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Savings and Implicit Debt in Pension Systems

SUMMARY: In recent years, there has been an ongoing debate on the restructuring of pay-as-you-go pension systems. The study, which models demographic effects, compares two types of pay-as-you-go pension systems (those with and without notional individual accounts) on the basis of two features (savings and implicit debt in the pension systems). There is a strong link between these features in a theoretical framework. According to one of the results featured in the study, in case of equal revenues and expenditures, a smaller population growth rate and a higher life expectancy together yield a smaller savings ratio in both pension systems, while the effect on implicit debt is unclear. Another result shows that a lower capital market yield may also be accompanied by greater implicit debt and higher savings ratio in both pension systems. The results also highlight the complexity of the effects of demographic changes on pension systems.

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In recent years, the transformation of pension systems has been the focus of professional debates in several countries. One the reasons for this is that certain demographic trends, such as population decline or the rise of life expectancy, could easily cause pension funding problems as in the pay-as-you-go pension systems widely used in the majority of developed countries, the coverage for pension expenditures is basically the amount of contribution payments. Solutions recommended for this problem include the application of the funded pension (pay-as-you-earn) principle in the pension system or for instance – along countless other possibilities – the introduction of notional individual accounts within the pension system while retaining the pay-as-you-go principle. The practical establishment of the 'optimal' pension system is made all the more difficult by the fact that even in theory, the 'optimum' may vary in the various models.

This is supported by the fact that a few decades ago, when comparing pay-as-you-go and funded pension systems, the literature primarily focused on the 'rate of return' achievable in the pension system [for instance, the literature based on the results of Samuelson (1958) and Aaron (1966)]. If beyond the 'rate of return', we also take riskiness into account, depending on model parameters, in theory, the optimal pension system could also be a combination of the funded and pay-as-you-go pension systems [for instance by employing the financial portfolio theory; the topic is dealt with, among others, by Dutta et al. (2000) and Szüle (2011)]. The theoretical results related to the optimal pension system may also change depending on whether a given model takes the criteria of one or more generations into account or the effects of implicit debt related to pension obligations and that of government bond markets.

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In summing up the theoretical results, therefore, we can conclude that in general, theoretical models do not provide clear answers regarding the optimal structure of pension systems. All in all, theoretical results as well as practical experiences indicate that this optimum could be identified by taking a variety of aspects into account. The study features a variety of aspects when comparing pension systems: the rate of substitution (the ratio of average pension and average wage), the rate of implicit pension debt and risk, as well as the ratio which shows what part of individuals' contribution payments can be considered 'savings' in the pension system. The study simply presents these aspects, and compares them using a theoretical model for various pension systems: achieving 'total social-level utility' would only be possible by weighting the significance of these and perhaps some other factors. This method of weighting, however, is not just an economic matter; therefore, the study will not deal with how this weighting is realised.

The structure and results of the model featured in the study also point out that within pay-as-you-go pension systems, there is a strong link between implicit pension debt and the savings of individuals within the pension system. This link is based on the fact that pension liabilities to be paid are determined on the basis of contribution payments made by individuals into the pension system (and by taking certain other factors into account) and according to one possible definition, the value of pension liabilities will in theory be equal to implicit pension debt at one point. If, for example, in the case of a given generation we define the values (expected present value) of contribution payments and pension benefits, the ratio of these two values may also be considered a 'savings rate' within the pension system [similarly to Sinn's (2000) analysis, but in a different scheme]. In the interest of contributing to earlier literature, with respect to implicit debt, our model deals with the topics of valuation as well as riskiness, applying the indicator of duration as an indicator of risk.

In the study, our analysis focuses on the correlations of pay-as-you-go pension systems and certain demographic changes: one of the objectives of the analysis is to highlight the differences arising from the existence of notional individual accounts with respect to these indicators. The impact of the population growth rate on the equilibrium of the pension system has been extensively analysed by earlier literature, however, the modelling of life expectancy has had relatively lower coverage until now. Besides these two demographic indicators, capital market yield also plays a priority role in the analyses of our study. One of the reasons for this is that the riskiness of pension liabilities and the payment of government bonds may be considered similar in the model and, furthermore, from another aspect, in theory, the returns on payments made into the pension system – as a type of savings – may also be comparable with capital market yield.

In our study, we have reviewed a number of results (that were calculated on the basis of the theoretical model). Some of the results reflect some findings already published in relevant literature, such as the result regarding the link between the rate of population growth and implicit debt (according to which, the greater the rate of population growth, the greater implicit debt is). Some of the other results, however, concern topics previously analysed to a lesser extent in literature. Interestingly for example, in the theoretical model, the values measuring the riskiness of implicit pension debt in the two types of pension systems are equal even if revenues and expenditures are different in the given year.

The study firstly deals with a few features of the notional individual account pension scheme. Chapter 2 presents the structure of the theoretical model, followed by the comparison of certain indicators of pension systems with and without notional individual accounts in cases where the revenues and expenditures of the given pension system are different. Assuming from here on that revenues and expenditures are equal, we will be dealing with the general characteristics of pay-as-you-go pension systems by analysing the effects of population growth rate, life expectancy and capital market yield. The study closes with a summary of the main findings and observations.

NOTIONAL INDIVIDUAL ACCOUNTS IN PAY-AS-YOU-GO PENSION SYSTEMS

The literature on pension systems typically mentions a variety of grouping types for pension schemes. Among the most widely used is the distinction between the defined contribution and defined benefit pension system, as well as the funded and pay-as-you-go pension scheme [for more detailed definitions of the classification of pension systems, see OECD (2005) for instance]. It is worth noting that the overlap between the two grouping categories is not complete: for instance, in the definition of *Palmer* (2006, p. 18), the notional individual account pension scheme is a pay-as-you-go scheme defined by contribution payments:

„An NDC scheme is a defined contribution, pay-as-you-go (PAYG) pension scheme."

In relevant literature, notional individual account pension systems are often referred to as NDC *(Non-financial Defined Contribution* or *Notional Defined Contribution)* pension models [Palmer (2006) and Holzmann (2012) also use both terms]. As the abbreviation shows, the notional individual account pension scheme is similar to the defined contribution scheme in its 'accumulation' phase (at the time of pension contribution payments) and the time of retiring when calculating pension benefits, however, funding does not have an unfunded approach here.

The papers related to the theory of notional individual account pension schemes are decades old, and the practical application of this particular pension scheme started in Europe in the 1990s (Palmer, 2006). The very first statute related to the notional individual account pension system was created in 1994 in Sweden (practical application followed, also in Sweden, a few years later). The NDC pension scheme was introduced in Europe in 1995 in Italy, and 1996 in Latvia, and introduction began in Poland in 1999 (Chłoń-Domińczak et al., 2012). Apart from these, in practice there are certain pension systems, the features of which show similarities with the NDC pension scheme. The pension point scheme functioning as part of the pension scheme in Germany and France (where the value of pensions depends on both the number of points calculable for a given individual and the 'value' of points) may be considered similar to NDC pension systems in many respects. (Legros, 2006) Theoretical similarities aside, there are a number of differences among practical NDC pension systems, some of which are shown in *Table 1*.

It is worth noting that within NDC pension schemes, it is possible to consider periods spent off work due to reasons significant from a social aspect (for instance childbirth) as benefit payment periods in the case of notional accounts (in an NDC pension scheme for example, these 'additional credits' may be funded from state revenues). In Latvia, Poland and Sweden, the option of booking additional credits to notional accounts is available when on maternity and

Table 1

COMPARISON OF THE FEATURES OF VARIOUS NDC PENSION SCHEMES

Various pension scheme features	Italy	Latvia	Poland	Sweden
Notional rate of return	GDP growth, Rolling three year average	Covered wage bill growth	Covered wage bill growth	Per capita wage growth
Assumed return value at calculation of benefits	1.5% real wage growth	none	none	1.6% real wage growth
Indexation of benefits (above inflation)	none	none	Characterised by varying methods in recent years	There may be adjustments besides the taking into account of inflation

Source: Chłoń – Domińczak et al., 2012

parental leave, while in Italy, contributions are accounted for the time spent on maternity leave, and the pensions of mothers may be preferentially calculated (Chłoń – Domińczak et al., 2012)

SCHEME STRUCTURE

The theoretical assumptions presented in the study make up a simplified model of practical situations. The assumptions regarding the survival of individuals are emphatic in the theoretical model as this particular topic has a clear link to the modelling of the development of life expectancy. In the model presented in relation to this, assumptions are formulated based on the actuarial (insurance mathematics) approach, yet simplified at the same time. Furthermore, in the interest of taking another significant demographic factor into account, the model also contains the data of several generations at the same time. According to the assumptions, the key characteristics of individuals belonging to the given generation are identical. In the model, the number of persons in a generation increases by n per cent every year (the value of n can also be negative, therefore, population decrease can also be modelled).

The individuals in the model all start work at the age of x. In case of survival, the individuals work for a maximum of m years and are entitled to pension for a maximum of m years. In this model, the essence of results would not be changed by integrating the various maximum terms into the model. This maximum term is typical for individuals in case of survival. The related parameter in the model is the probability of surviving until the next age, which is indicated by d in the case of each age. In order to facilitate clarity, in the model we are assuming that actual mortality progresses in line with survival probabilities.

According to the assumptions, the pension contribution to be paid can be calculated from income using the fact that the contribution is a constant (k) part of individuals' income. In order to calculate the total value of contributions paid by generations, we must assume that the ratio of the employed compared to the population is constant for all generations, therefore, the population's growth rate is equal to the growth rate of the number of the employed. In order to calculate simpler results, it is another assumption in the model that in the case of individuals, we only deal with calculations related to own-right pensions and do not model surviving dependents' and other possible pension types.

In this case, the model also disregards the characteristics of the transition between pension systems as well as those of the transitional period immediately following the introduction of a given pension system. The reason for this is that the study only aims to compare the main characteristics of the two pension schemes. The restructuring of pension systems raises several questions and the precise modelling of these systems, due to sheer analysis volume, deserves a separate study.

At the start of the operation of the pension system, the number of persons in the youngest working generation is indicated by N_0, and the value of the annual income generated by a single member of this generation by B_0 (instead of the term "income", the term "wage" may also be used in simpler model assumptions). A simplification, which does not impact the essence of results, is that the payment of annual incomes and pension benefits is due at the beginning of the year.

Within the model, we assume that in the case of the two pension systems, neither requires 'support' from the budget to pay pensions. In the 'traditional', i.e. the pay-as-you-go pension system without notional individual accounts, this assumption means that per capita pension payments in a given year can be calculated by dividing the total value of all contribution payments by the number of individuals currently receiving pension. Since, according to the assumptions, the characteristics of each individual (relevant in terms of pension calculations) are the same within a generation, therefore, this assumption, overall, can even be in line with the theoretical model of a pension system in which the pensions would be calculated on the basis of the contributions paid earlier, in fact, as a function of those payments. In the present model, however, the most important characteristic of this pension scheme with respect to the analysis is that it has no notional individual accounts (and requires no actuarial calculations), while the pay-as-you-go approach is enforced in pension funding.

Within the model, the other pension scheme applying the pay-as-you-go approach has (notional) individual accounts as well. Similarly to the previous chapter, according to the assumptions, we are defining individuals' pension in the notional individual account pension scheme as something that can be calculated using actuarial (insurance mathematics) methods from the capital 'notionally' accumulated by individuals. This means that in this scheme, contributions paid by individuals prior to retirement age are recorded on a 'notional' individual account, and this recorded amount can then be used, taking the given size return into account, how much the value of pension benefit is. According to model assumptions, the value of pension benefits for an individual in the notional individual account pension system increases per year at a rate equal to the income growth rate. [This definition of the notional individual account pension system in the model cannot be considered as conflicting with the definitions of Whitehouse (2010).]

Return value is featured in the case of two calculations in the notional individual account pension scheme, even in the simple model framework: firstly when calculating the value of 'accumulation' arising from contribution payments, and secondly when calculating benefits (using an actuarial term, we may also call this value technical interest). We assume (in the interest of generating clearer results) that the value of the two types of returns is the same in the model. The value of the thus defined 'notional rate of return' is indicated by z. (At the same time, it is also worth mentioning that – based on the contents of the previous chapter – practice may show that the notional rate of return applied for contribution payments and the value assumed for benefit calculation are not equal.)

Within the model separate calculations may be performed for various generations when evaluating the total contributions paid by the generation in question. One of the advantages of simple model assumptions, however, is that the various values typical of the generations are linked in certain ways and thus the differences between the two pension systems are easier to highlight. For this reason, when deducting the results in the coming chapter, we will only be dealing with a single generation and a given time as well as accurately presenting the calculations involved.

THEORETICAL RESULTS

The two very frequently surfacing topics when analysing pension systems are the ratio of the incomes of pensioners and non-pensioners and the development of the financial balance of the given pension system. The study first deals with these two topics, followed by the breaking down into components (according to various considerations) of implicit debt and contribution payments calculated in pension systems, and the analysis of financial 'riskiness'. The results will be deducted in the theoretical model presented in the previous chapter.

Balances and the rate of substitution

In the 'traditional', pay-as-you-go pension system without notional individual accounts, we shall examine the year $t=2\times m-1$, when all pensioners receive pensions of equal value, which can be calculated using formula (1):

$$\frac{B_0\times k\times(1+b)^{2\times m-1}\times(1+n)^m}{d^m} \qquad (1)$$

Since the income of non-pensioners is not different at various ages in the theoretical model, the quotient of 'average' pension and 'average' wage within the pay-as-you-go pension system without notional individual accounts is the same in the various periods. The value of this constant 'rate of substitution' is given by formula (2):

$$k\times\left[\frac{1+n}{d}\right]^m \qquad (2)$$

In the case of pay-as-you-go pension systems that operate with notional accounts, in the interest of clarity we will be introducing the indication shown in formula (3):

$$C=\frac{\left[\frac{1+b}{1+z}\right]^m-1}{\left[\frac{1+b}{1+z}\right]-1}\times\frac{\left[\frac{(1+b)\times d}{1+z}\right]-1}{\left[\frac{(1+b)\times d}{1+z}\right]^m-1} \qquad (3)$$

In the notional individual account pension scheme, the value featured in formula (3) can be found in several results. In year $t=2\times m-1$, the pension of one member of the eldest pensioner generation is $B_0\times k\times C\times(1+z)^m\times(1+b)^{m-1}$. In the case of this pension scheme, it is clear that the quotient of average pension and average wage in the various years is constant, which quotient can be calculated using formula (4):

$$k \times C \times \left[\frac{1+z}{1+b}\right]^m \qquad (4)$$

Based on formulae (2) and (4), we can relatively simply conclude that according to the assumptions of the theoretical model, the rate of substitution is greater in the notional individual account pension scheme, if:

$$(1+z)^m > \frac{\left[(1+n)\times(1+b)\right]^m}{d^m \times C} \qquad (5)$$

This result refers to an easy-to-interpret correlation, according to which the rate of substitution in the notional individual account pension system is more favourable for pensioners when applying a 'notional rate of return' that is higher than a certain value. In this particular pension scheme, however, the topic of the sign of balances (the difference of revenues and expenditures) in the given year also arises. The reason for this is that in the case of this pension scheme, the balance is not automatically zero; its value also depends for instance on notional rate of return. In the notional individual account pension scheme, based on theoretical model assumptions, expenditures are not greater than revenues in a given year, if:

$$(1+z)^m \le \frac{\left[(1+n)\times(1+b)\right]^m}{d^m \times C} \qquad (6)$$

By comparing correlations (5) and (6), therefore, we can conclude that within the simple theoretical model of the notional individual account pension system, it is not possible to simultaneously maintain a rate of substitution greater than the 'traditional' pay-as-you-go pension scheme and expenditures that do not exceed the revenues of the pension system. This is a relatively simple result, at the same time, it merits a brief mention (in other words, that in the notional individual account pension scheme the rate of substitution is lower than in the other pension scheme if contribution revenues exceed pension expenditures).

Implicit debt

In pay-as-you-go pension systems, the payment of future pensions is usually based on a certain rule (for example, a statute stipulates how to calculate starting pensions or the growth rate of already determined pensions). As a result, pensions to be paid in future years represent a sort of 'debt', which at the same time has not been generated through the issue of government bonds, therefore, its value cannot be explicitly calculated through the pricing of state-issued securities. Pension liabilities allow us to calculate 'implicit' debt. A number of studies have already dealt with this topic (such as van den Noord – Herd, 1993; Holzmann et al., 2004; Eichhorst et al., 2011), and one of the findings of these analyses is that the difficulty of calculating comparable indicators regarding implicit debt value in practical calculations is partly due to the complexity of pension regulations in the case of certain countries. *Eichhorst et al.* (2011) mentions three methods for calculating pension liability (of which implicit pension debt is an index-number):

- in the case of accrued-to-date liabilities, the calculations ignore future contribution payments and pension rights acquired after the cut-off date;
- when calculating *'projected' liabilities'* only current workers and pensioners are taken into account during calculation (i.e. no new entrants to the pension system are taken into account after a specified cut-off date);
- in the case of 'open-system' liabilities, entitlements of new workers acquired by paying contributions in the future are also taken into account.

The results of practical calculations may also be significantly impacted by the value of the rate applied for discounting [the fact that Holzmann et al. (2004) also present calculation results by taking more than one rate applied for discounting is also related to this]. *Holzmann et al.* (2004) distinguishes gross and net implicit pension debt: gross implicit debt basically refers to the present value of future pensions, while net implicit debt refers to the balance of gross implicit debt and related assets (for example, the value of 'assets' may be related to the present value of future contribution payments). The size of gross implicit debt may be significant in practice: Eichhorst et al. (2011) mention estimates where this value could even be between 160 and 300 per cent of the GDP for certain countries. On a related note, the topic of implicit debt due to pension liabilities may also be interesting from an economic policy aspect [the analysis by Banyár (2011) for instance is also related to this particular topic].

With respect to implicit debt, the present study solely aims to answer the question of which of the two pay-as-you-go pension schemes (with and without individual accounts) has greater value and risk of implicit debt due to pension liabilities. Similarly to relevant literature, the study also calculates the value of implicit pension debt as the present value concerning a given date of pension payments due at future dates. Of the gross and net implicit pension debts distinguished by Holzmann et al. (2004), the calculations of our study are similar to the calculation of gross implicit pension debt, as the model only deals with liabilities arising from pension payments during the calculation of implicit pension debt. Of the methodology groups described by Eichhorst et al. (2011) related to the calculation of pension liabilities, the method employed by the study is similar to the calculation of 'open-system' liabilities, during which the present value of a cash-flow series of infinite length is calculated during the calculation of implicit pension debt in relation to the construction of the theoretical model.

In order to calculate present value (and for the discounting of values due at future dates), an appropriate rate of return is required. The determination of this rate of return raises a number of questions in practice, however, for the purposes of this study we are assuming that the return of government papers traded on the capital market is an appropriate value in the calculations. This assumption can be explained with the fact that in theory, the rate applied for discounting should also be in line with the riskiness of discounted cash-flows, and according to the theoretical model featured in the study, in the case of pension payments it can be assumed that the risk of payment is the same as in the case of government securities that are part of explicit public debt. In this model, we are assuming equal government securities market rates of return for all periods, which is in line with the assumption of the horizontal yield curve. This level of capital market yield is indicated by r in further calculations.

Regarding the value of the (capital market) yield applied for discounting in the model, we can assume that $r>(1+b)\times(1+n)$. This assumption may be considered appropriate in the case of a dynamically effective economy.

Since, based on model assumptions, the annual growth rate of pension liabilities for both pension schemes is $(1+b)\times(1+n)-1$, the calculation of implicit debt in essence is the same as the valuation of growing perpetuity. In the 'traditional', i.e. the pay-as-you-go pension scheme without notional individual accounts, the value of implicit debt for example at the time $t=2\times m-1$ is:

$$\frac{(1+r)\times N_0\times B_0\times k\times d^{2\times m-1}\times\left[\left[\frac{1+n}{d}\right]^m-1\right]\times(1+b)^{2\times m-1}\times(1+n)^m}{\left[\left[\frac{1+n}{d}\right]-1\right]\times\left[(1+r)-(1+n)\times(1+b)\right]} \quad (7)$$

In the notional individual account pension scheme, the value of implicit debt can also be calculated at the $t=2\times m-1$ time. In this pension scheme, this value also depends on the value of the notional rate of return:

$$\frac{(1+r)\times N_0\times B_0\times k\times d^{2\times m-1}\times\left[\left[\frac{1+n}{d}\right]^m-1\right]\times C\times(1+z)^m\times(1+b)^{m-1}}{\left[\left[\frac{1+n}{d}\right]-1\right]\times\left[(1+r)-(1+n)\times(1+b)\right]} \quad (8)$$

In order to answer the question of which pension scheme has the greatest implicit debt, we are comparing the values featured in formulae (7) and (8). During this comparison, we can determine the ratio of implicit debts in the two pension schemes (based on model assumptions, this ratio is of identical value at given times). If the quotient of values featured in formulae (7) and (8) is lower than one, then the notional individual account pension scheme has lower implicit debt. This situation is typical when:

$$(1+z)^m<\frac{\left[(1+b)\times(1+n)\right]^m}{d^m\times C} \quad (9)$$

Comparing this result with correlation (6), we can conclude that implicit debt is lower in the notional individual account pension scheme than the 'traditional' pay-as-you-go pension scheme if the given annual revenues in the notional individual account pension scheme are greater than the expenditures.

Duration

The results presented above are relatively simple and are clearly in line with the results of relevant literature to date. The next section deals with modelling the risk of implicit pension debt as this topic has been less extensively covered by literature in the past. Based on the literature related to the theory of financial investment, the assessment of the riskiness of bonds in particular, in the case of certain securities, the value related to duration may be considered a possible indicator of riskiness (Bodie et al., 2005). On the bond market, duration measures how sensitive the price of a given bond is to yield level change and this indicator, with a few exceptions, is not the same as the due date of the last cash-flow belonging to the given liability. Based on this, the risk indicator could even be interpreted in the case of pension liabilities related to implicit debt – which liabilities in theory have infinite 'duration'. (Of course, infinite duration can only be assumed if we ignore the issue of the 'termination' of the pension system at a certain date.)

In the following section, we will be comparing duration values calculated for the two types of pension systems in the theoretical model, which values can also be considered indicators of risk. During the calculations, (similarly to the calculation of implicit debt) we are assuming that the first pension payment (indicated by the value P) is due immediately at the given date. At this point, the value of implicit debt depending on the rate of return applied for discounting (taking the formula for growing perpetuity into account):

$$A(r)=\frac{P\times(1+r)}{(1+r)-(1+n)\times(1+b)} \quad (10)$$

The value outlined in formula (10) is similar to the theoretical price of a bond in the sense that it represents the present value of cash-flows by assuming a given yield level. Applying the formula of duration (also called average time in financial literature) to implicit debt arising out of pension liabilities, we are in essence, therefore, defining risk in such a way that it indicates that the value of implicit debt could change when the value of market return changes. Let us take a look at the next indicator of yield level sensitivity:

$$\frac{\frac{\partial A(r)}{\partial r}}{A(r)} = -\frac{1}{1+r} \times \frac{(1+n)\times(1+b)}{(1+r)-(1+n)\times(1+b)} \qquad (11)$$

The negative sign in formula (11) indicates that the drop in yield level increases the value of implicit debt. Interestingly, the value of duration (expressed in years) is the same in both pension systems in the theoretical model:

$$\frac{(1+n)\times(1+b)}{(1+r)-(1+n)\times(1+b)} \qquad (12)$$

This result, therefore, indicates that the riskiness of the above defined implicit debt of the two pension systems is not different in the theoretical model (this is also related to the fact that the value of the notional rate of return is not featured in the formula of the duration). At the same time, this result does not exclude the possibility that upon the change of market yield level, the change (calculated at absolute value) in the value of implicit debt may vary depending on the market yield level value. On the whole, this result may also be interesting because it indicates – in contrast with earlier results – that a given indicator of risk is not different in the two types of pension systems even if the value of revenues and expenditures is not equal within the notional individual account model.

Savings rate

A few decades ago, pay-as-you-go pension systems were typically introduced in a manner where in the case of members of the first few generations to receive pensions, the value of earlier contribution payments at the time of retirement were significantly below the value of expected pension payments. This difference is termed as 'introductory gains' by Sinn (2000). In the interest of off-setting these *'introductory gains'*, part of the contribution payments of subsequent generations can be viewed as a sort of 'tax'. *Sinn's* (2000) relatively simple theoretical model, while taking two lifetimes and generations living together into account, also deals with how the 'tax' and 'savings' parts of contribution payments are transformed as a result of demographic changes. In the present study, we are analysing this question by assuming more than a two-period possible lifetime. The analyses allow for the study of the effects of population growth rate and capital market yields [there is similarity with Sinn's (2000) model in this respect], at the same time though, the analysis of the effect of life expectancy is a topic ignored by Sinn's (2000) model.

In the theoretical model, we can calculate the present and future values of contribution and pension payments at given times for each generation. Let us examine the generation, the members of which begin working at the launch of the pension system. At retirement, in other words at the time $t=m$, the total value of the contribution payments of the generation is (calculating future value with yield r):

$$B_0 \times N_0 \times k \times \frac{\left[\frac{(1+b)\times d}{1+r}\right]^m - 1}{\left[\frac{(1+b)\times d}{1+r}\right] - 1} \times (1+r)^m \qquad (13)$$

The value in formula (13) is the same in the theoretical models of both pension systems. In the 'traditional', i.e. the pay-as-you-go pension scheme without notional individual accounts, the present value of total pensions paid to members of the generation at the time $t=m$:

$$B_0 \times N_0 \times k \times (1+n)^m \times (1+b)^m \times \frac{\left[\frac{(1+b)\times d}{1+r}\right]^m - 1}{\left[\frac{(1+b)\times d}{1+r}\right] - 1} \qquad (14)$$

By dividing the value in formula (14) by the value in formula (13), we get a value that can be interpreted as a 'savings part'. Similarly to the findings of Sinn (2000) (although in a significantly different theoretical model framework), this value can be interpreted in terms of the whole of the generation that if this part of the contributions paid were invested on the capital market, 'bonds' could have been bought at market yield, the payments on which would correspond to the pension payments made to the whole of the generation. In the 'traditional' pension scheme, therefore, the value of the savings rate thus interpreted:

$$\left[\frac{(1+n)\times(1+b)}{(1+r)}\right]^m \qquad (15)$$

In the notional individual account pension scheme, the present value of total pension payments in case of the same generation at the $t=m$ time:

$$B_0 \times N_0 \times k \times d^m \frac{\left[\frac{(1+b)\times d}{1+r}\right]^m - 1}{\left[\frac{(1+b)\times d}{1+r}\right] - 1} \times (1+z)^m \qquad (16)$$

The savings rate within notional individual account pension schemes can be calculated by dividing the values in formulae (16) and (13) (the value of this savings rate compared to the other pension scheme is not different at the different times, according to the assumptions of the model):

$$\frac{(1+z)^m \times d^m \times C}{(1+r)^m} \qquad (17)$$

By comparing values (15) and (17), it can be established that the savings rate of the notional individual account pension scheme is smaller than that of the other pension scheme, if:

$$(1+z)^m \times d^m \times C < \left[(1+n)\times(1+b)\right]^m \qquad (18)$$

Comparing correlation (18) with correlation (6), we can conclude that based on assumptions, the savings rate is lower in the notional individual account pension scheme than the 'traditional' pay-as-you-go pension scheme if the given annual revenues in the notional individual account pension scheme are greater than the expenditures. This result is easy to interpret as the result attained in the case of the rate of substitution and implicit debt: if the revenues are greater in the notional individual account pension scheme than the expenditure, then (similarly to the rate of substitution and implicit debt) the savings rate will be lower than in pension scheme without notional individual accounts.

INTERPRETATION OF RESULTS

An interesting correlation can be deducted in the model in relation to the value of the notional rate of return. Based on correlation (6), in a notional individual account pension scheme, revenues in the given year are greater than expenditures if $d^m \times C \times (1+z)^m < \left[(1+n)\times(1+b)\right]^m$. In the preceding sections, we also took into consideration that the assumptions can be made in a dynamically efficient $r > (1+b)\times(1+n)$ economy. One of the interesting results of the model is that $d^m \times C < 1$, if $d<1$ then according to the assumptions of the model, theoretically, it may be possible for the notional yield to be the same in a pension scheme with notional individual accounts as it could be expected from government securities, by ensuring also that the revenues of the pension scheme are not lower than the expenditures (meaning that no government 'support' is needed to make the pension payments).

By analysing the demographic and capital market effects, additional interesting results may be obtained. In the interest of greater transparency, the next part is about how two demographic parameters and the value of capital market yields (government securities) influence the most important results, if revenues and expenditures are presumed to be equal in the case of both pension schemes (in this case, the rate of substitution, implicit debt and risk as well as the savings rate are the same in both types of pension schemes).

The effect of life expectancy

In terms of the insurance calculations, life expectancy and probabilities of survival values are closely related. The value of life expectancy in the model is represented by d, as life expectancy is a function of d in the model. This finding will be demonstrated using indications used for insurance calculations: for the purposes of the model, life expectancy is the life expectancy that is calculated at age x [considering that d is a constant in the model with an exception of the value corresponding to maximum age, which is also supported by the $d^{(\omega)}$ indication in formula (19)]:

$$(1-d)\times\left[\sum_{t=0}^{\omega-x-1}(x+t)\times d^{t}\right]+\omega\times d^{(\omega)} \qquad (19)$$

There is, therefore, a clear link between life expectancy and value d: a greater survival probability leads to higher age x life expectancy. Taking this into account, we can conclude that within the theoretical model, with higher life expectancy (assuming that the effects of all other factors remain unchanged):

- the rate of substitution drops,
- the value of implicit debt increases,
- the duration of implicit debt and the value of the savings rate are not impacted by the development of life expectancy.

Of these results, we must highlight that in the model, the savings rate is not linked to life expectancy (therefore, the savings rate may be the same in the model with lower or higher life expectancy), at the same time, the rate of substitution is lower in the case of higher life expectancy.

The effect of the population growth rate

The results related to the population growth rate are relatively easy to review in the model. In the case of a lower population growth rate, theoretical results show that the rate of substitution, i.e. the quotient of average pension and average wage, is lower as is implicit debt. The first derivative of the duration of implicit debt, according to the population growth rate:

$$\frac{(1+b)\times(1+r)}{\left[(1+r)-(1+b)\times(1+n)\right]^{2}} \qquad (20)$$

With respect to the value in formula (20), we can assume that it is greater than zero, as neither capital market (government securities market) yield, nor wage growth rate is typically negative in an economy, therefore, it can be established that in the case of a lower population growth rate, the duration of implicit debt is also lower. Based on formula (15), this is a correlation also typical of the savings rate: with a lower rate of population growth, the savings part is also lower within contribution payments. This result also means that within the contribution payments, the 'tax ratio' is greater if population growth rate is lower.

Based on the results, we can also establish that in the case of a higher population growth rate, the rate of substitution and the savings rate are also higher in the theoretical model (which may also be interpreted as being more favourable for individuals making payments to the pension system), at the same time, however, the value and risk of implicit debt are also higher (which could potentially be more unfavourable for the institution fulfilling pension liabilities).

The effect of capital market yield

In the theoretical model, capital market yield essentially means government security market yield. The results show that in the case of revenues and expenditures being equal, the capital market (government securities market) yield does not affect the value of the rate of substitution (in other words, the quotient of average pension and average wage) in the two pay-as-you-go pension schemes. The value of implicit debt changes in a contrary direction to capital market yield in the model, therefore, the value of implicit debt is greater with lower capital market yield. The first derivative, according to capital market yield, of the duration of implicit debt:

$$-\frac{(1+n)\times(1+b)}{\left[(1+r)-(1+n)\times(1+b)\right]^2} \qquad (21)$$

With respect to the value in formula (21), we can assume that its value is negative even in the case of a slight population growth rate. This result means that in the case of a lower capital market yield, the risk (in other words duration) of implicit debt is greater. Within contribution payments, the value of the savings rate is greater in the case of lower capital market yield. On the whole, it can be established that in the case of lower capital market (government securities market) yield (assuming all other factors are unchanged), greater value and greater risk implicit debt is simultaneously present with higher savings rate (which may be considered more favourable for individuals making up the various generations as a result of the presence of a lower 'tax ratio').

We should also emphasise some conclusions that can be drawn from the above and earlier results deducted when analysing the effects of the two demographic factors. If for example, compared to an initial situation, life expectancy and population growth are simultaneously higher in another situation and capital market yield is lower, then compared to this initial situation, the rate of substitution, the savings rate, implicit debt and its risk are greater. This may be interpreted as the situation resulting from the three changes is more favourable for individuals in the pension system than the original situation, while it is more unfavourable for the institution disbursing pensions.

SUMMARY

The professional debate regarding the optimal structure of pension systems has been continuously ongoing and active in recent years. The development of theoretical models that may be utilised in practice as well is no simple task, because, among other things, the operation of pension systems can be characterised by a number of indicators and optimal structure pension systems, as calculated on the basis of several aspects, are not necessarily identical. The analysis of the present study employed four indicators (the rate of substitution, the savings rate, implicit debt and its risk). We ignored the 'weighted' valuation of these indicators as well as the definition of the optimal pension scheme reached as result of such valuations, primarily because establishing the method of weighting is not exclusively an economic topic.

The comparison of indicator values in the case of various pension systems could also yield interesting results. This particular study featured pay-as-you-go pension schemes with and without notional individual accounts. The analysis was performed in a theoretical framework by the simplified modelling of practical characteristics, making sure that we can highlight the effects of the development of the two demographic processes significant in many countries (increase of life expectancy and decrease of population) as well as the effect of the changes of government securities market yield on certain indicators.

According to one of the interesting results, the key results calculated in the two types of pay-as-you-go pension schemes (with the exception of implicit pension debt) are different if the revenues and expenditures of the notional individual account pension system are not equal. Based on theoretical results, there may be cases where the notional rate of return is equal to capital market (government securities market) yield in the notional individual account pension scheme, where at the same time the revenues of the pension system are not lower than expenditures.

If we assume revenues and expenditures to be equal in the model, the effects of changes on demographic indicators and government securities market yield can be analysed more clearly. Of the many results, we should highlight that if for example, compared to an initial situation life expectancy is higher in another situation and simultaneously the population growth rate and the capital market (government securities market) yield are lower, then the value of the rate of substitution is lower, however, its effect on implicit debt, its risk and the savings rate is unclear. If, however, besides higher life expectancy and lower government securities market yield we are also assuming a greater population growth rate, the rate of substitution, the savings rate, implicit debt and its risk are also greater in the model. These results may become interesting when substantiating economic decisions related to the development of the population growth rate or government securities market yield. At the same time, it should be emphasised that the theoretical model featured in the study is based on simplified assumptions. There are a number of ways how these model assumptions may be modified even further, and thus become starting points for other research that could promote better understanding of the topic.

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