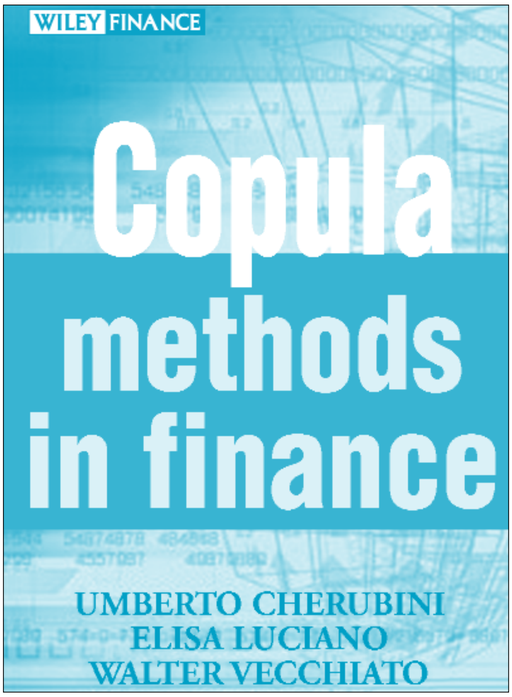
Umberto Cherubini, Elisa Luciano and Walter Vecchiato

Copula methods in finance



John Wiley & Sons, Ltd., 2004

The book focuses on the use of copula functions in financial applications. The copula function (*copula* – word of Latin origin, meaning band, rope, bond) connects the joint distribution functions of two or more random variables with the marginal distribution functions. Therefore, one may often read the definition that the copula is 'the world of distribution functions enclosed in a cube'. The name 'copula' was introduced by *Abe Sklar* in 1956, in his theorem that connects the copula function with the joint distribution function and the marginal distribution functions, and interprets copulas in the theory of probability. Put even more simply: the use of the copula function allows the formulation of new joint distributions, which also provides an opportunity to define new dependence indicators.

The book highlights three important areas that argue in favour of the use of copula functions: the first is the non-normality of returns, which questions the further use of the standard Black and Scholes option pricing model; the second is the choice of the right 'pricing kernel' in the pricing of financial products; and the third area is the applications in credit risk.

The tools used in the pricing and evaluation techniques of financial products originate from the theory of probability. The prices of derivative products may be described as the discounted expected values of their future pay-offs under a specific probability measure derived from 'non-arbitrage' arguments.

The copula methods used in finance are based on the standard hypothesis assumed for stochastic dynamics of the rates of returns on financial products. Until the 1987 crash, a normal distribution for these returns was held as a reasonable guess, and most of the modern finance theory was based on this assumption. In the field of pricing, it corresponds to the standard Black and Scholes approach to contingent claim evaluation. In risk management, assuming normality led to the standard parametric approach to risk measurement, which has been disseminated by J. P. Morgan, the leading global investment bank, under the trademark of RiskMetrics since 1994, and which is still being used by many financial institutions in their risk analysis. It is attributable to the assumption of normal distribution that the method relies on the volatility of the returns on the assets in the portfolio and on the correlation among them. However, the reality of this assumption was strongly questioned by market data, even in the case of standard financial products like stocks and bonds. At the same time, the latest products of financial innovation – for example plain *vanilla* options as well – show non-normal returns. This trend was further strengthened by the returns on credit derivatives and credit-linked products, the returns of which are also non-Gaussian. One of the values of this book is that from beginning to end it proves that tackling the issues of non-normality and non-linearity in portfolios composed of various financial products and securities would be a hopeless task without the use of copula functions.

The book consists of three main parts.

◘ The first one is the introductory chapter, which discusses those fundamental models and correlations that are essential for understanding the other chapters. They are as follows: derivative pricing basics: the binomial model; the Black–Scholes model; interest rate derivatives; 'smile and term' structure effects of volatility and credit risk models. In this part, the reader may learn about the foundations of copula methods and the definitions of copula functions, and may also gain insight into their use in financial analyses.

◘ The second part consists of the subsequent three chapters. These chapters are expressly theoretical. Chapter two introduces the concept of the copula function and its probability interpretation, which allows it to be considered a 'dependence' function. This latter can be understood on the basis of Sklar's theorem, i.e. in multivariate continuous distribution functions the marginal distribution functions of individual variables and the multivariate dependence structure can be separated and interpreted with copula. Another consequence of this theorem is that it points beyond the world of normal distribution. It examines the notions of survival copula and density as well, together with the canonical representation, and also mentions the use of copulas in determining the probability limits related to the sum of random variables. In addition to the theory, it collects numerous financial applications as well to present the usefulness of copulas, which are further developed in the subsequent chapters. Chapter three discusses market comovements and copula families. First it presents the correlations among copula functions, then the measures of association of randomly selected pairs with regard to market indicators, such as prices or returns.

Here it is worth briefly mentioning measures of association. Generally, random variables X and Y are 'associated', when they are not independent. However, there are numerous interpretations of association. They are as follows: concordance, linear correlation, marginal dependence, positive quadrant dependence and the related measures: Kendall τ, Spearman ρ, linear correlation coefficient, the indices of marginal dependence and the indices of the positive quadrant dependence. Each of these measures may be linked to the relevant copula, because in connecting the joint distribution function with the marginal distributions it 'captures certain (...) aspects of the relationship between the variates, from which it follows that (...) dependence concepts are properties of the copula' (Nelsen, 1991).

Thereafter, the authors explain the parametric families or classes of bivariate copulas, describe the density and conditional distribution through copulas, as well as discuss the concordance order and the so-called comprehensive properties of the family. They describe each family with a parameter or parameter vector which show the correlations between the measures of concordance and marginal dependence. The families or classes presented here are as follows: Gaussian copula, bivariate Student's copula, the Fréchet family, Archimedean copulas and the Marshall–Olkin copula.

Chapter four discusses the extensions of bivariate copula functions to the multivariate case.

■ The chapters that can be classified into the third and final part basically present empirical practical applications for the latest products of individual financial innovations.

Chapter five discusses estimation and calibration from market data. From a statistical point of view, the copula function – contrary to most multivariate statistical models – is a simple multivariate model, for which the classical statistical conclusion theory can be applied. (The only theory that can be applied to some extent is the asymptotic maximum likelihood estimation.) The authors devoted this chapter, inter alia, to the presentation of the statistical conclusion theory applied for the copula models. All the methods discussed here by the authors require the numerical optimisation of the objective function, as the copula is essentially a multivariate function, and its probability comprises mixed partial derivatives.

Copulas provide an efficient tool for describing joint distribution as well as modeling marginal distribution and joint distribution. Accordingly, for each data series one may select the marginal distribution that matches the sample best, and then, using the relevant copula, treat all of them together. The problem stems from the simple fact that the number of combinations is infinite, and it is easy to make a mistake in selecting the best combination of marginal distributions and the appropriate copula. As a remedy for the problem, the authors present some non-parametric methods, for example the canonical maximum likelihood method, the IMF method etc., for the modeling of marginal distributions and the copula, assuming the continuity of random variables. They give a non-parametric estimation for stock market data by introducing the empirical copula and the so-called kernel copula.

Chapter six presents simulations of market scenarios by applying the Monte Carlo and Marshall–Olkin methods. The *Clayton*, *Gumbel* and *Frank* n-copulas are used in conditional sampling.

Chapter seven is about credit risk applications. Credit derivatives, such as collateralised debt obligations (CDO) or credit default swaps (CDS), are financial contracts that allow the transfer of credit risk from one market participant to another. Undoubtedly, one of the main issues is the modeling of the joint distribution of default times. According to the proposal in *Li*'s (2000) study, the Gaussian copula may be an adequate tool for treating this problem. The key issue in this framework is how it is possible to shift the focus of the examination from the dependence modeling of default times to a fixed time horizon to a dependence between default times that are random variables and do not depend on the discretionally chosen time horizon.

Let's have a look at Li's (2000) copula method, namely the simplest case, when only single survival time is taken into account in modeling and calibration. Li describes the default with a survival function $S(t) = \Pr(t < \tau)$ that gives the probability that the security reaches age t. Survival time τ is called the time that elapsed until default or simply default time. If S is differentiable, and the risk rate can be given with the $h(u) = -S'(u)/S(u)$ function, and the survival function expressed in the risk rate is

$$S(t) = \exp\left(- \int_0^1 h(u)du \right),$$

then the occurrence of the default is an inhomogeneous Poisson process. A typical assumption is that the risk rate is constant, in which case survival time follows an exponential distribution with parameter *h*, and then the occurrence of the default is a homogeneous Poisson process. The authors show in their book that it is easy to generalise the distribution of the survival time for the case of Weibull distribution, and they also touch upon generalisation in the case of multiple survival times and multiple default times both in modeling and calibration.

The eighth and final chapter discusses option pricing with copulas. Here the authors demonstrate how copula functions are used in the pricing of multivariate contingent claims. Their primary objective is to exceed the standard Black–Scholes framework, and derive pricing formulae that are valid for very general distributions as well, namely with closed form solutions. It is known that the Black–Scholes model uses two assumptions that have been refuted by market data. One of them is that returns do not show a normal distribution, which was suggested by the 'smile and term' structure of volatility. The other is market imperfection, i.e. the difficulty arising in the selection of the exact replication strategy relating to all contingent claims and in giving the pricing kernel. Both problems are exacerbated in multivariate cases. Evaluating multivariate contingent claims in imperfect markets means the solving of two tasks: the selection of the pricing kernel for each asset in the basket separately and the selection of the copula function that is in compliance with the dependence structure between them. The authors also go into the details of special cases, for example, the pricing of 'two-color options' or barrier options. They present the application of the Monte Carlo method in the pricing of multivariate options, through the so-called basket option.

The book discusses in a clear and simple style many issues that are in the forefront of modern risk calculation. It is recommended for study by university students, researchers and value analysts dealing with risk calculation in various financial institutions.

József Móczár