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ORIGINAL PAPER

Discounted network Myerson value and an application in power grid management

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Abstract

The classical Myerson value for network games assigns payoffs to the players in a network based on their productivity in it. This often results in an unequal distribution of the resources generated by the players in the network. However, when link formation is costly but essential for generating resources in the network, one way of keeping the network stable is to incentivize those lesser-paid players with discounts. In this paper, we introduce a new allocation rule called the δ-discounted network Myerson value for network games, inheriting a similar idea from cooperative game theory. This value allows the designer to control the discounting amount by adjusting the discount parameter δ. We provide two characterizations of this value and discuss an application in the power grid management problem that specifies how this value can help sustainably transport green energy.

Keywords Network games · Myerson value · Cost allocation · Sustainable energy · Reduced carbon emission

1 Introduction

From social and professional relationships to biological systems, international relations, or infrastructure networks like transportation and power grids, networks form an indispensable part of our lives. Within these networks, cooperation is often seen to be more sustainable and beneficial than competition in the long run. This necessitates

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the study of networks from a cooperative standpoint. In this framework, players make links among themselves and generate resources or accrue costs: we call them values. The key concern is allocating benefits or burdens resulting from cooperation under binding agreements among the players involved in the network. Thus, given a network of players, an allocation rule assigns every player some payoff from the value (resource or cost) generated by the network. In this paper, we propose an allocation rule, namely the δ-discounted network Myerson value, and provide its characterizations based on some intuitive axioms. The idea inherits the notion of δ-discounted Shapley value due to (Joosten 1996) for cooperative games with transferable utilities or TU games in short. We provide a practical example of an energy grid where our model can be applied to determinate cost allocation pattern for the power plants connected to the grid.

(Myerson 1977) introduces the concept of communication situations restricting the notion of TU games, where players can generate non-zero values only if they are connected by a network. Jackson and Wolinsky (1996) argue that the value accrued by a network not only depends on whether players are connected in it or not but also on the structure of the network, i.e., whether the players are connected through direct or indirect links also matters in the generation of values. They (Jackson and Wolinsky 1996) propose the notion of network games based on this principle. Two important allocation rules, namely, the network Myerson value (Jackson and Wolinsky 1996) and the network position value (Slikker 2005) are player-based and link-based allocation rules respectively. Both these allocation rules adopt a marginalistic approach, and distribute payoffs to the players in the network, based on their contributions to the value-generating process. On the contrary, there may be situations, for example, where a player may not be able to contribute to the value-generation process, but she connects two very productive players in the network and therefore, her presence in the network is crucial for the other players to generate values. Thus, players occupying critical positions in the network even if they do not contribute to the value should be encouraged to keep the network stable. This is more important when link formation becomes costly. Van Den Brink and Funaki (2010) term this process as a consolidation between egalitarianism and marginalism.

In TU games, the Shapley value proposed by Shapley (1953) is regarded as the most prominent marginal solution for TU games. Its strong reliance on productivity means the unproductive (null) players receive zero payoff, representing an absence of solidarity. To address this lack of solidarity, Nowak and Radzik (1994) introduced the solidarity value by replacing the player's individual marginal contribution in the Shapley value with the average marginal contribution by each coalition being formed. Later, Joosten (1996) proposed two classes of solutions that use the Shapley value and the equal division rule as extremes. One of these is the egalitarian Shapley value i.e., the convex combination of the Shapley value and the equal division rule, where the parameter α allows the planner to adjust the level of marginality and egalitarianism. The second class of the solutions is the δ-discounted Shapley value, term later given by Driessen and Radzik (2002). This value is formally the Shapley value applied to a discounted game where the worth of the coalition is discounted based on the number of players outside the coalition. The δ-discounted Shapley value was characterized and implemented by Van Den Brink and Funaki (2010) introducing the

δ-reducing player property, which generalizes the null player property of the Shapley value. Calvo and Gutiérrez-López (2015) also provided a strategic implementation of the class of δ-discounted Shapley values. In later work, Calvo and Gutiérrez-López (2018) proposed a class of discounted solidarity value which interpolates between the solidarity value by Nowak and Radzik (1994) and the equal division rule. They showed that the discounted solidarity value is equivalent to applying the solidarity value to the discounted game. The effort to balance egalitarianism and marginalism led to several more generalized classes of values being proposed in (Borkotokey et al. 2022; Béal et al. 2017; Casajus and Huettner 2014, 2013; Choudhury et al. 2021).

Recent research has also extended these solidarity-based concepts to games with communication structures and to network-restricted games. Li and Meng (2023) introduced the α-egalitarian Myerson value for games with communication structure. This solution is a convex combination of the Myerson Value and the component wise equal division rule for communication situation. Borkotokey et al. (2025) extended this notion to network games by proposing the network egalitarian Myerson value (the NEM-value). In this allocation rule, a weight α is assigned to productivity-based allocation through the network Myerson value, while the remaining weight $(1-\alpha)$ captures egalitarian concerns via sharing of the value generated through the component wise equal division rule.

To motivate further, we present the following example, which is studied in detail in sect. 4.

Example 1 Consider the physical network formed by an energy grid that connects several renewable power plants with the assumption that each of them can have equitable access to electricity. In such networks, cooperation among the power plants is essential in improving network efficiency and grid stability. Here, by network efficiency, we mean the ability to transmit sustainable energy with minimal environmental impact, particularly through reduced carbon emissions. In order to achieve these objectives, the combined efforts and cooperation of different regions that house the power plants are essential. Therefore, different transmission links must be established among different regions (players) to make the transmission network more effective. However, this process is costly. Players forming multiple links are likely to incur more costs. Thus, players critically placed in the network and involved in costly link-formation processes need to be discounted[1]. The costs to be shared by the participating players should reflect these discounts. The existing Myerson value due to Jackson and Wolinsky (1996) does not address this issue, and rather considers only the marginal contribution of each player in generating resources after the network is established.

In a related application paper, Her et al. (2018) analyzed the stability and fair benefit sharing of the ASEAN Power Grid (APG) initiative, focusing on the Greater

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[1] In the context of cost sharing, solidarity means discounting the costs borne by players who incur high costs, with part of their cost burden being shared by other players. While in profit sharing, solidarity refers to allocating non-zero payoffs to unproductive players. That is players with zero productivity receive a non-zero payoff through redistribution from more productive players.

Mekong Subregion (Cambodia, Laos, Myanmar, Thailand, Vietnam) to ensure countries benefit and have no incentive to leave the integrated system. This study employs the Myerson value, to allocate cost savings, ensuring core stability and pairwise stability for the interconnected grid. However, in view of Example 1, a fairer approach could be the discounted network Myerson value rather than the Myerson value to explore stability of the grid network. Thus, the purpose of this paper is to show that the δ-discounted network Myerson value offers a balance between favouring individual contributions and equal pay for all, with δ determining the degree of emphasis on each preference. The proposed δ-discounted network Myerson value yields the network Myerson value for $\delta = 1$, and for $\delta = 0$, all non-isolated (connected) players in the network receive an equal payoff, following the principle of egalitarianism.

The rest of the paper proceeds as follows: Sect. 2 presents the necessary preliminaries for our model. Section 3 introduces and characterizes the δ-discounted network Myerson value. In Sect. 4, we demonstrate an example of cost sharing using our proposed value.

2 Preliminaries

2.1 Cooperative games with transferable utilities

Let $N = \{1, \ldots, n\}$ be a finite set of players. The set $S \subseteq N$ is called a coalition. N itself is called the grand coalition, while the empty coalition is denoted by $\emptyset$. A cooperative game with transferable utility (or TU game) is defined by the pair (N, v), where N is a finite set of players and $v : 2^N \longrightarrow \mathbb{R}$ is a characteristic function that assigns a real number $v(S)$ to each coalition $S \subseteq N$, indicating the worth generated by that coalition, with $v(\emptyset) = 0$. For a fixed N, we denote a TU game (N, v) as v and $|S|$, the cardinality of any coalition S as s. Let $\mathcal{G}^N$ denote the class of all TU games defined for the player set N.

For any $\emptyset \neq T \subseteq N$, the unanimity game (N, u_T) is given by:

$$u_T(S) = \begin{cases} 1, & \text{if } T \subseteq S \\ 0, & \text{otherwise} \end{cases}$$

(Shapley 1953) shows that the class $\{u_T | \emptyset \neq T \subseteq N\}$ of unanimity games constitute a basis for $\mathcal{G}^N$. Thus $\mathcal{G}^N$ is a vector space over the set of reals with dimension $2^{|N|} - 1$. Every $v \in \mathcal{G}^N$ can be uniquely expressed as $v = \sum_{T \subseteq N, T \neq \emptyset} \Delta_v(T) u_T$, where $\Delta_v(T) = \sum_{S \subseteq T} (-1)^{t-s} v(S)$ is the Harsanyi dividend (Harsanyi 1959).

A solution is an n-dimensional vector where each component of the vector represents the payoff assigned to a player in the set N. Single-point solutions are called values. We denote a value by the function $\Phi : \mathcal{G}^N \mapsto \mathbb{R}^n$ such that for every $v \in \mathcal{G}^N$, $\Phi(v) = (\Phi_i(v))_{i \in N}$.

Among various solution concepts in cooperative game theory, the Shapley value (Shapley 1953) is the most popular single-point solution or value, under which each player gets a payoff proportional to her marginal contributions. More specifically, the Shapley value assigns to every player the average expected marginal contributions across all possible orderings in which she joins the coalitions. The marginal contribution of player $i \in N$ to the coalition $S \subseteq N \backslash i$ is $m_i^v(S) = v(S \cup i) - v(S)$. Mathematically, the Shapley value can be expressed as follows:

$$\Phi_i^{Sh}(v) = \sum_{S \subseteq N \backslash i} \frac{s!(n-s-1)!}{n!}[v(S \cup i) - v(S)]$$

The Equal division rule is an egalitarian solution concept where for $v \in \mathcal{G}^N$ and a fixed player set *N*, every player receives an equal payoff. Mathematically,

$$\Phi_i^{ED}(v) = \frac{v(N)}{n} \tag{1}$$

For $\delta \in [0,1]$, Joosten (1996) proposes the δ-discounted Shapley value as the Shapley value of a special class of games denoted by v^δ given by $v^\delta(S) = \delta^{|N|-|S|}v(S)$ for each $S \subseteq N$. This game is known as the discount game, see also (Driessen and Radzik 2002). Thus, the δ-discounted Shapley value denoted by $\Phi^{\delta-Sh}$ is given by

$$\Phi_i^{\delta-Sh}(v) = \sum_{S \subseteq N \backslash i} \frac{s!(n-s-1)!}{n!}[v^\delta(S \cup i) - v^\delta(S)] \tag{2}$$

After some deductions, Eq. (2) can be expressed as follows:

$$\Phi_i^{\delta-Sh}(v) = \sum_{S \subseteq N \backslash i} \frac{s!(n-s-1)!}{n!}\delta^{n-s-1}[v(S \cup i) - \delta v(S)] \tag{3}$$

Note that when $\delta = 1$ and $\delta = 0$, $\Phi^{\delta-Sh}$ coincides with Φ^{Sh} and Φ^{ED} respectively.

Next, we mention some important definitions and axioms related to these values, which are also relevant to the development of our model.

- For any $v \in \mathcal{G}^N$, player $i \in N$ is said to be a null player in the game *v* if for each $S \subseteq N \backslash i$, $v(S \cup i) = v(S)$.
- Player $i \in N$ is said to be a nullifying player in the game *v* if $v(S \cup i) = 0$ for each $S \subseteq N \backslash i$.
- Two players $i, j \in N$ are said to be symmetric with respect to *v* if $v(S \cup i) = v(S \cup j)$ for all $S \subseteq N \backslash \{i, j\}$.
- Generalizing the null and nullifying player, Van Den Brink and Funaki (2010) define a player $i \in N$ in the game *v* as a δ-reducing player if $v(S \cup i) = \delta v(S)$ for $\delta \in [0,1]$.

A value $\Phi : \mathcal{G}^N \longrightarrow \mathbb{R}^n$ is said to satisfy:

- *Efficiency* if $\sum_{i \in N} \Phi_i(v) = v(N)$ for each $v \in \mathcal{G}^N$.
- *Null player property* if for every $v \in \mathcal{G}^N$, $\Phi_i(v) = 0$ for each null player $i \in N$ in the game *v*.
- *Symmetry* if $\Phi_i(v) = \Phi_j(v)$ for any pair of symmetric players $i, j \in N$ with respect to $v \in \mathcal{G}^N$.
- *Linearity* if $\Phi_i(\alpha v + \beta w) = \alpha \Phi_i(v) + \beta \Phi_i(w)$ for all $v, w \in \mathcal{G}^N$ and $\alpha, \beta \in \mathbb{R}$, and every player $i \in N$.
- *δ-Reducing player property* if for every $v \in \mathcal{G}^N$, $\Phi_i(v) = 0$ for every δ-reducing player *i* in the game *v*.

In his seminal work, Shapley (1953) characterizes the Shapley value using the initial four axioms outlined above. Meanwhile replacing the Null Player Property with the δ-Reducing Player Property alongwith Efficiency, Linearity, and Symmetry, Van Den Brink and Funaki (2010) provide an axiomatic characterization of the δ-discounted Shapley value.

2.2 Network games

In this subsection, we discuss the definitions and results of network games, taken mostly from Borkotokey et al. (2014), which are necessary for developing our model.

A network is a pair (*N*, *g*) where $N = \{1, \ldots, n\}$ is a finite set of players and *g* is a set of pairs $\{i, j\}$ known as links, representing a relationship between player $i, j \in N$. We will use the notation *ij* instead of $\{i, j\}$. $g^N = \{ij | i, j \in N, i \neq j\}$ is the set of all subsets of *N* with size 2, known as a complete network of *n* nodes. $\mathbb{G}^N = \{g | g \subseteq g^N\}$ denotes the set of all possible networks on *N*. For any subset $g \subseteq g^N$, *l*(*g*) denotes the number of links within *g*, and $l_i(g)$ denotes the number of links with player *i* in *g*. Let *N*(*g*) be the set of players having at least one link in *g*, and $n(g) = |N(g)|$. A path between players *i* and *j* in network (*N*, *g*) is a finite set $\{i_0, i_1, \ldots, i_K\}$ such that $i_{p-1}i_p \in g$[2] for $p \in \{1, \ldots, K\}$, with $i_0 = i$ and $i_K = j$. For any $g \in \mathbb{G}^N$ and coalition $S \subseteq N$, $g|_S$ represents the subnetwork induced by *g* among players in *S*, i.e., $g|_S = \{ij \in g | i, j \in S\}$. Let g_i be the set of links that involve player *i* in *g*, $g \setminus g_i$ is a network without player *i*.

A subnetwork $g' \subseteq g$ is said to be a component of *g* if (i) for any distinct pairs $i, j \in N(g')$, there is a path between *i* and *j* in g' and (ii) $i \in N(g')$ and $ij \in g$ then $ij \in g'$. The set of all components of *g* is denoted by *C*(*g*).

A value function *v* on $\mathbb{G}^N$ is a mapping $v : \mathbb{G}^N \longrightarrow \mathbb{R}$ such that $v(\emptyset) = 0$. The set of all possible value functions is denoted by $\mathbb{V}^N$. A value function may represent the resource generated or the cost accrued by the network under a cooperative setup

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[2] Here $i_{p-1}i_p$ denotes the link between players i_{p-1} and i_p. Whenever this shorthand notation may lead to ambiguity; for example, whether 114 refers to a link between players 11 and 4 or between 1 and 14, we will use the original notation $\{i_{p-1}, i_p\}$.

depending on the social or economic situation. Thus, we use the term “worth" to represent both resource and cost.

A value function *v* is component additive if, for any $g \in \mathbb{G}^N$, $v(g) = \sum_{g' \in C(g)} v(g')$.

A network game is a pair (*N*, *v*), where *N* is a finite set of players and *v* is a value function.

For a given $g' \in \mathbb{G}^N$, a basic value function $u_{g'} \in \mathbb{V}^N$ can be defined as,

$$u_{g'}(g) = \begin{cases} 1, & \text{if } g' \subseteq g \\ 0, & \text{otherwise} \end{cases}$$

This value function is called a unanimity value function for network games. The class of such unanimity value functions forms a basis for the space $\mathbb{V}^N$. Any $v \in \mathbb{V}^N$ can be uniquely expressed as $v = \sum_{\emptyset \neq g' \in \mathbb{G}^N} \lambda_{g'} u_{g'}$, where $\lambda_{g'} = \sum_{\emptyset \neq g'' \subseteq g'} (-1)^{l(g')-l(g'')} v(g'')$ is the Harsanyi dividend for the network game. Hence, $\mathbb{V}^N$ is a $2^{\frac{n(n-1)}{2}} - 1$ dimensional vector space over the set of real numbers. A network allocation rule is a function $Y : \mathbb{G}^N \times \mathbb{V}^N \longrightarrow \mathbb{R}^n$ that assigns a payoff $Y_i(g, v)$ to each player *i* for a given $g \in \mathbb{G}^N$ and $v \in \mathbb{V}^N$.

For any $v \in \mathbb{V}^N, g \in \mathbb{G}^N$ the network Myerson value (Jackson and Wolinsky 1996) is given by,

$$Y_i^{NMV}(g, v) = \sum_{S \subseteq N \setminus i} \frac{s!(n-s-1)!}{n!} [v(g|_{S \cup i}) - v(g|_S)]$$

The network Myerson value Y^{NMV} can also be expressed using the Harsanyi dividends $\lambda_{g'}$ as follows:

$$Y_i^{NMV}(g, v) = \sum_{\substack{\emptyset \neq g' \subseteq g : \\ i \in N(g')}} \frac{\lambda_{g'}(v)}{n(g')}$$

For any network game (*N*, *v*), a corresponding cooperative TU game (N, w_v^g) can be constructed such that $w_v^g(S) = v(g|_S) \ \forall S \subseteq N$. This formulation ensures that the network Myerson value is indeed the Shapley value of the network-restricted TU game i.e. $Y^{NMV}(g, v) = \Phi^{Sh}(w_v^g)$.

A network allocation rule *Y* is said to satisfy:

- *Component balance* if for any component additive $v \in \mathbb{V}^N$, $g \in \mathbb{G}^N$,

$$\sum_{i \in N(h)} Y_i(g, v) = v(h), \text{ for all } h \in C(g).$$

- *Equal bargaining power* if for any $g \in \mathbb{G}^N, v \in \mathbb{V}^N$ and $ij \in g$,

$$Y_i(g,v) - Y_i(g\backslash ij, v) = Y_j(g,v) - Y_j(g\backslash ij, v).$$

Theorem 1 *(Jackson 2005). An allocation rule Y satisfies Component Balance and Equal Bargaining Power if and only if* $Y(g,v) = Y^{NMV}(g,v)$ *for all g and component additive v.*

However, in this paper we do not restrict our study to component additive value functions instead, we consider the space $\mathbb{V}^N$ as such.

Below, we present several additional solution concepts for network games, which will later be used for comparison with our proposed value.

- *Component wise equal division rule*: For any component additive $v \in \mathbb{V}^N$ and any $g \in \mathbb{G}^N$, the component wise equal division rule (Jackson 2005) is defined as follows:

$$Y_i^{NCE}(g,v) = \begin{cases} \frac{v(h)}{n(h)}, & \text{if there exists } h \in C(g) such that i \in h \\ 0, & \text{otherwise} \end{cases}$$

- *Network position value*: For any $v \in \mathbb{V}^N$, $g \in \mathbb{G}^N$ the network position value (Slikker 2005) can be expressed as,

$$Y_i^{NPV}(g,v) = \sum_{g' \subseteq g} \frac{1}{2} \frac{l_i(g')}{n(g')} \lambda_{g'}(v)$$

where $\lambda_{g'}$ is the Harsanyi dividend for the network games.

- *Network egalitarian Myerson value*: For any $v \in \mathbb{V}^N$, $g \in \mathbb{G}^N$, the network egalitarian Myerson value (NEM-value) (Borkotokey et al. 2025) is given by,

$$Y_i^{\alpha-NEM}(g,v) = \alpha Y_i^{NMV}(g,v) + (1-\alpha) Y_i^{NCE}(g,v) \text{ , where } \alpha \in [0,1] \,.$$

3 The δ-discounted network Myerson value

This section introduces the δ-discounted network Myerson value, $Y^{\delta-NMV}$ where the parameter $\delta \in [0,1]$ represents the discount factor.

For any $g \in \mathbb{G}^N$,$v \in \mathbb{V}^N$, the δ-discounted network Myerson value can be defined as,

$$Y_i^{\delta-NMV}(g,v) = \sum_{S \subseteq N\backslash i} \frac{s!(n-s-1)!}{n!} \delta^{l(g)-l(g|_{S\cup i})} \left[v(g|_{S\cup i}) - \delta^{l_i(g|_S)} v(g|_S) \right], \delta \in [0,1]. \quad (4)$$

It follows from Eq. 4 that the δ-discounted network Myerson value gives discount on the payoffs to the players by scaling up/down the marginal contributions by the factor δ and its exponents.

Remark 1 For any fixed network *g*, the parameter δ, and every value function *v* defined on $\mathbb{V}^N$ there exists a value function v^δ on $\mathbb{V}^N$ given by,

$$v^\delta(g') = \begin{cases} \delta^{l(g)-l(g')}v(g'), \text{ if } g' \subseteq g \\ 0, \text{ otherwise.} \end{cases}$$

We call (N, v^δ) the discounted network game. It is easy to show that the network Myerson value of (N, v^δ) is equal to the δ-discounted network Myerson value of (*N*, *v*).

Thus, we formally state the following proposition.

Proposition 1 *The δ-discounted network Myerson value of v on any $g \in \mathbb{G}^N$ is equivalent to the network Myerson value of v^δ on the same $g \in \mathbb{G}^N$.*

Next, for any $\emptyset \neq g' \in \mathbb{G}^N$ and $\delta \in [0, 1]$, we define a value function $u^\delta_{g'}$ as follows:

$$u^\delta_{g'}(g) = \begin{cases} \delta^{l(g)-l(g')}, \text{ if } g' \subseteq g \\ 0, \text{ otherwise.} \end{cases}$$

We call $u^\delta_{g'}$, the δ-unanimity value function.

Proposition 2 *The class of δ-unanimity value functions $\{u^\delta_{g'} | g' \in \mathbb{G}^N\}$ forms a basis for $\mathbb{V}^N$.*

Proof We follow Lemma 2.2 due to Nowak and Radzik (1994) to prove the theorem as follows.

Note that $|\mathbb{G}^N \backslash \emptyset| = |2^{l(g^N)} - 1| = 2^{\frac{n(n-1)}{2}} - 1$. Let $k = 2^{\frac{n(n-1)}{2}} - 1$.

Consider the fixed sequence $g^N = g_1, g_2, g_3, \ldots, g_K$ of non-empty networks in $\mathbb{G}^N$ with $|g^N| = |g_1| \geq |g_2| \geq \cdots \geq |g_k| = 1$. Define $A = [a_{ij}]_{k\times k}$ such that $a_{ij} = u^\delta_{g_i}(g_j)$. By the definition of $u^\delta_{g'}$, *A* is an upper triangular matrix with diagonal entries equal to 1, hence $\det(A) = 1$, implying the linear independence of its rows. That is, the class $\{u^\delta_{g'} | g' \in \mathbb{G}^N\}$ constitutes a set of *k* linear independent vectors in the *k*-dimensional linear space $\mathbb{V}^N$ and hence forms a basis for $\mathbb{V}^N$. Thus, any $v \in \mathbb{V}^N$ can be expressed as $v = \sum_{\emptyset \neq g' \subseteq g^N} \lambda^\delta_{g'}(v) u^\delta_{g'}$. Specifically, for a given $g \in \mathbb{G}^N$, $v(g) = \sum_{\emptyset \neq g' \subseteq g} \lambda^\delta_{g'}(v) \times \delta^{l(g)-l(g')}$. Using Mobius inversion, we obtain

$$\lambda_g^\delta(v) = \sum_{\emptyset \neq g' \subseteq g} (-\delta)^{l(g)-l(g')} v(g'). \tag{5}$$

□

Following similar terminology in TU-games (Harsanyi 1959), we call the term $\lambda_g^\delta(v)$ in Eq. 5 the Harsanyi dividends on $\delta \in [0,1]$. Proposition 2 prompts us to represent the δ-discounted network Myerson value by means of the Harsanyi dividends on $\delta \in [0,1]$.

Proposition 3 *The δ-discounted network Myerson value $Y^{\delta-NMV}$ has the following alternative representation,* $Y_i^{\delta-NMV}(g,v) = \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')} \lambda_{g'}^\delta(v)$, *where $\lambda_{g'}^\delta(v)$ is given by Eq. 5.*

Proof The proof requires the fact that $Y_i^{\delta-NMV}(g,v) = Y_i^{NMV}(g, v_g^\delta)$. Therefore,

$$\begin{aligned}
Y_i^{\delta-NMV}(g,v) &= \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{\lambda_{g'}(v^\delta)}{n(g')} \\
&= \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{1}{n(g')} \sum_{\emptyset \neq g'' \subseteq g'} (-1)^{l(g')-l(g'')} v^\delta(g'') \\
&= \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{1}{n(g')} \sum_{\emptyset \neq g'' \subseteq g'} (-1)^{l(g')-l(g'')} \delta^{l(g)-l(g'')} v(g'') \\
&= \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')} \sum_{\emptyset \neq g'' \subseteq g'} (-1)^{l(g')-l(g'')} \delta^{l(g')-l(g'')} v(g'') \\
&= \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')} \sum_{\emptyset \neq g'' \subseteq g'} (-\delta)^{l(g')-l(g'')} v(g'') \\
&= \sum_{\substack{\emptyset \neq g' \subseteq g: \\ i \in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')} \times \lambda_{g'}^\delta(v).
\end{aligned}$$

□

Fig. 1 Network (N, g)

Table 1 Worth of subnetworks $g' \subseteq g$

g′	{12}	{23}	{34}	{12, 23}	{12, 34}	{23, 34}	{12, 23, 34}
$v(g')$	1	0	0	1	1	0	1

Table 2 Comparison of payoff distributions

Players	Y^{NCE}	Y^{NMV}	Y^{NPV}	$Y^{\alpha-NEM}$	$Y^{\delta-NMV}$
1	0.25	0.5	0.5	0.45	0.423
2	0.25	0.5	0.5	0.45	0.423
3	0.25	0	0	0.05	0.103
4	0.25	0	0	0.05	0.05

Next, we present a comparative example of δ-discounted network Myerson value ($Y^{\delta-NMV}$) with the component wise equal division rule (Y^{NCE}), the network Myerson value (Y^{NMV}), the network position value (Y^{NPV}), and the network egalitarian Myerson value ($Y^{\alpha-NEM}$).

Example 2 Consider the network in Fig. 1

For the network (N, g) in Fig. 1, the worth of each $g' \subseteq g$ is given below in the Table 1

Consequently, the total payoffs are allocated using several solution concepts, as reported in the Table 2.

We have already mentioned that under the component wise equal division rule, every player in the network receives equal pay regardless of their productivity or position in the network, which promotes free riding. In contrast, both the network Myerson value and the network position value allocate 0 payoff to the non-worth-generating players 3 and 4, reflecting strict marginalism. But in a society, it is often desirable to extend a degree of solidarity to players who do not directly generate value, while at the same time avoiding incentives for free-riding. Example 2 shows that the network egalitarian Myerson value[3] addresses this concern by allocating a non-zero payoff to players 3 and 4. Our value, the δ-discounted network Myerson value[4] further refines this idea by not only exhibiting solidarity towards null players 3 and 4 but also accounting for their structural roles within the network as well. Under our value, player 3 is given a higher payoff than player 4 due to its pivotal position in the network. Therefore, although player 3 does not contribute directly to value generation, its presence is essential for realizing the full potential of the network. In this manner, our value recognizes and rewards structural importance, thereby captur-

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[3] We have taken $\alpha = 0.8$ for the calculation of $Y^{\alpha-NEM}$; this choice of α is arbitrary

[4] We have taken $\delta = 0.8$ for the calculation of $Y^{\delta-NMV}$; this choice of δ is arbitrary

ing a sense of solidarity and appreciating players for the role they play in sustaining cooperation, even in the absence of direct value creation.

In the subsequent section, we put forward the characterizations of the δ-discounted network Myerson value.

3.1 Characterization

For the characterization of the δ-discounted network Myerson value, we first present the axioms of Efficiency, Linearity, and Anonymity in the context of network games. Then, we define the δ-Proportional Bargaining Power property, which can be treated as the generalization of the Equal Bargaining Property of the network Myerson value due to Jackson (2005).

An allocation rule $Y : \mathbb{G}^N \times \mathbb{V}^N \longrightarrow \mathbb{R}^n$ is said to satisfy:

Axiom 1 *Efficiency: if* $\sum_{i\in N} Y_i(g, v) = v(g)$ *for any* $g \in \mathbb{G}^N$ *and* $v \in \mathbb{V}^N$.

Axiom 2 *Linearity: if for* $v, w \in \mathbb{V}^N$, $g \in \mathbb{G}^N$ *and* $a, b \in \mathbb{R}$, $Y_i(g, av + bw) = aY_i(g, v) + bY_i(g, w)\ \forall i \in N$.

Axiom 3 *Anonymity: if for each permutation* π *on N,* $g \in \mathbb{G}^N$ *and* $v \in \mathbb{V}^N$ *such that* $\pi v(\pi g) = v(g)$, $Y_i(g, v) = Y_{\pi i}(\pi g, \pi v)\ \forall i \in N$.

Axiom 4 *δ-Proportional bargaining power: if for any* $g \in \mathbb{G}^N$, $v \in \mathbb{V}^N$, *and* $ij \in g$,

$$Y_i(g, v) - Y_j(g, v) = \delta\Big[Y_i(g\backslash ij, v) - Y_j(g\backslash ij, v)\Big]. \tag{6}$$

Recall from Theorem 1 that the network Myerson value is characterized by Component Balance and Equal Bargaining Power. The Equal Bargaining Power requires that the difference in payoffs to any two players in the network is due to the removal of their common link is same. On the contrary, the δ-Proportional Bargaining Power requires that the difference in the payoffs to any two players before and after removing their link is balanced by δ. However, notice that for $\delta = 1$, the δ-Proportional Bargaining Power coincides with the Equal Bargaining Power given in Jackson (2005).

Before proceeding with the characterization of our allocation rule, we first establish the following lemmas.

Lemma 1 *The δ-discounted network Myerson value* $Y^{\delta-NMV}$ *satisfies Efficiency.*

Proof This can be seen from the following set of arguments.

$$\begin{aligned}\sum_{i\in N} Y_i(g,v) =& \sum_{i\in N} \sum_{\substack{\emptyset \neq g' \subseteq g:\\ i\in N(g')}} \frac{\lambda_{g'}^{\delta}(v)}{n(g')} \times \delta^{l(g)-l(g')}\\ =& \sum_{i\in N(g)} \sum_{\substack{\emptyset \neq g' \subseteq g:\\ i\in N(g')}} \frac{\lambda_{g'}^{\delta}(v)}{n(g')} \times \delta^{l(g)-l(g')}\\ =& \sum_{\emptyset\neq g'\subseteq g} \lambda_{g'}^{\delta}(v) \times \delta^{l(g)-l(g')}\\ =& v(g).\end{aligned}$$

□

Lemma 2 *The δ-discounted network Myerson value $Y^{\delta-NMV}$ satisfies δ-Proportional Bargaining Power.*

Proof For given $\delta \in [0,1]$, we have

$$\begin{aligned}&Y_i^{\delta\text{-NMV}}(g,v) - \delta Y_i^{\delta}(g\backslash ij, v)\\ =& \sum_{\substack{g' \subseteq g:\\ i\in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')}\lambda_{g'}^{\delta}(v) - \delta \sum_{\substack{g' \subseteq g\backslash ij:\\ i\in N(g')}} \frac{\delta^{l(g\backslash ij)-l(g')}}{n(g')}\lambda_{g'}^{\delta}(v)\\ =& \sum_{\substack{g' \subseteq g:\\ i\in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')}\lambda_{g'}^{\delta}(v) - \sum_{\substack{g' \subseteq g\backslash ij:\\ i\in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')}\lambda_{g'}^{\delta}(v)\\ =& \sum_{\substack{g' \subseteq g:\\ i,j\in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')}\lambda_{g'}^{\delta}(v).\end{aligned}$$

Similarly,

$$Y_j^{\delta-NMV}(g,v) - \delta Y_j^{\delta}(g\backslash ij) = \sum_{\substack{g' \subseteq g:\\ i,j\in N(g')}} \frac{\delta^{l(g)-l(g')}}{n(g')}\lambda_{g'}^{\delta}.$$

Hence, we get, $Y_i(g,v) - \delta Y_i(g\backslash ij,v) = Y_j(g,v) - \delta Y_j(g\backslash ij,v)$. This further implies, $Y_i(g,v) - Y_j(g,v) = \delta\Big[Y_i(g\backslash ij,v) - Y_j(g\backslash ij,v)\Big]$. □

In the following, we have our first characterization theorem.

Theorem 2 *The δ-discounted network Myerson value $Y^{\delta-NMV}$ is the unique allocation rule satisfying Efficiency and δ-Proportional Bargaining Power for $\delta \in [0,1]$.*

Proof It is already shown in Lemma 1 and 2 that $Y^{\delta-NMV}$ satisfies Efficiency and δ -Proportional Bargaining Power. To establish uniqueness, suppose Y and $Y^{\delta-NMV}$ both satisfy Efficiency and δ-Proportional Bargaining Power. Let, g be a network with the least number of links such that $Y \neq Y^{\delta-NMV}$. Then for any $ij \in g$, we have

$$Y(g\backslash ij,v) = Y^{\delta-NMV}(g\backslash ij,v). \tag{7}$$

The δ-Proportional Bargaining Power yields,

$$Y_i(g,v) - Y_j(g,v) = \delta\left[Y_i(g\backslash ij,v) - Y_j(g\backslash ij,v)\right]. \tag{8}$$

Using Eq. 7 in Eq. 8 we get,

$$Y_i(g,v) - Y_j(g,v) = \delta\left[Y_i^{\delta-NMV}(g\backslash ij,v) - Y_j^{\delta-NMV}(g\backslash ij,v)\right]. \tag{9}$$

Since, $Y^{\delta-NMV}$ also satisfies δ-Proportional Bargaining Power we have

$$Y_i^{\delta-NMV}(g,v) - Y_j^{\delta-NMV}(g,v) = \delta\left[Y_i^{\delta-NMV}(g\backslash ij,v) - Y_j^{\delta-NMV}(g\backslash ij,v)\right]. \tag{10}$$

Using Eqs. 9 and 10 we obtain,

$$Y_i(g,v) - Y_i^{\delta-NMV}(g,v) = Y_j(g,v) - Y_j^{\delta-NMV}(g,v), \forall i,j \in N(g). \tag{11}$$

Now, Efficiency implies $\sum_{i=1}^{n}\Big[Y_i(g,v) - Y_i^{\delta-NMV}(g,v)\Big] = 0$. Therefore, Efficiency and Eq. 11 imply $Y_i(g,v) = Y_i^{\delta-NMV}(g,v)$ for any $i \in N(g)$. □

Observe that in the above characterization of the δ-network Myerson value, we use Efficiency instead of Component Balance. Component Balance is commonly used for most of the network allocation rules along with a variance of Bargaining Power.

Next, we propose our second characterization. Van Den Brink and Funaki (2010) introduce $\delta-$reducing players for TU games, extending the notions of null and nullifying players. Extending this notion to network games, we have the following definition.

Definition 1 For any $g \in \mathbb{G}^N$ and $\delta \in [0,1]$, the δ-reducing player in network games can be defined as a player $i \in N$ in the game (N, v) such that $v(g) = \delta^{l_i(g)} v(g\backslash g_i)$.

From Definition 1 notice that the δ-reducing player reduces the value of a network in powers of δ when she joins this network. However, the δ-reducing player in TU games introduced by Van Den Brink and Funaki (2010) differs from the one we define here in the sense that in TU games when a player leaves a coalition, it is unilateral, meaning only her contributions are lost, but, in case of a network, the contributions of all her links are also lost. In this sense, the δ-reducing player, we have defined is a stronger version of the same by Van Den Brink and Funaki (2010). Moreover, observe that, for any $S \subseteq N\backslash i$, explicitly focusing on subnetworks $g|_S$ of g, if $i \in N$ is a δ-reducing player, then $v(g|_{S\cup i}) = \delta^{l_i(g|_S)} v(g|_S)$ for any $v \in \mathbb{V}^N$.

Since the δ-reducing player reduces the value of a network, it is natural to award her zero payoff from the network situation. Thus, we have the corresponding axiom as follows.

Axiom 5 *For any $g \in \mathbb{G}^N$ and $v \in \mathbb{V}^N$, an allocation rule Y satisfies the δ-Reducing Player Property, namely $Y_i(g, v) = 0$ for every δ-reducing player $i \in N$ in the game (N, v).*

Prior to our next characterization, we provide the following lemmas.

Lemma 3 *The δ-discounted network Myerson value $Y^{\delta-NMV}$ satisfies Linearity.*

Proof This follows directly from the Linearity of the Harsanyi dividend on δ given by Eq. 5. ☐

Lemma 4 *The δ-discounted network Myerson value $Y^{\delta-NMV}$ satisfies Anonymity.*

Proof The proof of Lemma 4 is straightforward. It directly follows from the definition of Anonymity. ☐

Lemma 5 *The δ-discounted network Myerson value $Y^{\delta-NMV}$ satisfies δ-Reducing Player Property.*

Proof The proof follows directly from the definition of δ-Reducing Player Property, and therefore omitted. ☐

Next, we present our second axiomatic characterization of the δ-discounted network Myerson value, which is a network counterpart of the δ-discounted Shapley value characterization given by Theorem 3.2 (Van Den Brink and Funaki 2010).

Theorem 3 *For $\delta \in [0, 1]$, an allocation rule Y satisfies Efficiency, Linearity, δ -Reducing Player Property and Anonymity if and only if $Y = Y^{\delta-NMV}$.*

Proof This proof is exactly similar to Theorem 3.2 in Van Den Brink and Funaki (2010), following an analogous procedure within the network framework, and therefore, omitted here. ☐

4 Example of a cost allocation problem using the δ-discounted network Myerson value

Revisiting the example mentioned in the introduction, we observe that the climate crisis is at a more critical stage than ever. Thus, cutting down on carbon emissions is one of the primary focuses of the 21st century and also is a key strategy in the United Nations agenda for sustainable development[5]. A major amount of energy-related carbon emissions is due to fossil-fueled electricity generation, so addressing the power generation industry is crucial for reducing these emissions. Therefore, the transition of the power industry from fossil fuel to renewable energy by integrating renewable energy plants into the power grid is necessary. This requires an efficient and well-connected transmission network.

Moreover, renewable energy sources are not equally distributed across regions. A well-connected transmission network is essential to mitigate these regional differences in renewable energy. It is also typical to observe fluctuations in electricity demand and supply. A robust transmission network redistributes electricity from areas with surplus generation to areas experiencing higher demands, providing grid stability. In addition, for rural electrification in a vast country like India with its diverse geography and remote settlements, a well-connected transmission network is indispensable.

Even though establishing and maintaining a comprehensive transmission network is expensive, it is required to ensure grid stability, maximize renewable energy usage, and distribute electricity across all regions.

Now, considering that forming a network involves significant costs, and players differ in resources, capacities, and efficiency in establishing connections, the challenge, then, is how to fairly allocate the cost per unit of the transmission network or power grid among the players involved. While numerous parameters contribute to the cost of these systems, we focus on a simple cost-sharing model of the power grid transmission network. The core concept is to incentivize players with a greater link formation cost which results in higher overall costs for these players by offering them discounts using δ-discounted network Myerson value compared to the standard cost allocation procedure using the marginalistic framework. This approach recognizes the dual role of such players - they not only incur higher costs due to more connections but also play a vital role in maintaining the grid's stability. Disrupting connections from these players could significantly jeopardize grid stability. Conversely, players with notably higher costs due to other parameters are similarly incentivized within this framework. The level of incentive can be decided on mutual agreement among players. In allocating costs within an electricity transmission network, relying solely on extreme marginalism can unfairly burden certain players with higher costs, while extreme egalitarianism is also not advisable as it might lead to free-riding activities.

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[5] United Nations (2015) Transforming our world : The 2030 agenda for sustainable development, https://sdgs.un.org/2030agenda

Next, we define a cost function representing per unit electricity usage cost. While developing our model for electricity grid cost-sharing, we reference several key studies in the field. Although our problem statement differs significantly, we have drawn inspiration from some ideas and methods in these works. Notable references include (Her et al. 2018), Qin et al. (2020) etc.

Let g denote the transmission network. For each region, $i \in N(g)$, suppose m_i and d_i denote the maximum clean energy production capacity and electricity demand of *i* at period *t*, respectively. If $m_i - d_i > 0$, region i has unused clean energy, denoted CE_i^{Unused}; if $m_i - d_i < 0$, it indicates a deficit in clean energy at *i*, denoted $CE_i^{Deficit}$.

For any transmission network *g*, $m(g) = \sum_{i \in N(g)} m_i$ is the maximum clean energy production capacity, and $d(g) = \sum_{i \in N(g)} d_i$ is the demand at period *t*. The assumption is that each region in the network prioritizes using its clean energy sources to meet the electricity demand. If a region's clean energy supply is insufficient to meet its demand, it will use surplus clean energy from other connected regions via transmission links in the network. Fossil-fueled energy in the network is only used as a last resort when clean energy sources are insufficient to fulfill demand.

Next, clean energy produced in $g' \in C(g)$ is given by the following formula.

$$CE_g(g') = \begin{cases} m(g') \text{ if } m(g') < d(g') \\ d(g') \text{ otherwise.} \end{cases}$$

Clean energy produced in *g*:

$$CE_g^{Produced} = \sum_{g' \in C(g)} CE_g(g')$$

If $m(g) - CE_g^{Produced} > 0$, then there is unused clean energy in *g* and is denoted by CE_g^{Unused}, and if $d(g) - CE_g^{Produced} > 0$, there is a deficit, denoted by $CE_g^{Deficit}$.

Let CE_i^j denote the amount of clean energy used by region *j* that is produced in region *i* at period *t*[6]. For $ij \in g$, let c_{ij} be the cost per unit of electricity transmitted between *i* and *j* and t_{ij} be the operations and maintenance cost of the transmission line *ij*. α is the parameter for penalizing fossil energy usage, while β represents the parameter associated with clean energy wastage cost. Next, we have η and γ representing the per unit production cost of fossil fuel and clean energy respectively. Next, we formulate our cost function, $c : \mathbb{G}^N \longrightarrow \mathbb{R}$ that represents the cost of functioning of network (power grid) *g*.

At period *t*,

---

[6] Note that regions *i* and *j* need not be directly connected. In such cases, clean energy is transmitted via intermediate regions. For simplicity, no storage or any other local costs at intermediate regions are considered.

$$
\begin{aligned}
c(g) = {} & \frac{\min\left\{\sum_{\substack{i,j \in N(g)\\ i \neq j}} CE_i^j \times (c_{i_1 i_2} + c_{i_2 i_3} + \cdots + c_{i_{p-1} i_p})\right\}}{d(g)} \\
& + \frac{(\alpha + \eta) CE_g^{\text{Deficit}} + \beta CE_g^{\text{Unused}} + \gamma CE_g^{\text{Produced}} + \sum_{ij \in g} t_{ij}}{d(g)} \\
& \text{where } i_k, i_l \in g \text{ and } i_1 = i, i_p = j
\end{aligned}
\tag{12}
$$

Subject to the constraints:

$$
\begin{aligned}
& 0 \leq CE_i^j \leq \min\{CE_i^{\text{Unused}}, CE_j^{\text{Deficit}}\}, \\
& \forall i \in N(g), \sum_{j \in N(g), j \neq i} CE_i^j \leq CE_i^{\text{Unused}}, \\
& \forall j \in N(g), \sum_{i \in N(g), i \neq j} CE_i^j \leq CE_j^{\text{Deficit}}, \\
& \sum_{i,j \in N(g), i \neq j} CE_i^j = \min\{\sum_{i \in N(g)} CE_i^{\text{Unused}}, \sum_{i \in N(g)} CE_i^{\text{Deficit}}\}
\end{aligned}
\tag{13}
$$

The path with the minimum cost is chosen to transmit electricity from *i* to *j*. $\{i_1 i_2, \ldots, i_{p-1} i_p\}$, where $i_1 = i$ and $i_p = j$ represents a path from *i* to *j*.

To formally build the model, we take the following network.

Example 3 Consider a transmission network as in Fig. 2 and Table 3.

For the subnetwork $g = \{13, 26\}$, using the above formulae, we have, $m(\{13, 26\}) = 43$ units and $CE_{\{13,26\}}^{Produced} = 35$ units. Subnetwork $\{13\}$ has an excess capacity of 8 units of clean energy, while subnetwork $\{26\}$ has a deficit of 9 units. However, as they are disconnected, energy transmission from $\{13\}$ to $\{26\}$ is not possible. Thus,

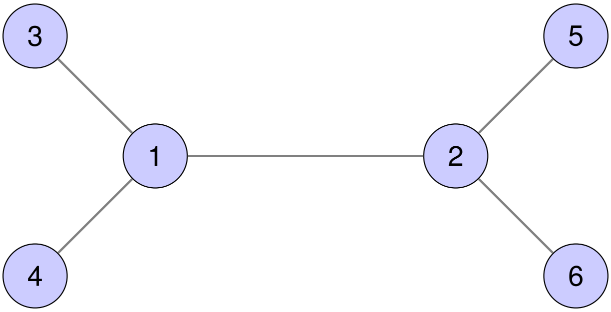

Fig. 2 Network (*N*, *g*).

Table 3 Clean energy unused/deficit for players

Player	m_i	d_i	$CE_i^{Unused}(+)/CE_i^{Deficit}(-)$
1	21	10	+11
2	11	7	+4
3	3	6	− 3
4	5	9	− 4
5	7	4	+3
6	8	21	− 13

Table 4 Cost per unit of electricity transmission for transmission lines

Transmission line (ij)	Cost per unit of electricity transmission(c_{ij})
12	1 units
13	2 units
14	1 units
25	1 units
26	2 units

Table 5 Transmission line operations/maintenance costs

Transmission line (ij)	Operations/maintenance cost (t_{ij})
12	20 units
13	10 units
14	15 units
25	10 units
26	10 units

$$CE_{\{13,26\}}^{Unused} = m(\{13,26\}) - CE_g^{Produced}(\{13,26\}) = 8$$

and

$$CE_{\{13,26\}}^{Deficit} = d(\{13,26\}) - CE_g^{Produced}(\{13,26\}) = 9$$

and

$$CE_{\{13,26\}}^{Produced} = CE_{13}^{Produced} + CE_{26}^{Produced} = 35.$$

Next, we compare the cost allocation using the network Myerson and the δ-discounted network Myerson values for the cost function specified in Eqs. 12 and 13. The following example demonstrates their differences.

Example 4 Let (N, g) be as in Example 3 and define $c \in \mathbb{V}^N$ as in Eqs. 12 and 13. We present in Table 3, the maximum clean energy production capacity (m_i), the energy demand (d_i), and the amount of clean energy that is unused or deficit for the players, Table 4 presents the per unit cost of electricity transmissions in each transmission line and Table 5 presents the maintenance and operations cost of each transmission line respectively in a given time period t. For computing the cost of operating the trans-

Table 6 δ-discounted network Myerson value for different δ Values

Player	$Y^{\delta-NMV}(\delta=1)$	$Y^{\delta-NMV}$ $(\delta=0.8)$	$Y^{\delta-NMV}$ $(\delta=0.5)$	$Y^{\delta-NMV}$ $(\delta=0.2)$
1	3.393	3.060	2.529	1.995
2	3.797	3.247	2.567	1.997
3	0.860	1.039	1.300	1.546
4	0.710	0.961	1.274	1.539
5	0.895	1.041	1.280	1.531
6	0.450	0.757	1.155	1.497

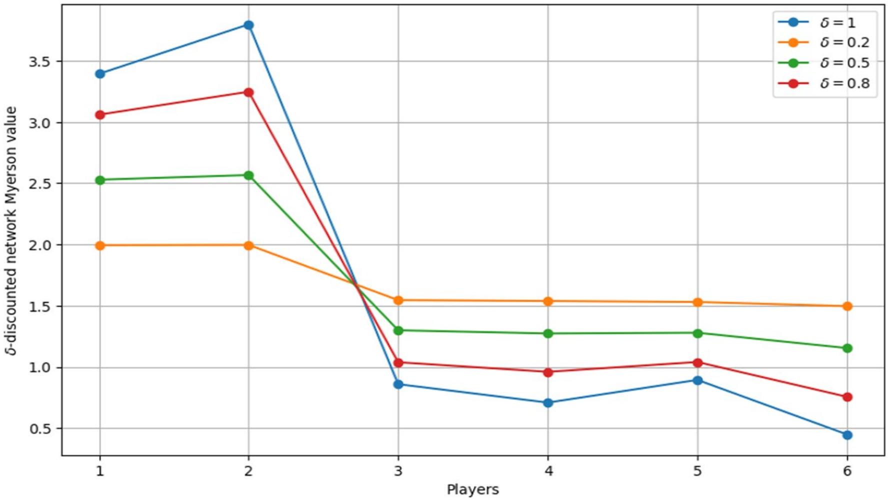

Fig. 3 The δ-discounted network Myerson values in Table 6 for different δ Values

mission network g, we set $\alpha=6$, $\beta=4$, $\eta=10$, and $\gamma=8$. The numerical values reported in Tables 3, 4, and 5, as well as all parameter values used in this example, are chosen arbitrarily and are considered solely for illustrative and computational purposes.

When the per-unit electricity usage costs are allocated using the network Myerson value, we observe a disproportionate burden placed on players 1 and 2. Despite their critical role in maintaining the grid and providing clean energy to the network, these players incur significantly higher costs than others.

To address these concerns, we apply the δ-discounted network Myerson value with $\delta\in[0,1]$, which gives a more equitable distribution of costs by recognizing the importance of players 1 and 2. By discounting the costs these key players bear, we ensure that the peripheral players share some of their burden in the network.

Next, we present the per unit electricity usage cost allocation using the δ-discounted network Myerson value ($Y^{\delta-NMV}$). When $\delta=1$, it coincides with the network Myerson value. The cases $\delta=0.8, 0.5,$ and 0.2 are choosen arbitrarily to illustrate how different levels of solidarity can be incorporated. In general, the planner may select any $\delta\in[0,1]$ to determine the desired degree of solidarity (Fig. 3).

For better visualization, we provide a plot below.

By adjusting the discount factor δ, peripheral players, who rely on the grid and clean energy provided by central players, take on a fairer share of the costs. This approach reduces the burden on central players while promoting a more balanced cost distribution across the network.

5 Conclusion

In this paper, we proposed the δ-discounted network Myerson value for network games as an allocation rule, suitable to situations, where players in the network need incentives to maintain their links even though they are less productive. We apply our model to an electric grid network based on the sustainable utilization of green energy. We have shown that in such a grid network, the players in the key positions need to be discounted while allocating the share of the costs incurred by the whole network so as to maintain the network. To maintain green energy transmission, the maintenance of the network is essential. Alternate allocation rules and the characterization for incentivizing players in similar situations is an interesting open problem. Moreover, we have already mentioned that Her et al. (2018) used real time data from the ASEAN Powergrid initiatives and employ the Myerson value to ensure that the partnering countries benefit and have no incentive to leave the integrated power grid system. However, in Example 1, we illustrate why and how it is important to study the stability of the network in light of the discounted Myerson value instead of the Myerson value alone. Our current model is theoretical and the δ values in the given example are chosen arbitrarily for illustrative purposes only. They are intended to demonstrate how different levels of solidarity can be modelled within the proposed framework. In a follow-up paper, we plan to reframe the model of Her et al. (2018) and apply our framework to study how stability measures can be affected by shifting from Myerson to the discounted Myerson value.

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