)$.

Consider $\alpha$ satisfying

$$\frac{bm}{bm+(a+b)(v_L-m)} < \alpha < \frac{bm}{a(v_L-m)}. \tag{A.25}$$

Then, the profit is maximized at either $\delta_1 = (1-\alpha)m/(a+b)$ or at $\delta_1 = m/a$. The corresponding profits are

$$\begin{aligned} \pi_1^f\left(\frac{(1-\alpha)m}{a+b}\right) &= 4(v_L-m) + \frac{(1-\alpha)^2 m^2}{a+b}, \\ \pi_1^f\left(\frac{m}{a}\right) &= 4(v_L-m) + \frac{m^2}{a} - \frac{\alpha^2(v_L^2-m^2)}{b}. \end{aligned} \tag{A.26}$$

Then, $\delta_1 = m/a$ maximizes profit if $\pi_1^f(m/a) \geq \pi_1^f((1-\alpha)m/(a+b))$. This is satisfied if

$$\alpha \leq \frac{bm}{bm^2+(a+b)(v_L^2-m^2)}\left(m + v_L\sqrt{\frac{a+b}{a}}\right). \tag{A.27}$$

Observe that if $bm^2 > a(v_L^2-m^2)$, then $\delta_1 = m/a$ is the profit maximizer given any $\alpha < 1$.

**Partial market coverage**

Suppose that the monopolist partially covers the market in period 1. Plugging the cost functions in (7) into the period 2 profit under partial coverage in (A.6), the derivative with respect to $\delta_2$ is

$$\frac{\partial \pi_2^p(\delta_2)}{\partial \delta_2} = -\alpha(v_L-m) - C_2(\delta_1,\delta_1) = -\alpha(v_L-m) + 2b(\delta_1-\delta_2). \tag{A.28}$$

Let $\delta_2^p(\delta_1)$ be the profit-maximizing durability in period 2 when the market is partially covered in period 1. Then,

$$\delta_2^p(\delta_1) = \begin{cases} 0 & \text{if } \delta_1 \leq \frac{\alpha(v_L-m)}{2b}, \\ \delta_1 - \frac{\alpha(v_L-m)}{2b} & \text{otherwise.} \end{cases} \tag{A.29}$$

Using the functional forms for the cost functions in (7) and the expression for $\delta_2^f(\delta_1)$ in the monopolist's total expected profits (A.9) we get

$$\pi_1^p(\delta_1) = \begin{cases} 2(v_L-m)+(v_H-m)+(1-\alpha)\delta_1 m-(a+b)(\delta_1)^2 & \text{if } \delta_1 \leq \frac{\alpha(v_L-m)}{2b}, \\ 2(v_L-m)+(v_H-m)-\frac{\alpha^2(v_L^2-m^2)}{4b}+\delta_1 m - a(\delta_1)^2 & \text{otherwise.} \end{cases} \tag{A.30}$$

The derivative with respect to $\delta_1$ is

$$\frac{\partial \pi_1^p(\delta_1)}{\partial \delta_1} = \begin{cases} (1-\alpha)m - 2(a+b)\delta_1 & \text{if } \delta_1 < \frac{\alpha(v_L - m)}{2b}, \\ m - 2a\delta_1 & \text{if } \delta_1 > \frac{\alpha(v_L - m)}{2b}. \end{cases} \tag{A.31}$$

The second derivative is negative and so the profit function is concave. It is not difficult to verify (one can show) that $\delta_1 = (1-\alpha)m/(2(a+b))$ is local maximizer if $\alpha$ satisfies condition (A.22). Similarly, $\delta_1 = m/(2a)$ is a local maximizer if $\alpha$ satisfies condition (A.23).

For intermediate values of $\alpha$, i.e. satisfying Condition (A.25), the corresponding profits are

$$\begin{aligned} \pi_1^p\left(\frac{(1-\alpha)m}{2(a+b)}\right) &= 2(v_L - m) + (v_H - m) + \frac{(1-\alpha)^2 m^2}{4(a+b)} \\ \pi_1^f\left(\frac{m}{2a}\right) &= 2(v_L - m) + (v_H - m) + \frac{m^2}{4a} - \frac{\alpha^2(v_L^2 - m^2)}{4b}. \end{aligned} \tag{A.32}$$

Then, $\delta_1 = m/(2a)$ maximizes profit if $\pi_1^p(m/(2a)) \geq \pi_1^p((1-\alpha)m/(2(a+b)))$. This is satisfied for the same condition as under the full coverage case, given by (A.27). Observe that if $bm^2 > a(v_L^2 - m^2)$, then $\delta_1 = m/(2a)$ is the profit maximizer given any $\alpha < 1$.

**Comparison of profits**

Suppose that condition (A.27) holds. The difference in profits is

$$\pi_1^f\left(\frac{m}{a}\right) - \pi_1^p\left(\frac{m}{2a}\right) = 2v_L - v_H - m + \frac{3m^2}{4a} - \frac{3\alpha^2(v_L^2 - m^2)}{4b}. \tag{A.33}$$

Because $2v_L \geq v_H + m$, to show that the difference is positive, it suffices to show that

$$\frac{3m^2}{4a} - \frac{3\alpha^2(v_L^2 - m^2)}{4b} \geq 0 \Longleftrightarrow \alpha \leq \sqrt{\frac{bm^2}{a(v_L^2 - m^2)}}. \tag{A.34}$$

It turns out any $\alpha$ satisfying condition (A.27) will also satisfy the above inequality. Thus, fully serving the market is more profitable in this case.

Suppose that $\alpha$ violates condition (A.27). It is easy to verify that whenever $2v_L \geq v_H + m$, $\pi_1^f\left((1-\alpha)m/(a+b)\right) > \pi_1^p\left(((1-\alpha)m)/(2(a+b))\right)$. Thus, the monopolist wants to fully cover the market in this case, as well.

Combining all the results, we conclude that the monopolist maximizes his profit when he chooses to fully cover the market in period 1.